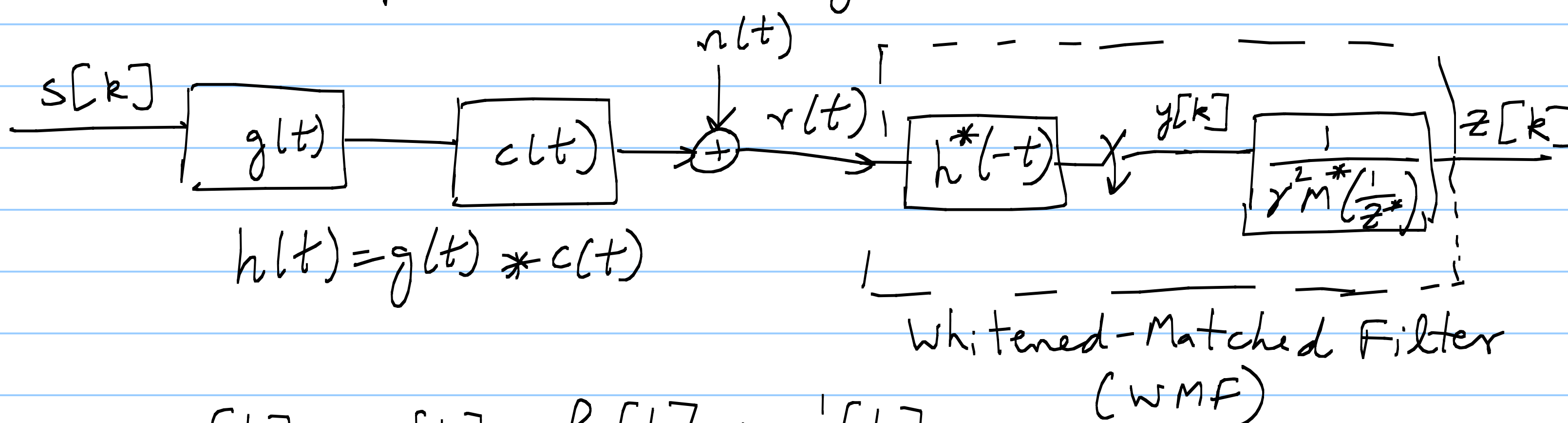
Lecture 25

Note Title 9/18/2008

Equalization

- → Rx signal processing to tackle ISI
- → Chapter 8, Barry, Lee et al



$$y[k] = s[k] * \rho_h[k] + n'[k]$$

$$\rho_h[k] = \gamma^2 \, m[k] * m^*[-k]$$

$$S_h(z) = \gamma^2 M(z)\, M^*(1/z^*)$$

$$S_h(e^{j2\pi fT}) = \gamma^2 M(e^{j2\pi fT})\, M^*(e^{j2\pi fT})$$

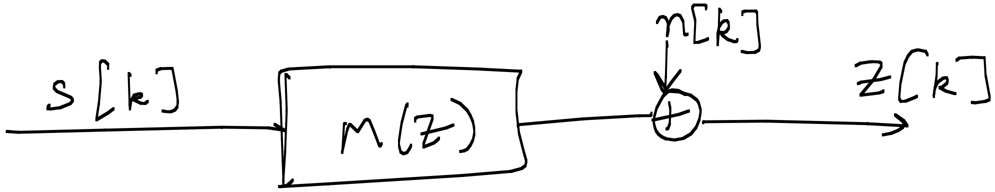
$$\gamma^2 = \exp\left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \log S_h(e^{j\theta})\, d\theta \right\}$$

$$= \langle S_h \rangle_G$$

$$\langle S_h \rangle_A = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_h(e^{j\theta})\, d\theta$$

Result: $\langle S_h \rangle_A \geq \langle S_h \rangle_G$

Equality: iff $S_h(e^{j\theta})$: flat.



$$z[k] = s[k] * m[k] + n[k]$$

White, Gaussian

Variance of real (or imaginary) part of $n[k] = \frac{N_0}{2\gamma^2}$

$\longrightarrow$ R_x has to <u>know</u> $h(t) = g(t) * c(t)$

<u>Figure of Merit</u>:

$$\Pr\{\text{Symbol error}\} \approx (\text{const})\, Q\left(\sqrt{\Gamma}/2\right)$$

Γ : figure of merit.

Non-ISI: $$\Gamma = \frac{d_{min}^2(\mathcal{X})}{\sigma^2}$$

ISI case: bounds on Γ

① Matched-filter bound:

$\longrightarrow$ only one symbol is transmitted.

$$z[k] = \underbrace{\underset{\in \mathcal{X}}{s[0]}\, m[k]} + n[k] \qquad k = 0, 1, 2, \ldots$$

Recvd. symbols:

$$\left[\, a\, m[0] \quad a\, m[1] \quad a\, m[2] \quad \ldots \ldots \,\right]$$

for $a \in \mathcal{X}$.

$$d_{min}^2 = \min_{a, a' \in \mathcal{X}} \sum_{k=0}^{\infty} \left| a\, m[k] - a' m[k] \right|^2$$

$$= \left(\min_{a, a' \in \mathcal{X}} |a - a'|^2 \right) \sum_{k=0}^{\infty} |m[k]|^2$$

$$d_{min}^2 = \underbrace{d_{min}^2(x)}_{\checkmark} \cdot \underbrace{\sum_{k=0}^{\infty} |m[k]|^2}_{= \frac{E_h}{\gamma^2}}$$

$$\Gamma_{MF} = \frac{d_{min}^2}{\sigma^2} = \underbrace{\frac{1}{\sigma^2}}_{\frac{N_0}{2\gamma^2}} \cdot \frac{d_{min}^2(x) E_h}{\gamma^2}$$

$$\Gamma_{MF} = d_{min}^2(x) \frac{2E_h}{N_0}$$

$$\Pr_{\text{with ISI}}(\text{Error}) \geq (\cdot)\, Q\left(\frac{\sqrt{\Gamma_{MF}}}{2}\right)$$

② MLSD bound

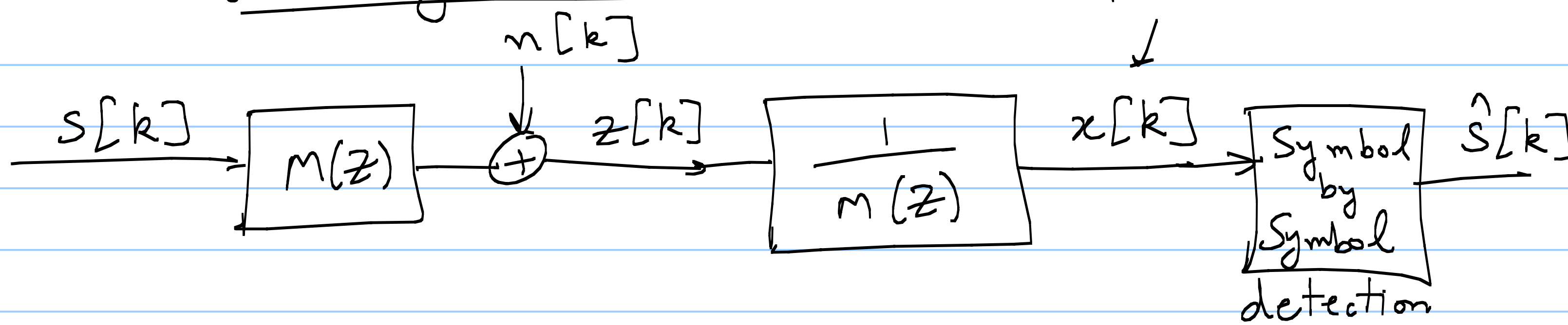
$$\Gamma_{MLSD} = \frac{d_{min}^2}{\sigma^2} \qquad \sigma^2 = \frac{N_0}{2\gamma^2}$$

$$d_{min}^2 = \min_{\text{Error events } E} d(E)$$

$$\geq d_{min}^2(\mathcal{X})$$

$$\frac{d_{min}^2(\mathcal{X})}{(N_0/2\gamma^2)} \leq \Gamma_{MLSD} \leq \Gamma_{MF}$$

Zero-forcing Linear Equalizer



$$x[k] = \underline{\underline{s[k]}} + n'[k]$$

$n'[k] \rightarrow$ Gaussian

$$\text{PSD} = \frac{N_0}{S_h(e^{j\theta})}$$

σ_v^2 = Variance of real part of $n'[k]$

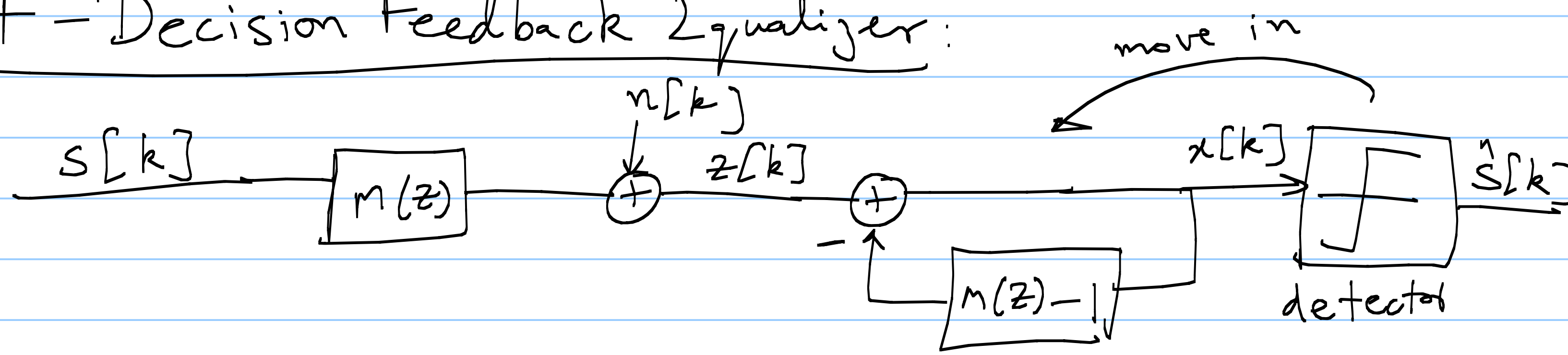
$$= \frac{N_0}{2}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{S_h(e^{j\theta})}\, d\theta\right) = \frac{N_0}{2}\left\langle \frac{1}{S_h} \right\rangle_A$$

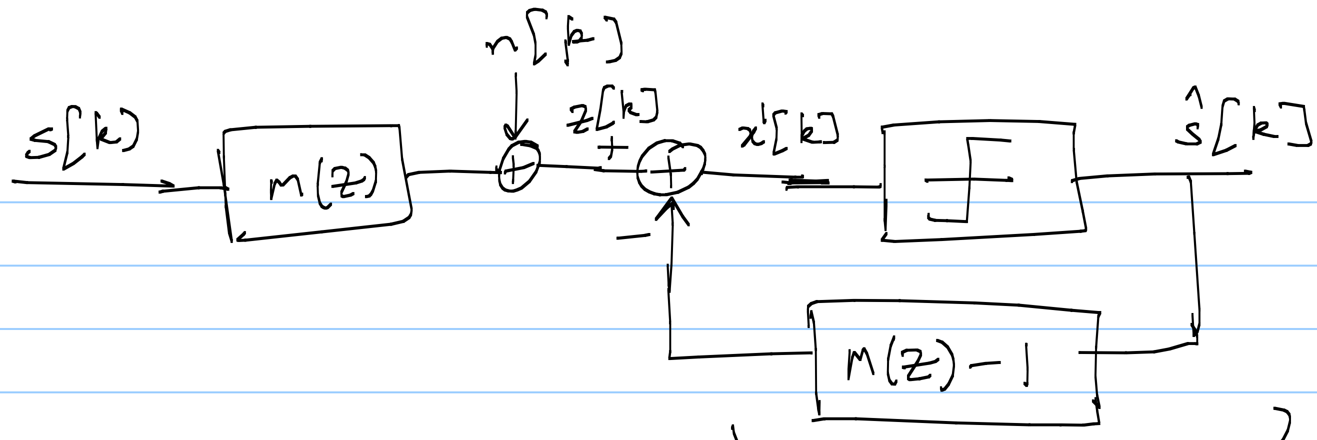
$\longrightarrow$ Noise enhancement.

$$\Gamma_{ZF-LE} = \frac{d_{min}^2(x)}{\sigma_v^2}$$

$$= \frac{d_{min}^2(x)\;\;2}{N_0 \left\langle \frac{1}{S_h} \right\rangle_A}$$

ZF-Decision Feedback Equalizer:





Non-linear

ZF-DFE