Lecture 24

Note Title 9/17/2008

Computations:

↳ additions, multiplication, comparisons

L stages, X, μ-tap $M(z)$

↓

1 stage → $|X|^{\mu+1}$ branches, $|X|^{\mu}$ states, N_s

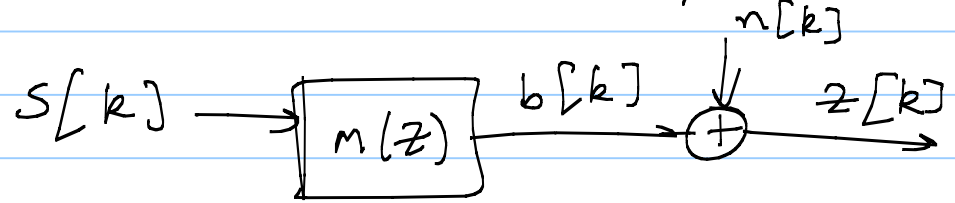
↓

$\sim 3-4N_s$ computations

Total $\sim 3L|X|^{\mu+1}$

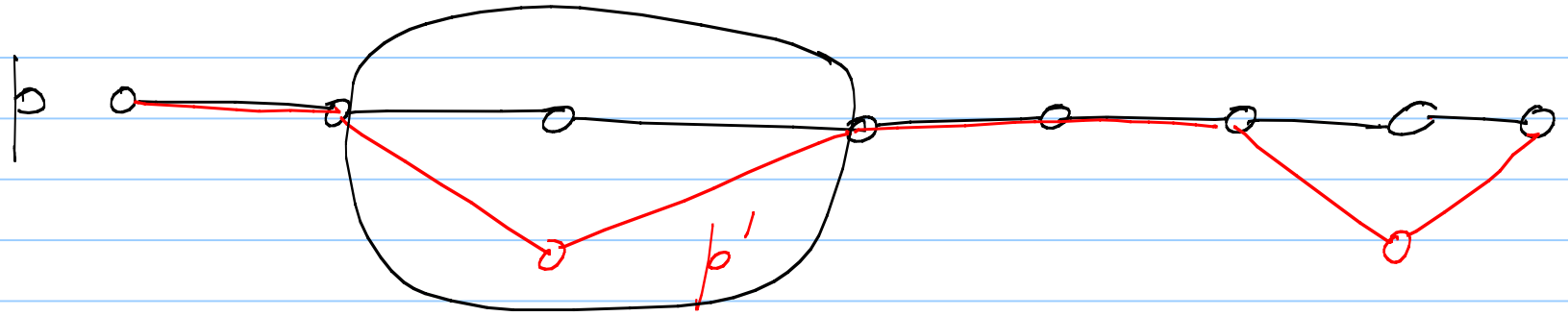
Estimating Prob{error}:

→ MLSD: $|\mathcal{X}|^L$ data symbol sequences



$$\underline{\hat{s}} = \arg \min_{\underline{a} \in \mathcal{X}^L} \| \underline{z} - \underline{b}(\underline{a}) \|^2$$

$$= \arg \min_{\text{path} \in \text{Trellis}} \text{Path metric.}$$



Error event: $E = (\phi, \hat{\phi})$

$\phi, \hat{\phi}$: paths on the trellis.

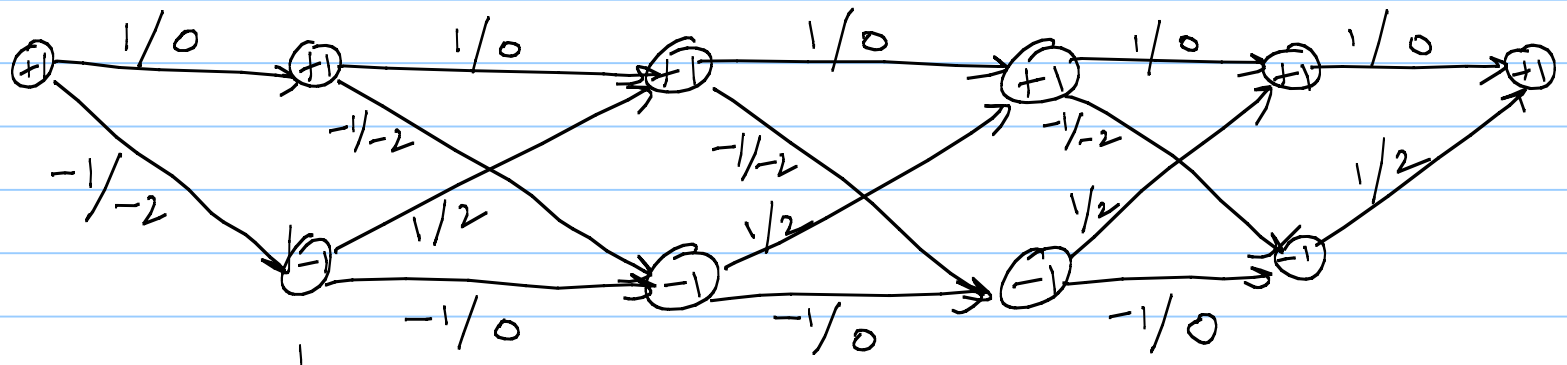
$\phi = \psi_1 \psi_2 \cdots \psi_{n-1} \psi_n$ "state trajectory"

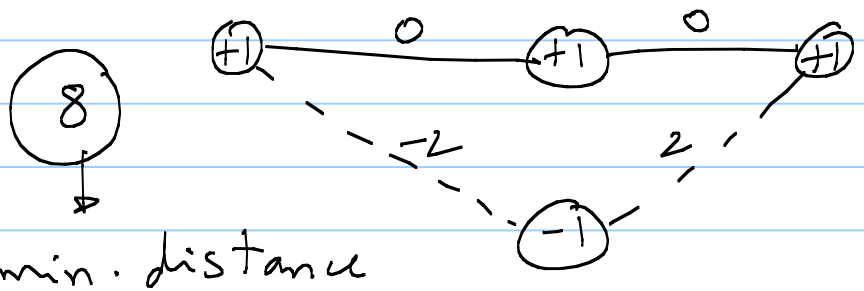
$\hat{\phi} = \psi_1 \underbrace{\psi_2' \cdots \psi_{n-1}'} \psi_n$

$\psi_i' \neq \psi_i \quad 2 \leq i \leq n-1$

$$d(E) = \sum_{i=1}^{n-1} \left| \phi(\psi_i \to \psi_{i+1}) - \phi(\psi_i' \to \psi_{i+1}') \right|^2$$

Ex: $M(z) = 1 - z^{-1}$, $\mathcal{X} = \{\pm 1\}$





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min. distance in the "large" constellation.

→ enumerating least-weight error events. (set-up a Viterbi!)

→ too many of them

Prob{Sequence error} versus Prob{Symbol Error}?

$$\underline{\hat{s}} = \arg \min_{\underline{a} \in \mathcal{X}^2} \|\underline{z} - \underline{b}(\underline{a})\|^2$$

$\Pr(\underline{s} \neq \underline{\hat{s}})$ $\qquad$ $\Pr\{s[k] \neq \hat{s}[k]\}$

"Minimum Distance" $\quad d_{min}^2 = \min_{\text{Error events } E} d(E)$

$$\Pr(\underline{s} \neq \underline{\hat{s}}) \simeq K_2 \, Q\left(\frac{d_{min}}{2\sigma}\right)$$

very large. $\rightarrow$ small

$$\Pr(s[k] \neq \hat{s}[k]) \simeq C \, Q\left(\frac{d_{min}}{2\sigma}\right)$$

Soft Detector for ISI:

Non-ISI: Hard detector

versus

Soft detector → outputs probabilities.

ISI: Viterbi is a hard detector

BCJR algorithm → Soft detector

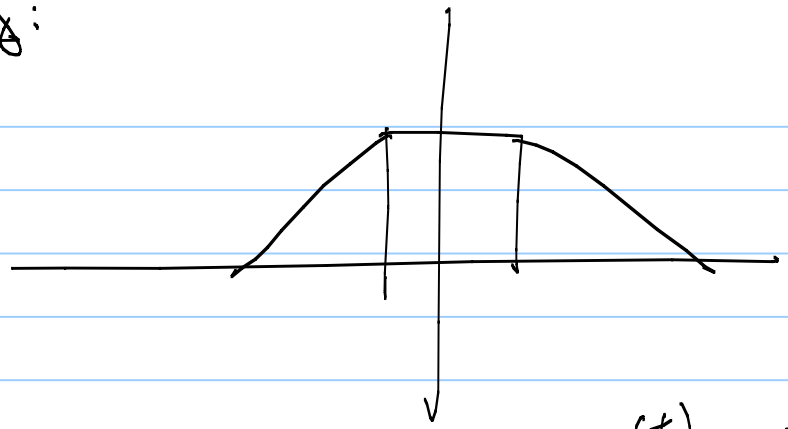
↓

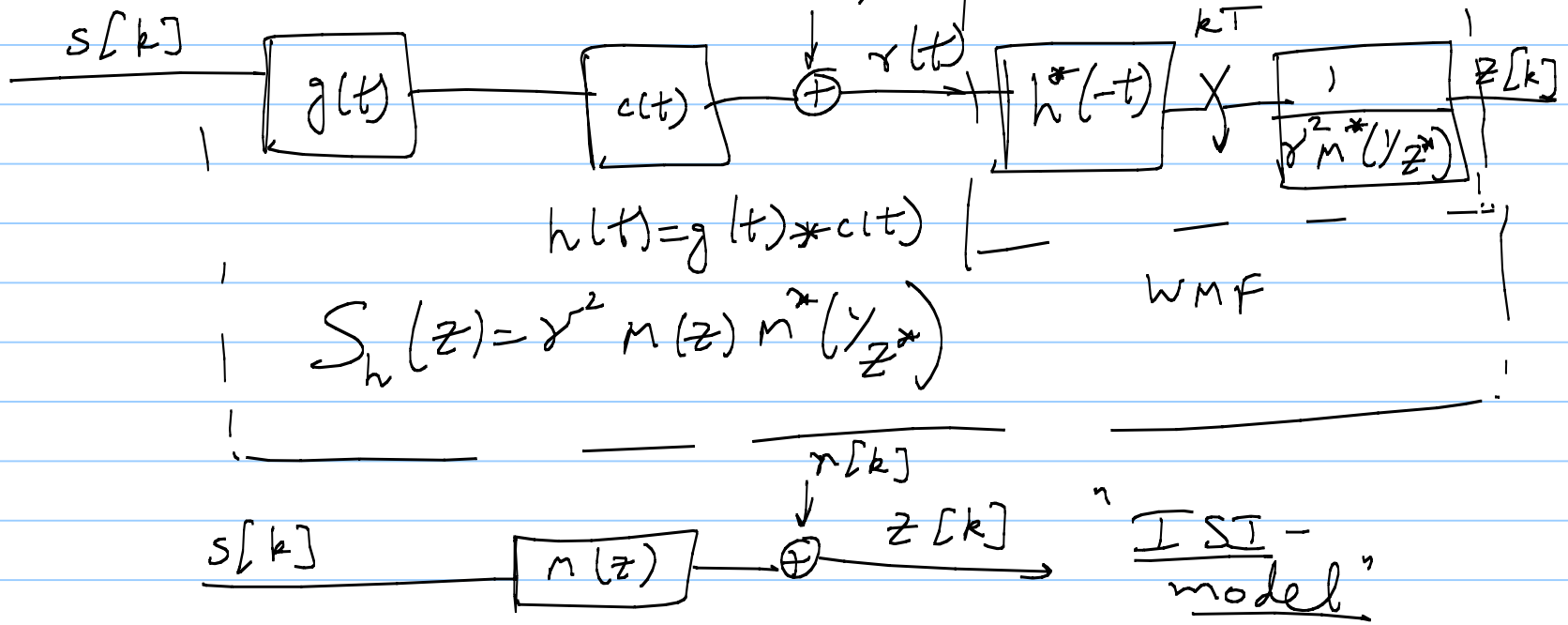
Bahl-Cocke-Jelinek-Raviv

log-MAP (or) algorithm

$Pr(s[k]=+1 \mid \underline{z})$

Summary:




whitened-Matched-filter
s[k]
g(t)
c(t)
n(t)
r(t)
h*(-t)
kT
z[k]
h(t)=g(t)*c(t)
WMF
$S_h(z)=\gamma^2 H(z) H^*(1/z^*)$
n[k]
s[k]
H(z)
z[k]
"ISI-model"