Textbook : "RF Microelectronics" by Behzad Razavi (2nd ed.)

Grading : Final exam : 50%

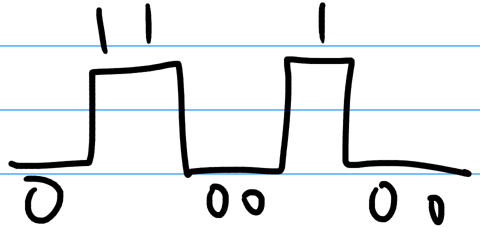
Assignments : up to 10%

Projects : 40% $\longrightarrow$ LNA, mixer, VCO, PA

Pre-req : EE3002/5310 $\longrightarrow$ MOS operation

Signals & Systems

RF = radio freq = wireless



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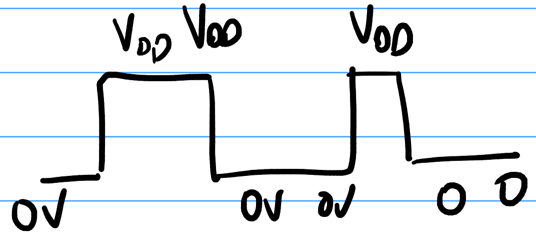


<u>Optical</u> - fibre

<u>Wireline</u> - Cu wire

dedicated medium

<u>Wireless</u>

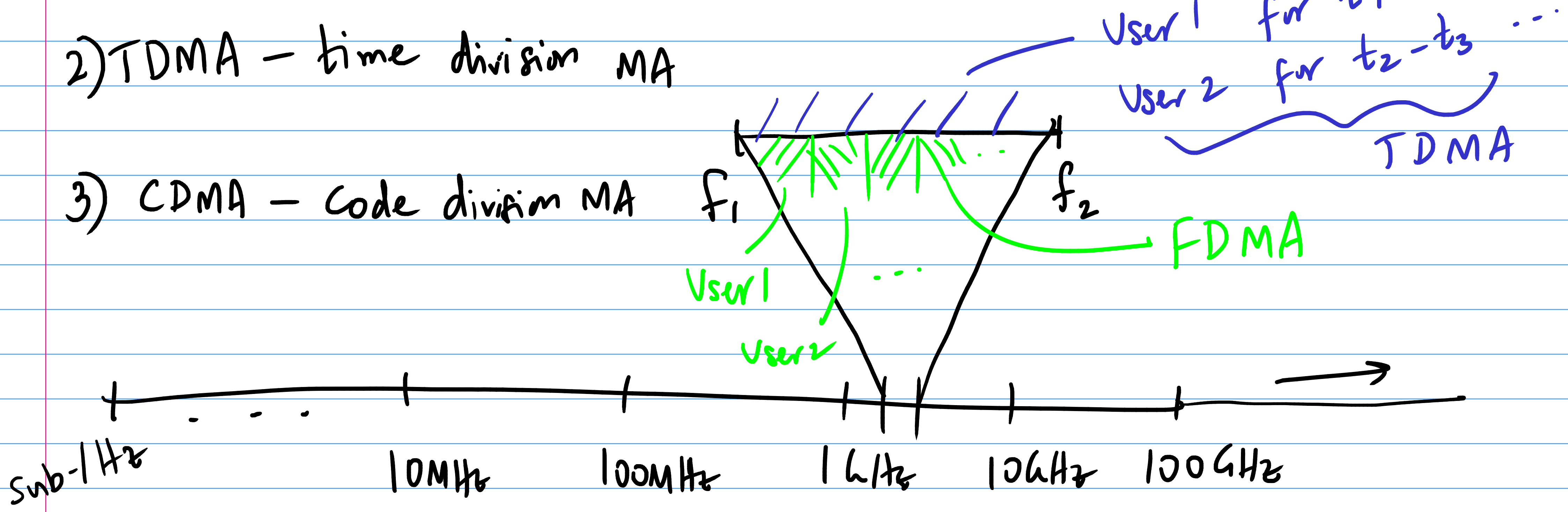
free space

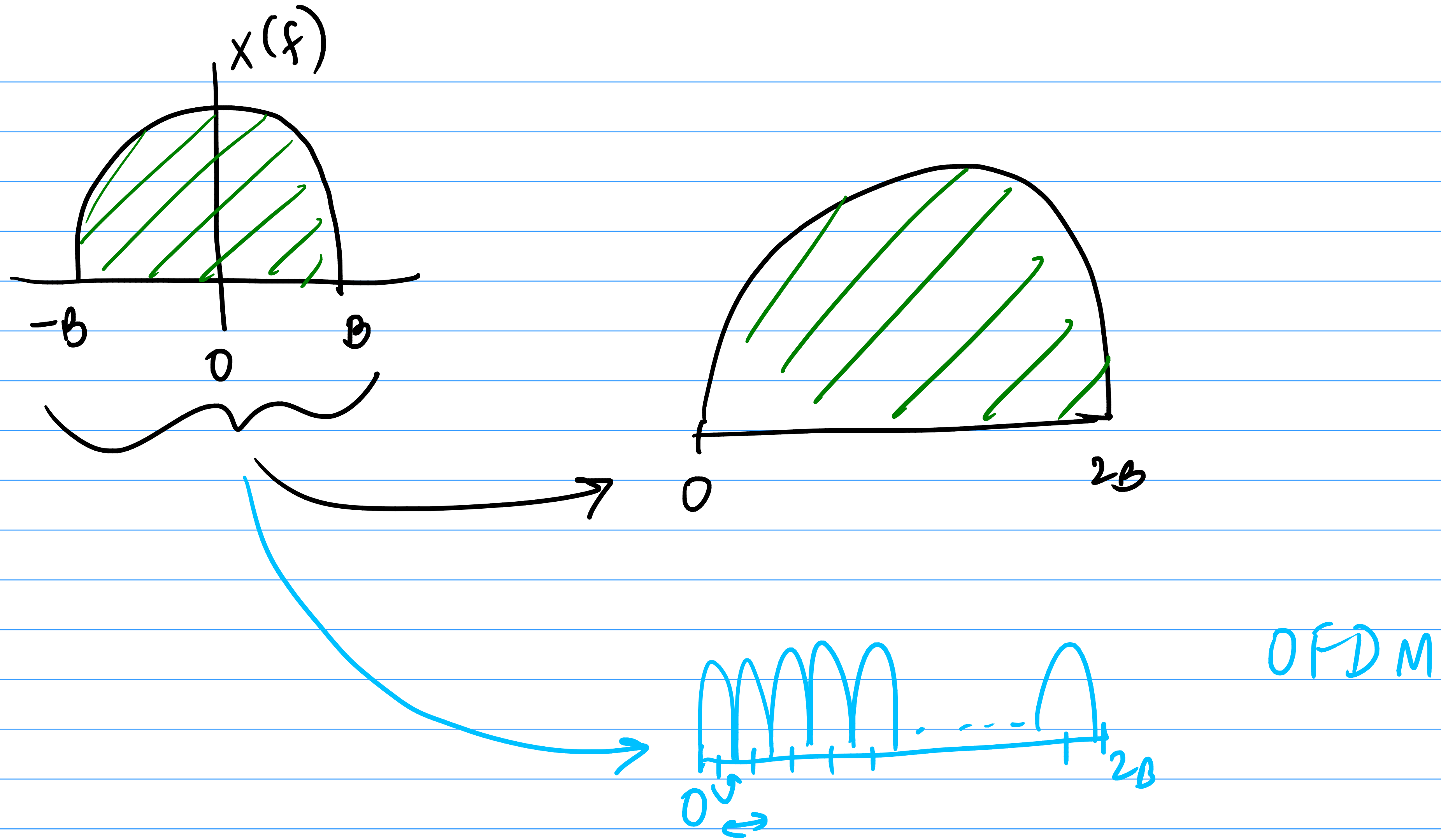
shared medium

Multiple Access → 1) FDMA Frequency Division Multiple Access

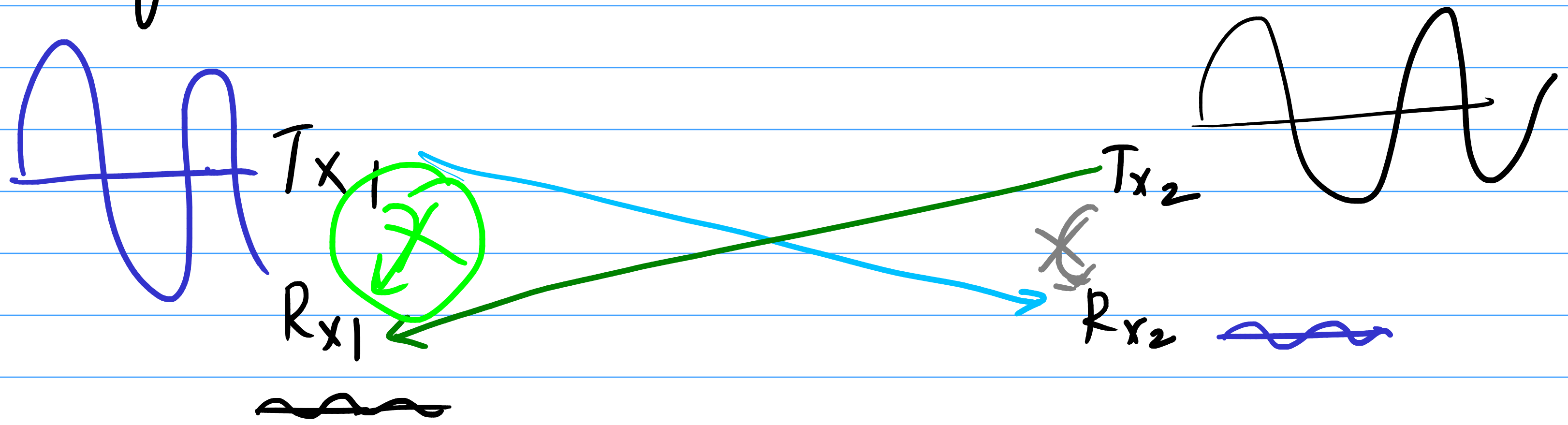
2) TDMA – time division MA

3) CDMA – Code division MA



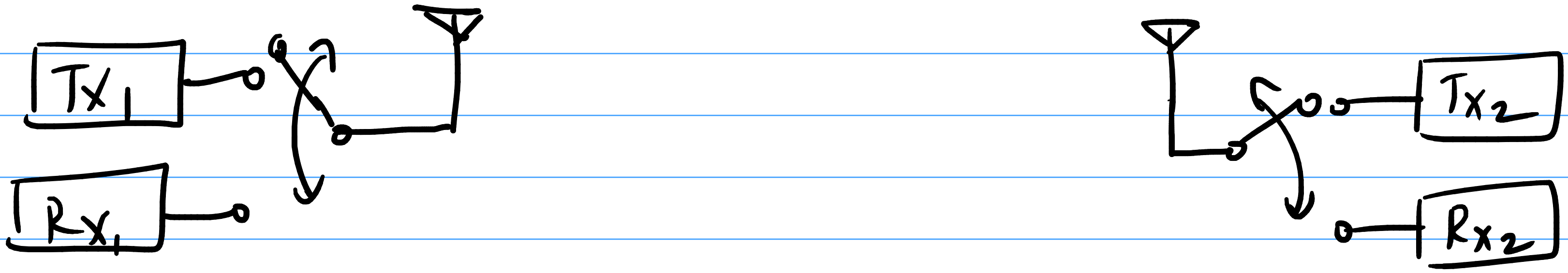

X(f)
-B
0
B
0
2B
0
2B
OFDM

Duplexing

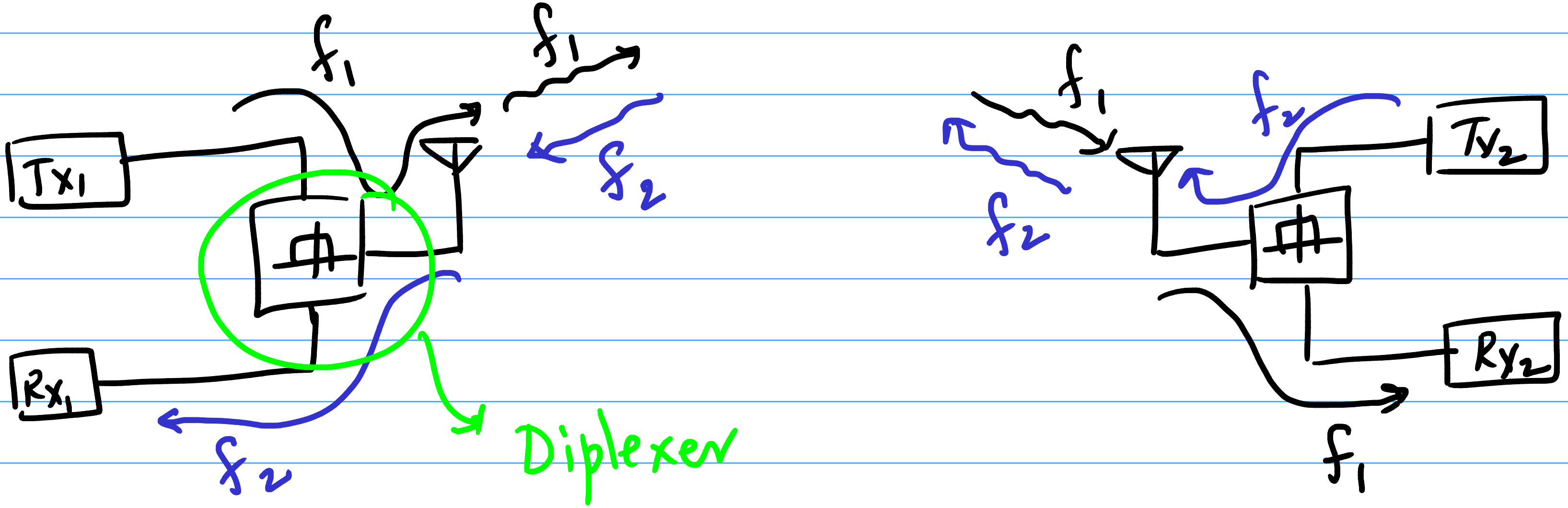


1) TDD Time division Duplex

2) FDD Freq. " "

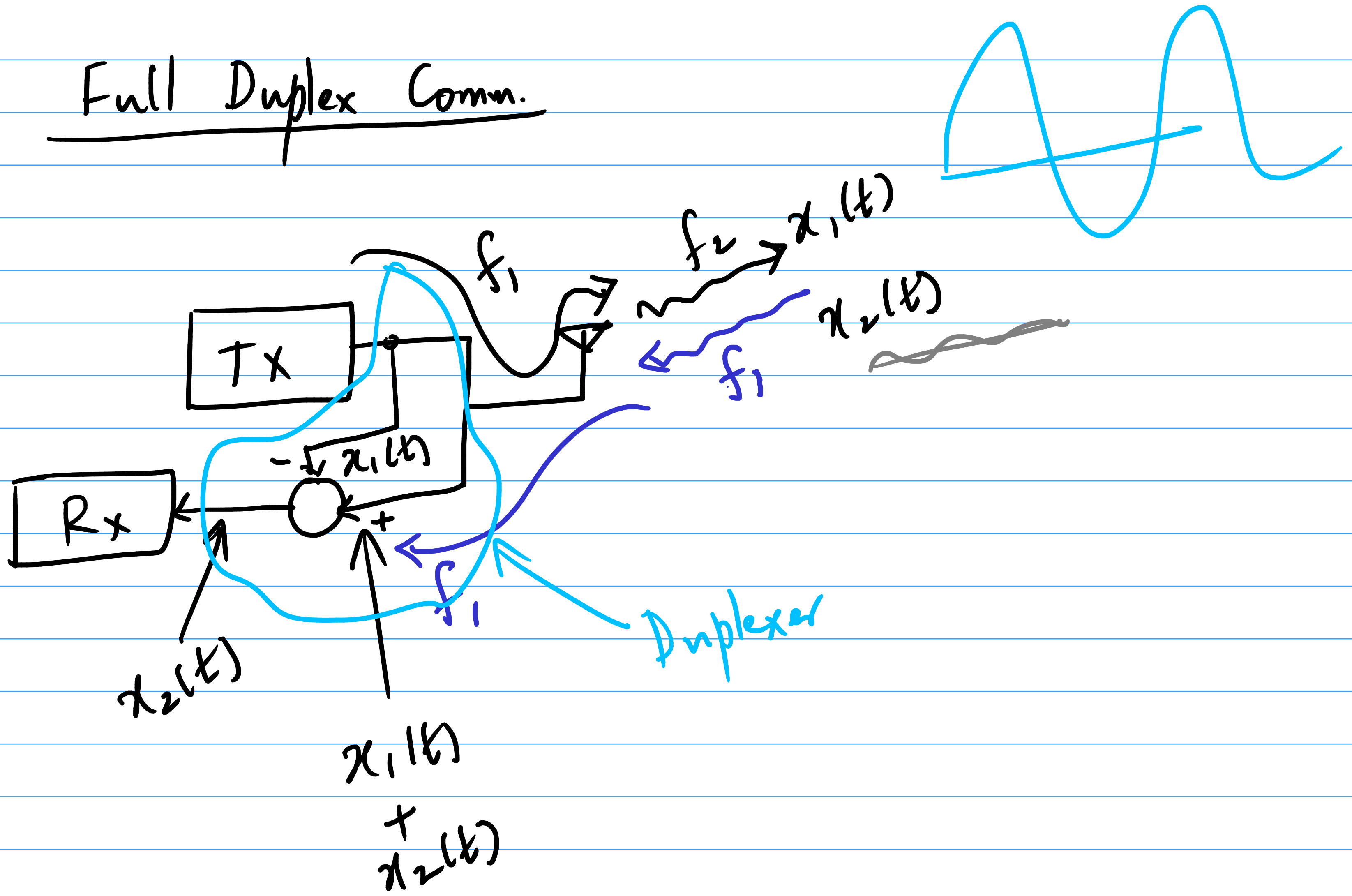


TDD

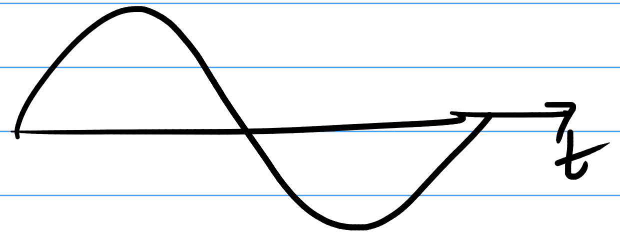


FDD

3) Full Duplex Comm.

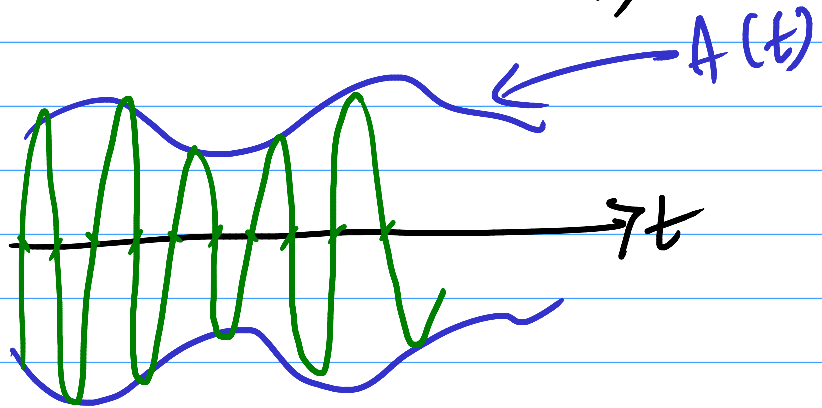


sine

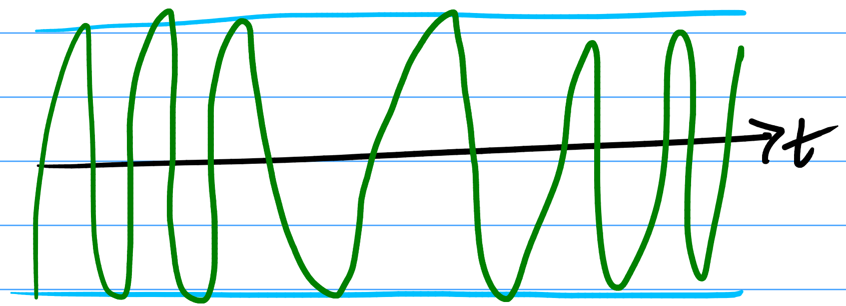


$A\cos(\omega t + \varphi)$

1) AM $\rightarrow$ $A(t)\cos(\omega_0 t + \varphi_0)$



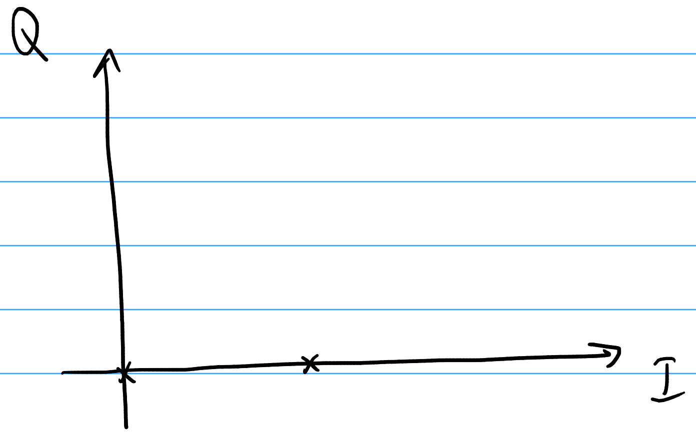
2) FM : $x(t) = A_0 \cos(\omega(t) \cdot t + \varphi_0)$

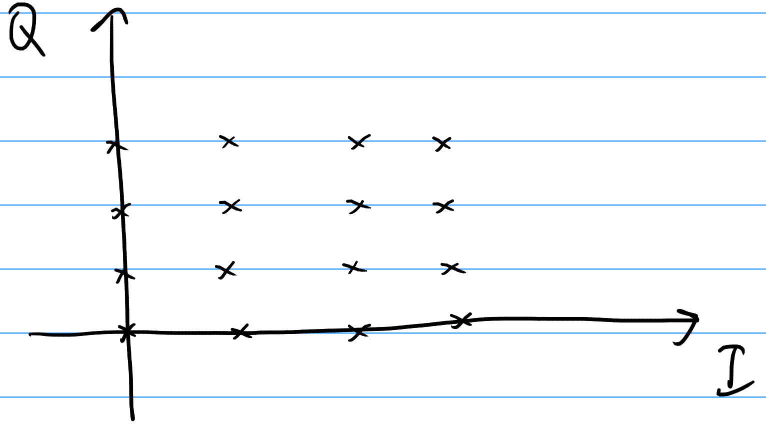


3) PM : $x(t) = A_0 \cos(\omega_0 t + \varphi(t))$

In general $\rightarrow$ Information in Amplitude & Freq.

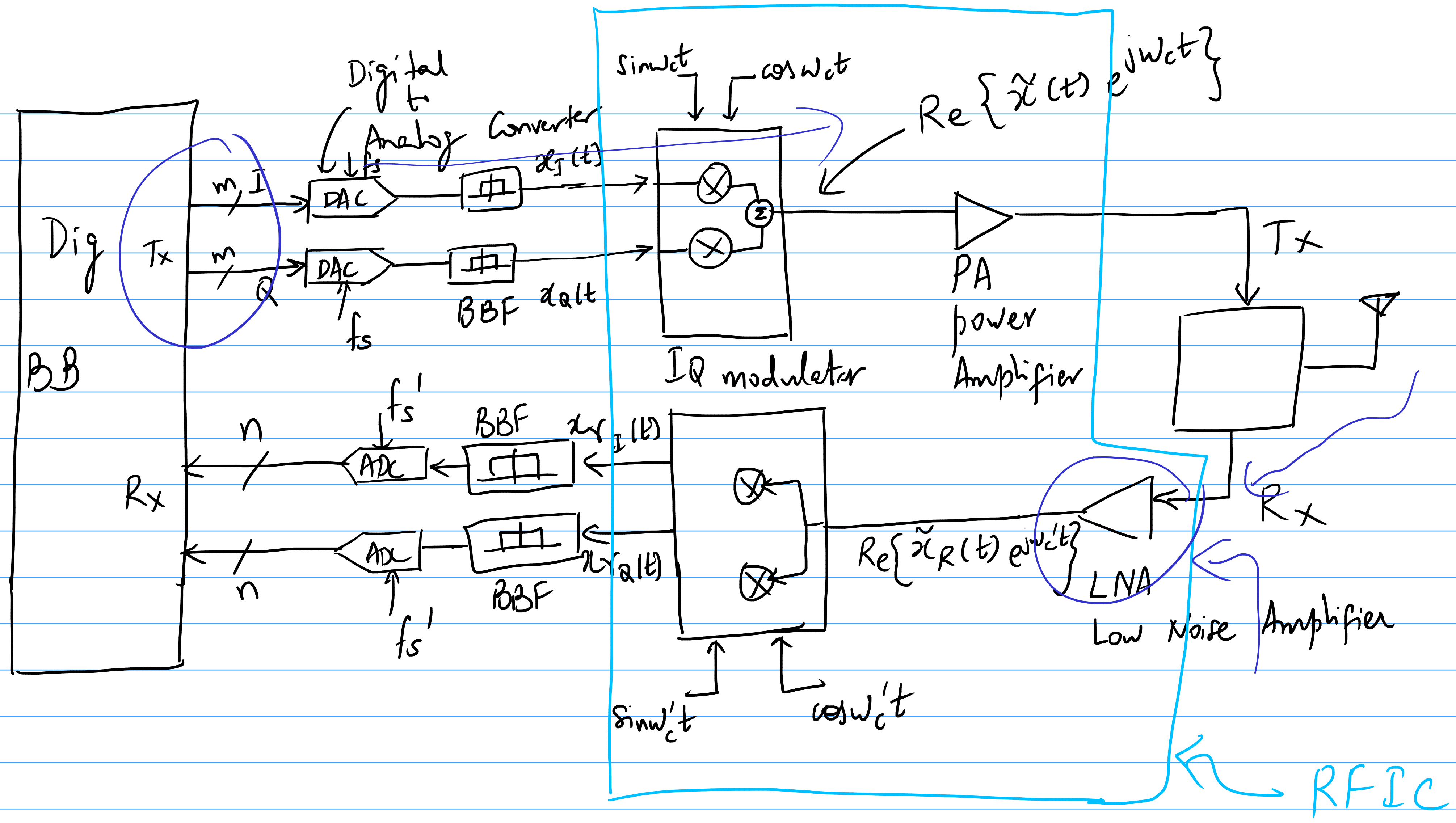
(or) Amp. & phase





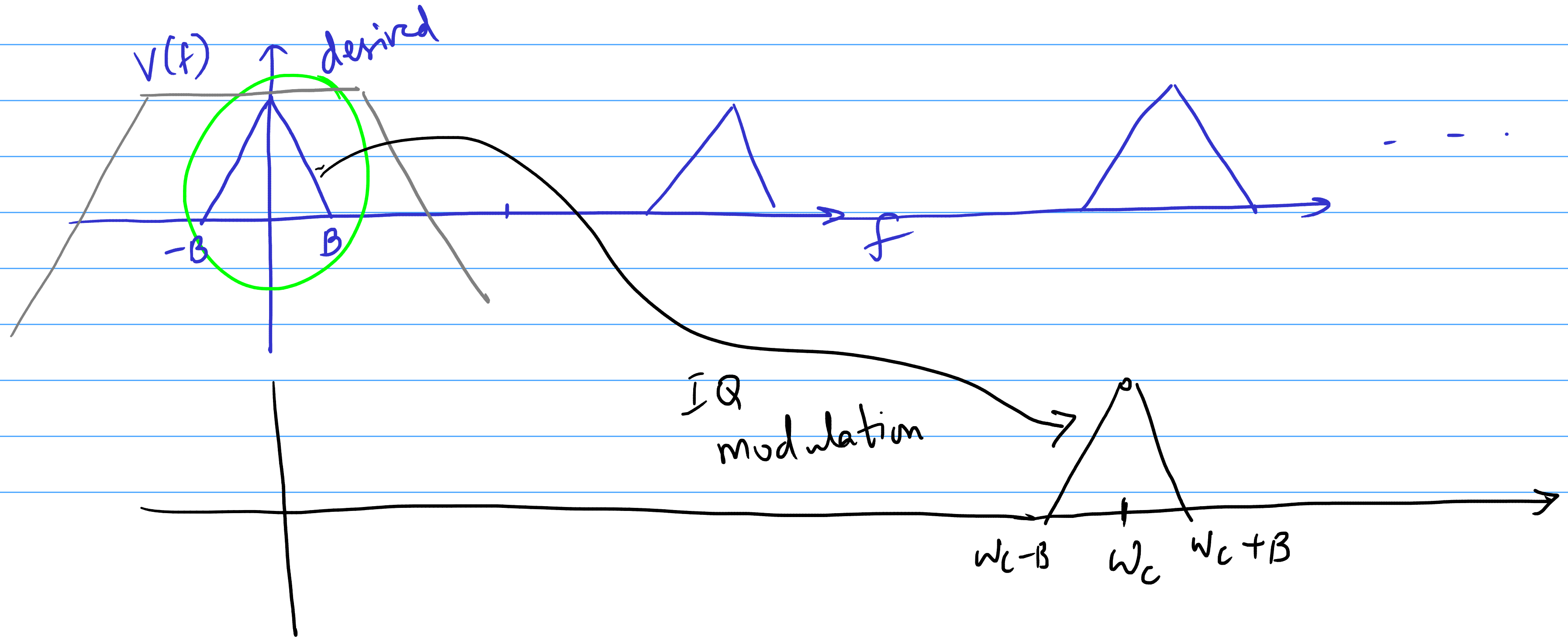
QAM - 16

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t)$$

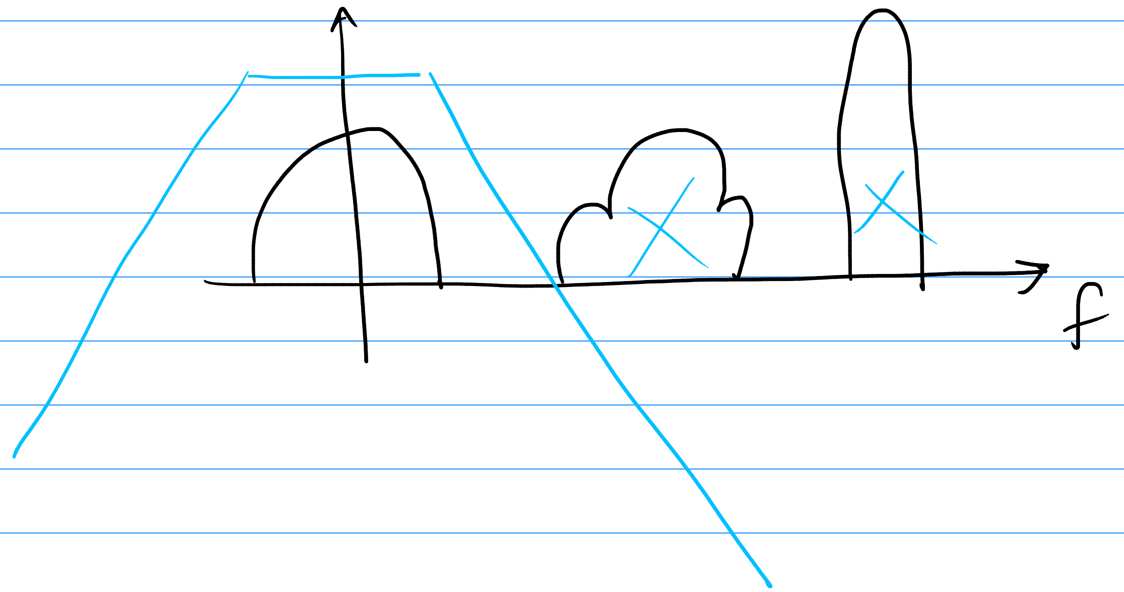


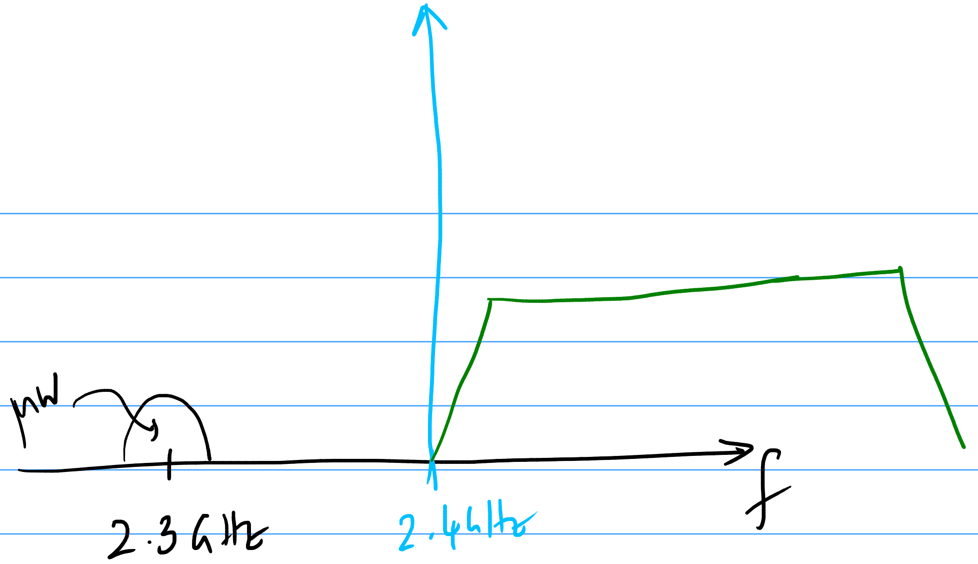
Digital to Analog Converter
Dig
BB
Tx
Rx
m, I
m
Q
DAC
DAC
fs
fs
BBF
$x_I(t)$
$x_Q(t)$
sin $w_c t$
cos $w_c t$
IQ modulator
$Re\{\tilde{x}(t)\, e^{jw_c t}\}$
PA
power Amplifier
Tx
Rx
n
n
ADC
ADC
fs'
fs'
BBF
BBF
$x_{rI}(t)$
$x_{rQ}(t)$
sin $w_c' t$
cos $w_c' t$
$Re\{\tilde{x}_R(t)\, e^{jw_c' t}\}$
LNA
Low Noise Amplifier
RFIC

DAC

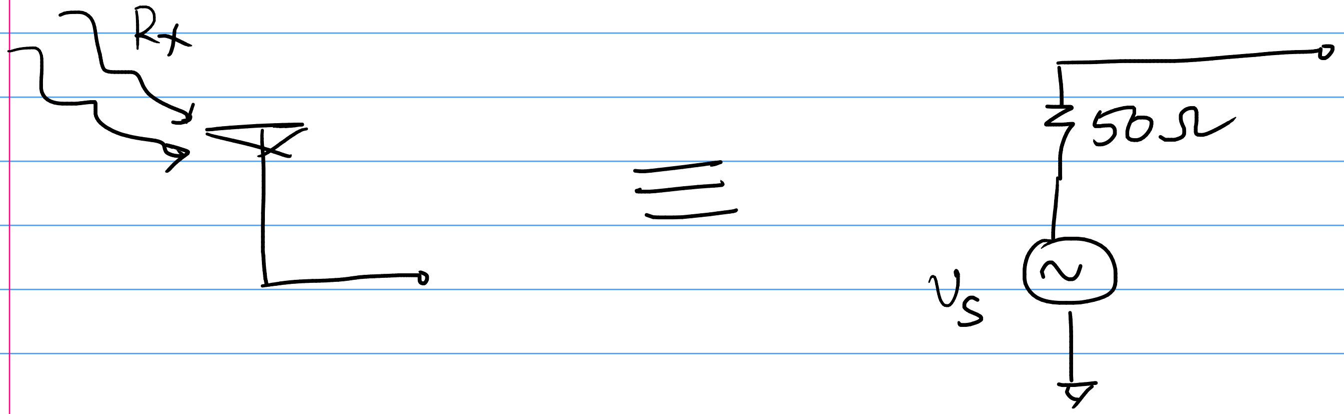


ADC



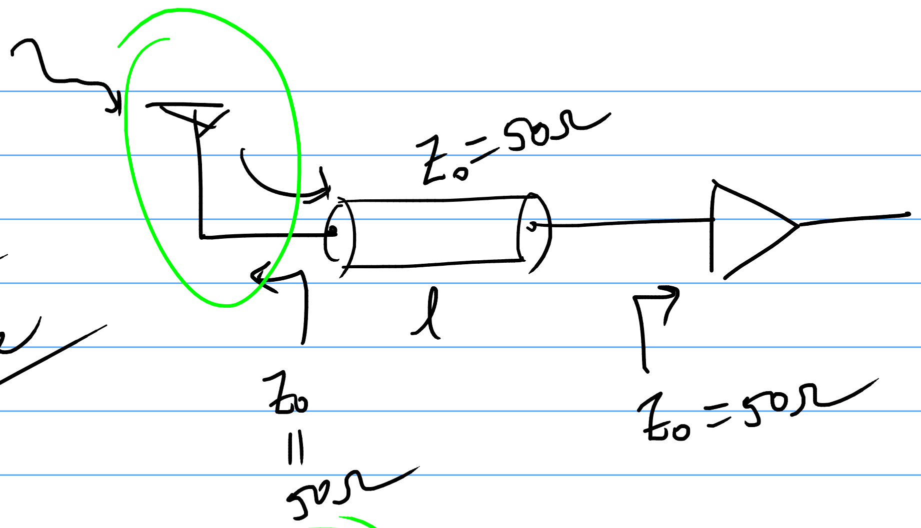


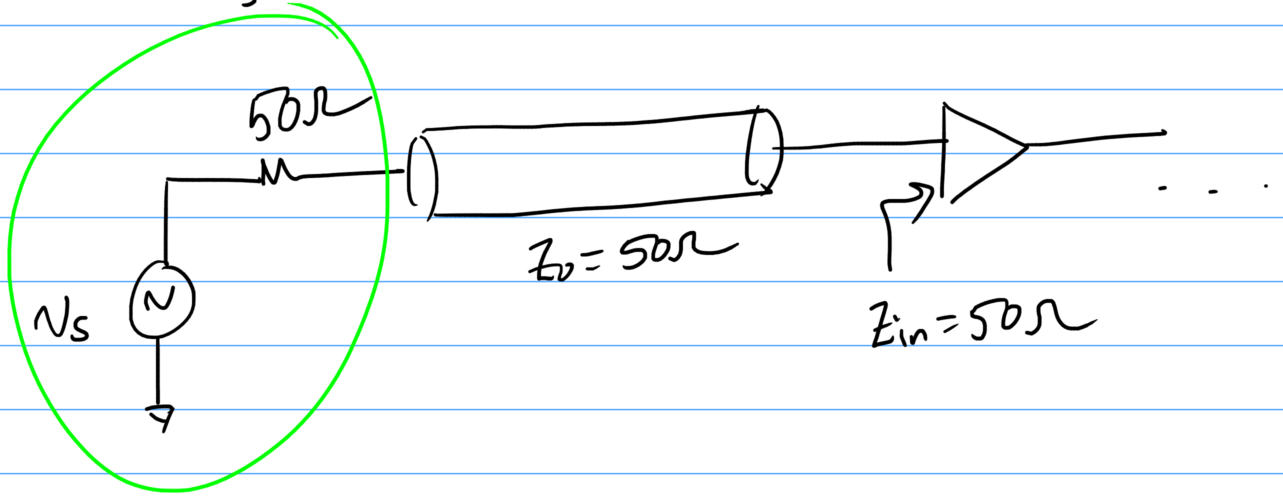
μW
2.3 GHz
2.4 GHz
f



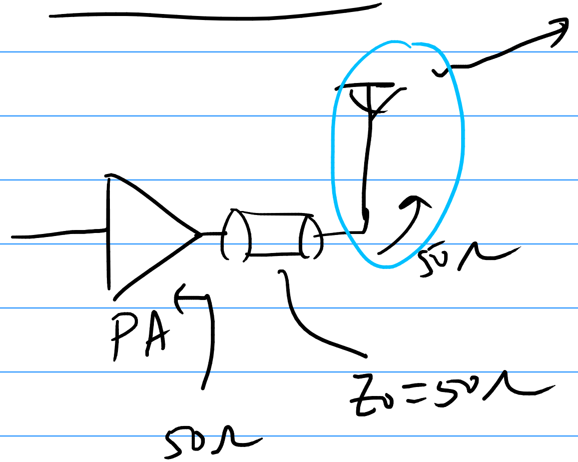
Rx
≡
50Ω
Vs

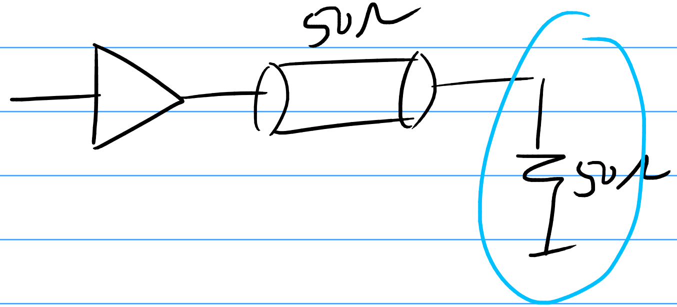
Rx mode

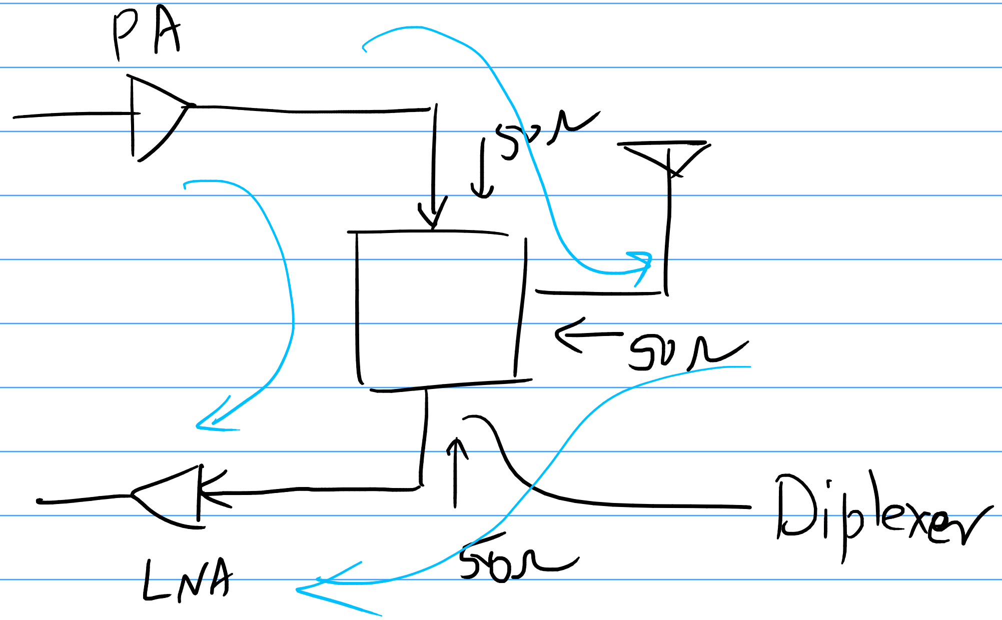




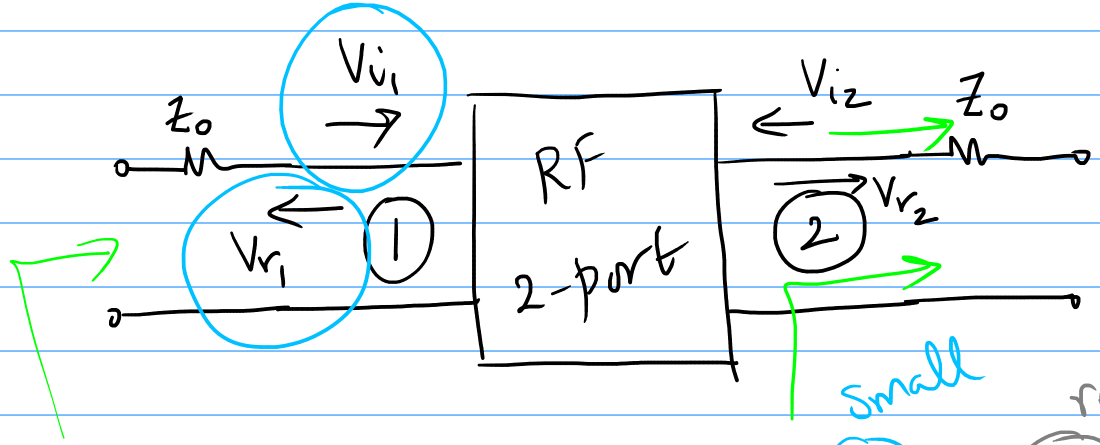
Tx Mode






PA
50Ω
50Ω
50Ω
LNA
Diplexer

S-parameters



Γ = reflection coeff.

small   reverse transmission gain

$$\begin{bmatrix} V_{r_1} \\ V_{r_2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{i_1} \\ V_{i_2} \end{bmatrix}$$

power gain (in dB)   small

Units

1) **dBm** Power P mW

$$P_{dBm} = 10 \log_{10} \left(\frac{P \text{ in mW}}{1 mW} \right)$$

$P_1 = 1 mW$

$P_{1\,dBm} = 0 \text{ dBm}$

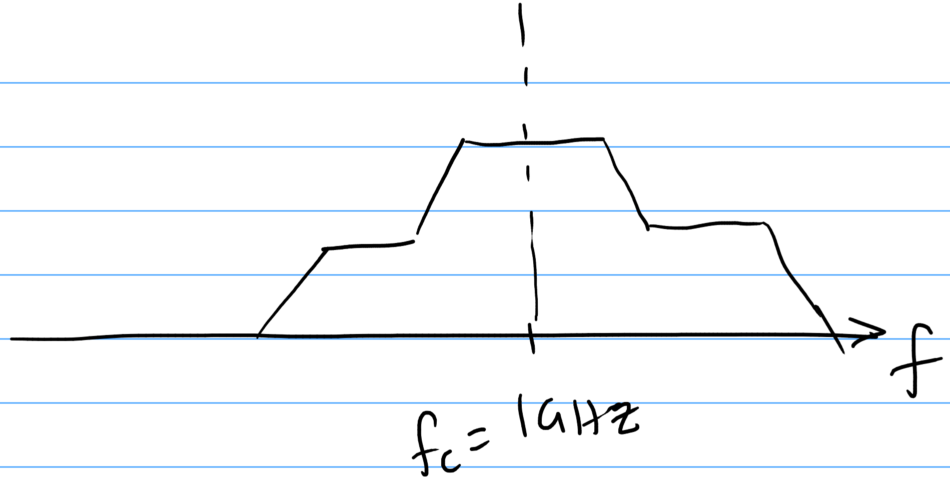
$P_2 = 10 mW$

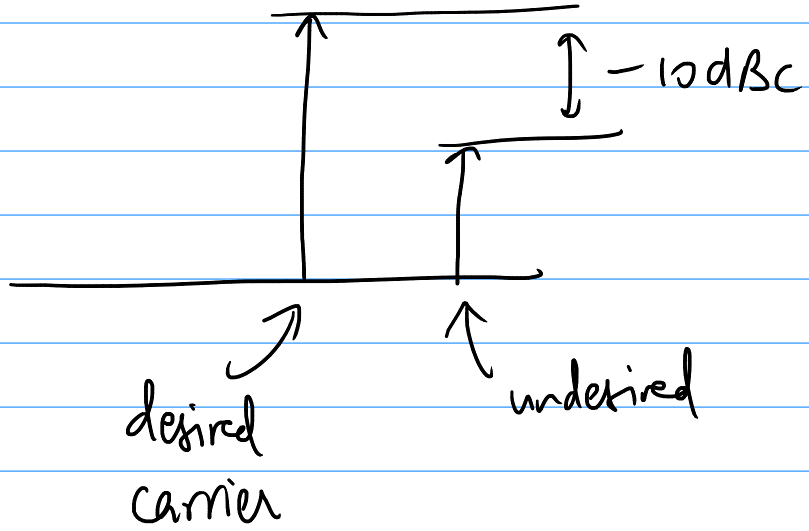
$P_{2\,dBm} = 10 \text{ dBm}$

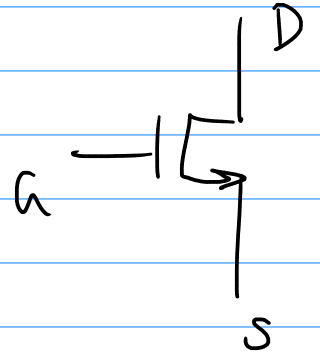
$P_3 = 100 \mu W$

$P_{3\,dBm} = -10 \text{ dBm}$

2) dBc







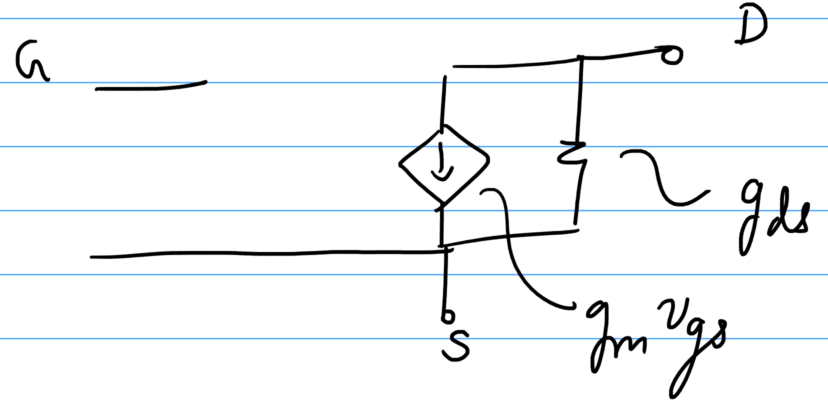
$V_{GS} > V_T$

Sat.

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

linear

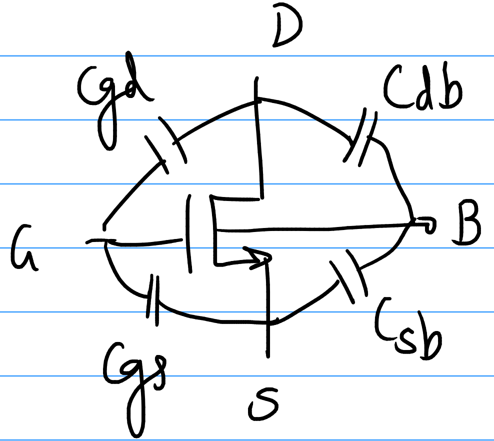
$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$



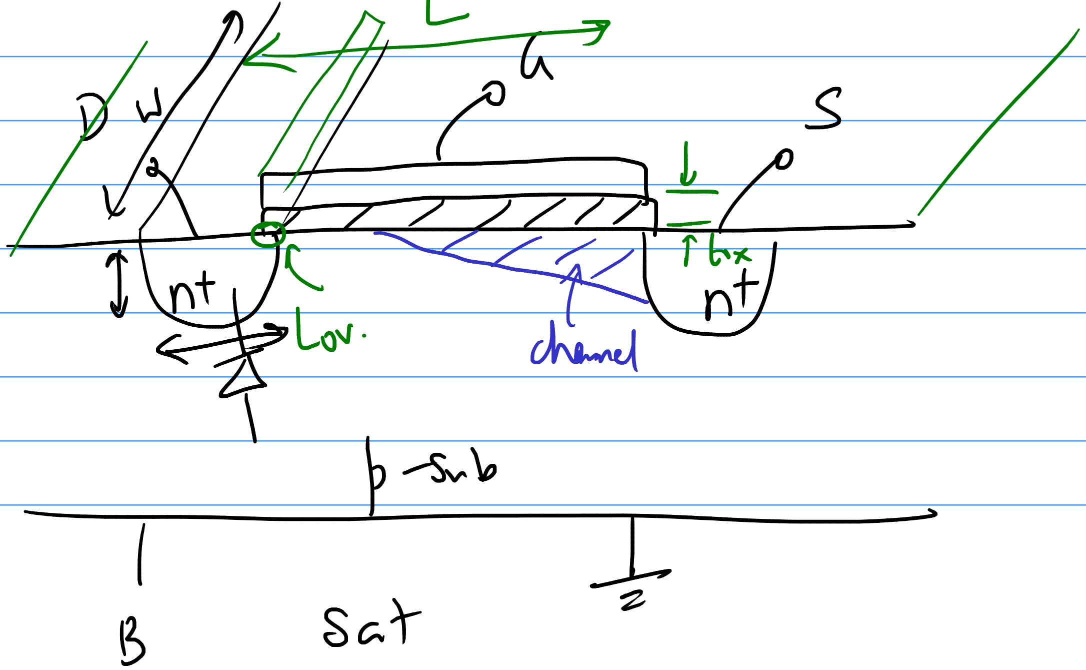
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)$$

$$g_{ds} = \lambda I_D$$

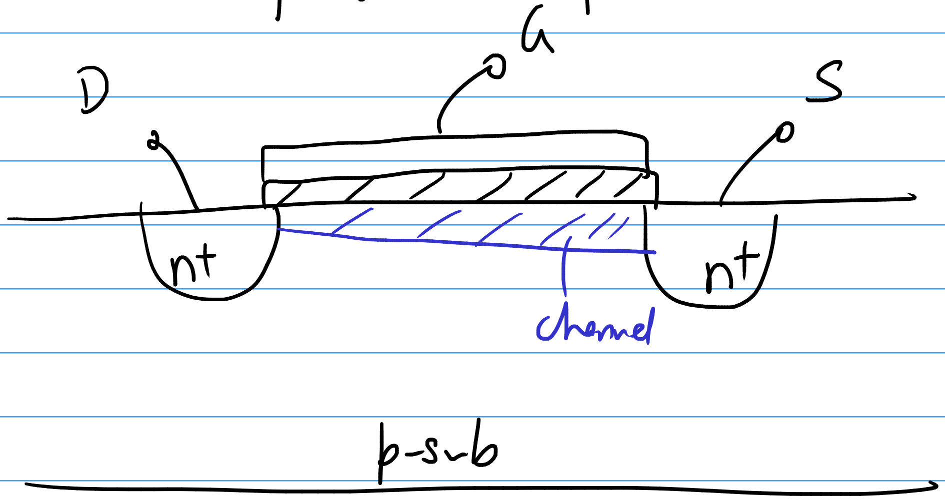
$$g_{mbs} = ?$$



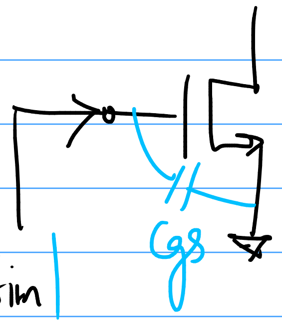
	Sat.	linear
C_{gd}	$W \cdot L_{ov} \cdot C_{ox}$	$\frac{1}{2} W L C_{ox}$
C_{gs}	$\frac{2}{3} W \cdot L \cdot C_{ox}$	$\frac{1}{2} W L C_{ox}$
C_{db}	$C_{jdb} + C_{swdb}$	$C_{db_{lin}}$
C_{sb}	$C_{jsb} + C_{swsb}$	$C_{sb_{lin}}$



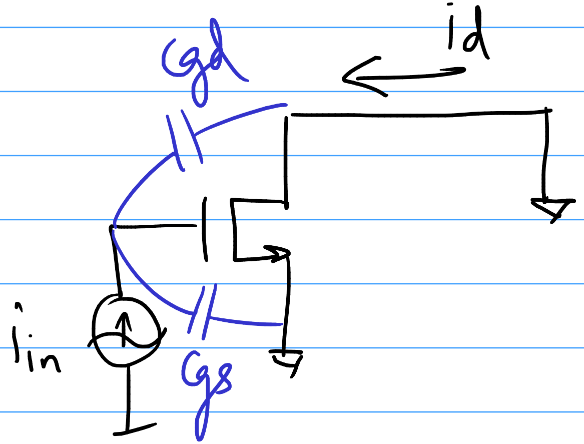
Sat



linear



$$\left| Z_{in} \right| = \left| \frac{1}{\omega C_{gs}} \right|$$



$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$$

$$\left| \frac{i_d}{i_{in}} \right| = \left| \frac{i_d}{v_{gs}} \right| \cdot \left| \frac{v_{gs}}{i_{in}} \right|$$

$$= g_m \cdot \frac{1}{\omega C_{gs}}$$

$\omega_T \equiv$ freq. @ which $\left| \frac{i_d}{i_{in}} \right| = 1 \implies \omega_T = \frac{g_m}{C_{gs}}$

$$f_T = \frac{\omega_T}{2\pi}$$

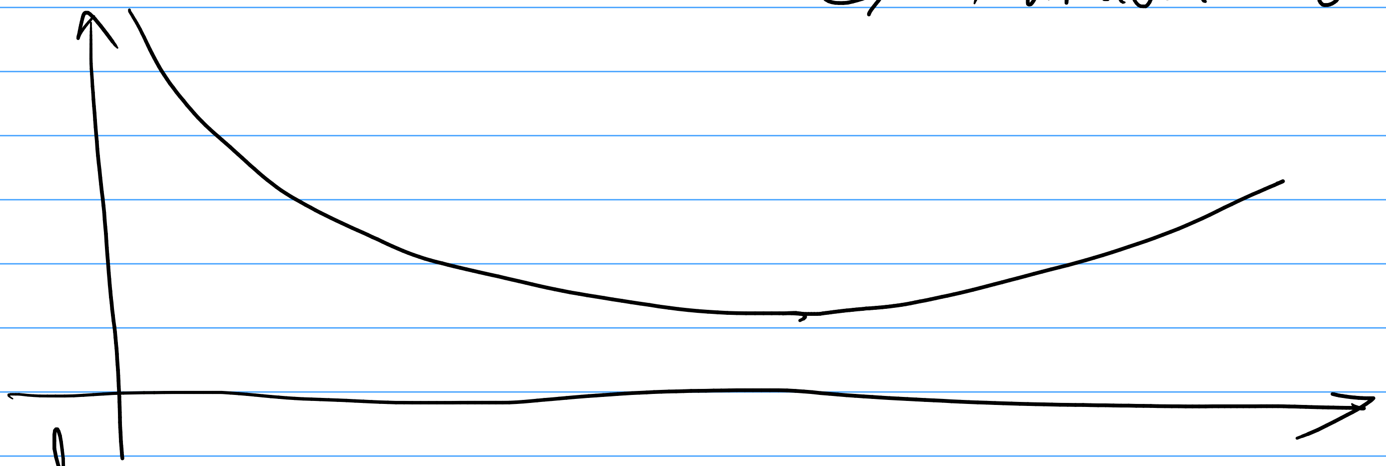
Noise – Unwanted signal?

↳ Random sources

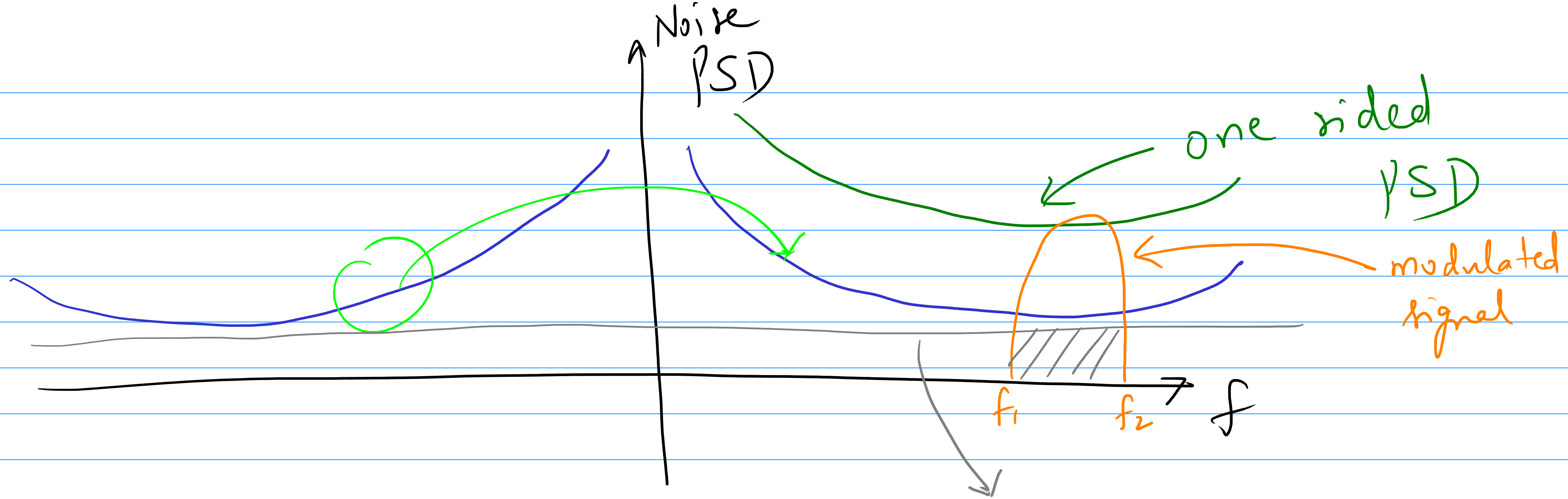
Noise PSD

V^2/Hz or A^2/Hz

power spectral density



f (Hz)

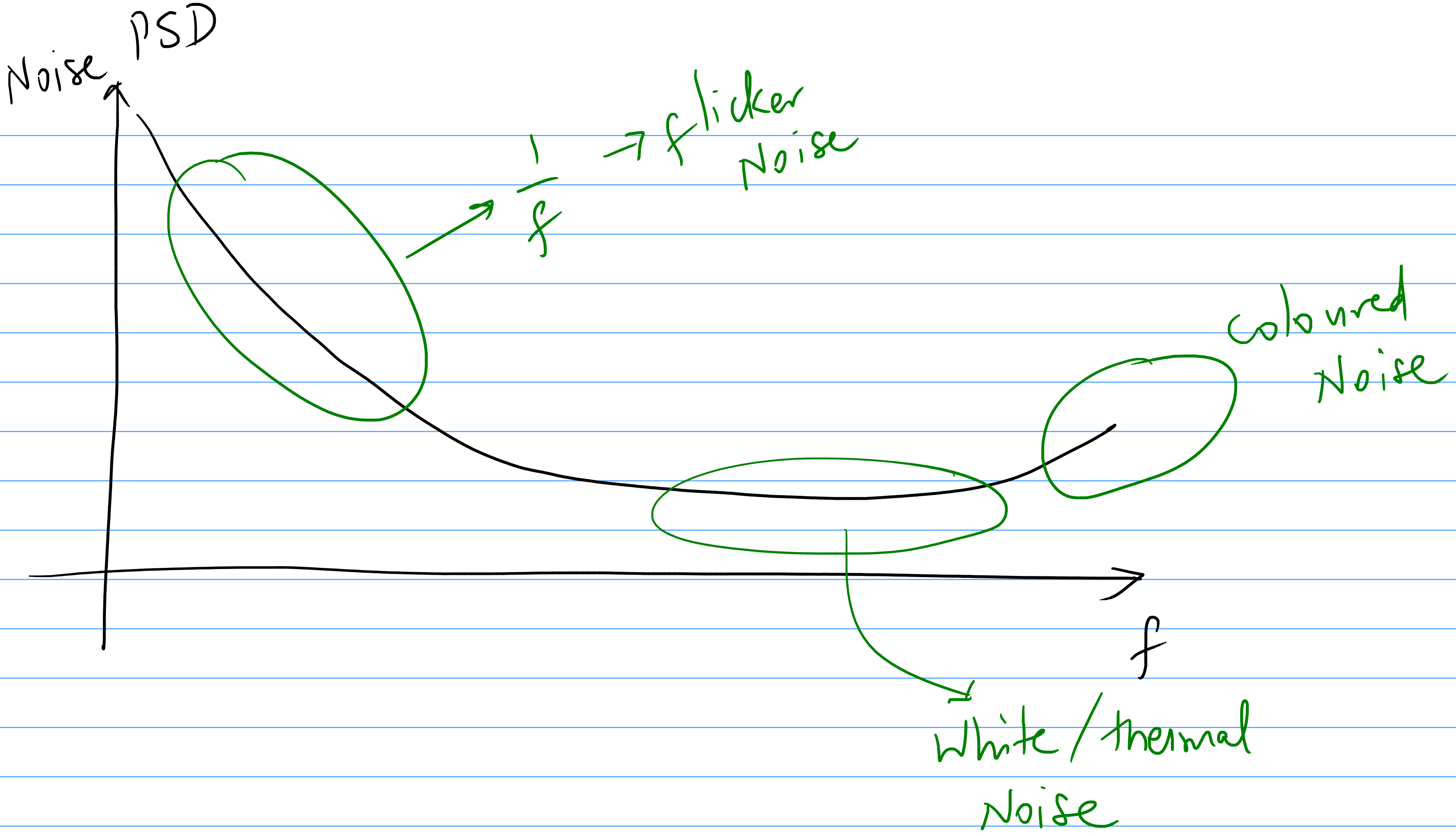


AWGN — additive white Gaussian Noise

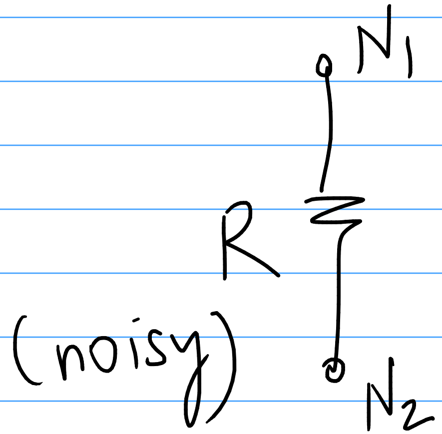
degrade BER

$\updownarrow$

SNR


Noise PSD
1/f → flicker Noise
Coloured Noise
f
White/thermal Noise

Resistor thermal noise



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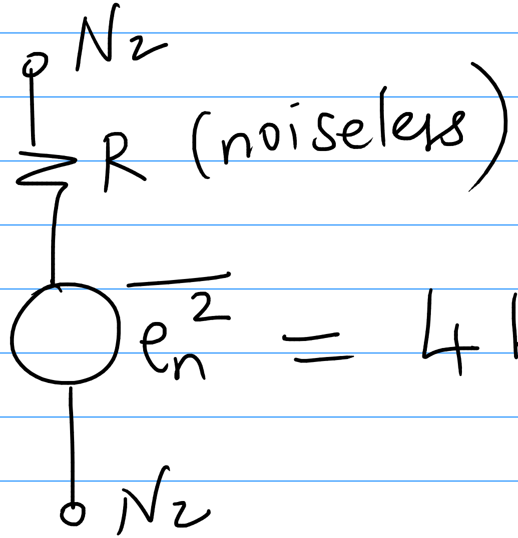
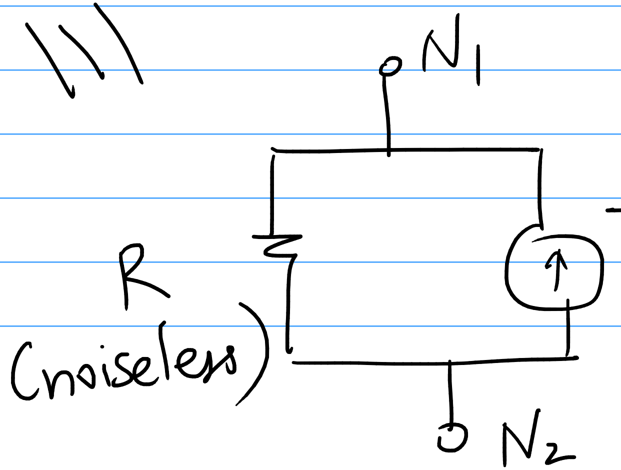


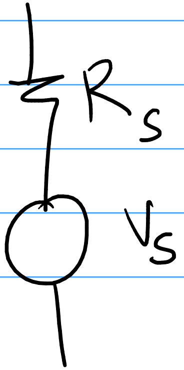
$$\overline{e_n^2} = 4kTR\,\Delta f \quad V^2$$

$\equiv$

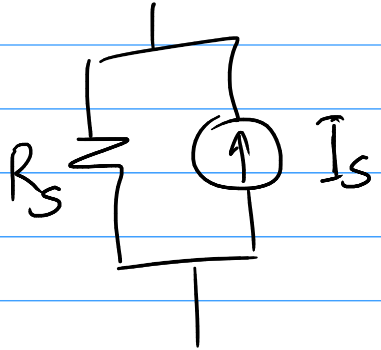


$$\overline{i_n^2} = \frac{4kT}{R}\Delta f$$

$$= 4kTG\,\Delta f$$



$\equiv$



e.g. $1k\Omega \rightarrow \dfrac{\overline{e_n^2}}{\Delta f} = 1.6 \times 10^{-17} \dfrac{V^2}{Hz}$

ms

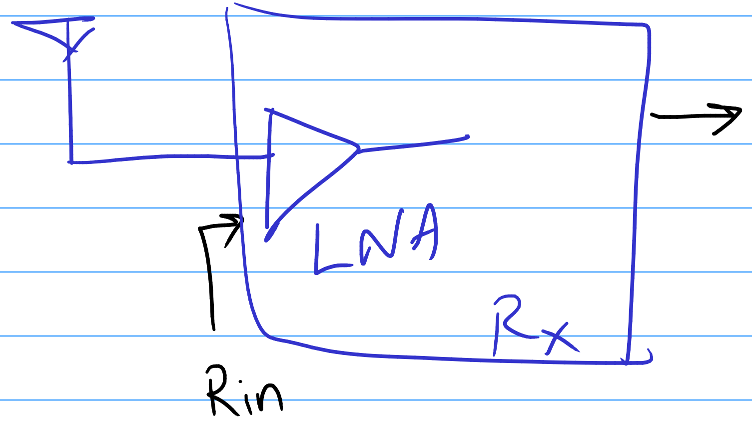
$\dfrac{\overline{i_n^2}}{\Delta f} = 1.6 \times 10^{-23}$ A^2/Hz

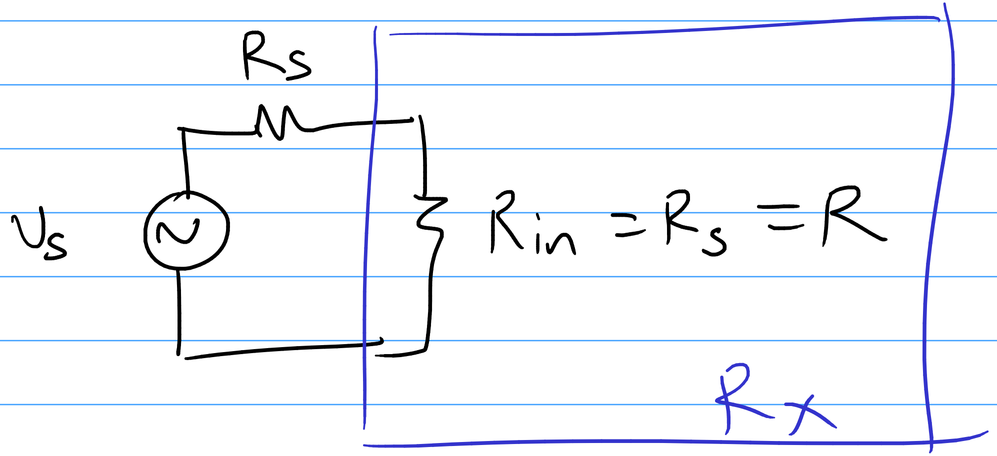
rms $\longrightarrow \dfrac{\overline{e_n}}{\sqrt{\Delta f}} = 4nV/\sqrt{Hz}$

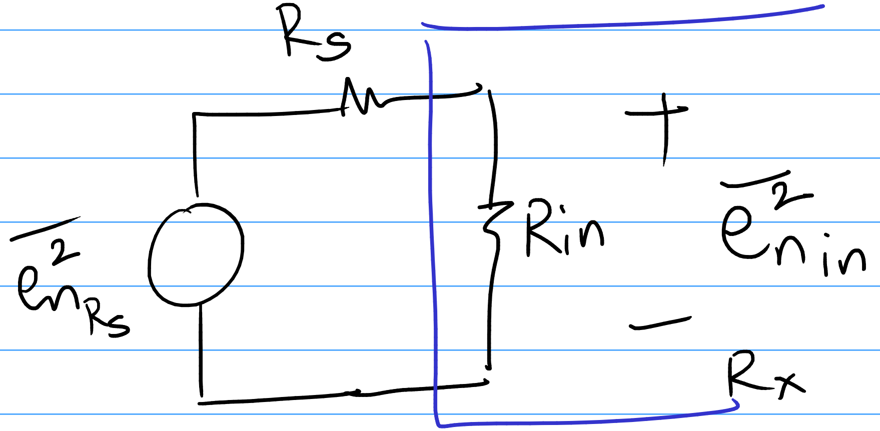
ms

$50\Omega \rightarrow \dfrac{\overline{e_n^2}}{\Delta f} = 8 \times 10^{-19}\ V^2/Hz$

rms $\longrightarrow \overline{e_n}/\sqrt{\Delta f} = 0.9nV/\sqrt{Hz}$







$$\overline{e_{n_{in}}} = \overline{e_{n_{R_s}}} \cdot \frac{R_{in}}{R_s + R_{in}}$$

$$= \frac{\overline{e_{n_{R_s}}}}{2}$$

$$\overline{e^2_{n_{in}}} = \frac{1}{4}\,\overline{e^2_{n_{R_s}}}$$

$$\overline{e_{n_{in}}^2} = \frac{4kTR \cdot B}{4}$$ ← bandwidth

V^2

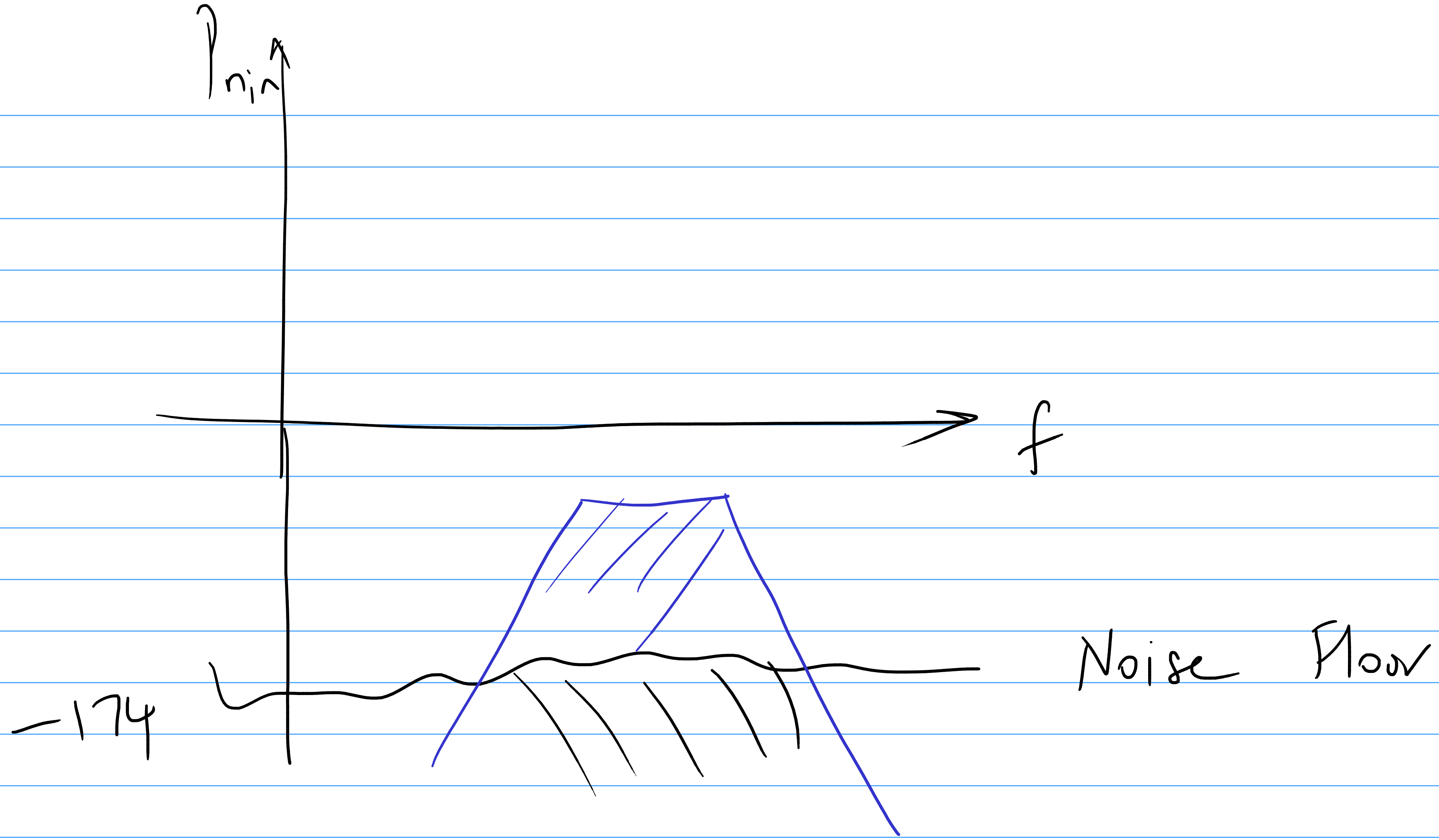
$$\overline{P_{n_{in}}} = \frac{4kTR \cdot B}{4R_{in}} = kTB \quad \text{(Watts)}$$

available noise power

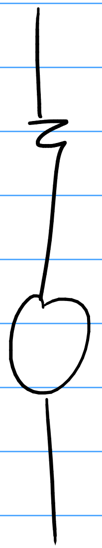
Input Noise PSD $\Big\} = kT$ W/Hz

$= 4.1 \times 10^{-21}$ W/Hz @ RT

$= -174$ dBm/Hz

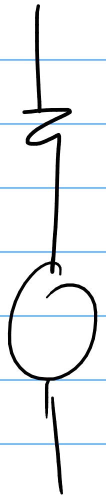


P_{min}
f
-174
Noise Floor



$R_1 = R$

$$\frac{\overline{e_{n_1}^2}}{\Delta f} = 4kTR$$

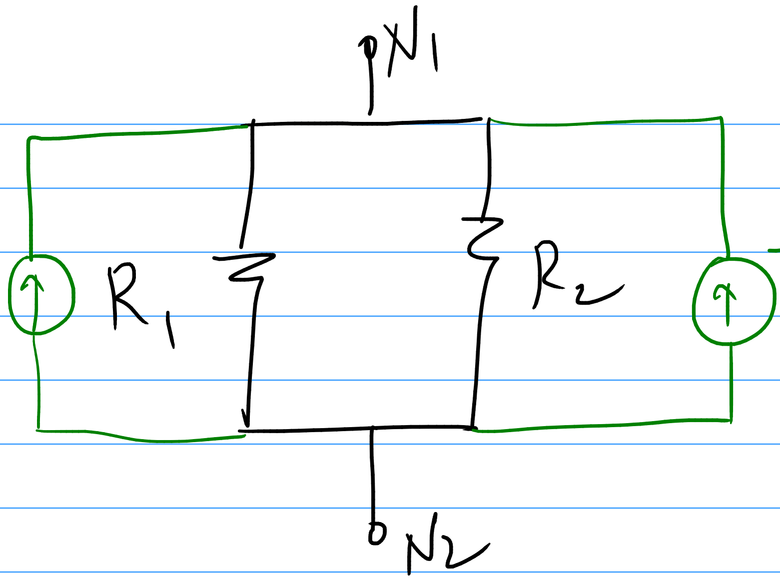


$R_2 = R$

$$\frac{\overline{e_n^2}}{\Delta f} = 4kTR$$

un- correlated

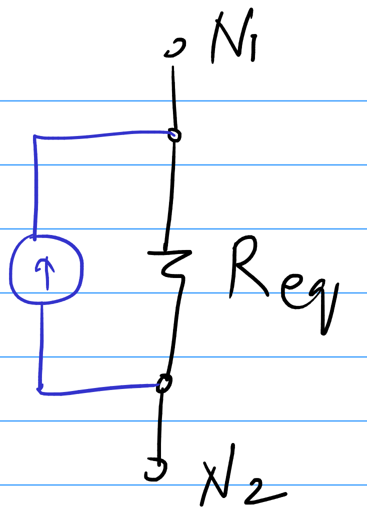
$\overline{i_{n1}^2} = \frac{4kT}{R_1}$



$\overline{i_{n2}^2} = \frac{4kT}{R_2}$

$\equiv$

$\overline{i_{neq}^2}$

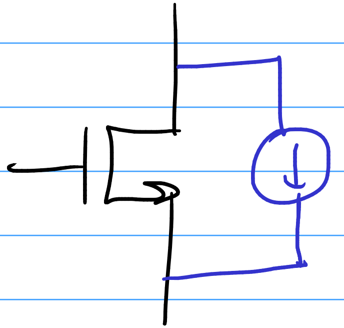


$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\overline{i_{neq}^2} = 4kT\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{4kT}{R_{eq}}$$

ideal L   ideal C $\rightarrow$ No noise

Thermal Noise in MOSFETS



$\overline{i_{nd}^2}$

1) Triode : $\overline{i_{dn}^2} = 4kT\, g_{do}\, \Delta f$

$\left\{ \text{or } 4kT\gamma g_{do} \Delta f \,;\ \gamma = 1 \right\}$

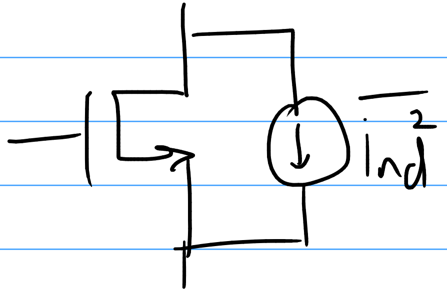
$$g_d = \frac{\partial I_D}{\partial V_{DS}}$$

$$= \frac{\partial}{\partial V_{DS}}\left[\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)\cdot V_{DS} - V_{DS}^2/2 \right]$$

$$g_d = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_T - V_{DS}\right)$$

$$g_{d0} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_T\right)$$

<u>In Sat. region</u>

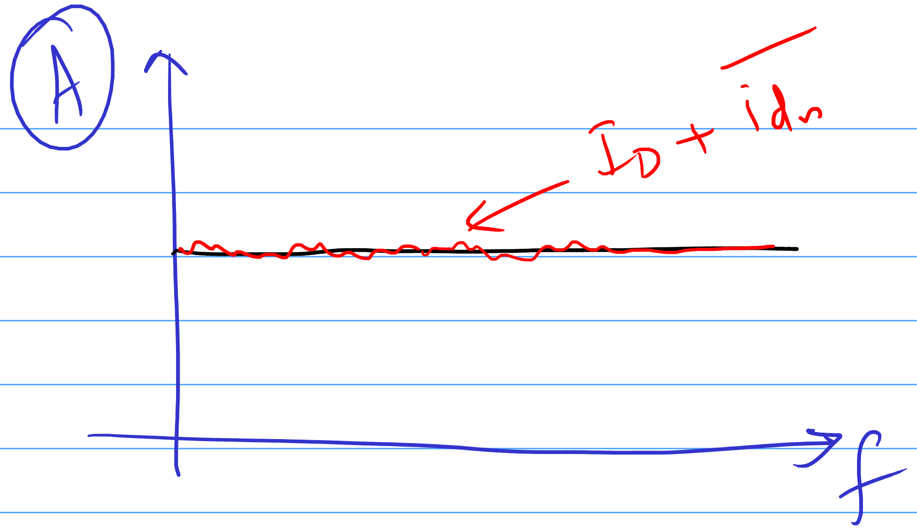


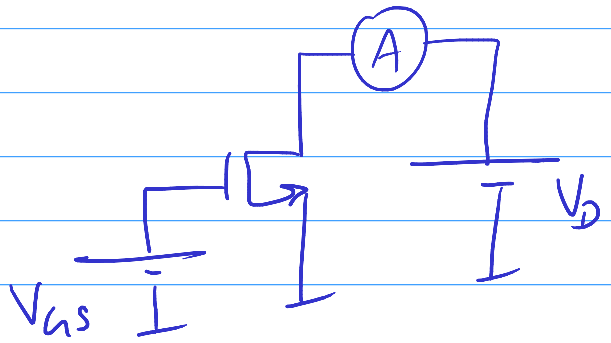
$$\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f \qquad = g_{m\,sat.}$$

$\gamma = \frac{2}{3}$ for long channel device

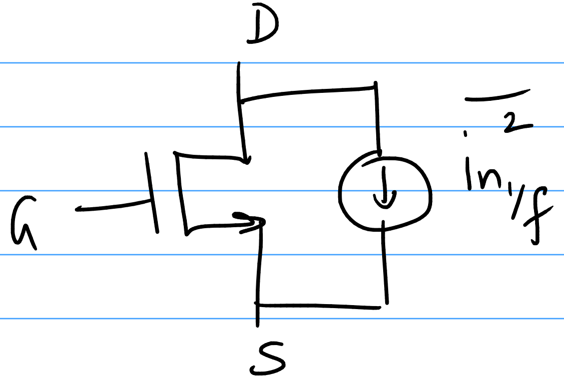
$$\overline{i_{nd}^2} = \frac{8kT}{3} g_m \Delta f$$

short channel devices $\rightarrow$ $\gamma = 2-3$


A
$\overline{I_D + i_{dn}}$
f

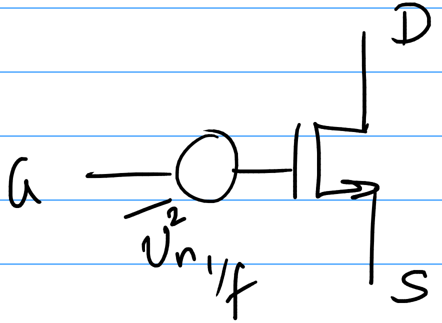

A
V_{GS}
V_D

Flicker Noise

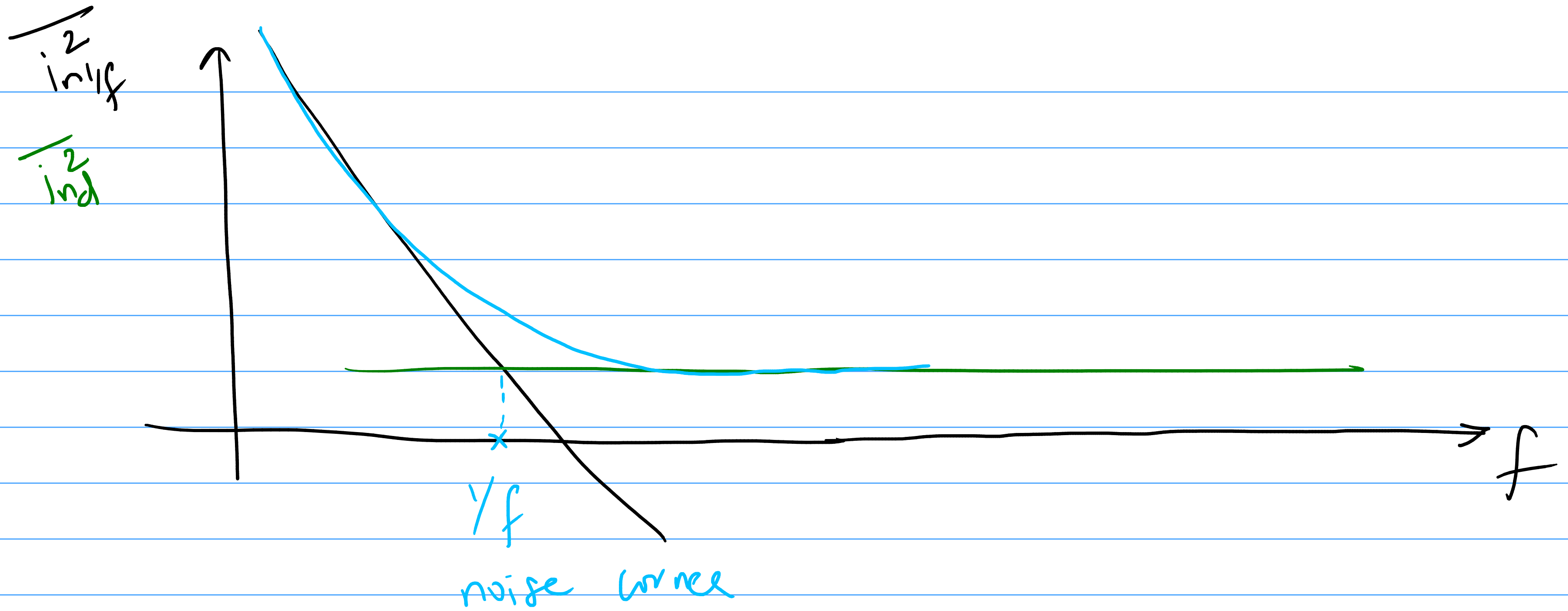


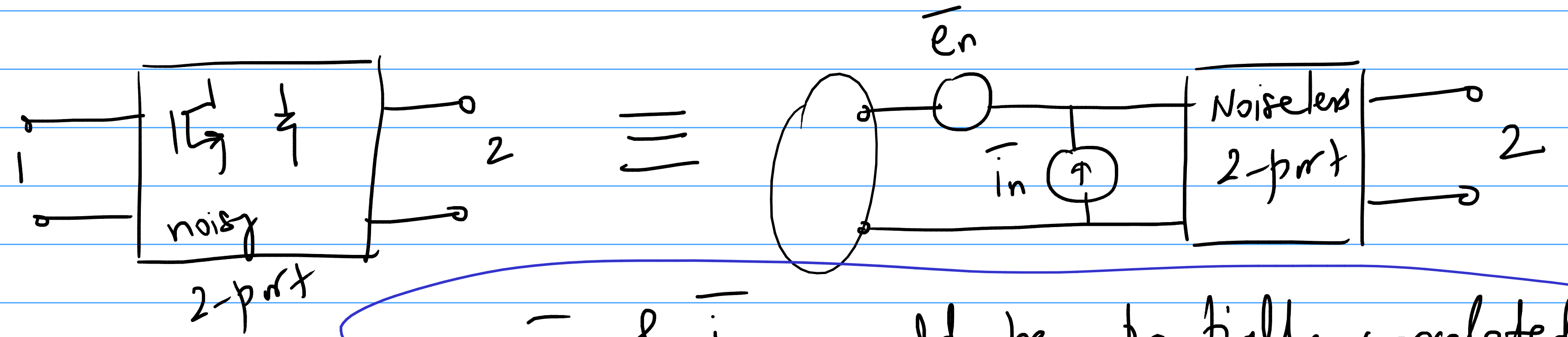
$$\overline{i_{n_{1/f}}^2} = \frac{K_f}{WLC_{ox}} g_m^2 \cdot \frac{1}{f} \cdot \Delta f$$

Lower flicker noise if you increase gate area

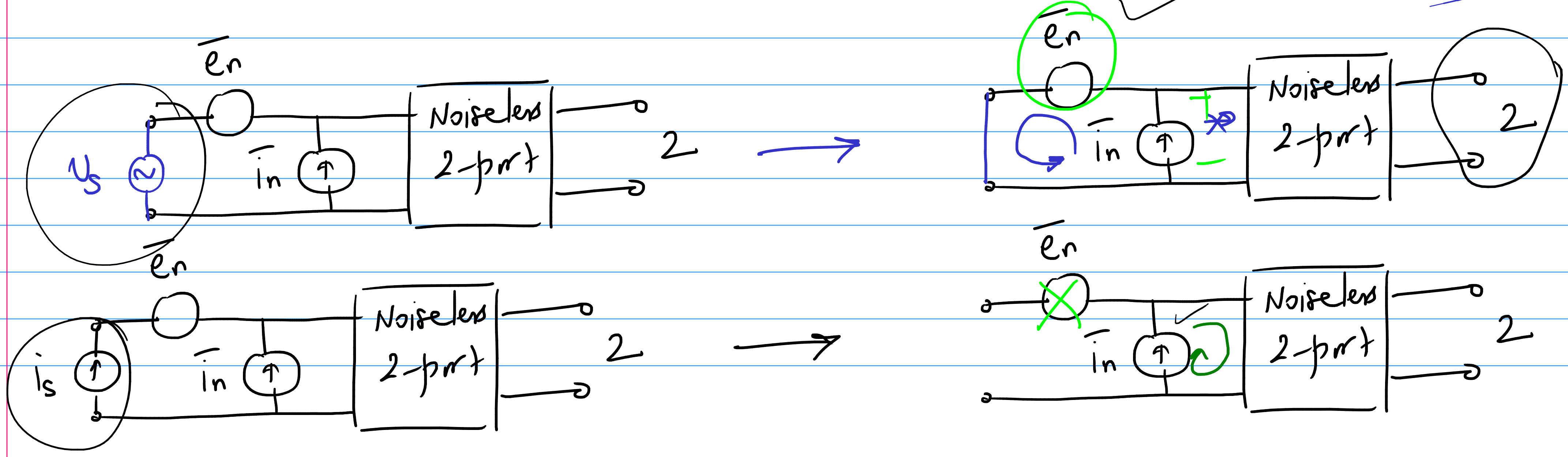


$$\overline{v_{n_{1/f}}^2} = \frac{K_f}{WLC_{ox}} \cdot \frac{1}{f} \cdot \Delta f$$


$\overline{i^2_{n,1/f}}$
$\overline{i^2_{nd}}$
f
1/f
noise corner

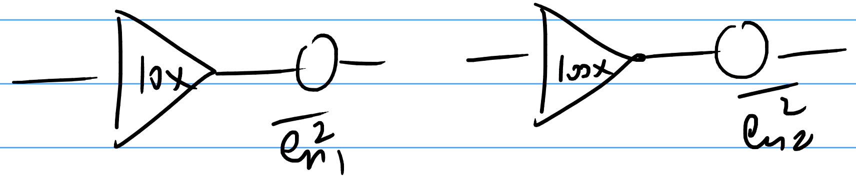


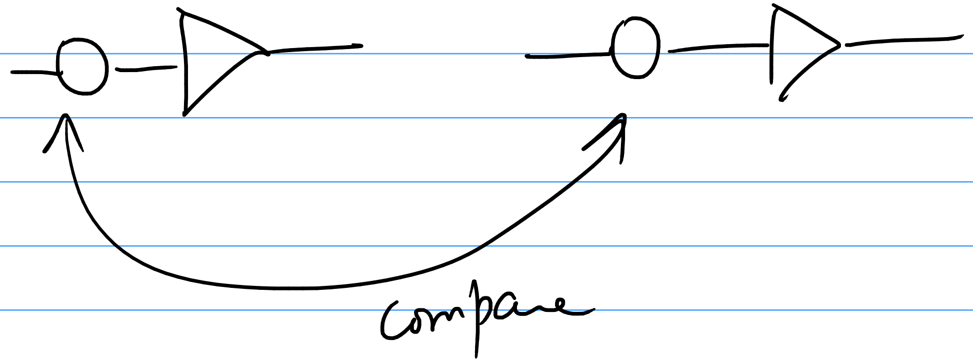
$\bar{e}_n$ & $\bar{i}_n$ could be partially correlated

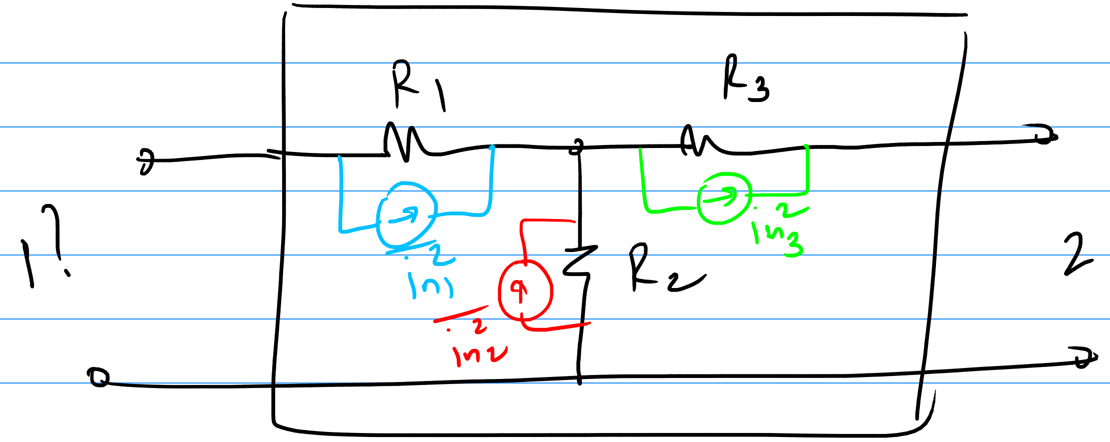


* Need both $\overline{e_n}$ & $\overline{i_n}$ → esp. for RF circuits

* $\overline{e_n}$ & $\overline{i_n}$ are fictitious sources



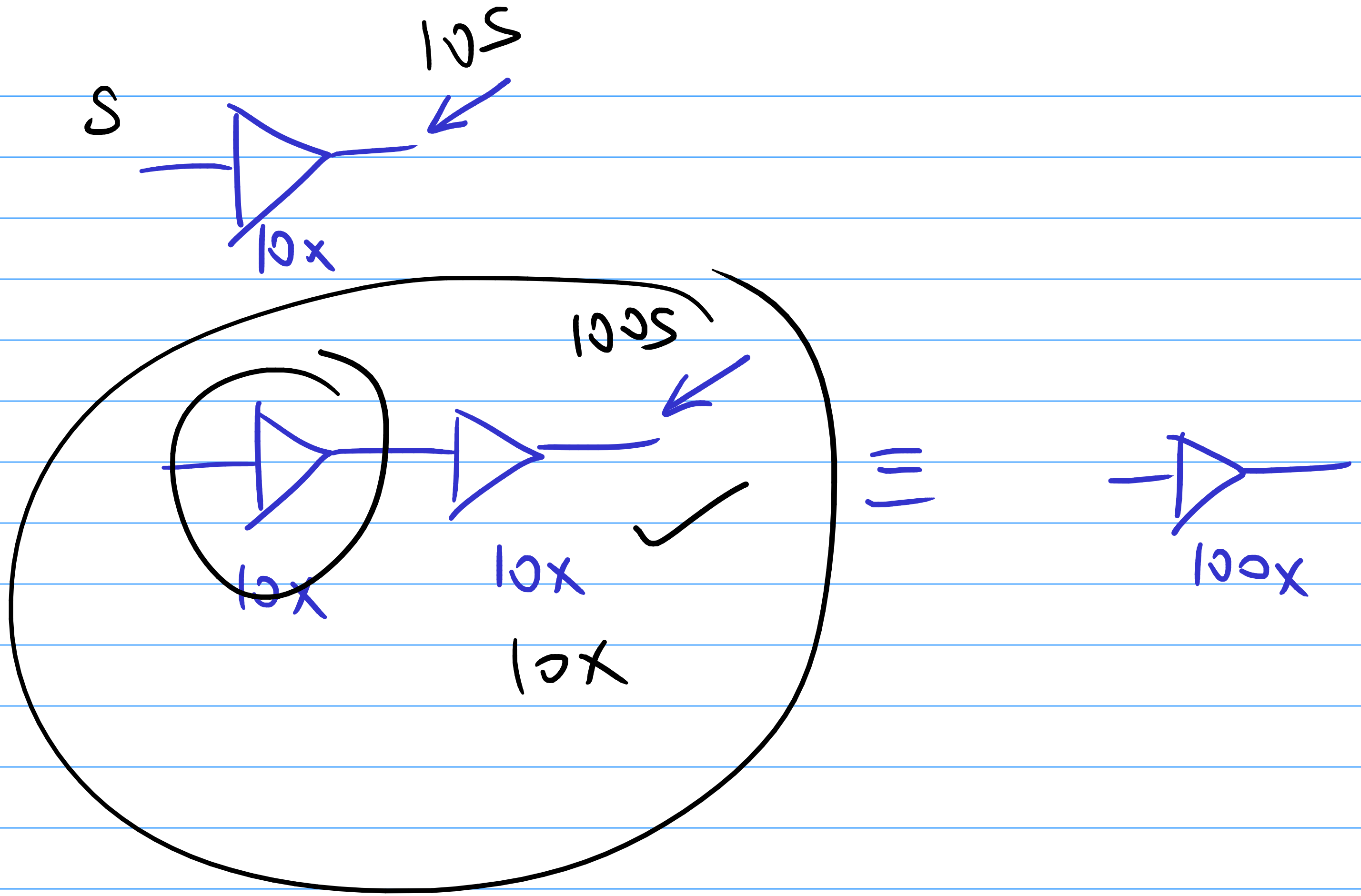




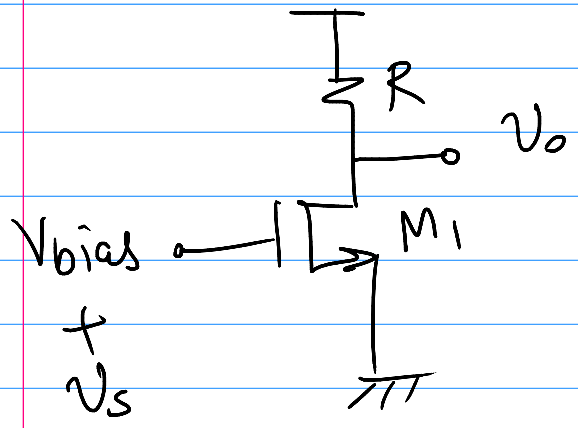
?

* Apply a "convenient" load @ port 2 e.g. S.C. or O.C.
* Apply ideal v_s @ port 1
* calculate noise due to $\overline{e_n}$ @ port 2 (A)
* Do the same for actual network (B)
* Apply $\overline{i_{n1}}$, $\overline{i_{n2}}$, $\overline{i_{n3}}$ in turn
* Equate (A) & (B) → gives us $\overline{e_n}$

* apply ideal i_s @ port 1
* ...
* calculate $\overline{i_n}$


S
10S
10x
100S
10x
10x
10x
≡
100x

Example #1

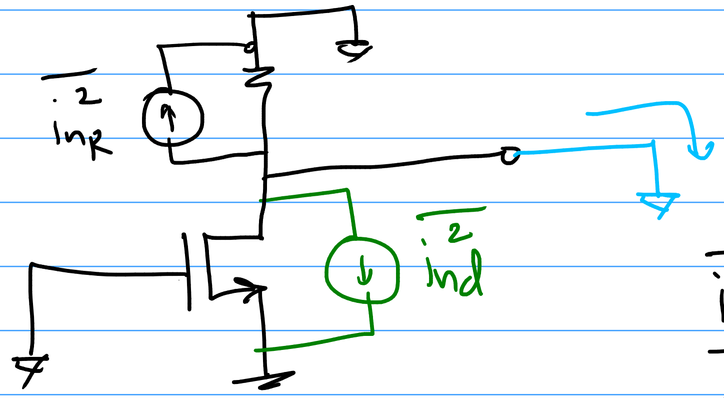


$\overline{e_n}, \overline{i_n} = ?$ @ low freq.

$Z_{in} = \infty$ @ low freq.

$\exists\ \overline{i_n} = 0$

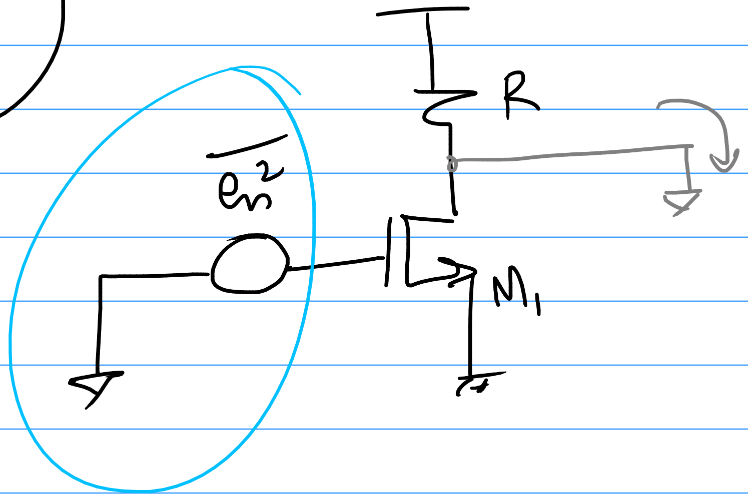
I



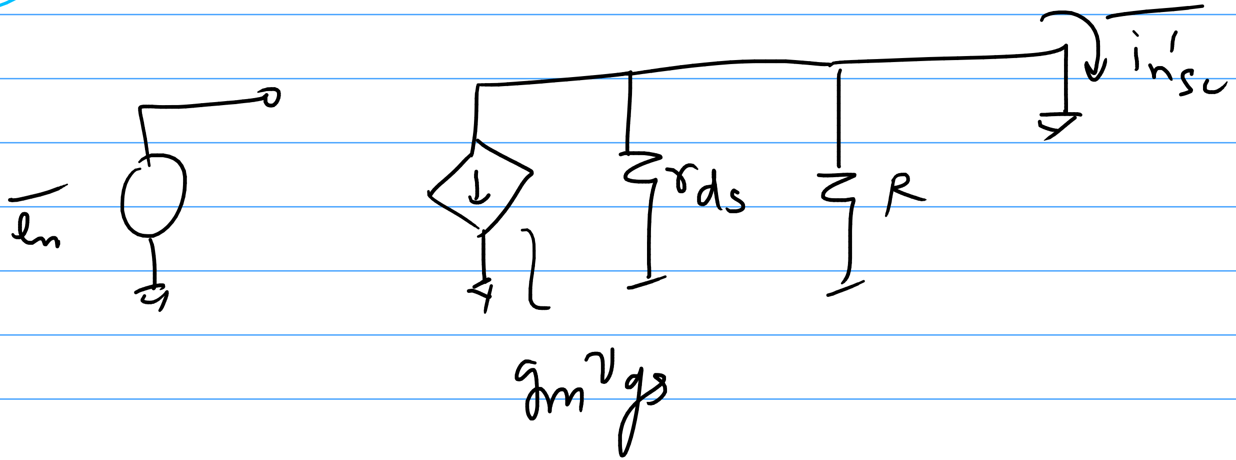
$\overline{i^2_{nsc_1}} = \overline{i^2_{nR}} + \overline{i^2_{nd}}$

$$\frac{\overline{i^2_{nsc_1}}}{\Delta f} = \frac{4kT}{R} + 4kT\gamma g_m$$

II



$$\overline{i'^2_{nsc}} = \overline{e_n^2} \cdot g_m^2$$



Equate

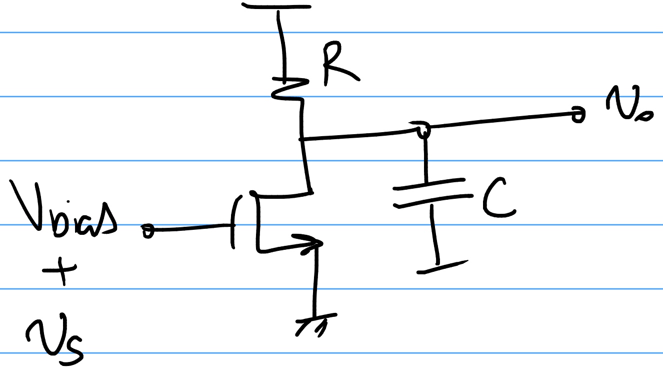
$$\overline{i'^2_{nsc}} = \overline{i^2_{nsc}}$$

$$g_m^2 \frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{R} + 4kT\gamma g_m$$

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT}{g_m^2 R} + \frac{4kT\gamma}{g_m}$$

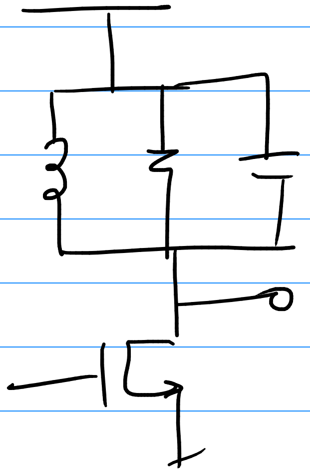
$\uparrow g_m \Rightarrow \downarrow \overline{e_n}$

Ex 2

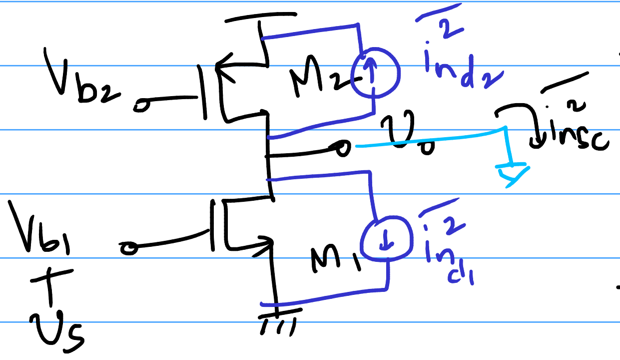


$\overline{i_n^2} = 0$

$\overline{e_n^2}$ = same as before



Ex 3



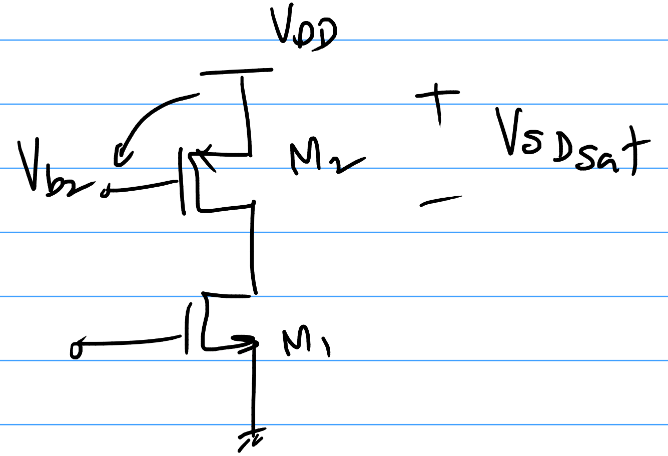
$\overline{i_n^2} = 0$

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{\overline{i_{nsc}^2}}{g_{m_1}^2} = \frac{4kT\gamma_1 g_{m_1} + 4kT\gamma_2 g_{m_2}}{g_{m_1}^2}$$

$$\frac{\overline{e_n^2}}{\Delta f} = \frac{4kT\gamma_1}{g_{m1}} + 4kT\gamma_2 \cdot \frac{g_{m2}}{g_{m1}^2}$$

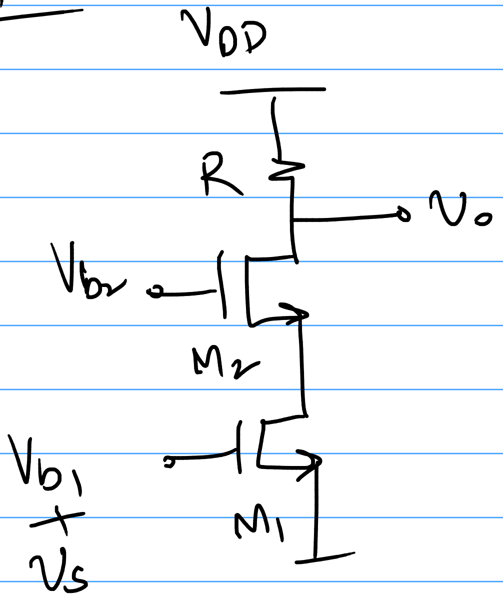
for low noise $\rightarrow$ $\uparrow g_{m1}$, $\downarrow g_{m2}$ $\rightarrow$ $V_{GS}-V_T \uparrow$

keep I_{bias} constant

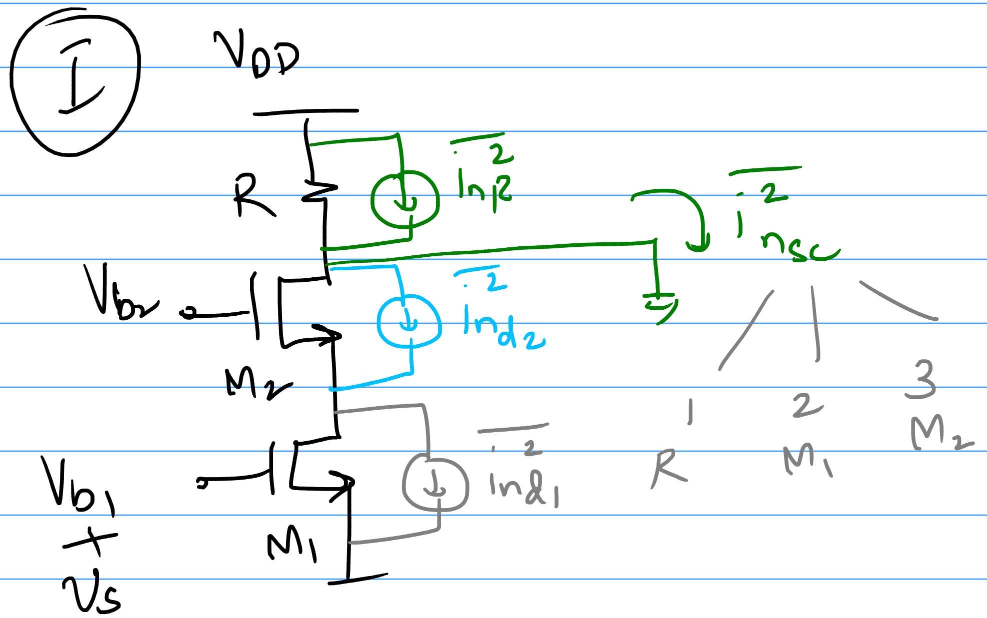


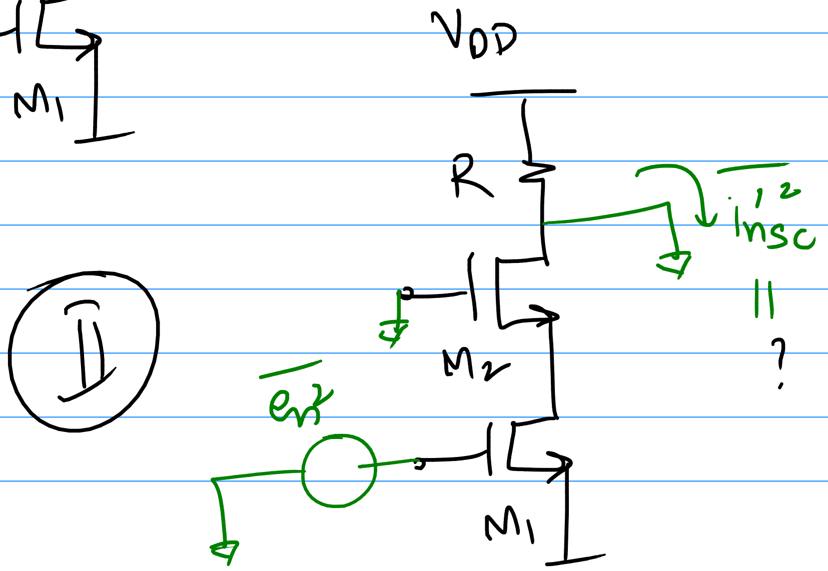
$\uparrow \left(\frac{W}{L}\right)_1 \rightarrow (V_{GS}-V_T)_1 \downarrow$

Ex 4

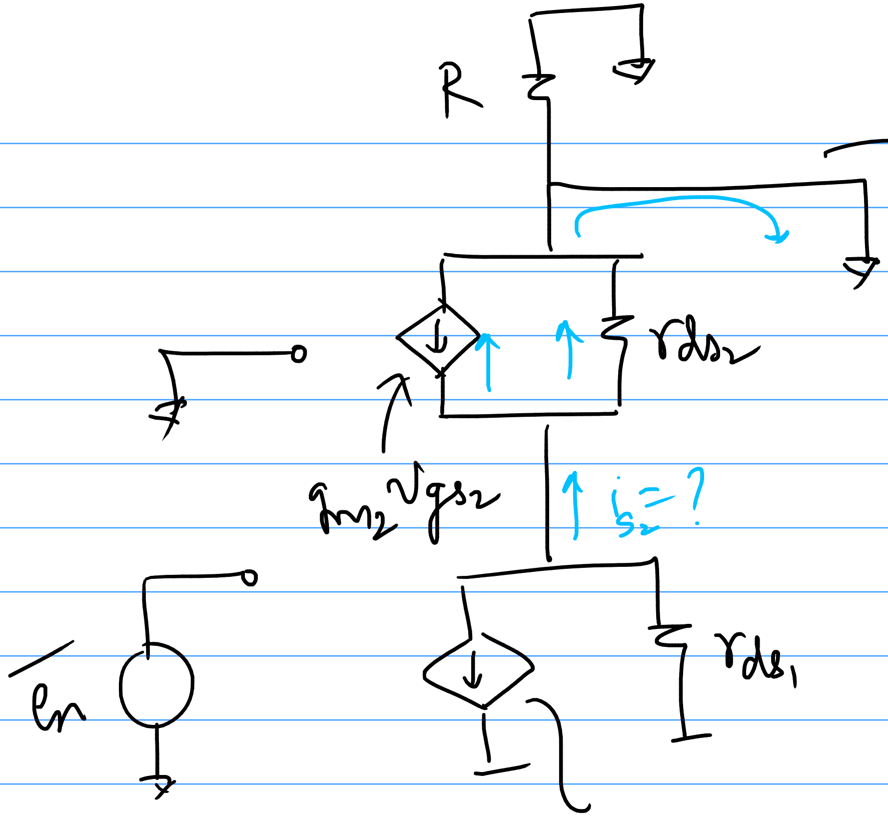


$\overline{e_n^2} = ?$



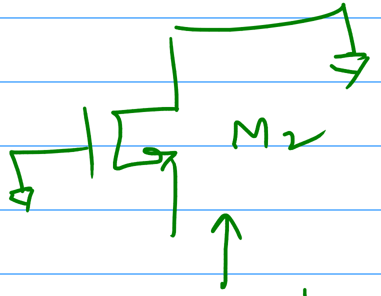


II



$$\overline{i_{nsc}'} = g_{m1}\overline{e_n}$$

$$\overline{i_{nsc}'^2} = g_{m1}^2\overline{e_n^2}$$



$$R_{in} = \frac{1}{g_{m2}+g_{ds2}}$$

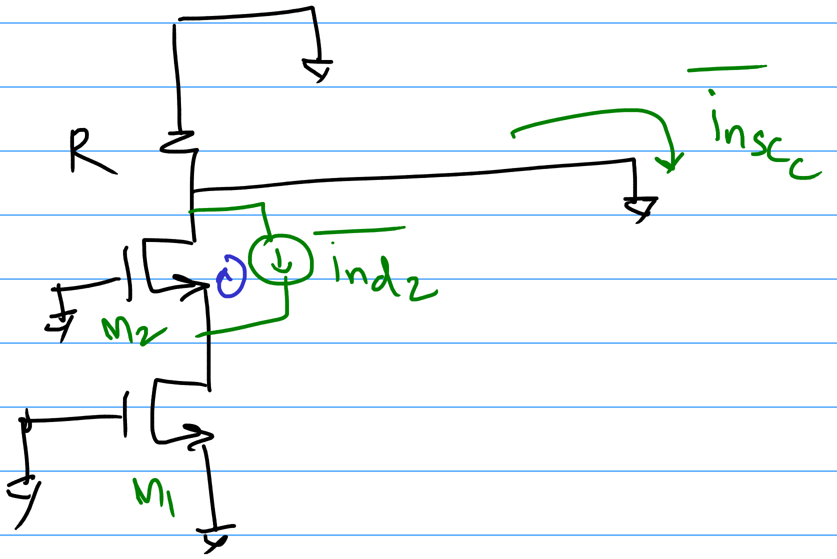
$$g_{m1}v_{gs1} = g_{m1}\overline{e_n}$$

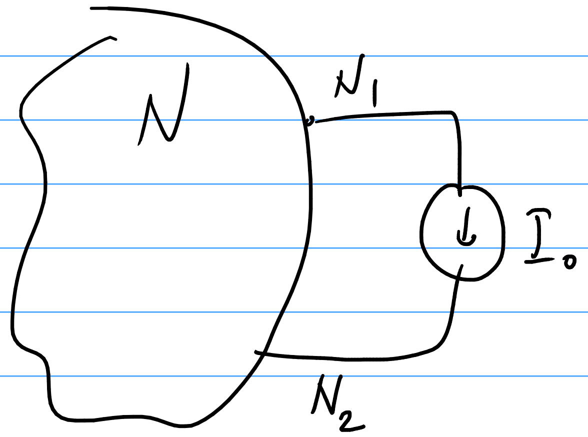
$$\overline{i_{s2}} = g_{m1}\overline{e_n} \times \frac{r_{ds1}}{r_{ds1}+\frac{1}{g_{m2}+g_{ds2}}} \simeq g_{m1}\overline{e_n}$$

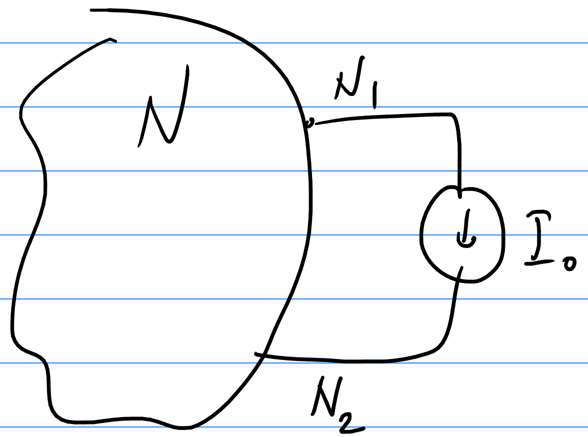
I a) $\overline{i_{nR}^2}$: $\overline{i_{nsc_a}^2} = \overline{i_{nR}^2}$

b) $\overline{i_{nd_1}^2}$: $\overline{i_{nsc_b}^2} = \overline{i_{nd_1}^2}$

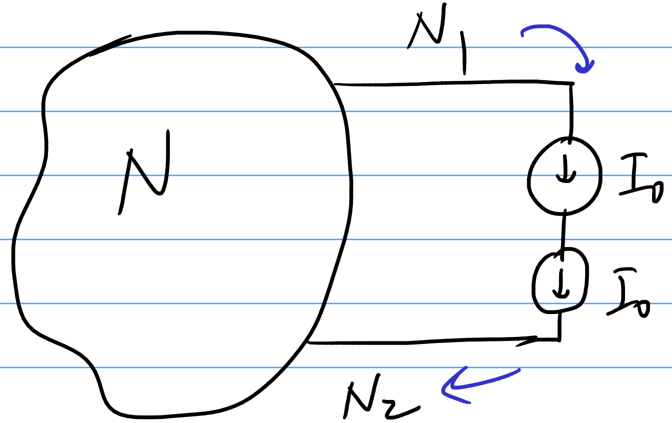
c) $\overline{i_{nd_2}^2}$:



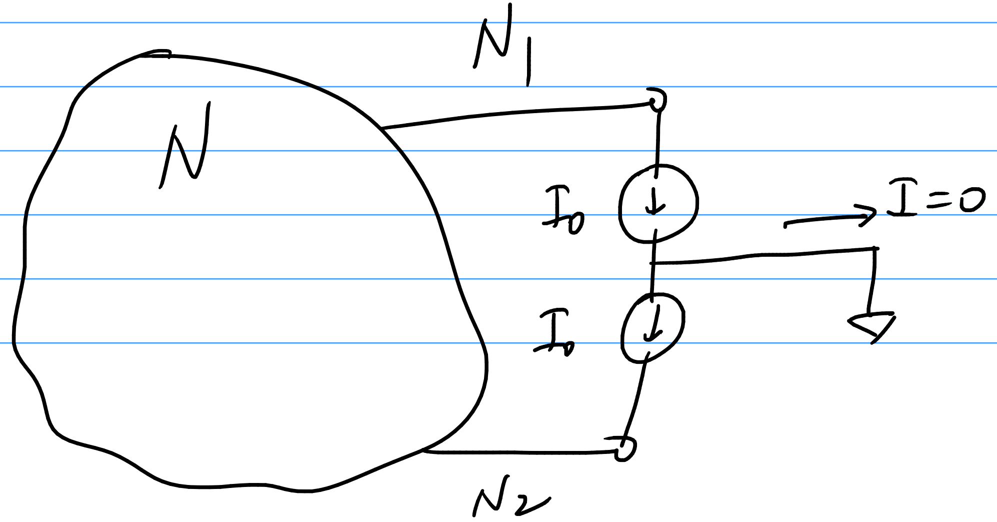


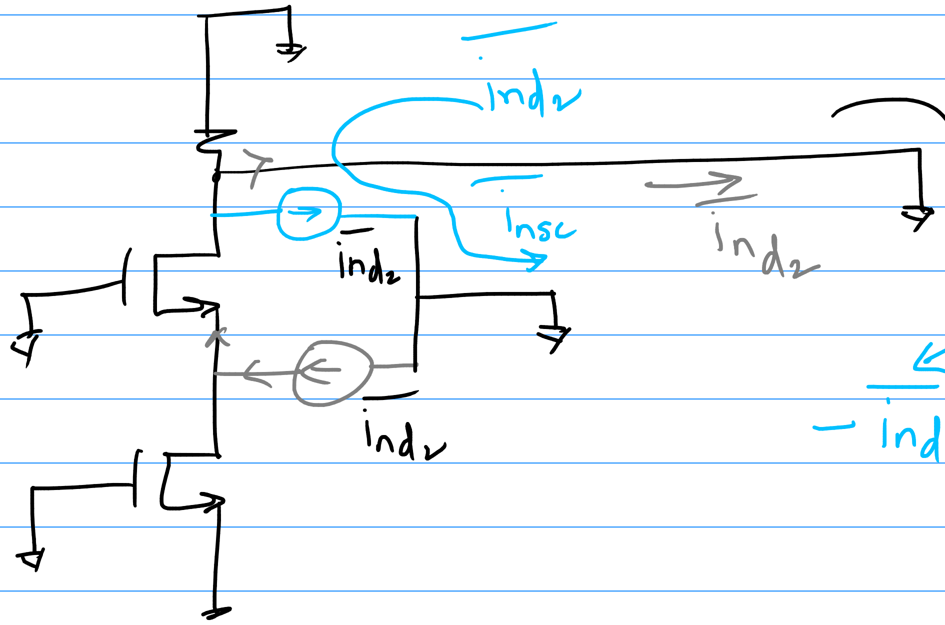


$\equiv$



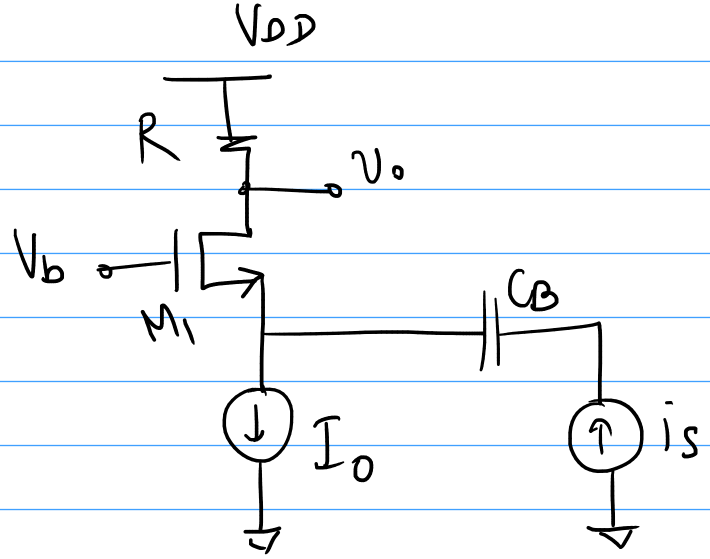
$\equiv$





$$\overline{i_{nsc}} = \overline{i_{nsc}} + \overline{i_{nsc}}$$

$$-\overline{i_{nd_2}} + \overline{i_{nd_2}} = 0$$

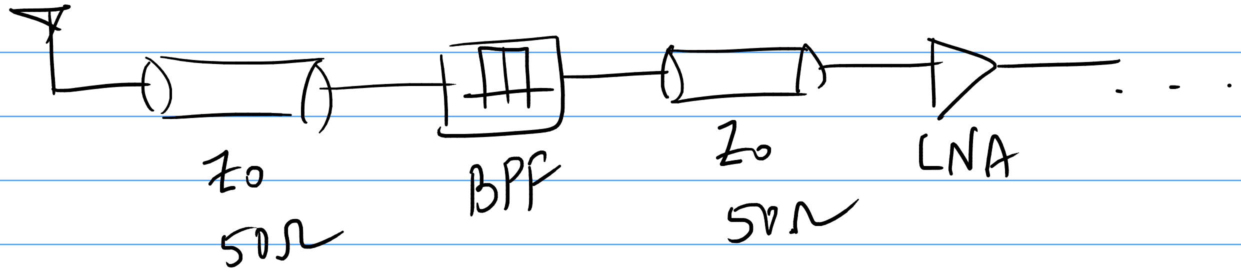


$\overline{e_n} = ?$ (HW)

$\overline{i_n} = ?$ (HW) $\longrightarrow$ Output referred noise due to $\overline{i_n}$

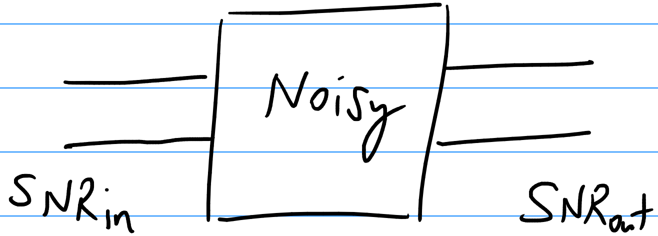
$=$

Output referred noise due to R & M_1

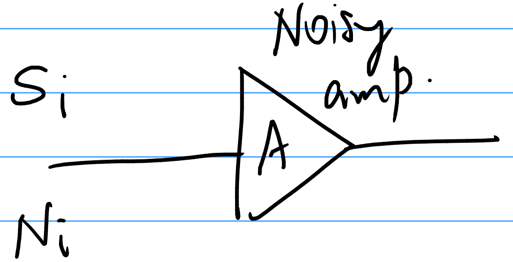


In RF Systems → Use "Noise Figure"

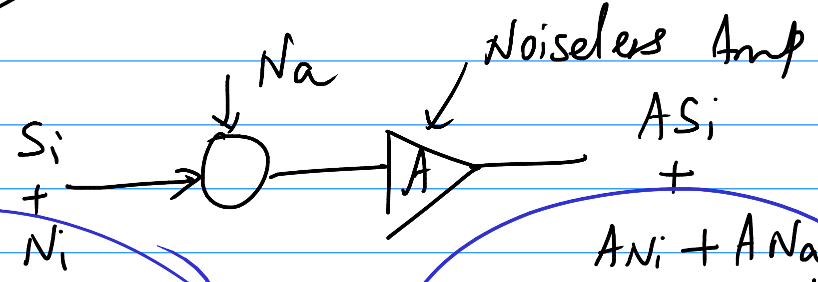
$$\boxed{\text{Noise Factor "F"} = \frac{SNR_{in}}{SNR_{out}}}$$



$SNR_{in} > SNR_{out}$



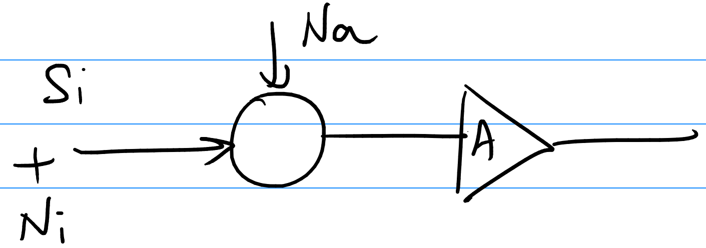
→



$SNR_{in} = \dfrac{S_i}{N_i}$

$SNR_{out} = \dfrac{AS_i}{A(N_i + N_a)}$

Noise Figure "NF" (dB) $= 10\log_{10}(F) = SNR_{in}(dB) - SNR_{out}(dB)$



$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{S_i / N_i}{A S_i / A(N_i + N_a)}$$

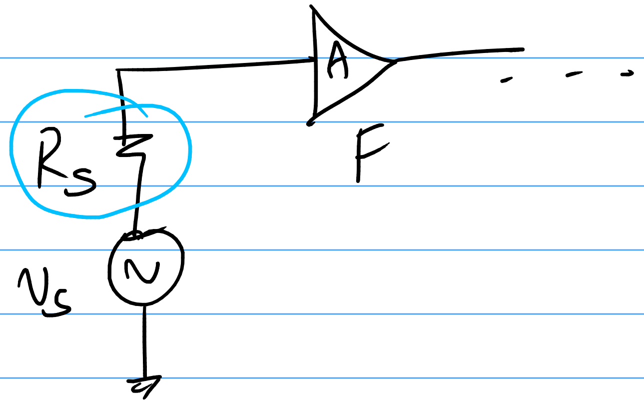
$$F = \frac{N_i + N_a}{N_i} = \frac{A(N_i + N_a)}{A N_i}$$

$$\boxed{F = \frac{\text{Total output-referred noise}}{\text{Output noise due to source alone}}}$$

$$F = 1 + \frac{N_a}{N_i}$$

depends on R_s, & N_i

Noise Temperature



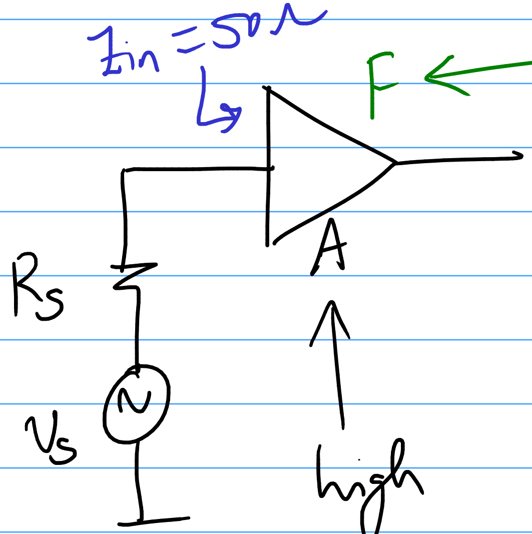
Noise temp. T_N

|||

increase in temp of R_s to account for F i.e. Na

$$F = 1 + \frac{T_N}{T_{ref}}$$

Low - Noise Amplifiers



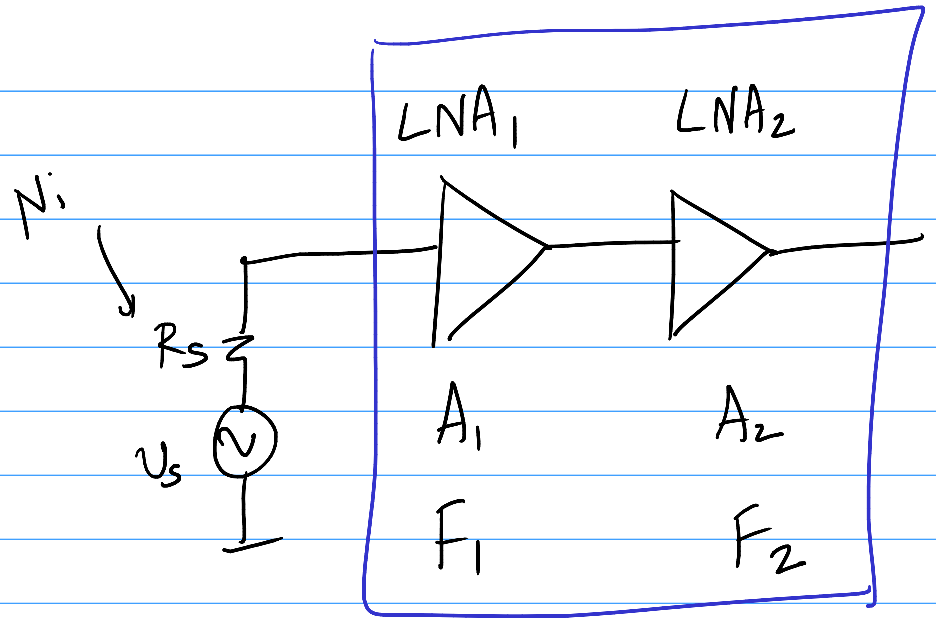
low NF

high gain

freq. : $f_0 \pm \Delta f$

(Narrow band LNA)

1) CSA ✓ $Z_{in} = \infty$ / C_{in} high gain
2) CGA ✓ Z_{in} is finite lower gain
3) CDA ✗
4) Cascode



A, F

$\hookrightarrow A = A_1 \cdot A_2$

$F = ?$

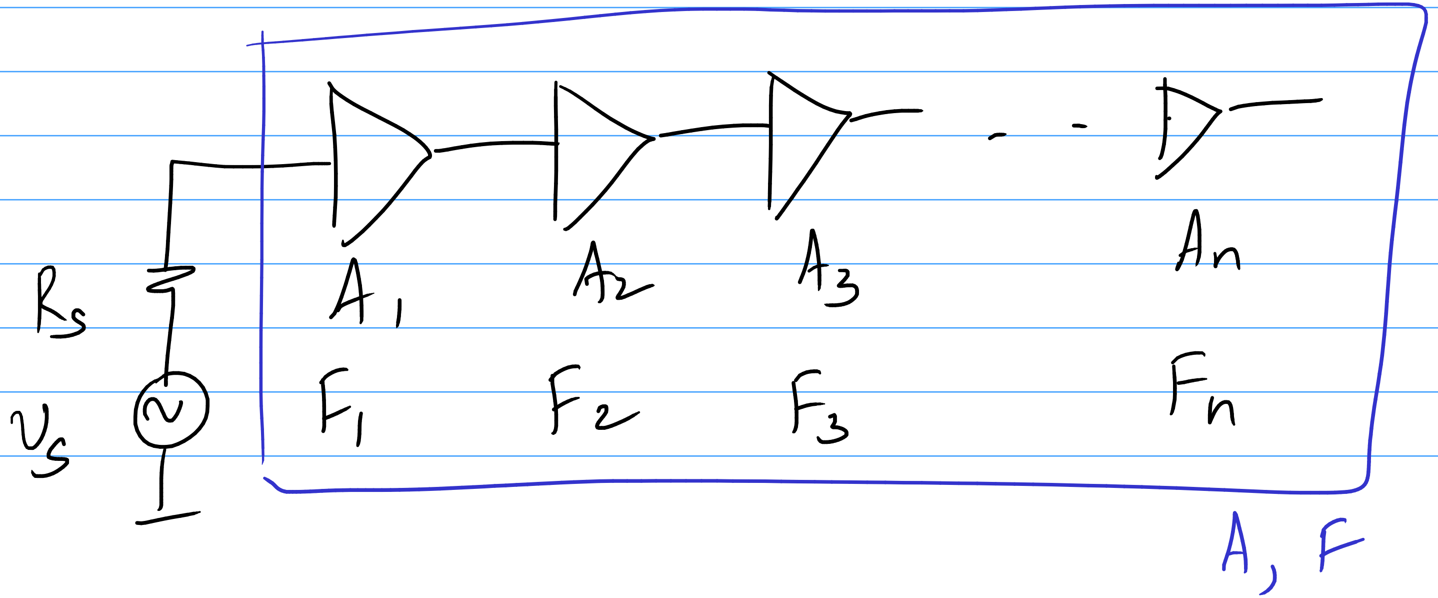
$$F = \frac{N_{out}}{A_1 \cdot A_2 \cdot N_i}$$

$$= \frac{A_1 A_2 N_i + A_1 A_2 N_{a1} + A_2 N_{a2}}{A_1 A_2 N_i}$$

$$= 1 + \frac{N_{a1}}{N_i} + \frac{N_{a2}}{A_1 N_i}$$

$$F = 1 + (F_1 - 1) + \frac{F_2 - 1}{A_1}$$

$$\boxed{F = F_1 + \frac{F_2 - 1}{A_1}}$$

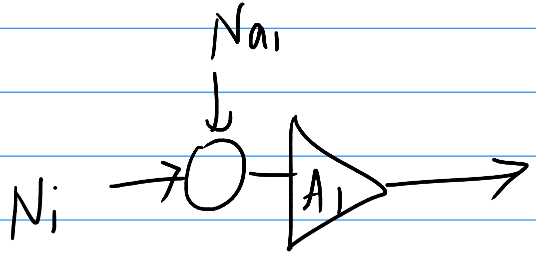


$$\boxed{F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1 A_2} + \cdots \frac{F_n - 1}{A_1 A_2 \cdots A_{n-1}}}$$

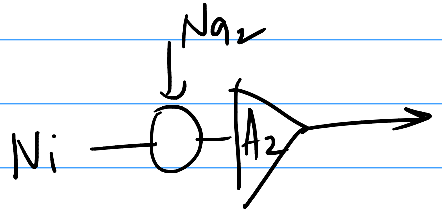
FRIIS EQUATION

low NF
LNA

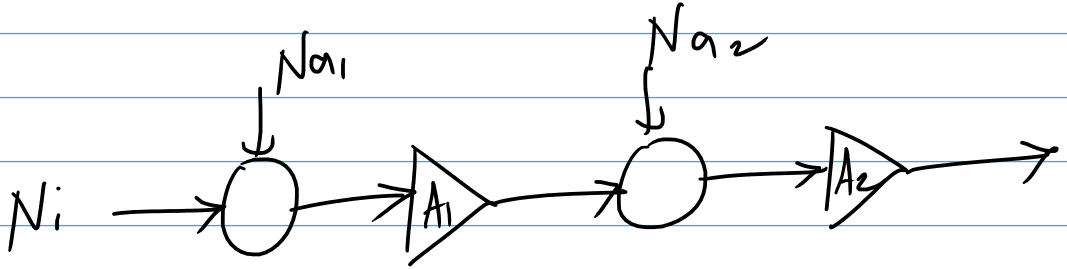
High gain LNA



$$F_1 = 1 + \frac{N_{a1}}{N_i}$$

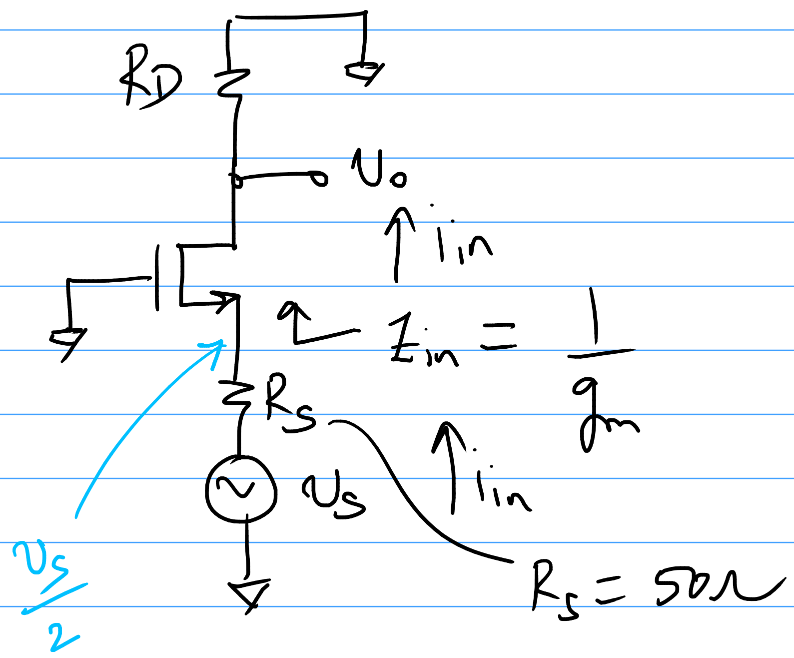


$$F_2 = 1 + \frac{N_{a2}}{N_i}$$



$$N_{out} = A_1 A_2 N_i + A_1 A_2 N_{a1} + A_2 N_{a2}$$

Common-gate Amp.

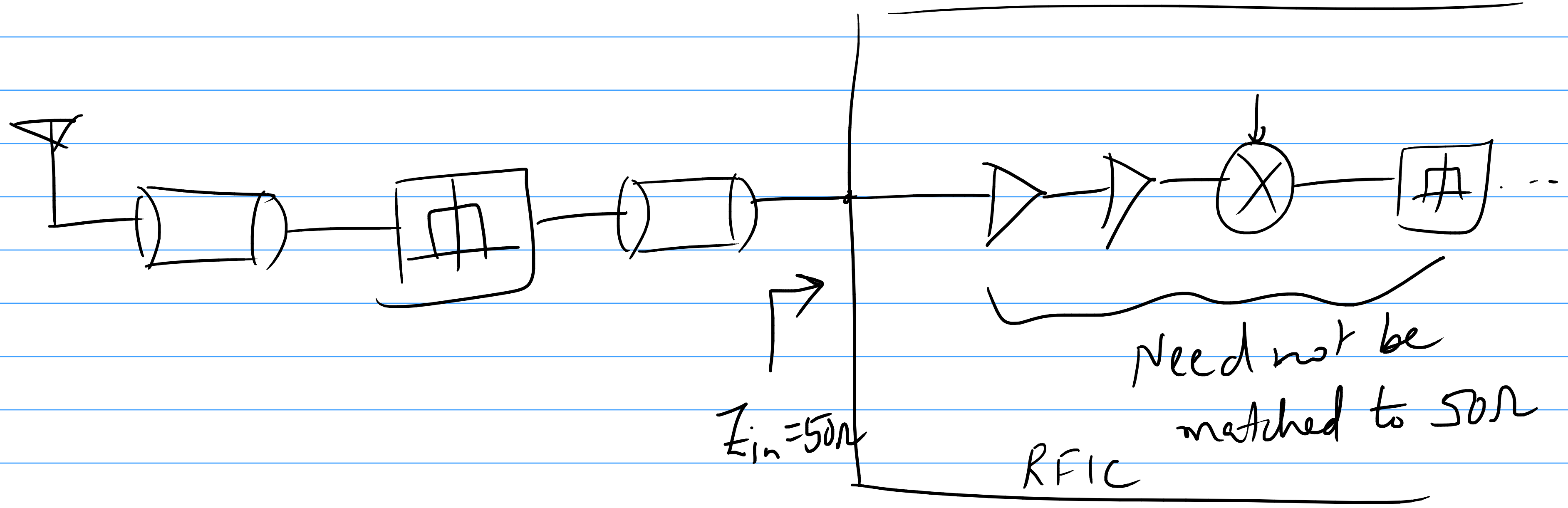


$$\frac{1}{g_m} = 50\Omega \Rightarrow g_m = 20\,mS$$

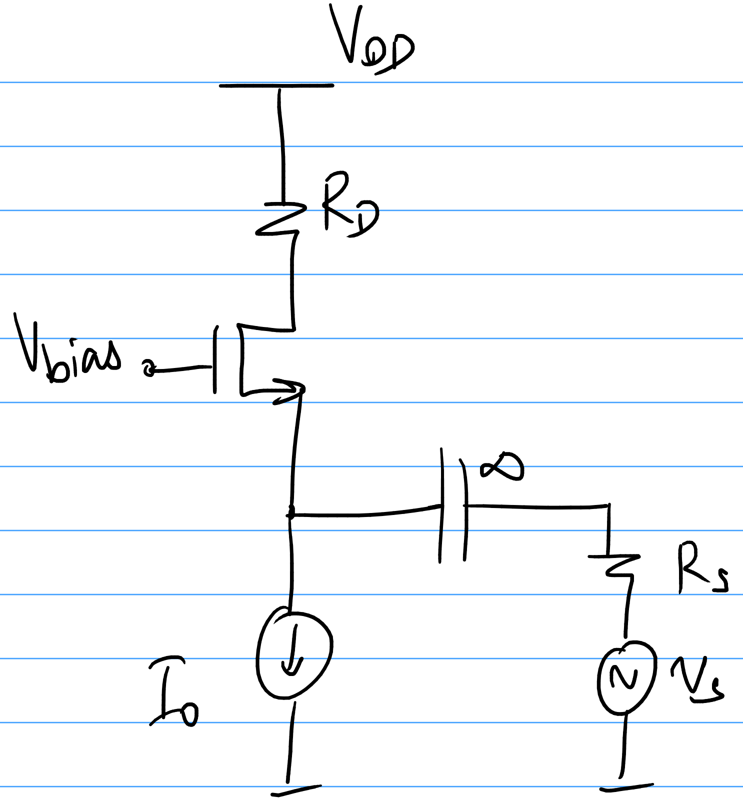
$$v_o = ?$$

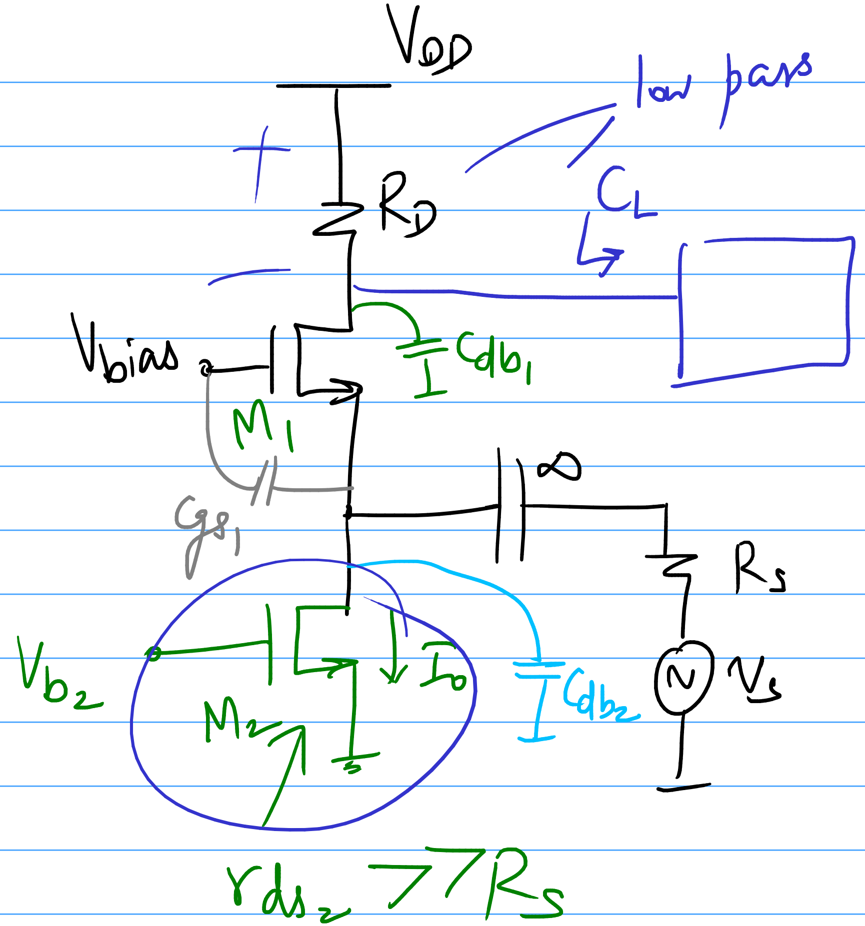
$$i_{in} = \frac{v_s}{2R_s} \Rightarrow \boxed{\frac{v_o}{v_s} = \frac{R_D}{2R_s}}$$

$$\frac{v_o}{v_s} = \frac{R_D}{100}$$

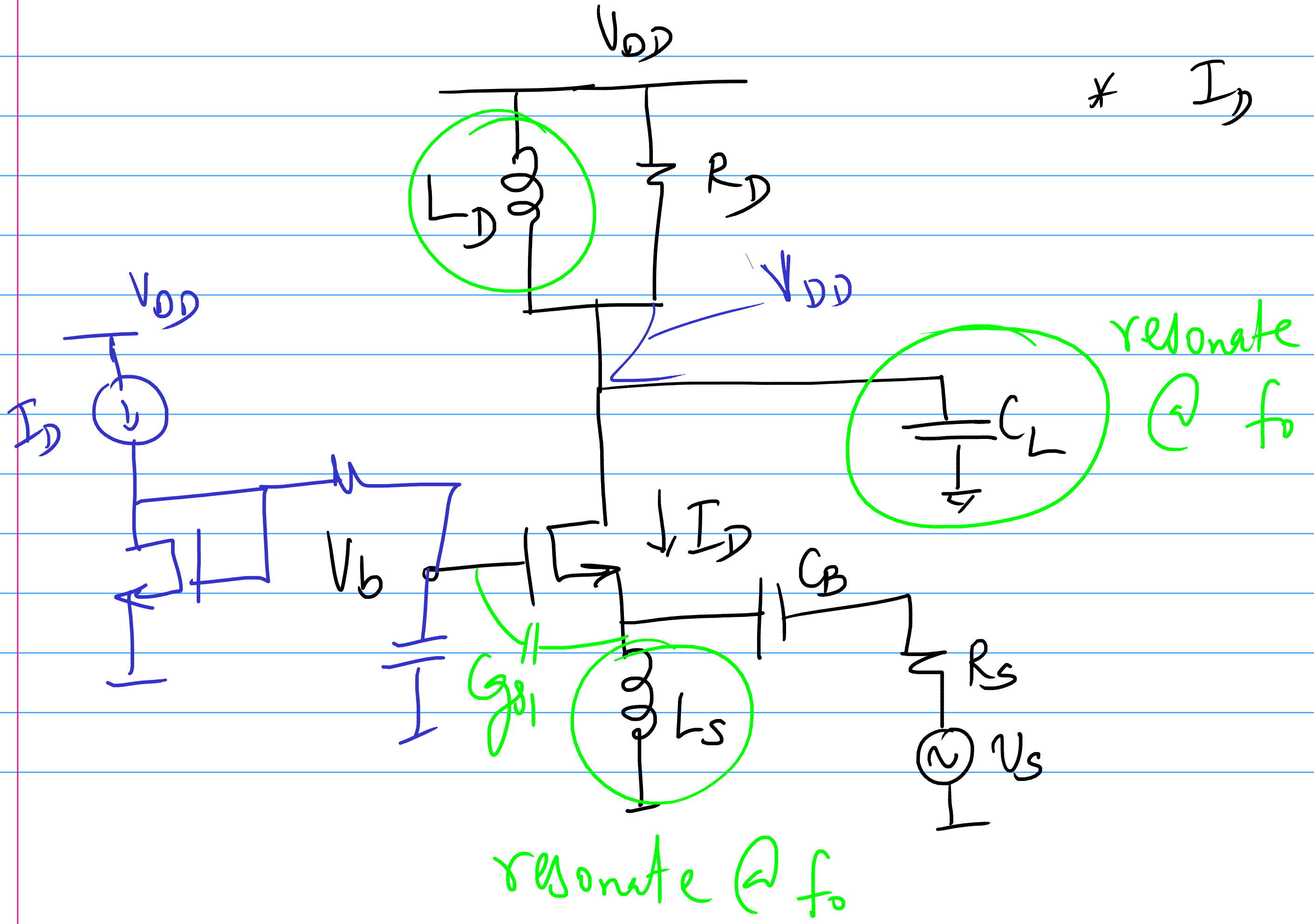


$Z_{in} = 50\Omega$
RFIC
Need not be matched to 50Ω

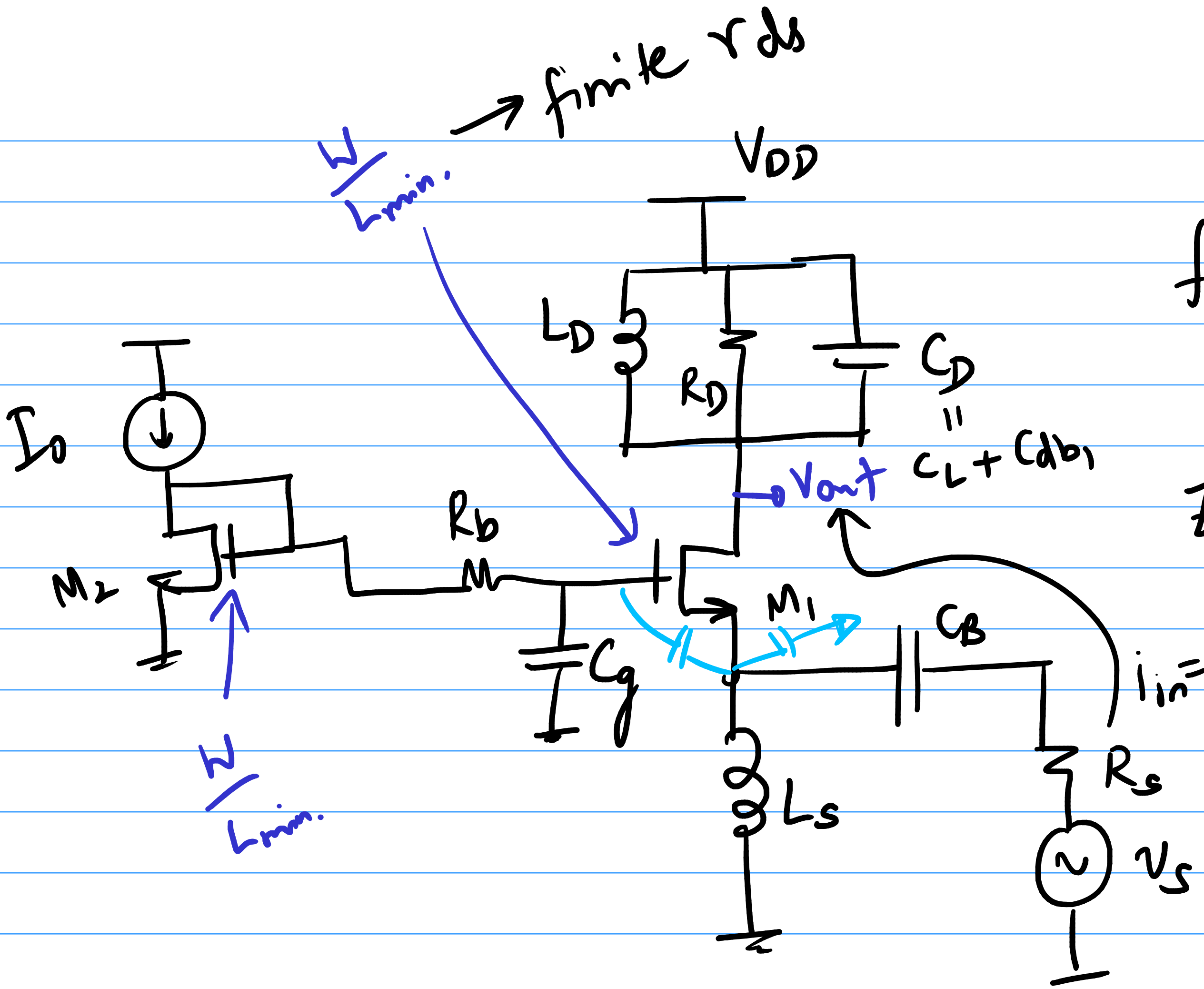

VDD
RD
Vbias
∞
Rs
Io
Vs


VDD
low pass
+
RD
CL
−
Cdb1
Vbias
M1
∞
Cgs1
Rs
Vb2
Io
Cdb2
Vs
M2
rds2 >> Rs

Narrowband CGA



* I_D is set by V_b & $\frac{W}{L}$

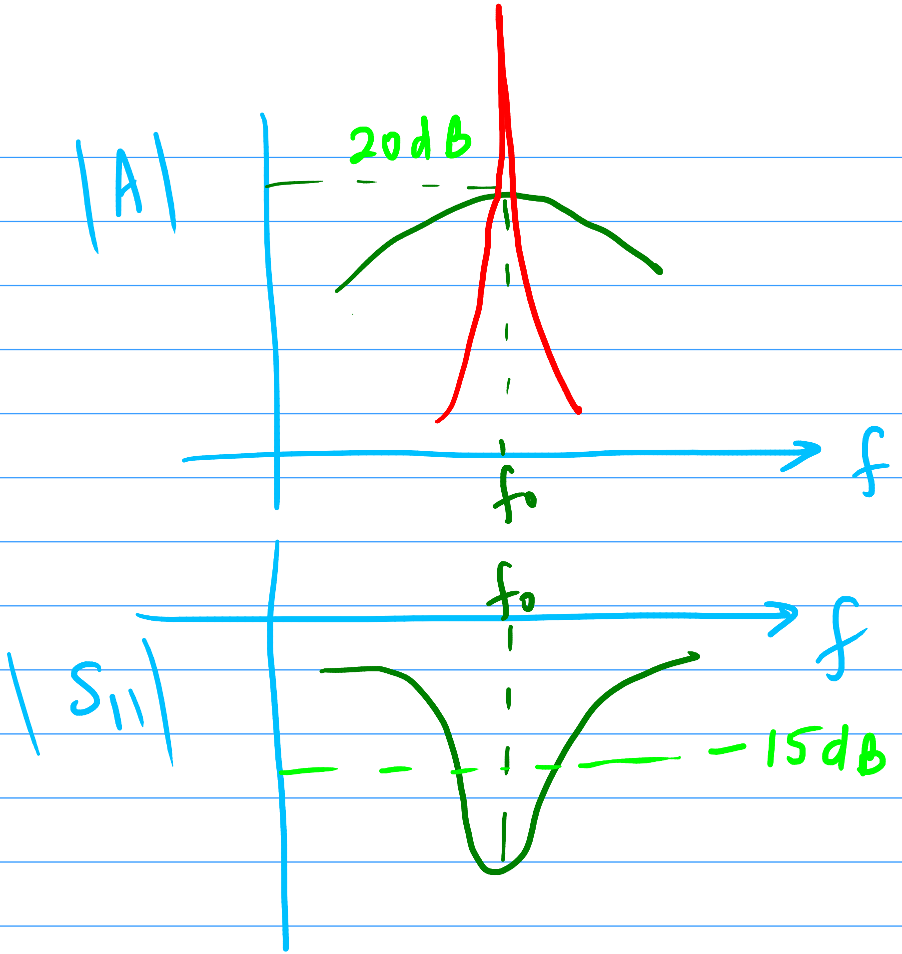


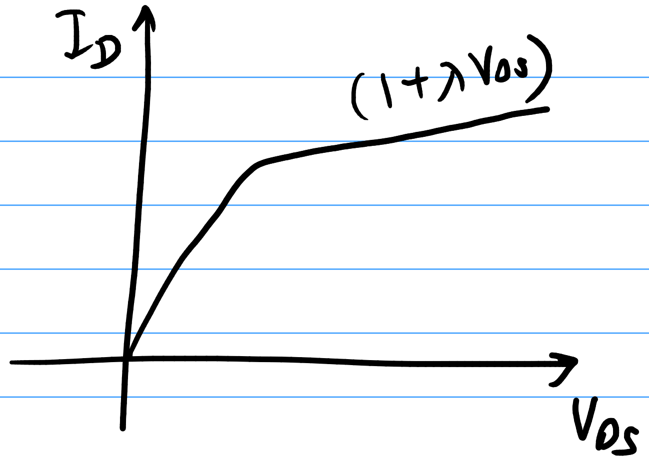
$$f_o = \frac{1}{2\pi\sqrt{L_s \cdot (C_{gs_1} + C_{sb_1})}}$$

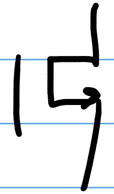
$$Z_{in}(f_o) = \frac{1}{g_{m_1}} = R_s \quad (=50\Omega)$$

$$i_{in} = \frac{v_s}{2R_s} \qquad g_{m_1} = 20\,mS$$

$$f_o = \frac{1}{2\pi\sqrt{L_D C_D}}$$

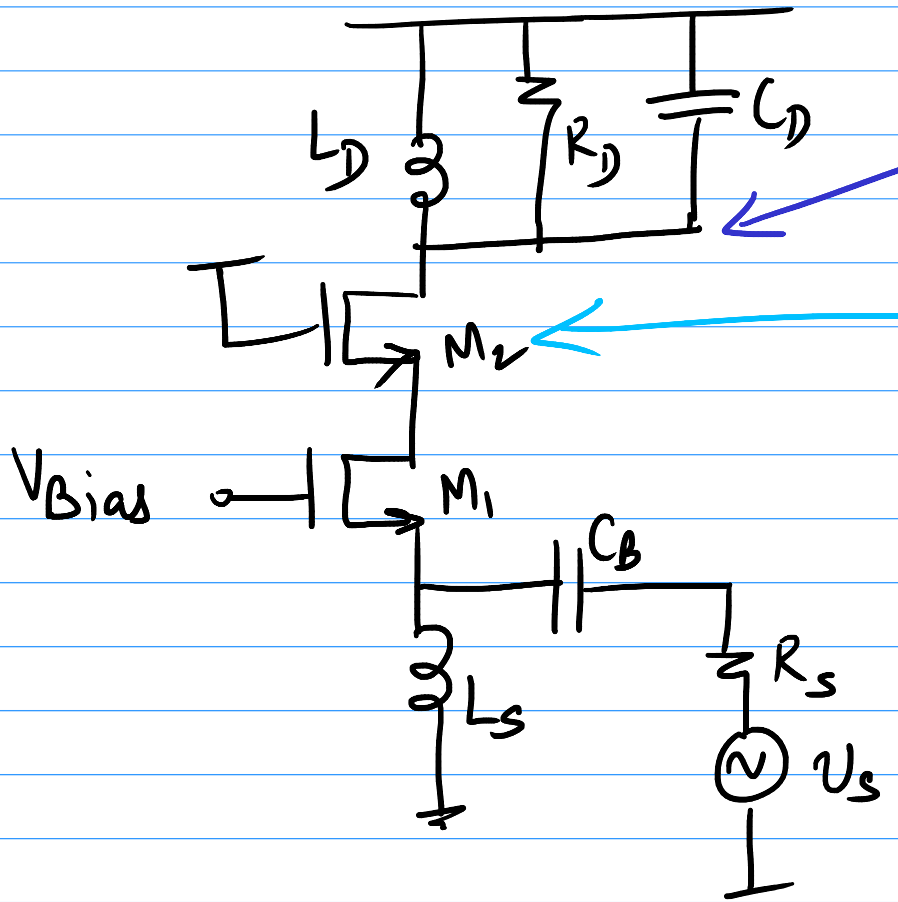






$$I_D = \frac{1}{2}\mu C_{ox} \frac{W}{L_{actual}} (V_{GS}-V_T)^2$$

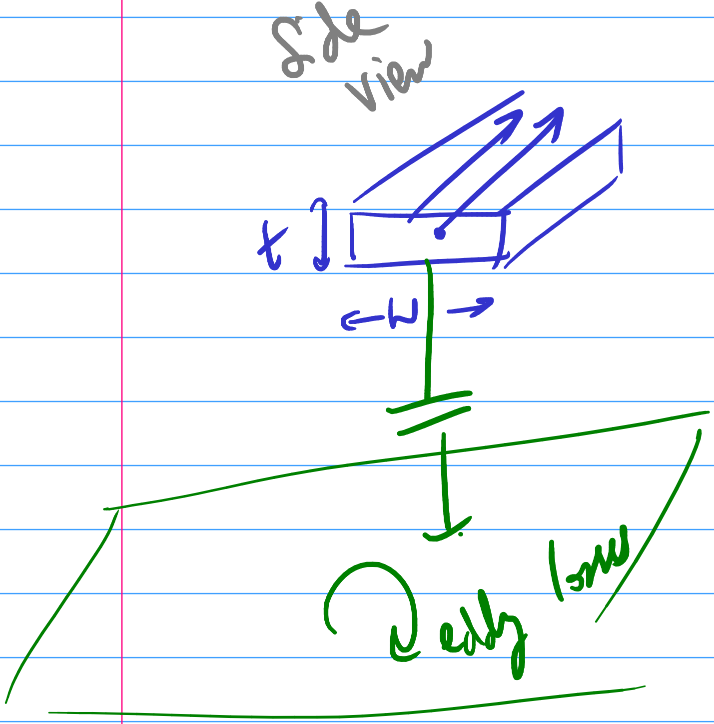
$$L\left(1-\frac{\Delta L}{L}\right)$$

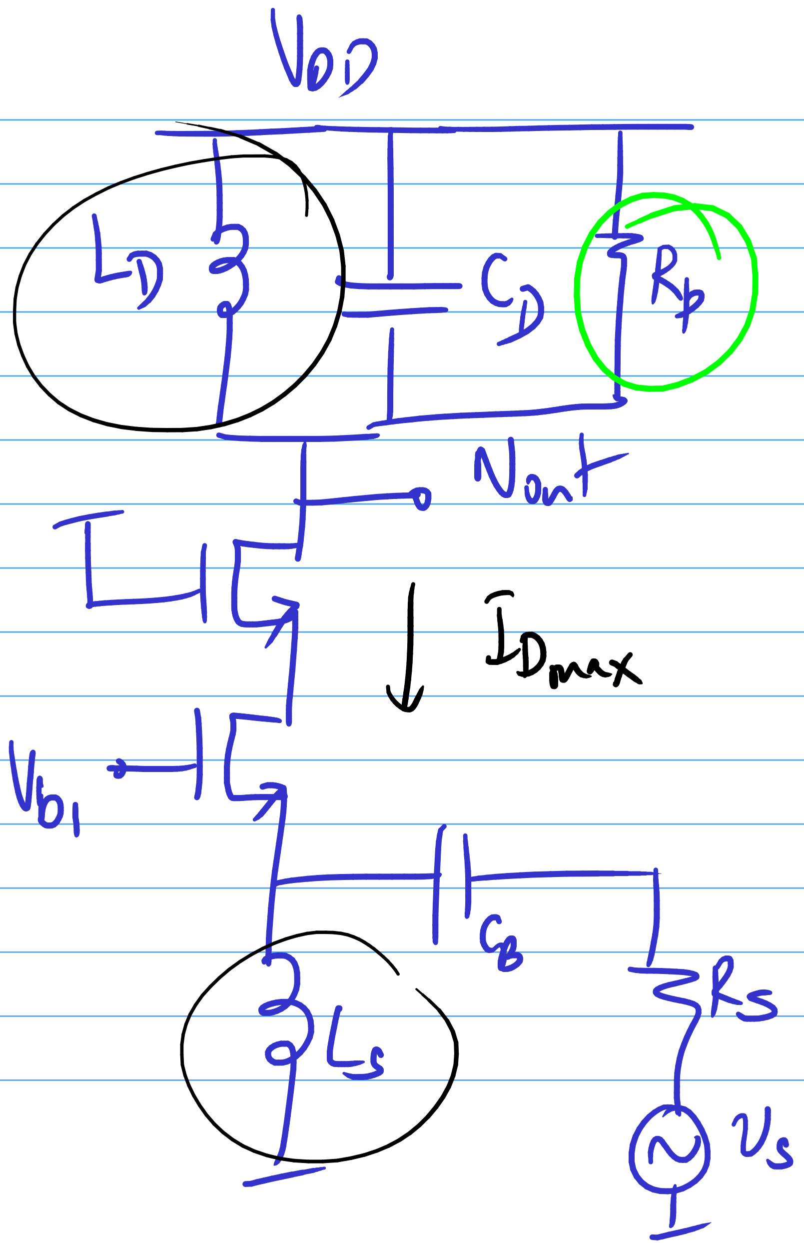


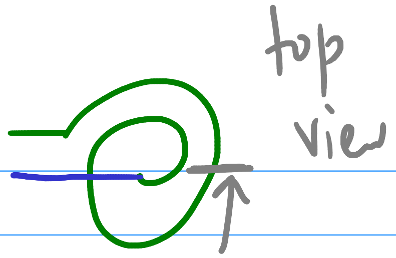
V_{out} is biased @ V_{DD}

Add Cascode to make the LNA unilateral

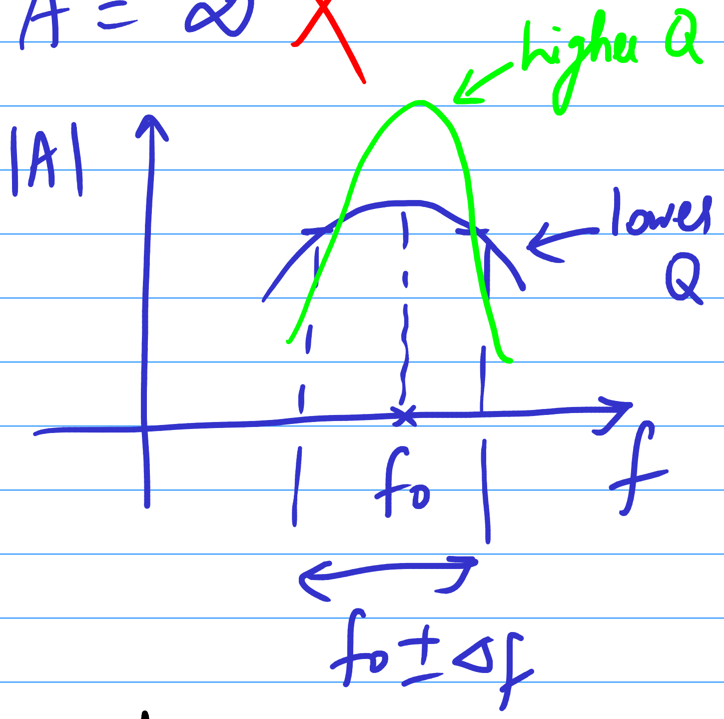
$$A = \frac{R_D}{2R_S}$$







$\cancel{A = \infty}$



$$A = \frac{R_p}{2R_s}$$

1) $\omega_T = \dfrac{g_m}{C_{gs}}$ $g_m \rightarrow \propto \sqrt{W}$, $C_{gs} \rightarrow \propto W$ $\Big\}$ $\omega_T \downarrow$

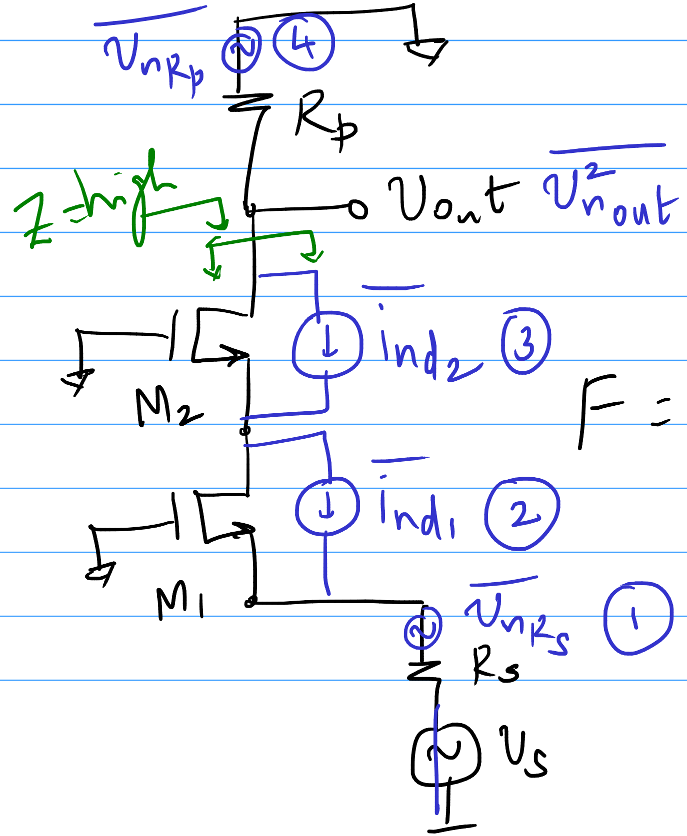
2) Device into moderate inversion

3) $L_D, L_S \downarrow$ $\leftarrow$ layout parasitics

EM sim. limits

NF of CG LNA

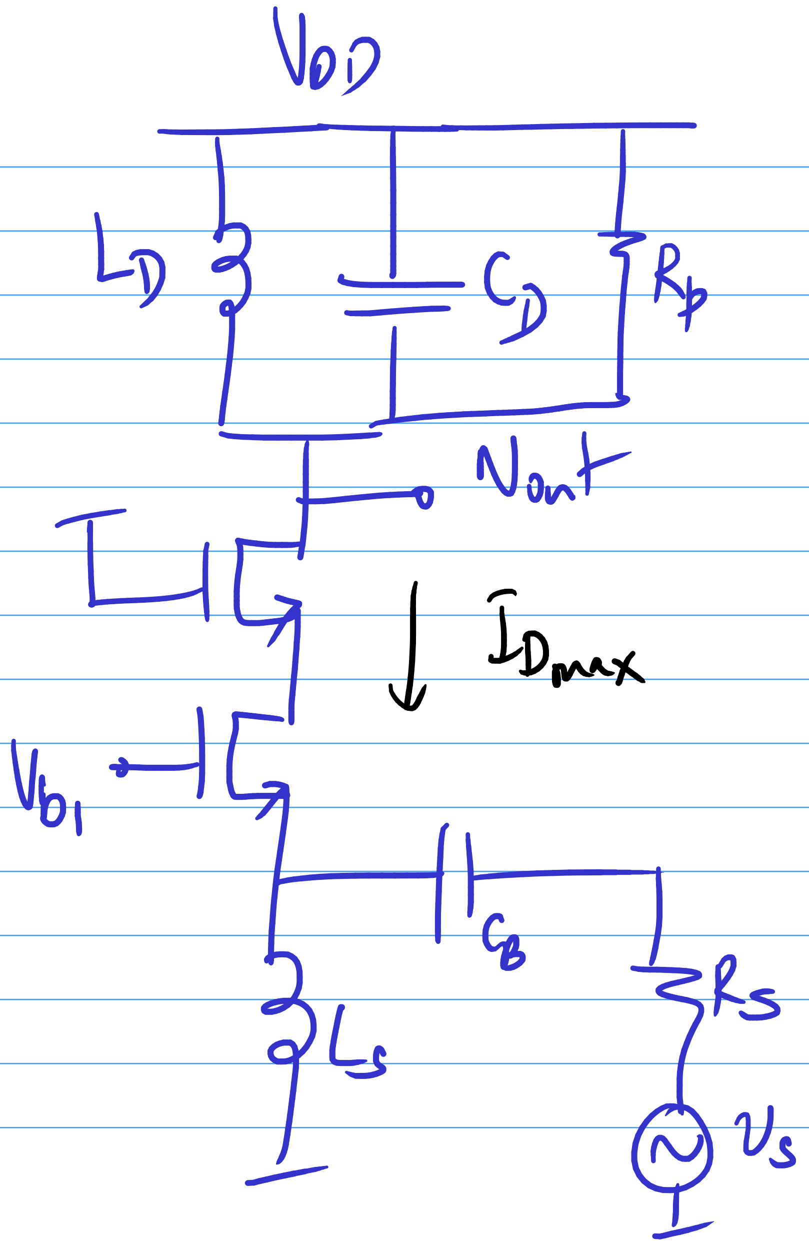
@ f_o



$$|A| = \frac{R_p}{2R_s}$$

$$Z_{in} = \frac{1}{g_m} = R_s$$

$$F = \frac{\text{Total } N_{out}}{N_{out}(R_s)}$$

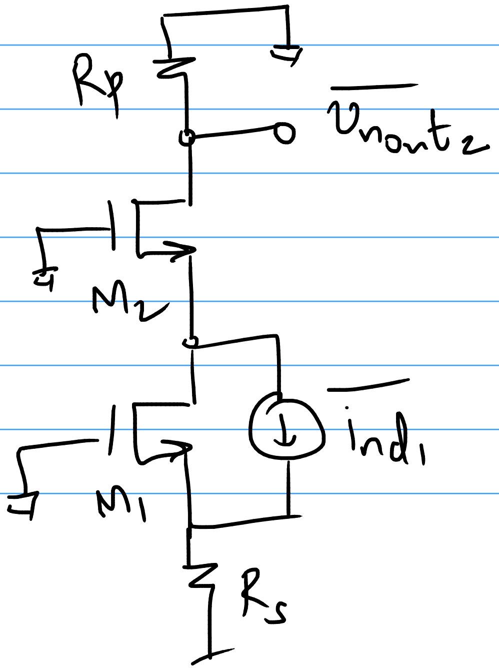


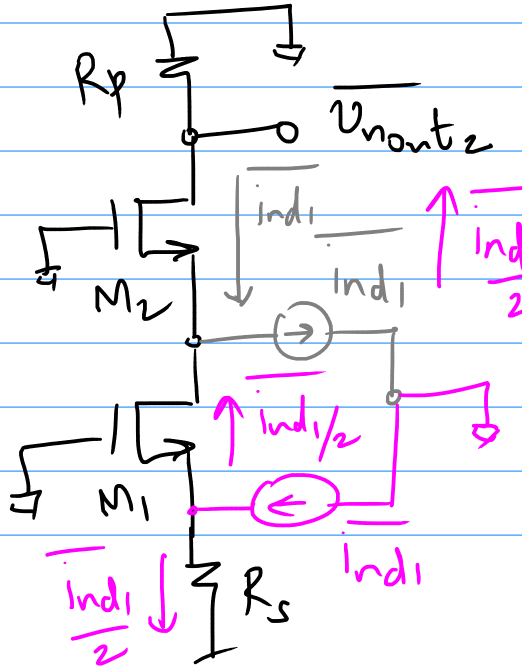
1) $\overline{v^2_{nout_1}}$ due to R_s $= \dfrac{R_p^2}{4R_s^2} \cdot \overline{v^2_{nR_s}}$

1

2) $\overline{v^2_{nout_2}}$ due to M_1 $= \overline{i^2_{nd_1}} \cdot \dfrac{R_p^2}{4}$

2



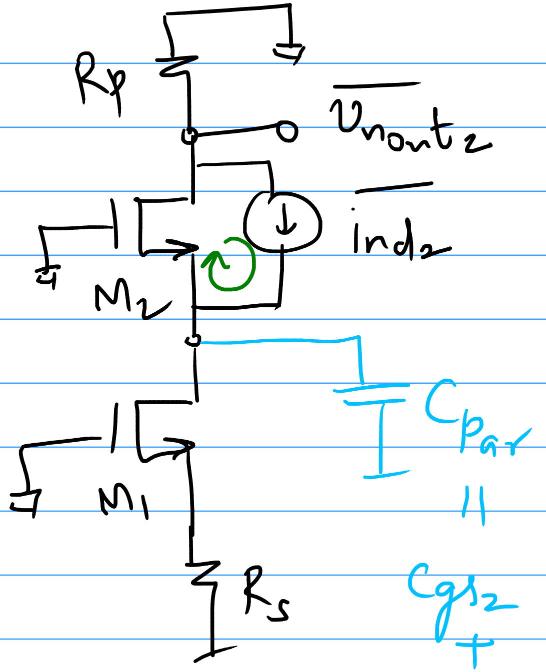


$$= -\overline{i_{nd_1}}\,R_p + \frac{\overline{i_{nd_1}}}{2} \cdot R_p$$

$$= -\frac{\overline{i_{nd_1}}}{2} \cdot R_p$$

3) $\overline{v^2_{nout_3}}$ due to $M_2 \approx 0$

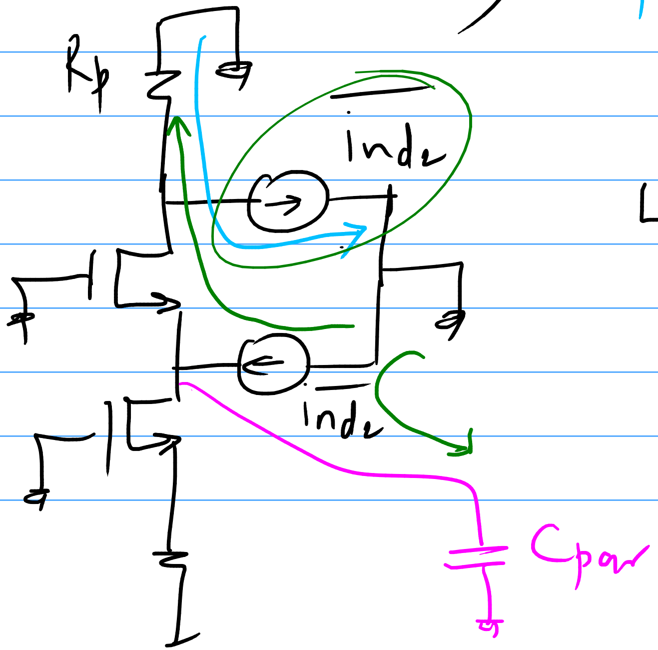
3



$C_{par} \,\|\, C_{gs_2} + C_{db_1} + C_{sb_2}$

exceptions: 1) $r_{ds_{1,2}}$ too low

2) C_{par}



4) $\overline{v^2_{nout_4}}$ due to $R_p = \overline{v^2_{n_{Rp}}}$

4

HW

$\overline{ind_2}$ with C_{par}

Noise Factor

$$F = 1 + \frac{\overline{i_{nd_1}^2} \cdot \frac{R_p^2}{4} + \overline{v_{n_{R_p}}^2}}{\frac{R_p^2}{4R_s^2} \cdot \overline{v_{n_{R_s}}^2}}$$

$$\overline{v_{n_{R_s}}^2} = 4kT R_s \Delta f$$

$$\overline{v_{n_{R_p}}^2} = 4kT R_p \Delta f$$

$$\overline{i_{nd_1}^2} = 4kT \gamma g_{m_1} \Delta f$$

$$F = 1 + \frac{\cancel{4kT}\,\gamma\, g_{m1} \cdot \frac{R_p^2}{4}\cancel{\Delta f} + \cancel{4kT}\, R_p \cancel{\Delta f}}{\frac{R_p^2}{4R_s^2} \cdot \cancel{4kT}\, R_s \cancel{\Delta f}}$$

$$= 1 + \gamma\, \overbrace{g_{m1} R_s}^{=1} + \frac{4R_s}{R_p}$$

$$\boxed{F = 1 + \gamma + \frac{4R_s}{R_p}}$$

Noise Factor of CGLNA

Upper limit : $R_p >> R_s$

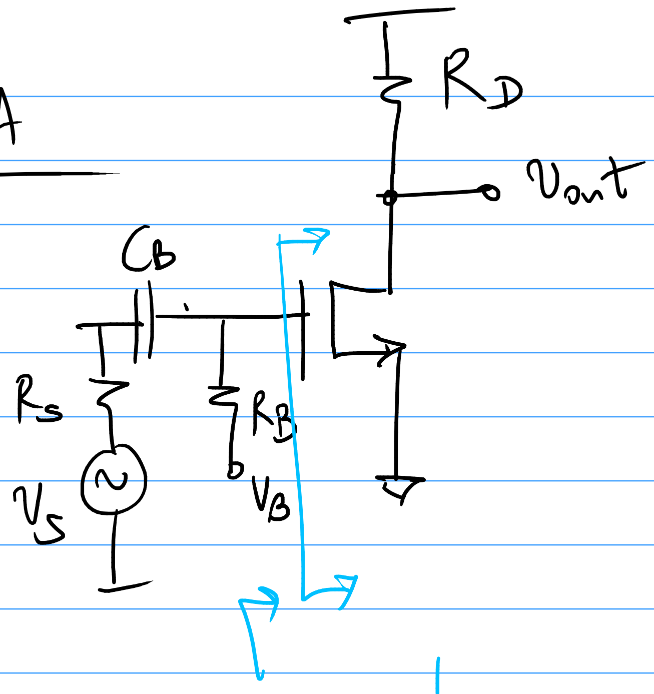
$$\gamma = \frac{2}{3}$$

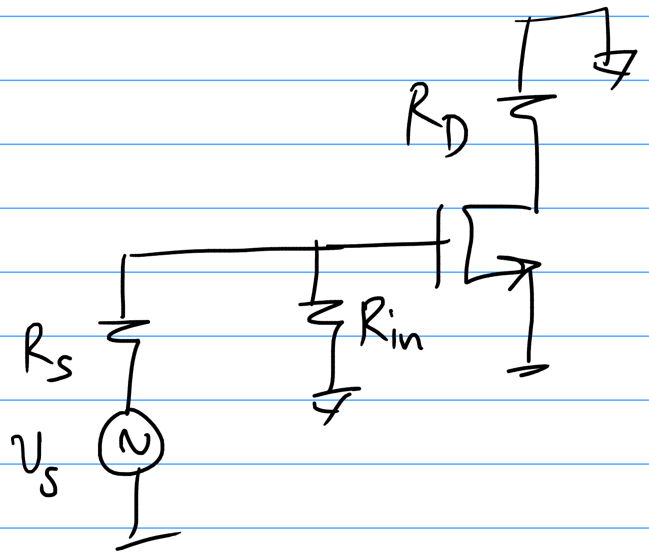
$$F = 1 + \frac{2}{3} = 1.67$$

$$NF = 10 \log_{10}(F) = 2.2\,dB$$

Upper limit for CGLNA

CS LNA





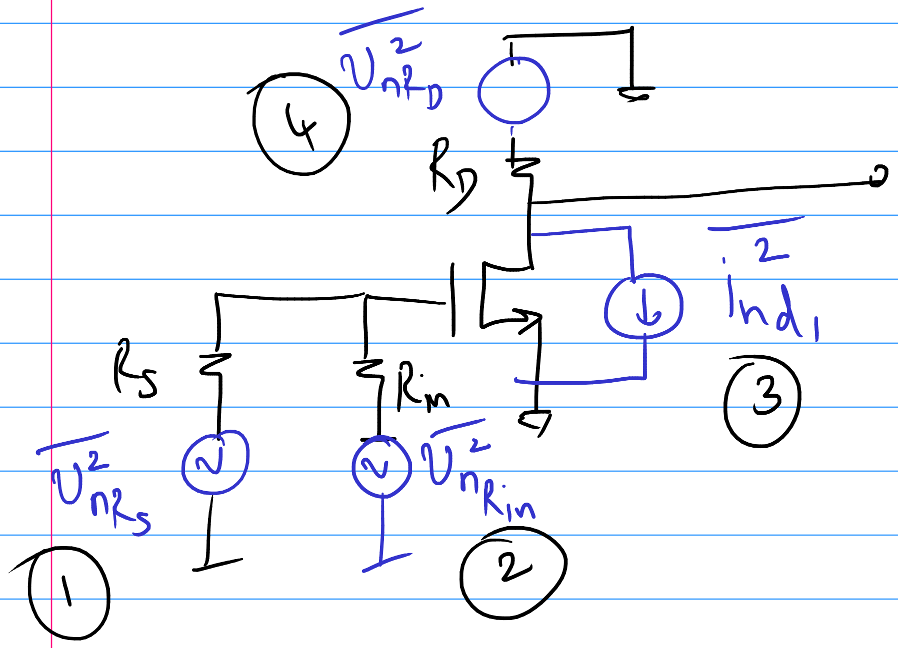
$$Z_{in} = \frac{1}{j\omega C_{gs}}$$

$$A = -g_m R_D \times \frac{R_{in}}{R_s + R_{in}}$$

$$= -\frac{1}{2} g_m R_D$$

either $R_b = 50\Omega$

(or) add $R_{in} = 50\Omega$ in shunt across gate



$\overline{v^2_{nout}} = ?$ $\quad F = ?$

1 $\Rightarrow$ $\overline{v^2_{nout_1}} = \frac{1}{4} g_m^2 R_D^2 \cdot \overline{v^2_{nR_s}}$

2 $\Rightarrow$ $\overline{v^2_{nout_2}} = \frac{1}{4} g_m^2 R_D^2 \cdot \overline{v^2_{nR_{in}}}$

3 $\Rightarrow$ $\overline{v^2_{nout_3}} = \overline{i^2_{nd_1}} \cdot R_D^2$

4 $\Rightarrow$ $\overline{v^2_{nout_4}} = \overline{v^2_{nR_D}}$

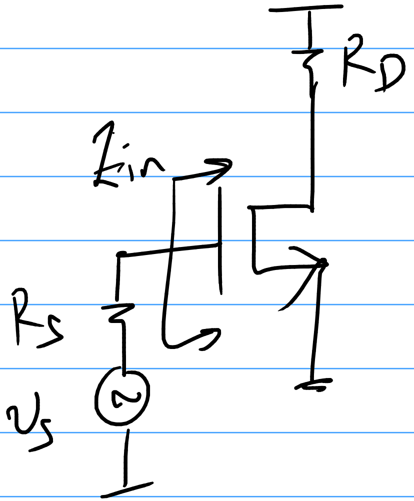
$$F = 1 + \frac{\frac{1}{4} g_m^2 R_D^2 \cdot \overline{v_{n R_{in}}^2} + \overline{i_{n D_1}^2} R_D^2 + \overline{v_{n R_D}^2}}{\frac{1}{4} g_m^2 R_D^2 \cdot \overline{v_{n R_s}^2}}$$

$$F = 2 + \overline{\qquad\qquad}$$

$\rightarrow$ min $NF = 3\,dB$

$$F = 2 + \frac{4\cancel{kT}\gamma \cancel{g_m} \cancel{R_D^2}}{\frac{1}{\cancel{4}} g_m^{\cancel{2}} \cancel{R_D^2} \cdot \cancel{4kT} R_s} + \frac{4\cancel{kT} \cancel{R_D}}{\frac{1}{\cancel{4}} g_m^2 R_D^{\cancel{2}} \cdot \cancel{4kT} R_s}$$

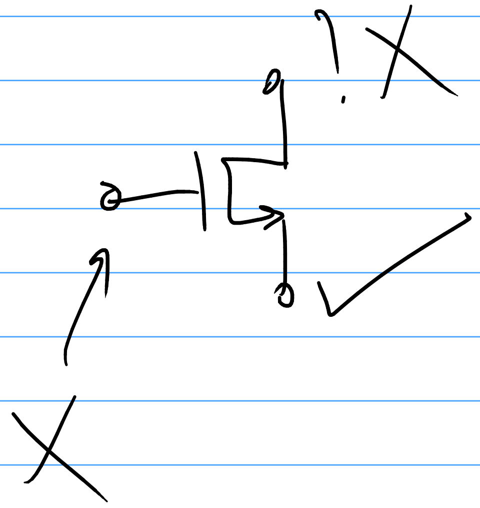
$$F = 2 + \frac{4\gamma}{g_m R_S} + \frac{4}{g_m^2 R_D R_S}$$

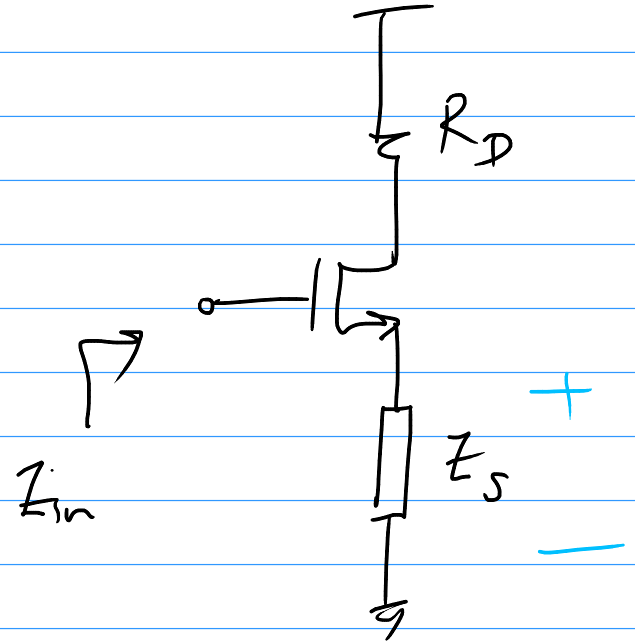


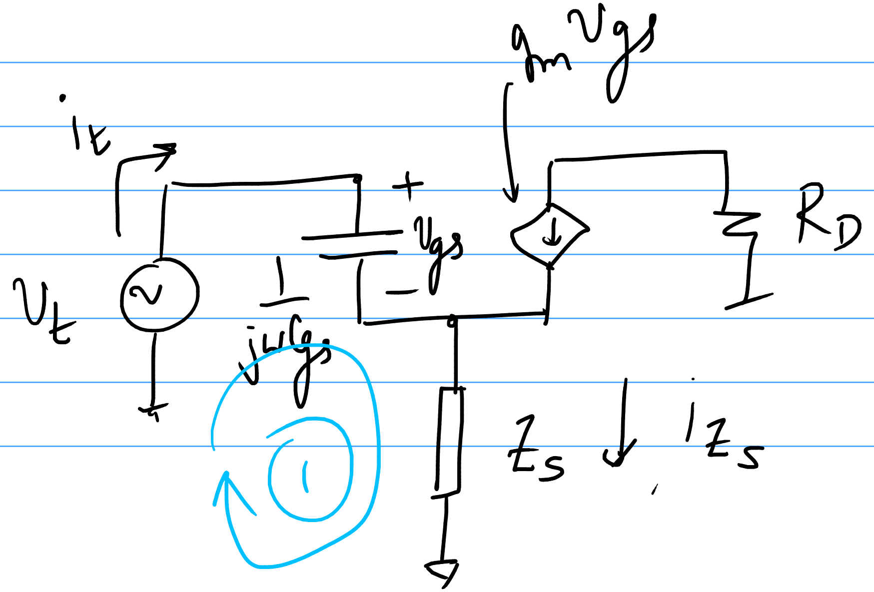
high gain ✓

low $\overline{e_n}$ ✓

$Z_{in} = \frac{1}{j\omega C_{gs}}$ ✗







$$Z_{in} = \frac{v_t}{i_t}$$

KCL @ Source of M_1

$$i_{Z_s} = i_t + g_m v_{gs}$$

$$v_{gs} = i_t \cdot \frac{1}{j\omega C_{gs}}$$

$$\frac{v_t - v_{gs}}{Z_s} = i_t + g_m v_{gs}$$

$$v_t - v_{gs} = Z_s i_t + g_m Z_s \cdot v_{gs}$$

KVL @ loop 1

$$v_t = v_{gs} + i_{Z_s} \cdot Z_s$$

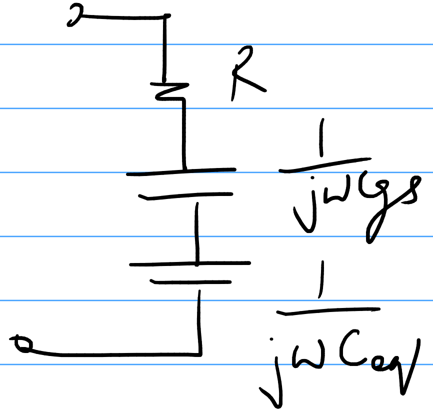
$$v_t = Z_s i_t + v_{gs}(1 + g_m Z_s)$$

$$= Z_s i_t + (1 + g_m Z_s) \cdot \frac{i_t}{j\omega C_{gs}}$$

$$\frac{v_t}{i_t} = Z_{in} = Z_s + \frac{1}{j\omega C_{gs}} + \frac{g_m}{j\omega C_{gs}} \cdot Z_s$$

Case 1 $\quad Z_s = R$

$$Z_{in} = \underbrace{R}_{\text{noise}} + \frac{1}{j\omega C_{gs}} + \frac{g_m R}{j\omega C_{gs}}$$



$$C_{eq} = \frac{1}{\omega_T R}$$

case 2 $\quad Z_s = \frac{1}{j\omega C}$

$$Z_{in} = \frac{1}{j\omega C} + \frac{1}{j\omega C_{gs}} + \frac{-g_m}{\omega^2 C_{gs} \cdot C}$$

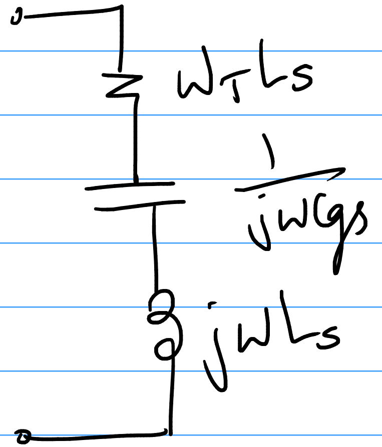
-ve resistance

instability

case 3 $\quad Z_s = j\omega L_s$

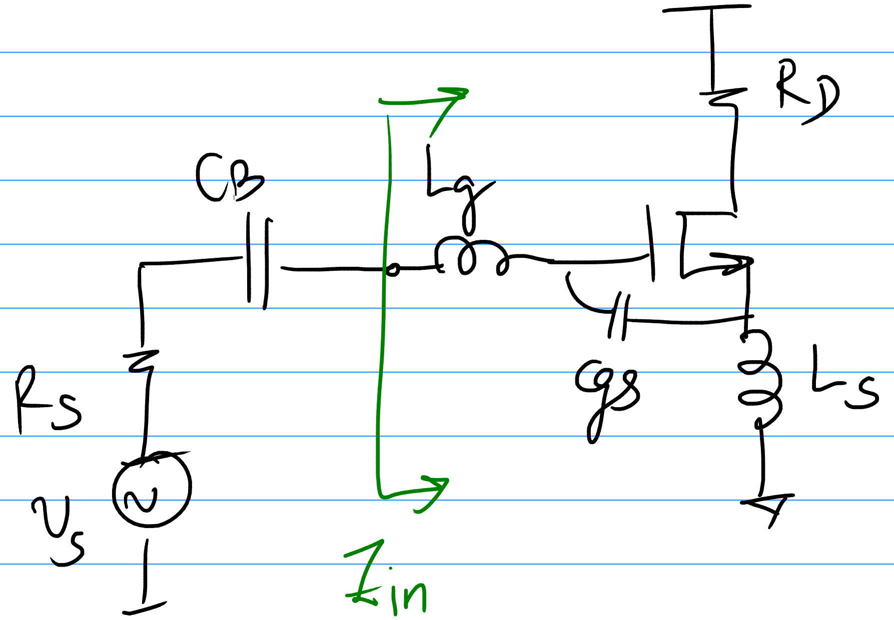
$$Z_{in} = j\omega L_s + \frac{1}{j\omega C_{gs}} + \frac{g_m \cdot j\omega L_s}{j\omega C_{gs}}$$

$$\frac{g_m}{C_{gs}} \cdot L_s = \omega_T L_s \quad ✓$$



resonance $\neq f_0$

$$Z_{in} = j\omega L_s + \frac{1}{j\omega C_{gs}} + \underbrace{\omega_T L_s}_{R_s}$$

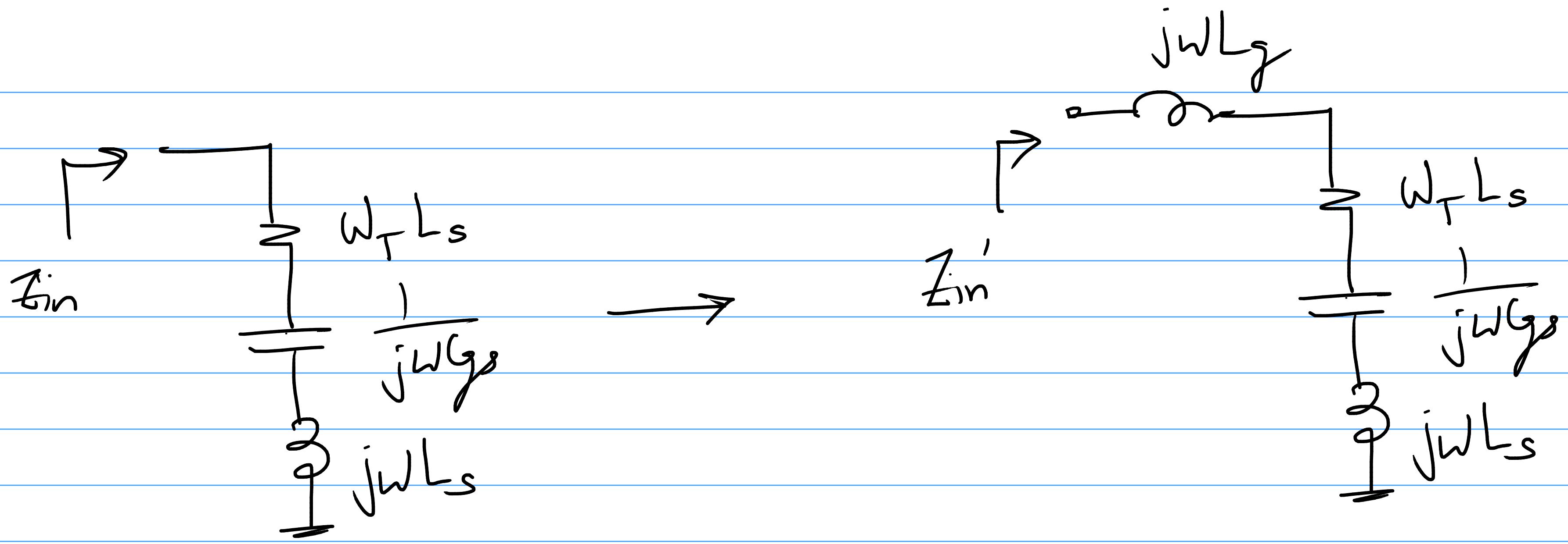


$$Z_{in} = j\omega L_g + j\omega L_s + \frac{1}{j\omega C_{gs}}$$

$$+ \ \omega_T L_s$$

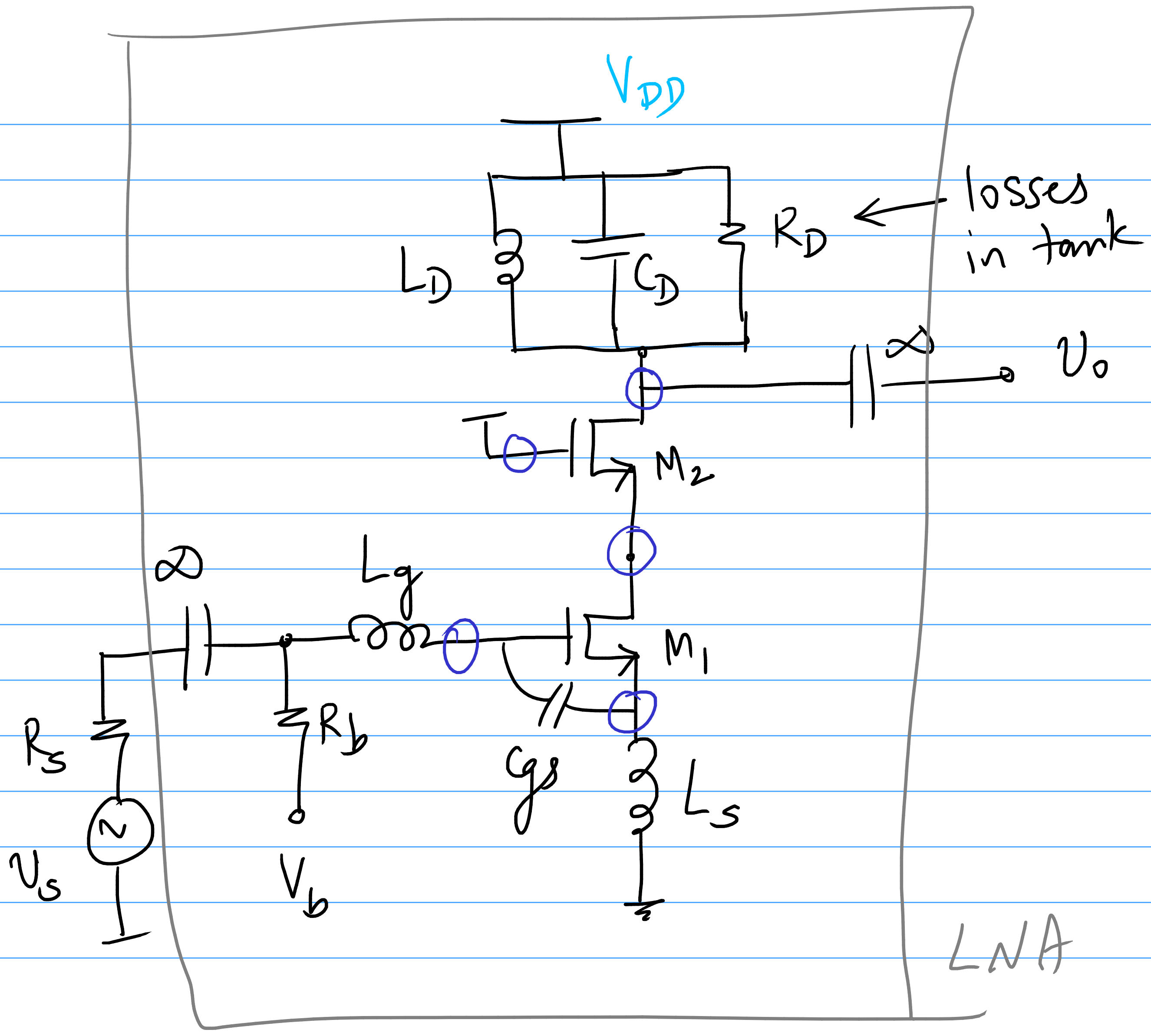
$$f_0 = \frac{1}{2\pi\sqrt{(L_s + L_g)C_{gs}}}$$

$\downarrow \omega_T \rightarrow \ \uparrow L$; add $C \parallel C_{gs}$;

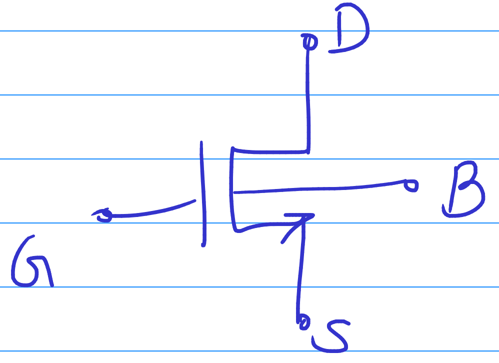


$$Z_{in}'(\omega_0) = \omega_T \cdot L_s = R_s$$

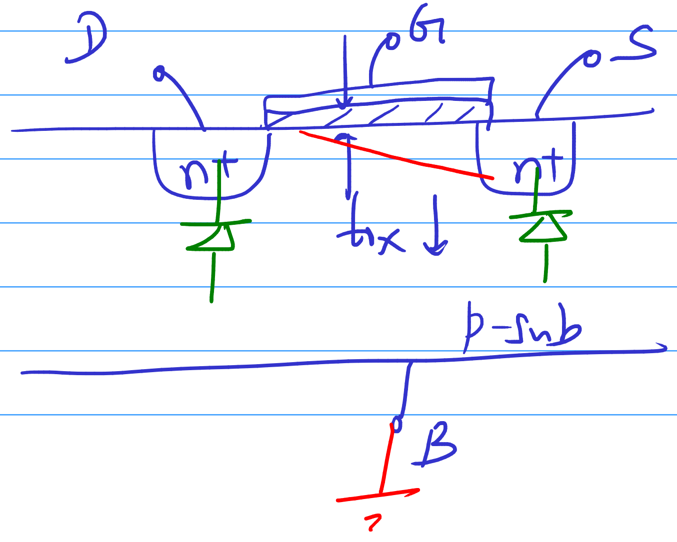
$$\omega_0 = \frac{1}{\sqrt{C_{gs}(L_g + L_s)}} \leftarrow \text{desired freq. of operation}$$



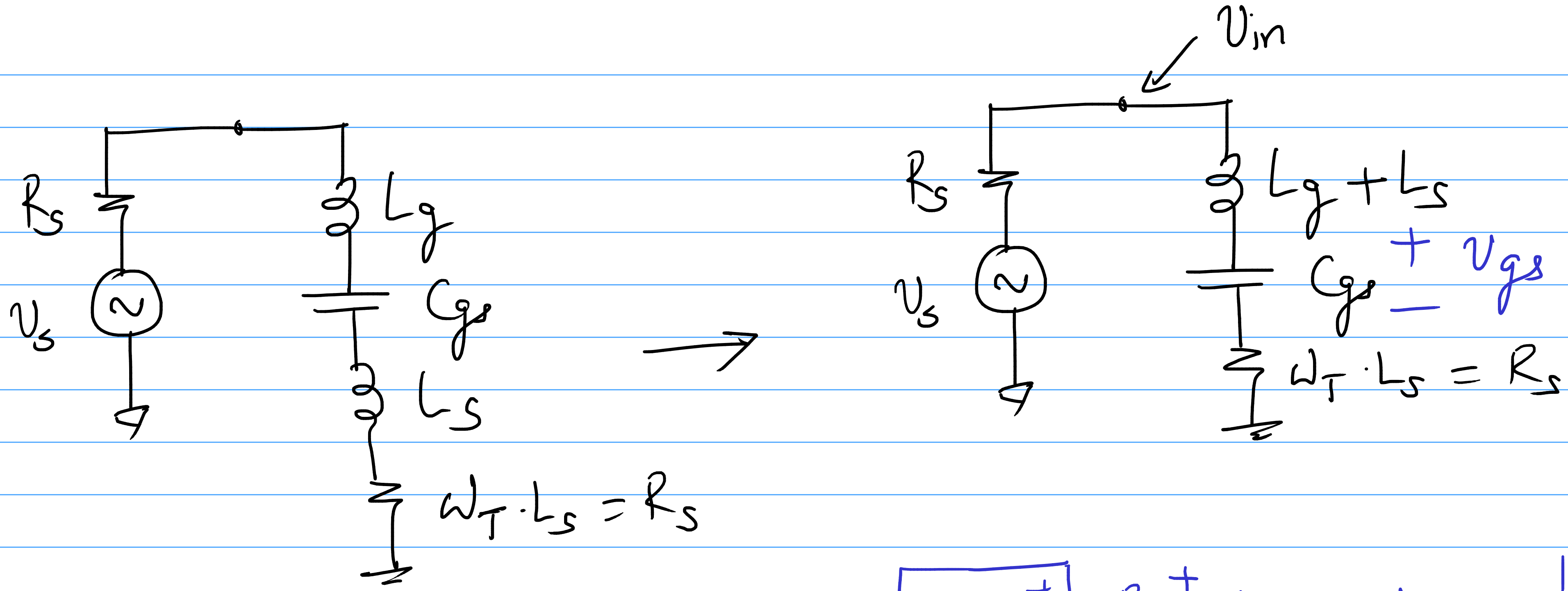
V_{DD}
L_D
C_D
R_D
losses in tank
∞
v_o
M_2
L_g
M_1
R_s
R_b
C_{gs}
L_s
v_s
V_b
LNA



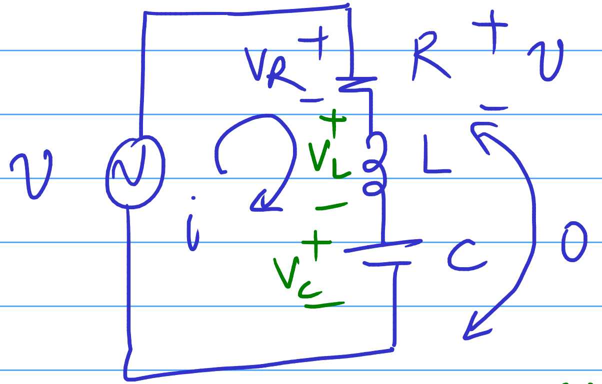
D
G
B
S



D
G
S
n+
n+
t_{ox}
p-Sub
B
?



$$@\ \omega_0 : \quad v_{in} = \frac{v_s}{2}$$

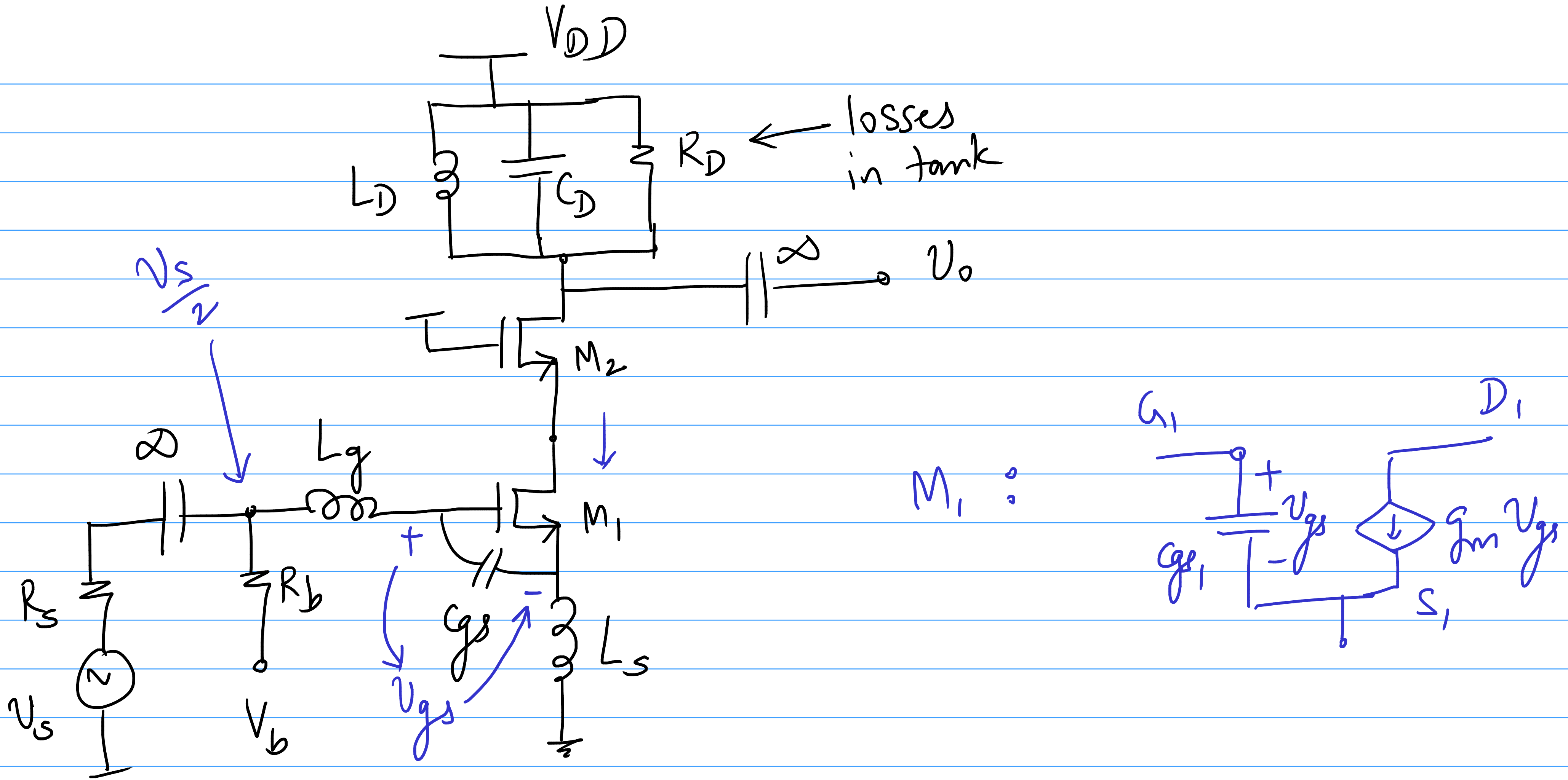


$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$i = \frac{v}{R}$$

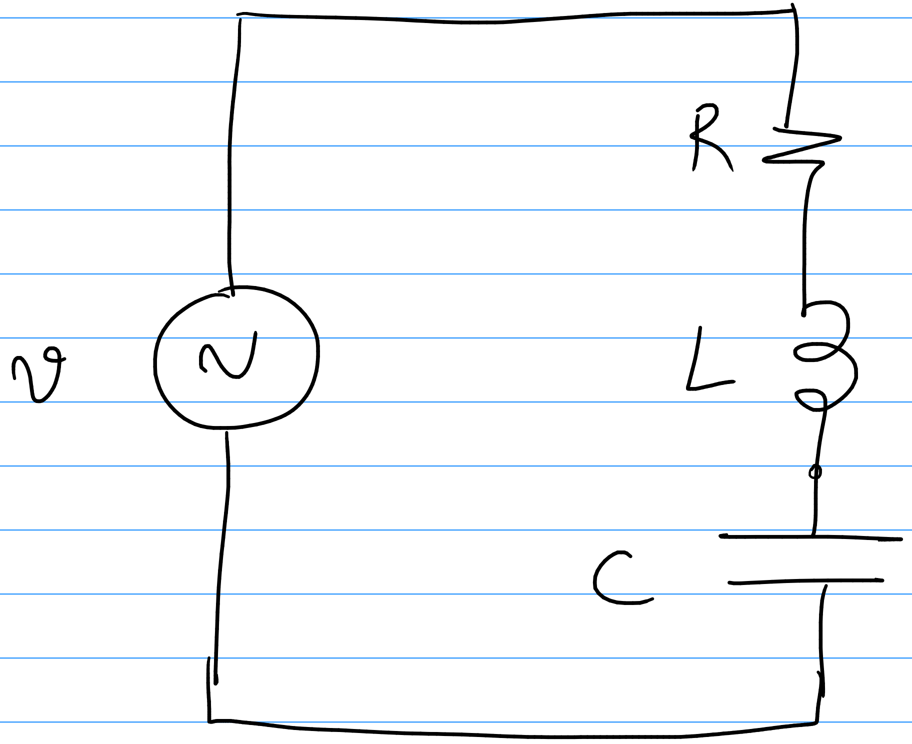
$$V_L = \frac{v}{R} \cdot j\omega L$$

$$V_C = \frac{v}{R} \cdot \frac{1}{j\omega C} = \frac{v}{R} \cdot \frac{-j}{\omega C}$$



VDD
L_D
C_D
R_D
losses in tank
∞
v_o
M_2
$v_s/2$
∞
L_g
M_1
R_s
R_b
C_{gs}
L_s
v_s
V_b
v_{gs}
M_1 :
G_1
D_1
C_{gs1}
v_{gs}
$g_m v_{gs}$
S_1

$$Q = \frac{\omega L}{R} = \frac{1}{\omega RC}$$



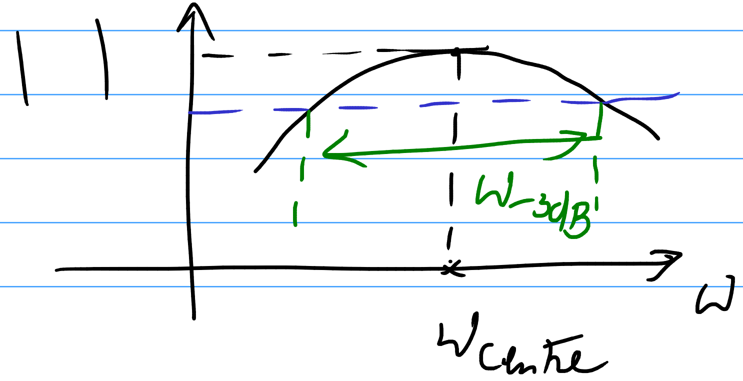
$$v_R = v$$

$$|v_L| = v \cdot \frac{\omega L}{R}$$

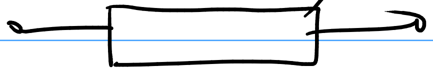
$$|v_C| = v \cdot \frac{1}{\omega CR}$$

1) $Q = \dfrac{\omega_{centre}}{\omega_{-3dB}}$

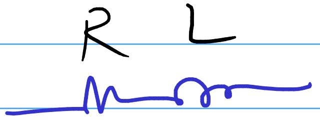
quality factor



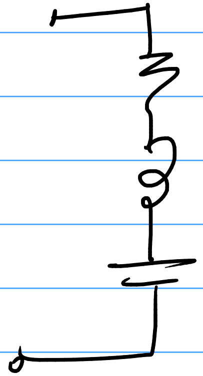
2) $$Z(\omega) = R + jX$$

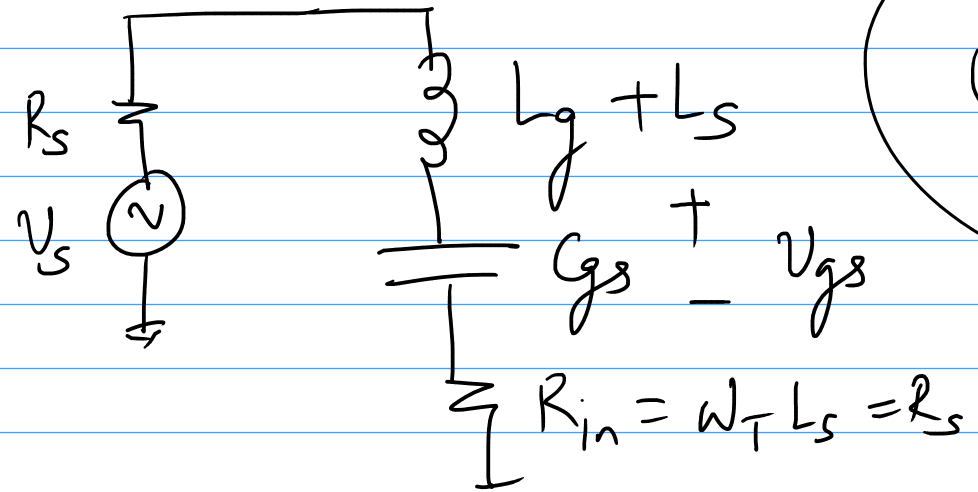


$$Q = 2\pi \cdot \frac{\text{peak energy stored}}{\text{energy dissipated/cycle}}$$



3) $$Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)}$$



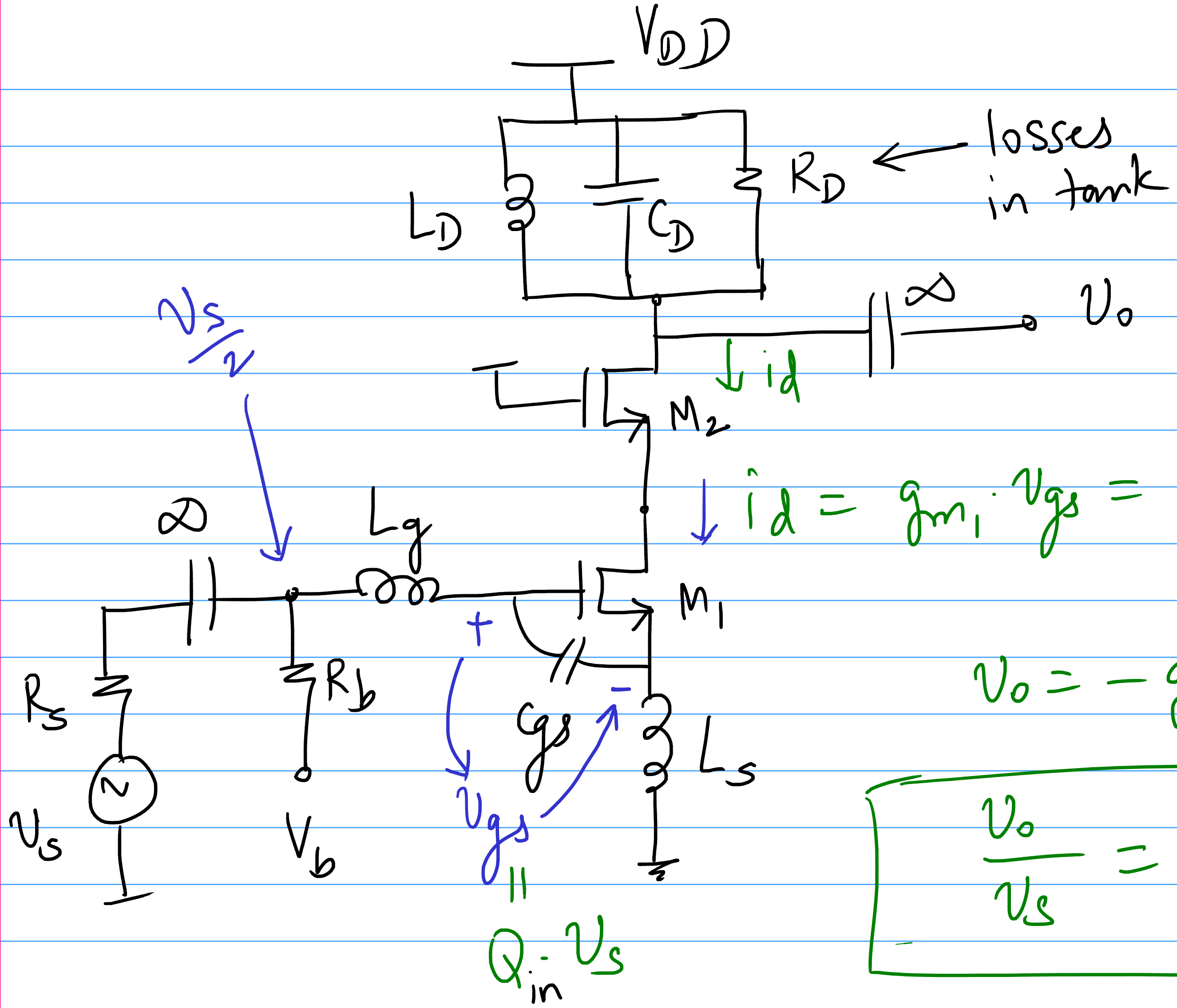


$$Q = \frac{1}{\omega_0 C_{gs} (2R_s)}$$

$$v_{gs} = Q \cdot v_s = \frac{v_s}{2\omega_0 C_{gs} R_s}$$ can be $> v_s$

if $Q > 1$

$\Rightarrow$ low NF possible



$$\omega_0 = \frac{1}{\sqrt{L_D C_D}}$$

$$i_d = g_{m_1} \cdot v_{gs} = g_{m_1} \cdot Q_{in} v_s$$

$$v_{gs} = Q_{in} v_s$$

$$v_o = -g_{m_1} R_D \cdot Q_{in} \cdot v_s$$

$$\boxed{\frac{v_o}{v_s} = -g_{m_1} R_D \cdot Q_{in}}$$

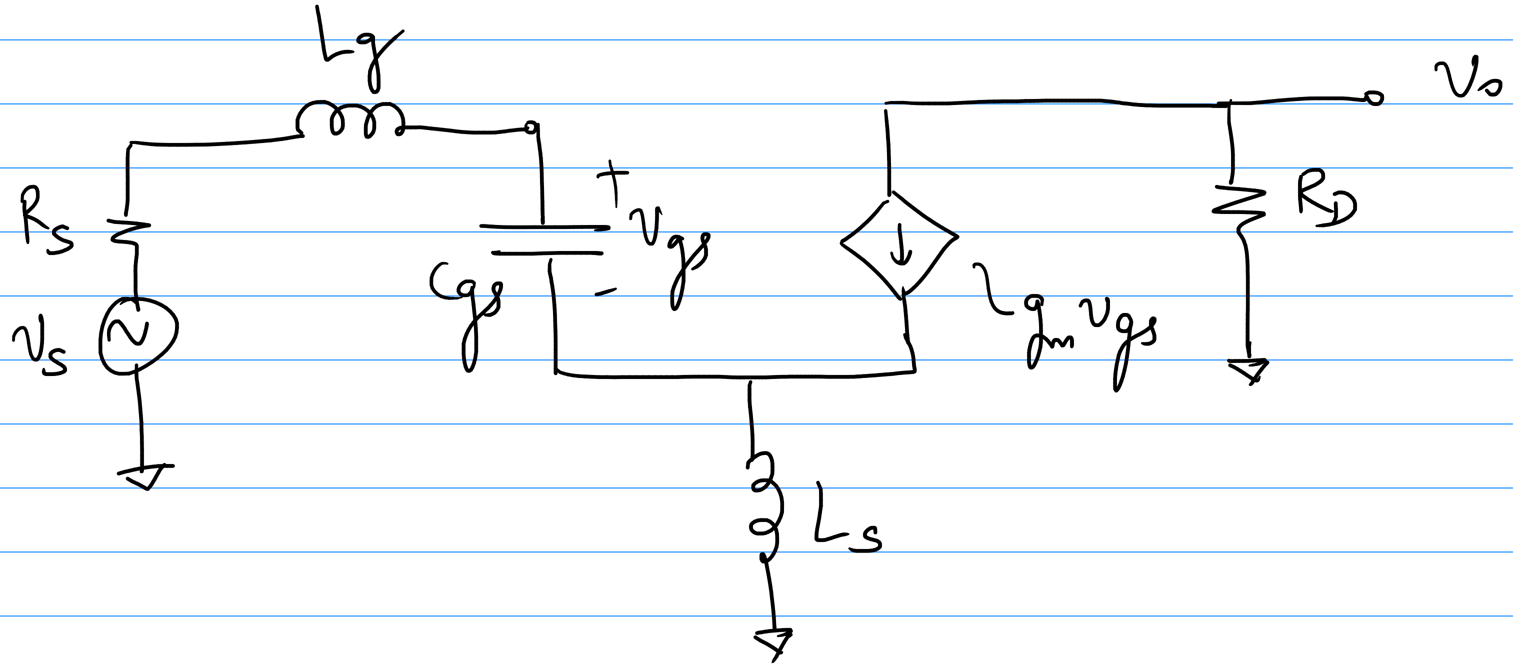
$$\frac{V_o}{V_s} = -g_{m1} R_D \cdot Q$$

$$\frac{V_o}{V_s} = \frac{-g_{m1} R_D}{2 \omega_o C_{gs} R_s}$$

$$\frac{g_{m1}}{C_{gs}} = \omega_T$$

$$\Rightarrow \boxed{\frac{V_o}{V_s} = \frac{-1}{2} \frac{\omega_T}{\omega_o} \cdot \frac{R_D}{R_s}}$$

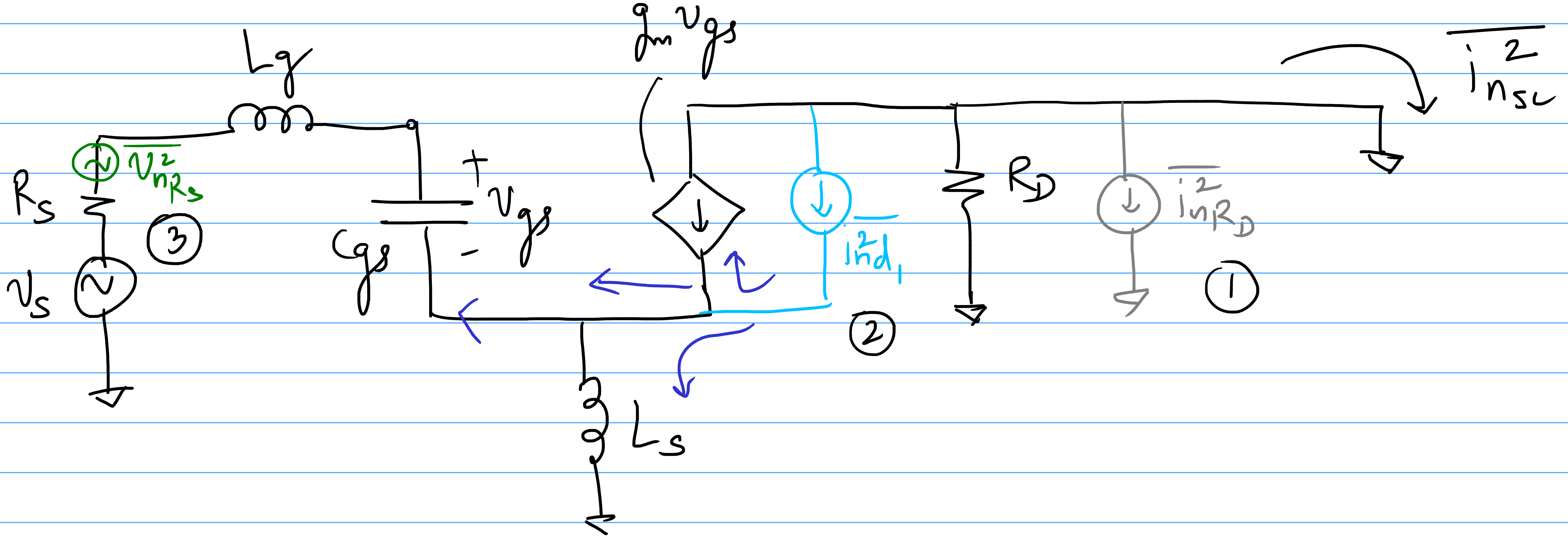
$$G_{m_{LNA}} = \frac{1}{2R_s} \cdot \frac{\omega_T}{\omega_o}$$



* Ignore effect of cascode for gain & noise
↳ only for reverse isolation

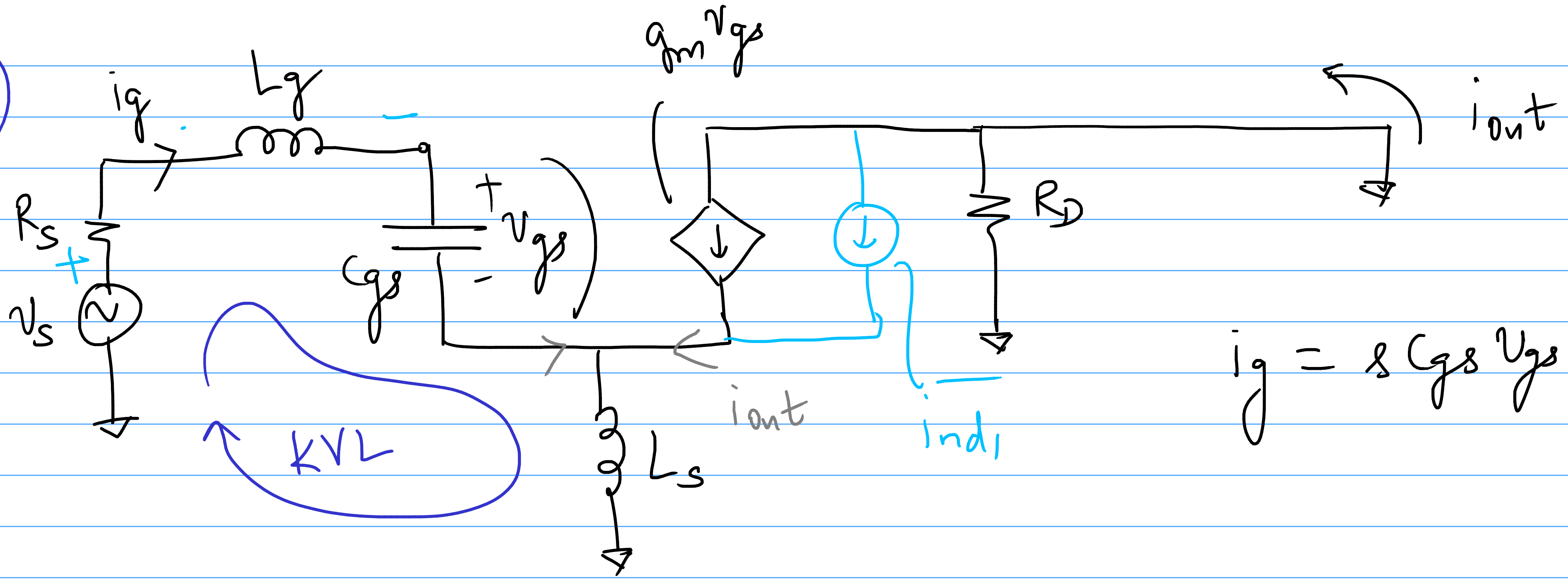
Noise Figure [Section 5.3.4 Razavi's textbook]

$$F = \frac{\text{Total } N_{out}}{N_{out}(R_s)}$$



$$* \quad \overline{i^2_{nsc_1}} = \overline{i^2_{nR_D}}$$

2)



$$i_g = s C_{gs} v_{gs}$$

$$i_{out} = \overline{i_{nd_1}} + g_m v_{gs} \Rightarrow v_{gs} = \frac{1}{g_m}\left(i_{out} - \overline{i_{nd_1}}\right)$$

$$v_s = \underbrace{(R_s + sL_g)\, i_g} + v_{gs} + sL_s\left(i_g + i_{out}\right)$$

$$v_s = (R_s + sL_g)\, sC_{gs} v_{gs} + v_{gs}$$

$$+ sL_s \left(sC_{gs} v_{gs} + i_{out} \right)$$

$$= sL_s i_{out} + \left[R_s \cdot sC_{gs} + s^2 L_g C_{gs} + s^2 L_s C_{gs} + 1 \right] v_{gs}$$

$$\underbrace{s^2 L_g C_{gs} + s^2 L_s C_{gs} + 1}_{= 0 \text{ @ } \omega_0}$$

$$s = j\omega \Big|_{\omega = \omega_0} \Rightarrow \omega_0^2 = \frac{1}{C_{gs}(L_s + L_g)}$$

$$v_s = j\omega_0 L_s \cdot i_{out} + j\omega_0 C_{gs} R_s \cdot v_{gs}$$

$$= j\omega_0 L_s \, i_{out} + j\omega_0 C_{gs} R_s \cdot \frac{1}{g_m}\left(i_{out} - \overline{i_{nd,1}} \right)$$

$$= i_{out}\left[j\omega_0 L_s + \frac{j\omega_0 C_{gs} R_s}{g_m} \right] - \overline{i_{nd,1}} \cdot \left[\frac{j\omega_0 C_{gs} R_s}{g_m} \right]$$

$$\frac{C_{gs}}{g_m} = \frac{1}{\omega_T}$$

$$v_s = i_{out}\left[j\omega_o L_s + j\frac{\omega_o R_s}{\omega_T} \right] - \overline{i_{nd_1}}\left[j\frac{\omega_o R_s}{\omega_T} \right]$$

(A)

If $\overline{i_{nd_1}} = 0$ for output signal component only

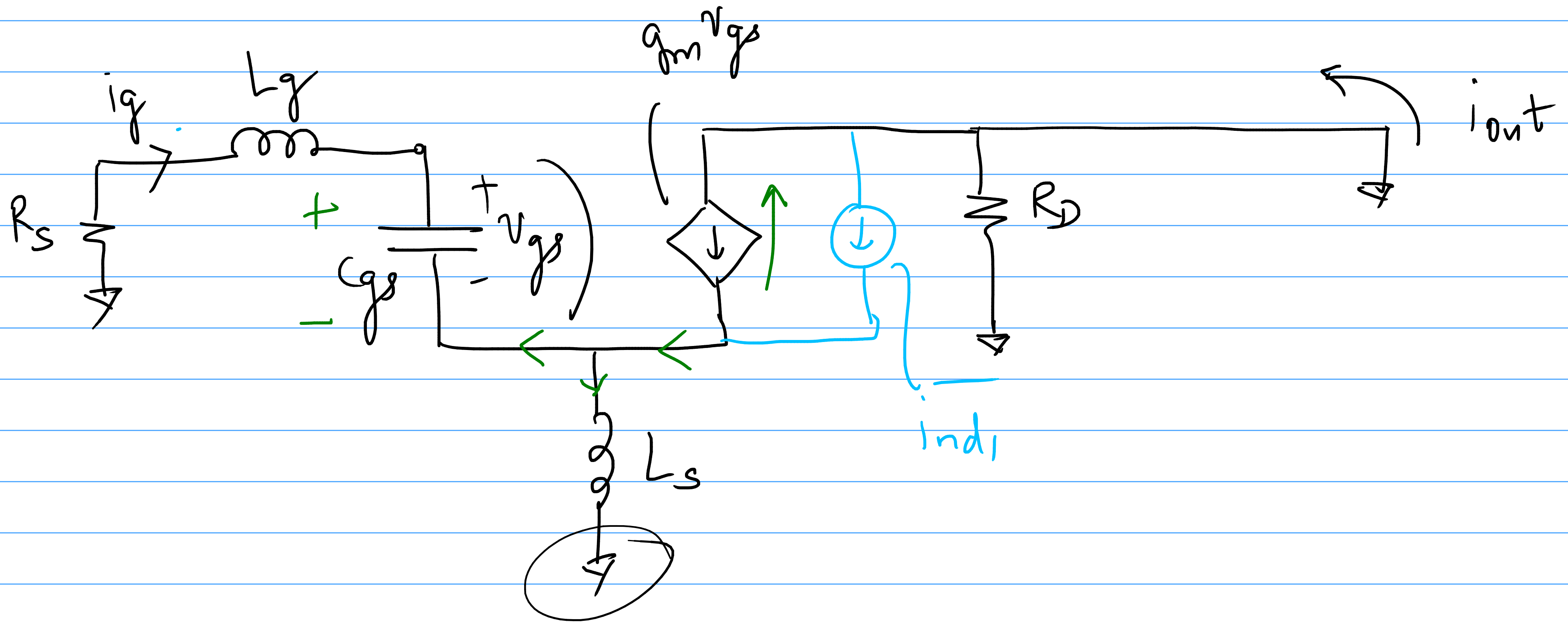
$$G_{m_{LNA}} = \left| \frac{i_{out}}{v_s} \right| = \left| \frac{\omega_T}{j\omega_o \omega_T L_s + j\omega_o R_s} \right| = \frac{\omega_T}{\omega_o} \cdot \frac{1}{2R_s}$$

(B) Set $v_s = 0 \Rightarrow i_{out}$ Noise component only ($= \overline{i_{nsc}}$ due to M_1)

Set $v_s = 0$

$$0 = \overline{i_{nsc}}\left[\frac{\overbrace{j\omega_0 L_s \omega_T + j\omega_0 R_s}^{=2j\omega_0 R_s}}{\omega_T}\right] - \overline{i_{nd1}}\left[\frac{j\omega_0 R_s}{\omega_T}\right]$$

$$\overline{i_{nsc}} = \frac{1}{2}\,\overline{i_{nd1}}$$



$$* \quad \overline{i_{nsc_2}^2} = \frac{1}{4}\overline{i_{nd_1}^2}$$

$$* \quad \overline{i_{n_{SC_3}}^2} = G_m^2 \cdot \overline{v_{n_{R_S}}^2}$$

$$F = \frac{G_m^2 \, \overline{v_{n_{R_S}}^2} + \frac{1}{4} \overline{i_{nd_1}^2} + \overline{i_{n_{R_D}}^2}}{G_m^2 \, \overline{v_{n_{R_S}}^2}}$$

$$= 1 + \frac{\frac{1}{4} \cdot 4kT\gamma g_m + \frac{4kT}{R_D}}{G_m^2 \cdot 4kT R_S}$$

$$F = 1 + \frac{\gamma g_m}{4 G_m^2 R_s} + \frac{1}{G_m^2 R_s R_D}$$

$$G_m = \frac{\omega_T}{\omega_0} \cdot \frac{1}{2 R_s}$$

$$F = 1 + \frac{\gamma g_m}{4 \left(\frac{\omega_T}{\omega_0}\right)^2 \times \frac{1}{4 R_s^2} \times R_s} + \frac{1}{\left(\frac{\omega_T}{\omega_0}\right)^2 \times \frac{1}{4 R_s^2} \times R_s R_D}$$

$$\boxed{F = 1 + \gamma g_m R_s \left(\frac{\omega_0}{\omega_T}\right)^2 + \frac{4R_s}{R_D}\left(\frac{\omega_0}{\omega_T}\right)^2}$$

* F can be small if $\omega_0 \ll \omega_T$

$$\omega_T \propto \frac{1}{L^2}$$

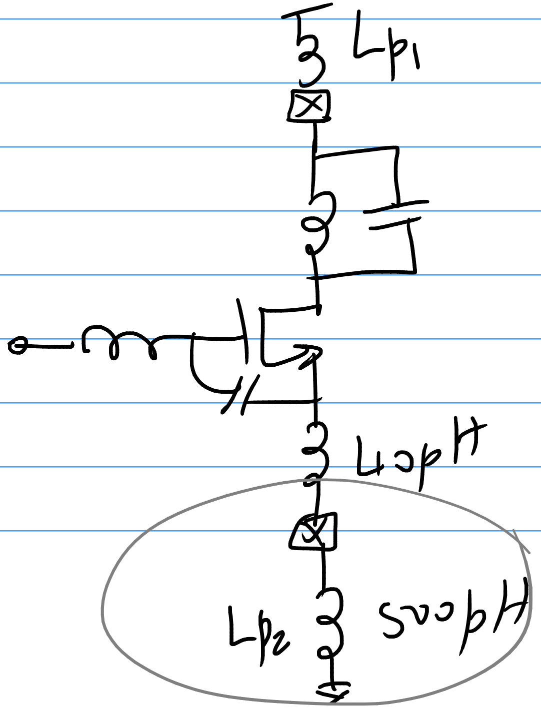
Max gain, Min NF $\Rightarrow$ max ω_T $\Rightarrow$ $L = L_{min}$.

Example

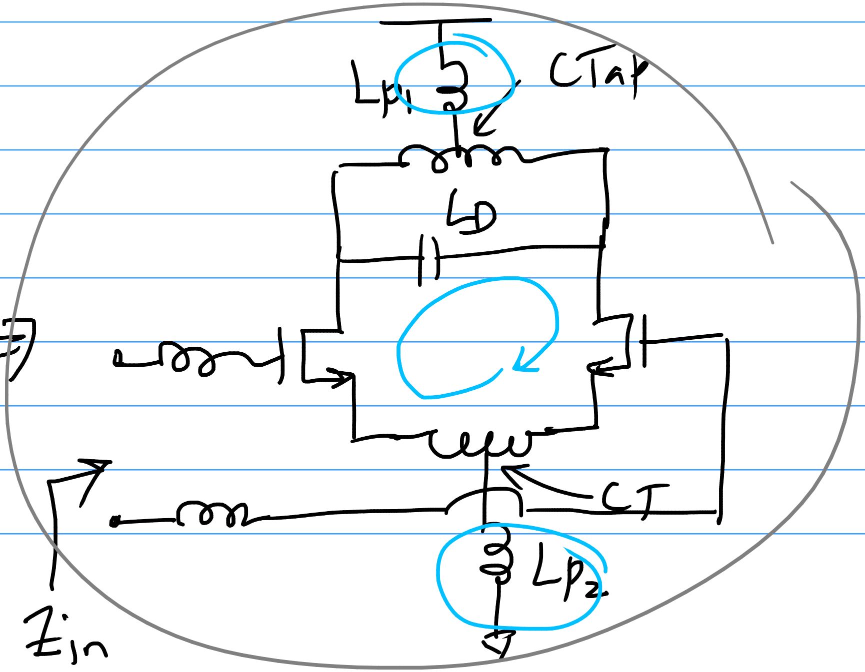
$f_T = 200\,GHz$

$L_s = ?$

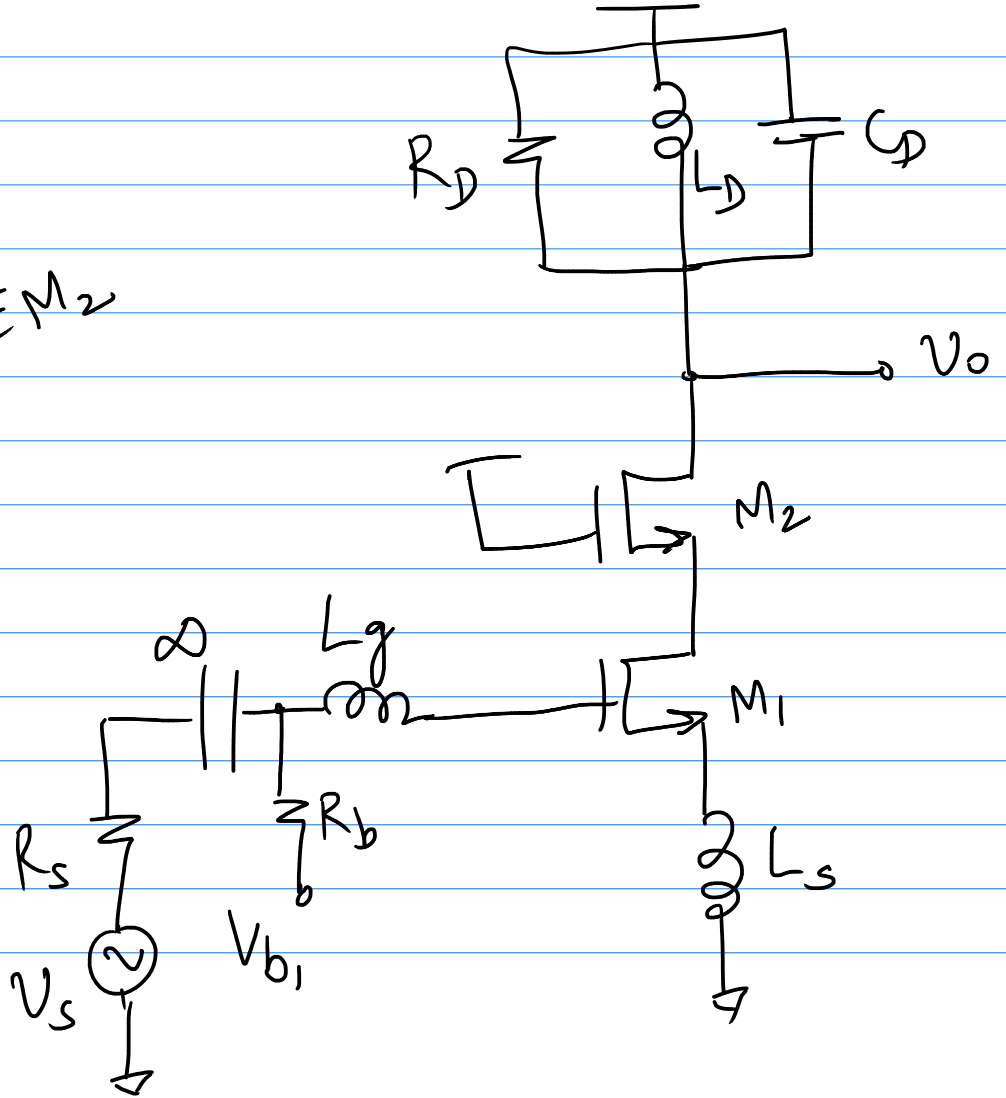
$2\pi f_T \cdot L_s = 50\Omega \Rightarrow L_s \approx 40pH$



$\Rightarrow$



$M_1 \equiv M_2$



Set L_D, C_D, W_{12}, L_g, L_s,

$(V_{GS} - V_T)$

1) L_D, C_D, R_D → depends on Q of L_D

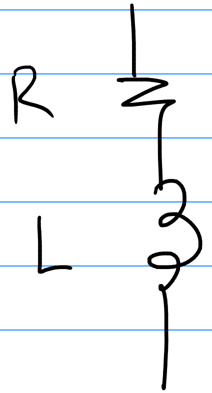
C_L, C_{db2}, C_{fixed}

$$@\ \omega_0:\ Q_{L_D} = \frac{R_D}{\omega_0 L_D}$$

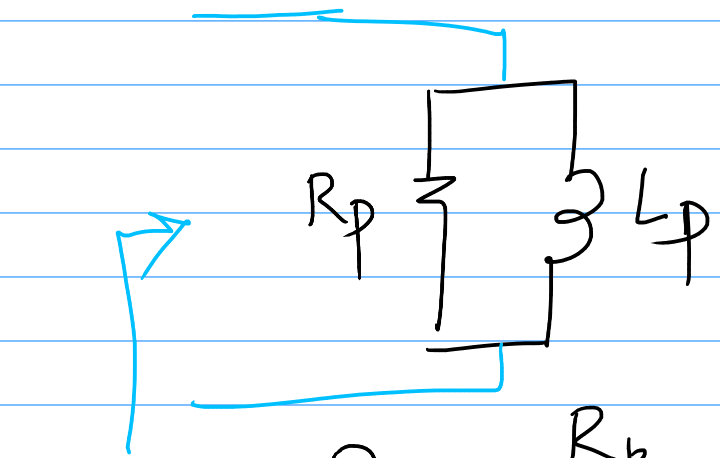
maximise Q_{L_D} ✓

maximise L_D with $C_{D,min}$.

$R_D = Q_{L_D} \cdot \omega_0 L_D$



$$Q = \frac{\omega L}{R}$$



$$Q = \frac{R_p}{\omega L_p}$$

$$Q = \frac{Im(Z)}{Re(Z)}$$

$$f_0 = \frac{1}{2\pi\sqrt{L_D C_D}}$$

2) $P_{max} \rightarrow$ bias current I_0

3) $$\frac{2I_0}{V_{GS}-V_T} = g_{m_1}$$

$$I_0 = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$

4) Min gain $= A_0$

$$|\text{gain}| = g_{m_1} R_D \cdot Q_{in}$$

$g_{m_1} \rightarrow k_1\sqrt{W_1}$

$R_D \rightarrow$ fixed by L_D

$Q_{in} \rightarrow \dfrac{k_2}{W_1}$

gives max W_1 to meet gain

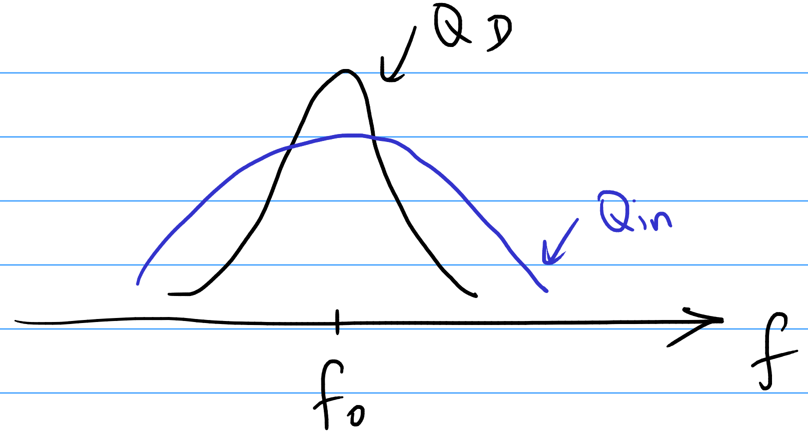
5) calculate ω_T, C_{gs}

6) $R_S = \omega_T \cdot L_S$

(calculate L_S)

7) $$\omega_0 = \frac{1}{\sqrt{C_{gs}(L_g + L_s)}}$$

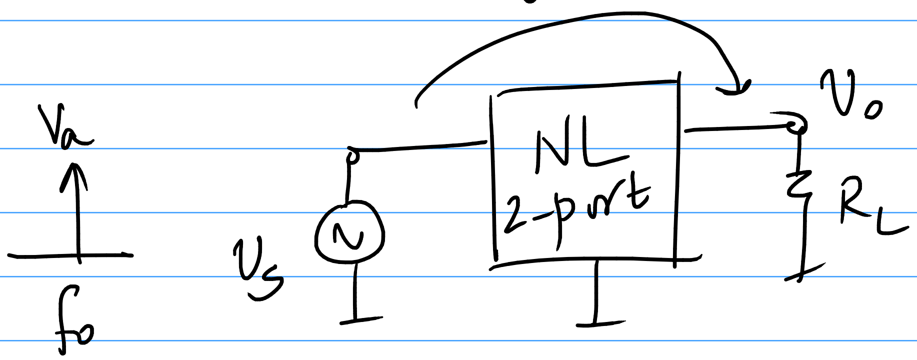
(calculate L_g)

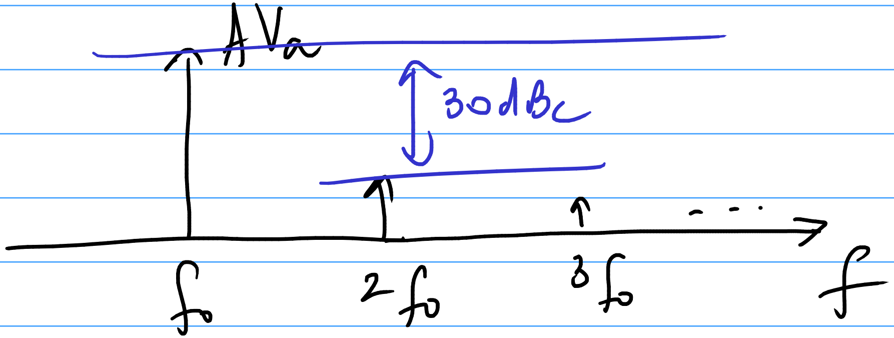


$Q = \frac{\omega_0}{BW} \approx$ limited by narrower of the two resonances

---

Non-linearity





THD = total harmonic distortion

small signal gain

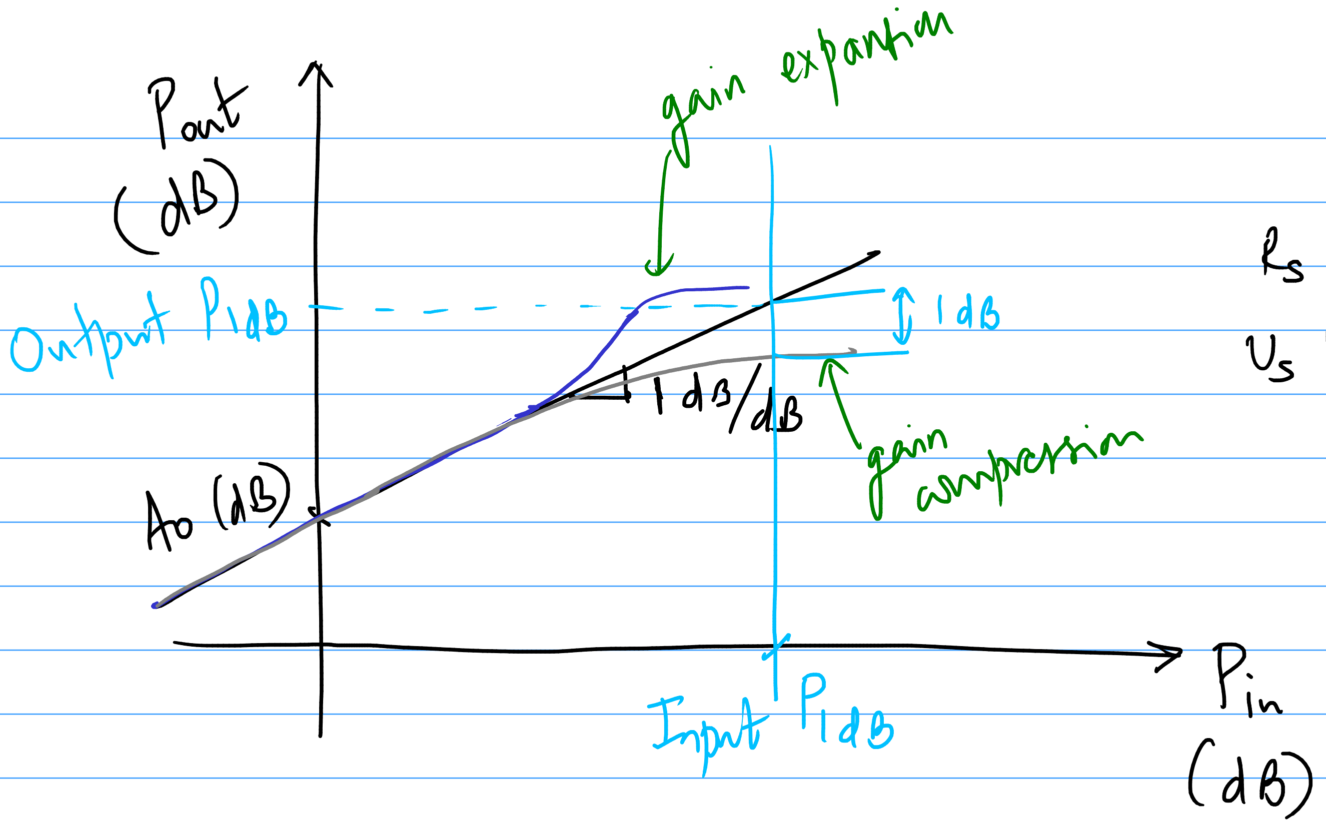
$$x(t) \longrightarrow \boxed{\text{NL circuit}} \longrightarrow y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

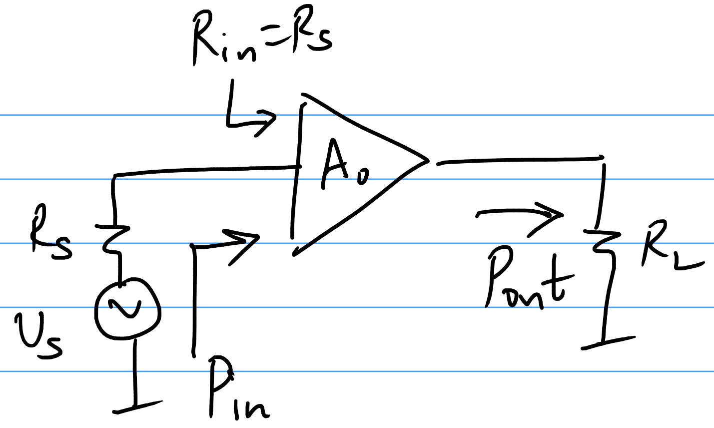
$$x(t) = V_a \cos \omega_0 t$$

$$y(t) = \alpha_0 + \alpha_1 V_a \cos \omega_0 t + \alpha_2 V_a^2 \cos^2 \omega_0 t + \alpha_3 V_a^3 \cos^3(\omega_0 t)$$

$$\alpha_2 V_a^2 \cos^2 \omega_0 t = \frac{\alpha_2 V_a^2}{2}\left(1 + \underbrace{\cos 2\omega_0 t}_{\text{2nd harmonic}}\right)$$

$$\alpha_3 V_a^3 \cos^3(\omega_0 t) = \frac{\alpha_3 V_a^3}{4}\left(\underbrace{\cos 3\omega_0 t}_{\text{3rd harmonic}} + 3 \cos \omega_0 t\right)$$





$$P_{out} = A_0 \cdot P_{in}$$

$$P_{out}(dB) = A_{0dB} + P_{in}(dB)$$

NL amplifiers:

small

$$y(t) = \alpha_0 + \frac{\alpha_2 V_a^2}{2} + \left\{ \alpha_1 V_a + \frac{3\alpha_3 V_a^3}{4} \right\} \cos \omega_0 t$$

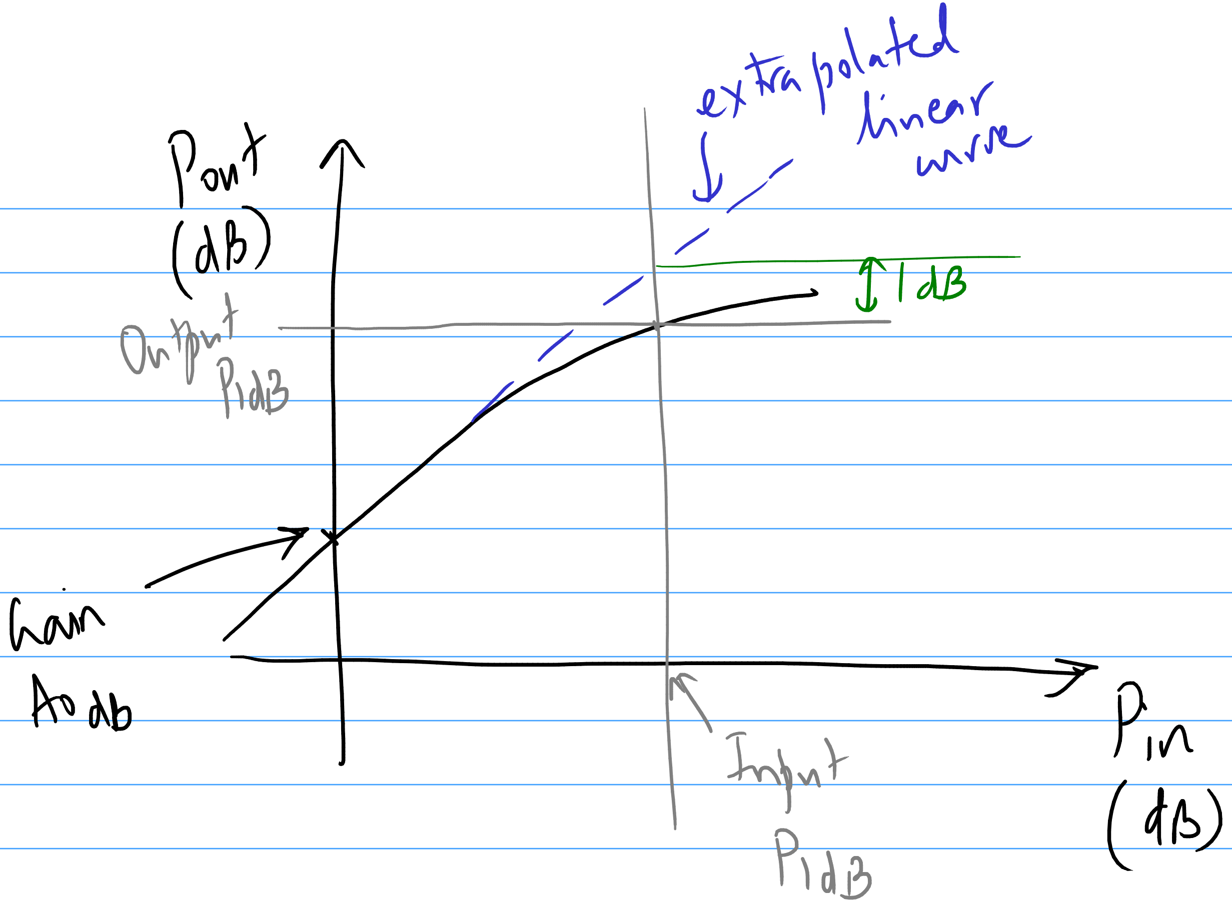
$$+ \frac{\alpha_2 V_a^2}{2} \cos(2\omega_0 t) + \frac{\alpha_3 V_a^3}{4} \cdot \cos 3\omega_0 t + \ldots$$

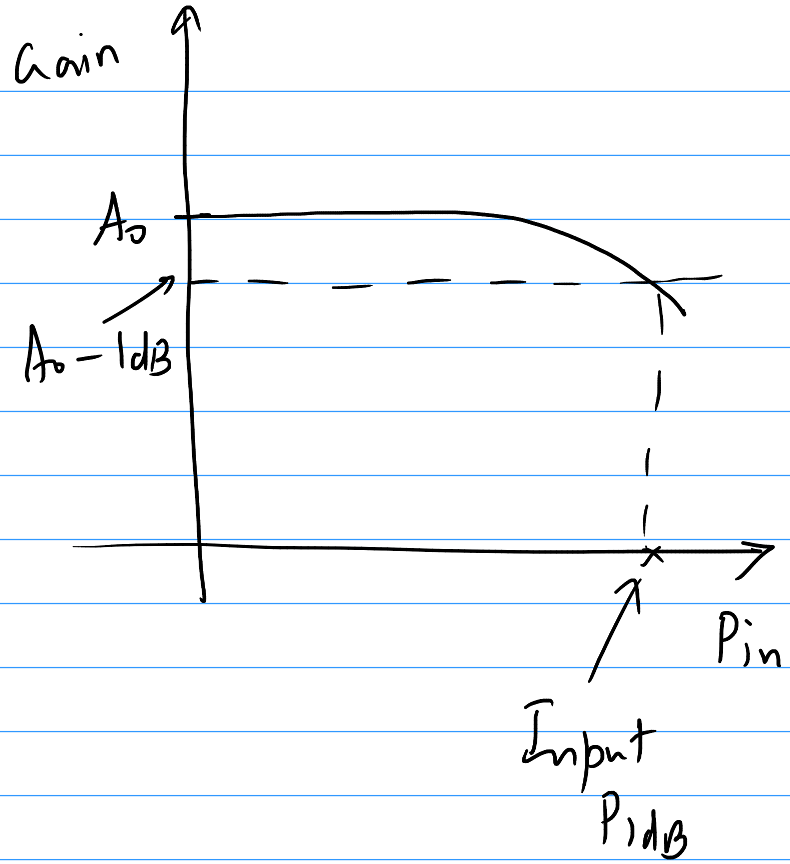
$$20 \log |\alpha_1| - 1\,dB = 20 \log \left| \alpha_1 + \frac{3\alpha_3 V_a^2}{4} \right|$$

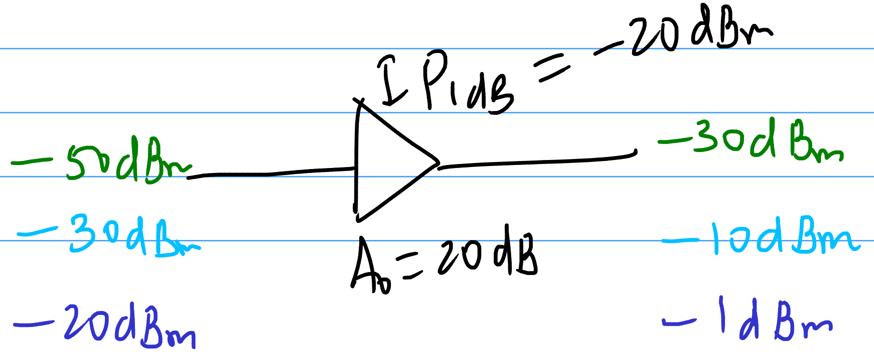
$V_a = V_{a_{1-dB}}$

Input P_{1dB} = Input-referred 1-dB compression point

= P_{in} where the actual P_{out} vs P_{in} curve falls 1dB below the extrapolated small-signal linear plot.

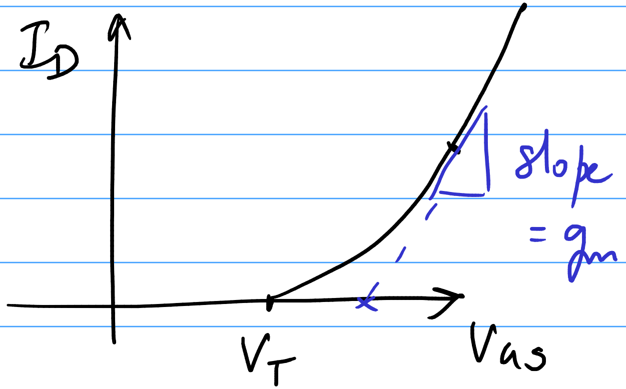

extrapolated linear curve
Pout (dB)
1dB
Output P1dB
Gain
A0 dB
Input P1dB
Pin (dB)

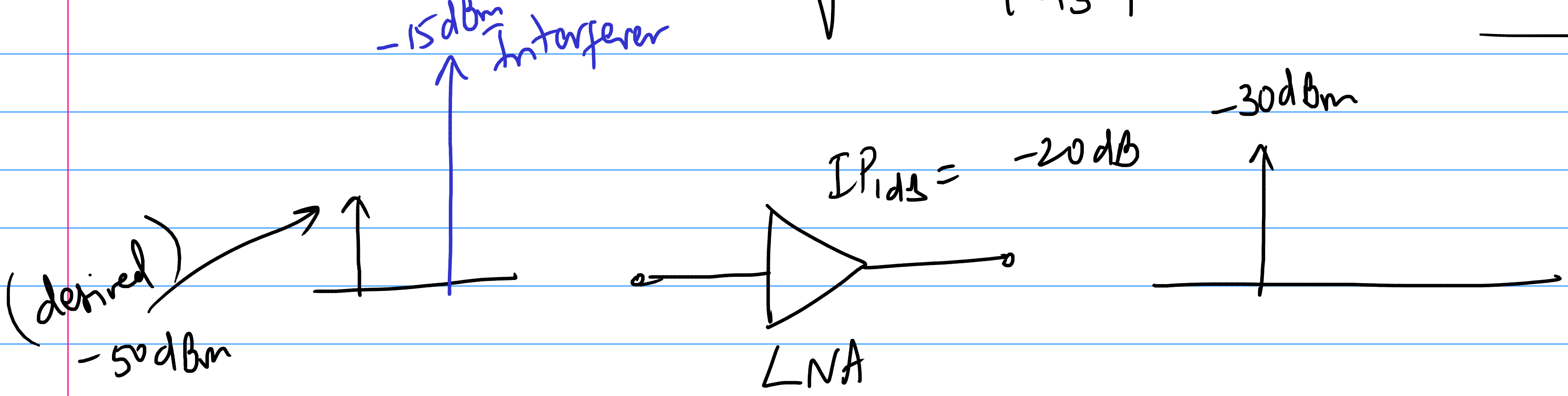

Gain
A0
A0 − 1dB
Pin
Input P1dB


IP1dB = −20dBm
−50dBm
−30dBm
−20dBm
A0 = 20dB
−30dBm
−10dBm
−1dBm

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{1-dB}^2\right| = 20\log|\alpha_1| - 1\,dB$$

$$A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

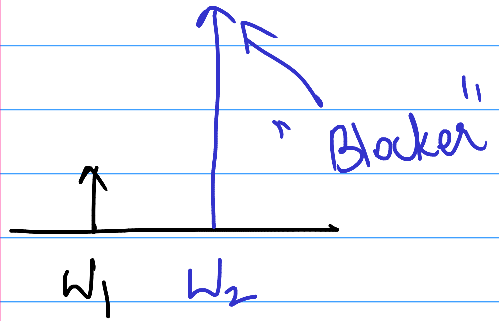




$A_0 = 20dB$

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t \qquad A_1 \ll A_2$$

(desired) (Int.)



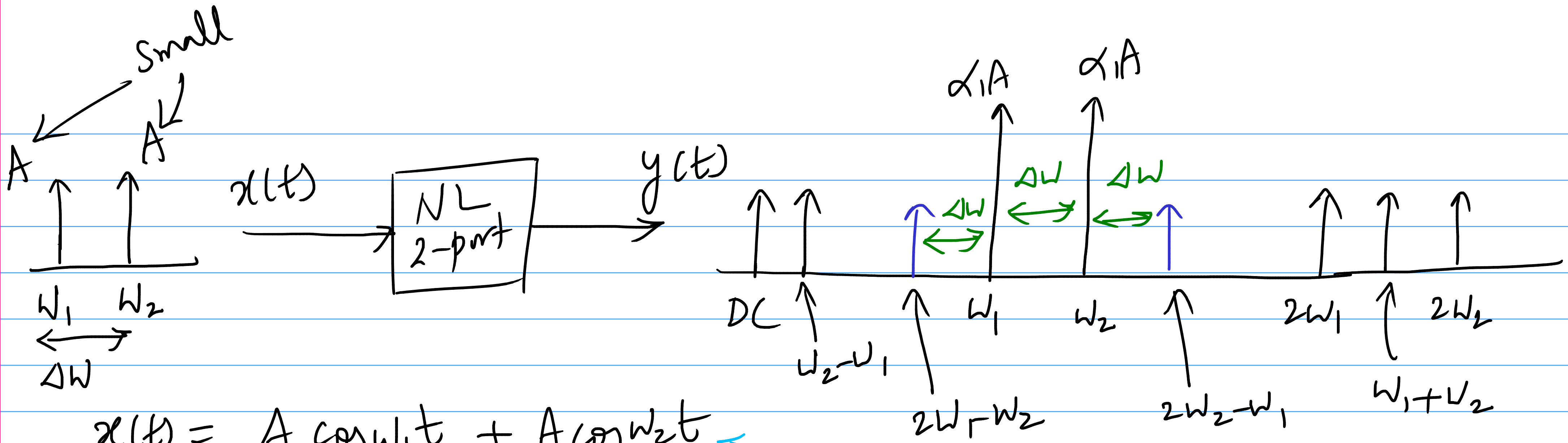
$$x(t) \longrightarrow \boxed{\text{Weakly NL Amp.}} \longrightarrow y(t)$$

HW

$$y(t) = \Big(\alpha_1 + \underbrace{\tfrac{3}{2}\alpha_3 A_2^2}_{} + \dots\Big) A_1 \cos \omega_1 t + \dots$$

α_3 – small & negative but A_2 is large

$\Rightarrow$ Gain is reduced drastically $\Rightarrow$ "Desensitiz"



$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

$$y(t) = \alpha_0 + \alpha_1 x(t) + \underbrace{\alpha_2 x^2(t)} + \underbrace{\alpha_3 x^3(t)}$$

$\alpha_1 A\cos\omega_1 t$

$+$

$\alpha_1 A\cos\omega_2 t$

DC + $2\omega_1 + 2\omega_2$

$+ \; 2\alpha_2 A^2 \cos\omega_1 t \cos\omega_2 t$

$3\omega_1, \; 3\omega_2$

$2\omega_1 \pm \omega_2 \; ; \; 2\omega_2 \pm \omega_1$

$$p\omega_1 \pm q\omega_2 = \text{"intermodulation" terms} \rightarrow IM_{p+q}$$

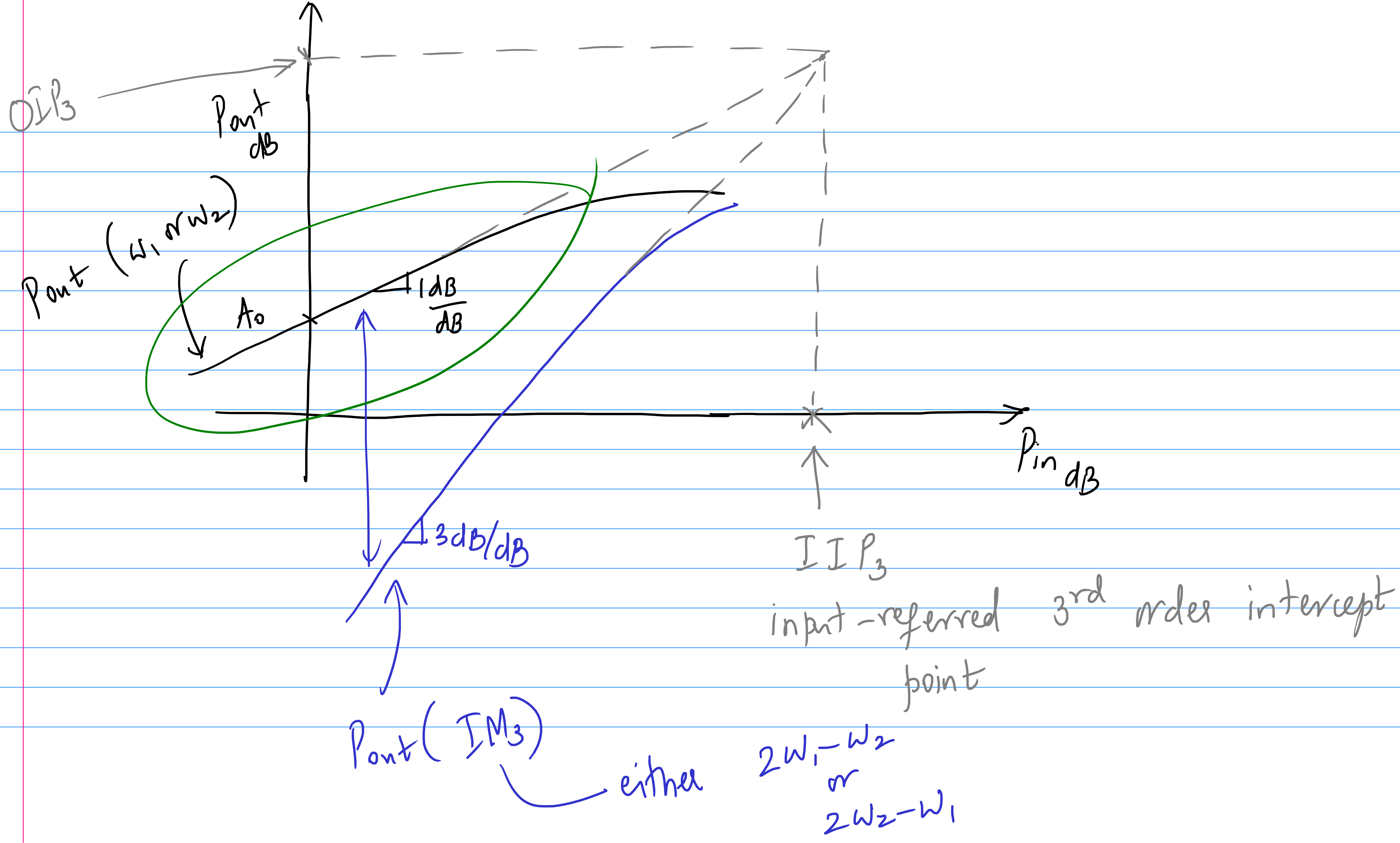
$$\left.\begin{matrix} 2\omega_2 \pm \omega_1 \\ 2\omega_1 \pm \omega_2 \end{matrix}\right\}$$ 3^{rd} order intermodulation (IM_3) terms

$$\left.\begin{matrix} 2\omega_2 - \omega_1 \leftarrow \omega_2 + \Delta\omega \\ 2\omega_1 - \omega_2 \leftarrow \omega_1 - \Delta\omega \end{matrix}\right)$$ If you choose small $\Delta\omega \rightarrow IM_3$ terms fall in band

IM_3 terms are: $\frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2)t$

$+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1)t$

HW


OIP3
Pout dB
Pout (ω1 or ω2)
A0
1dB/dB
3dB/dB
Pin dB
IIP3
input-referred 3rd order intercept point
Pout(IM3)
either 2ω1−ω2 or 2ω2−ω1

$$\left| \alpha_1 A_{IP_3} \right| = \left| \frac{3}{4} \alpha_3 A_{IP_3}^3 \right| \quad @ \quad IIP_3$$

$$A_{IP_3}^2 = \frac{4}{3} \frac{\alpha_1}{\alpha_3}$$

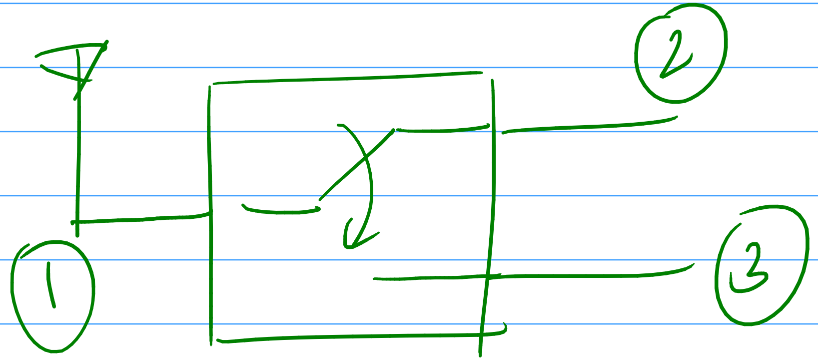
$$IIP_3 = \frac{A_{IP_3}^2}{2 R_S} = \frac{2}{3} \frac{\alpha_1}{\alpha_3} \cdot \frac{1}{R_S}$$

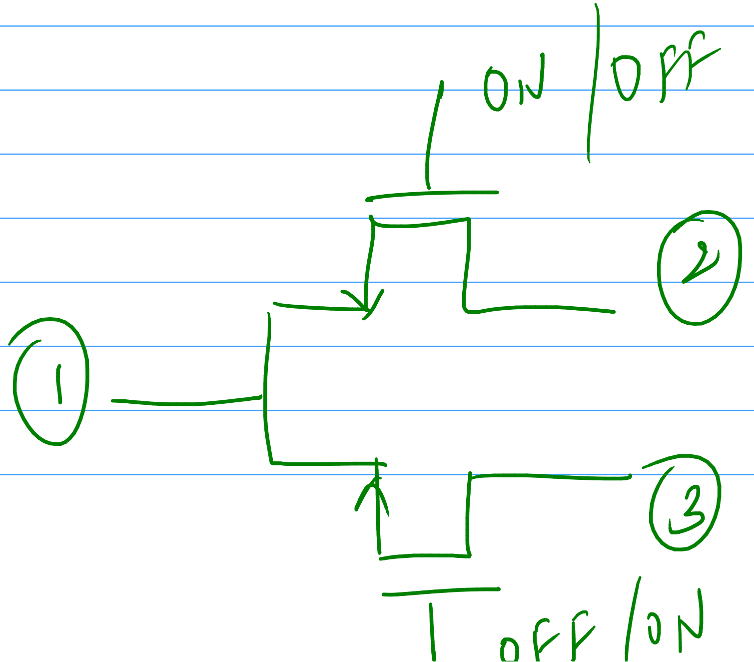
$$A_{1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

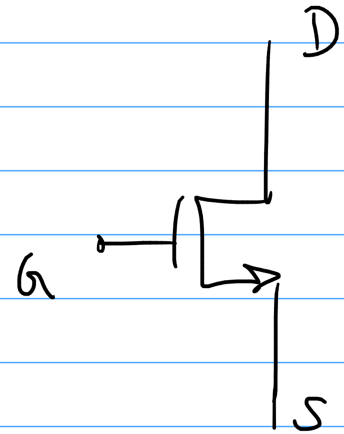
$$A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$\frac{A_{IP_3}}{A_{1dB}} = \sqrt{\frac{4}{3} \times \frac{1}{0.145}} = 3.03$$

$$IIP_{3\,dB} = P_{1dB} + 9.63\,dB$$

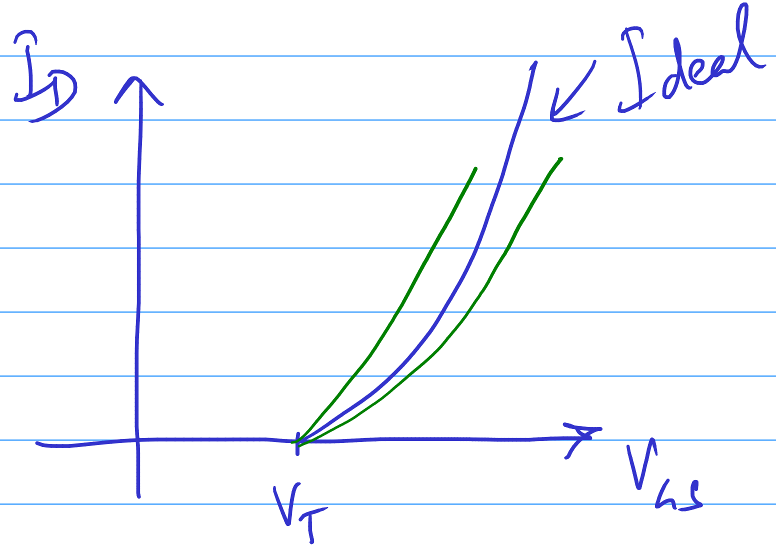

1
2
3

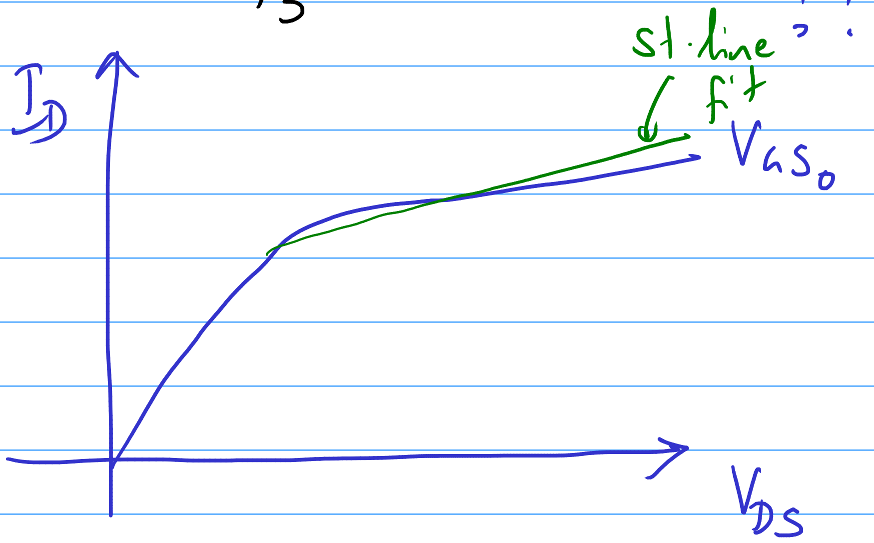

ON/OFF
1
2
3
OFF/ON



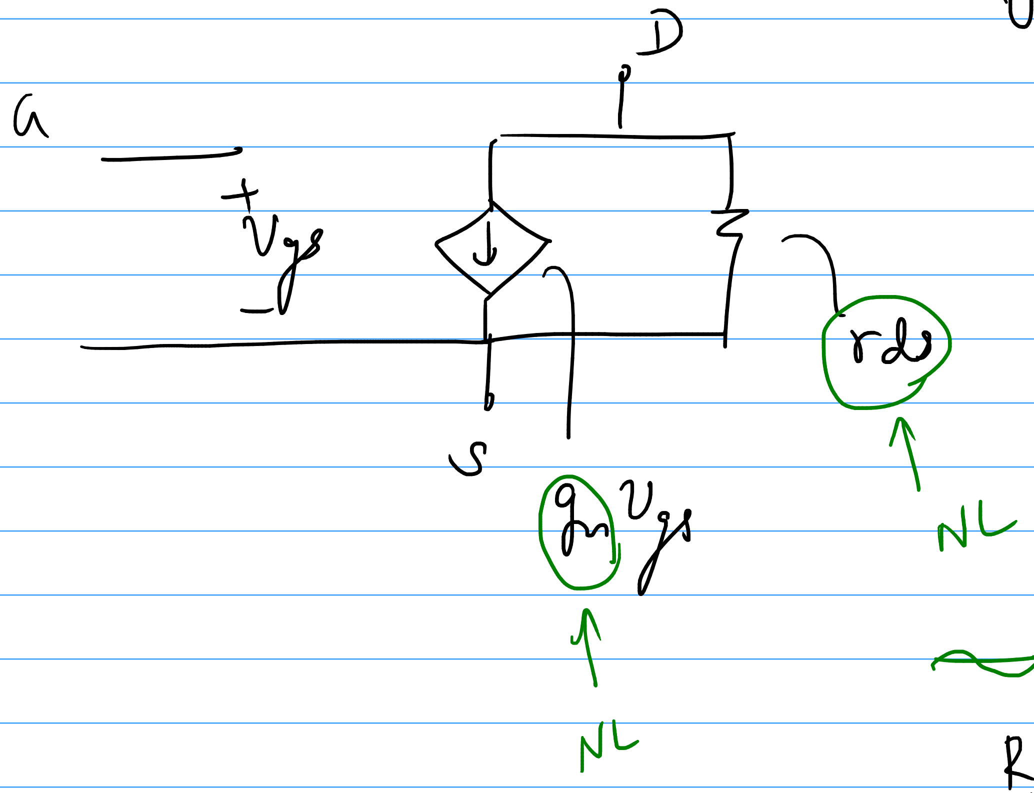
$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)\left(V_{GS}-V_T\right)^2\left(1+\lambda V_{DS}\right)$$

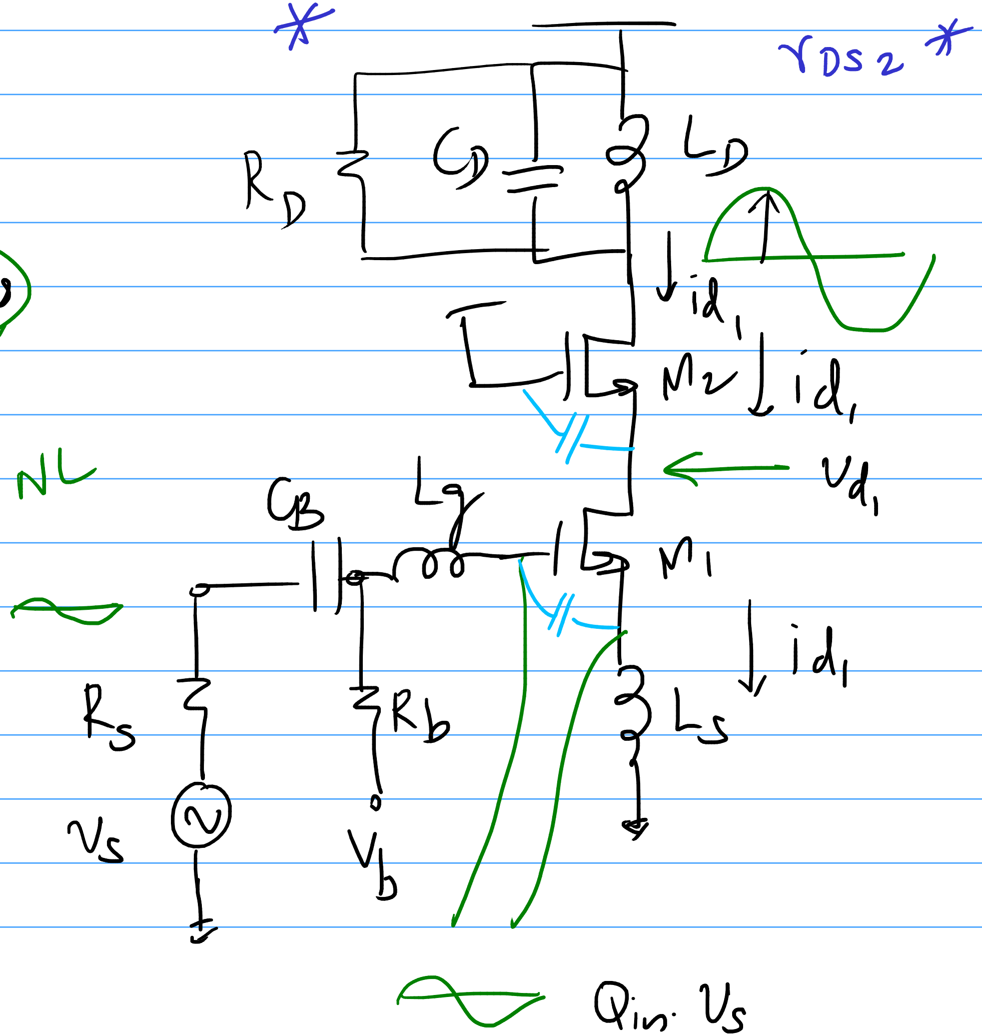
↳ $IIP_3 = \infty$??





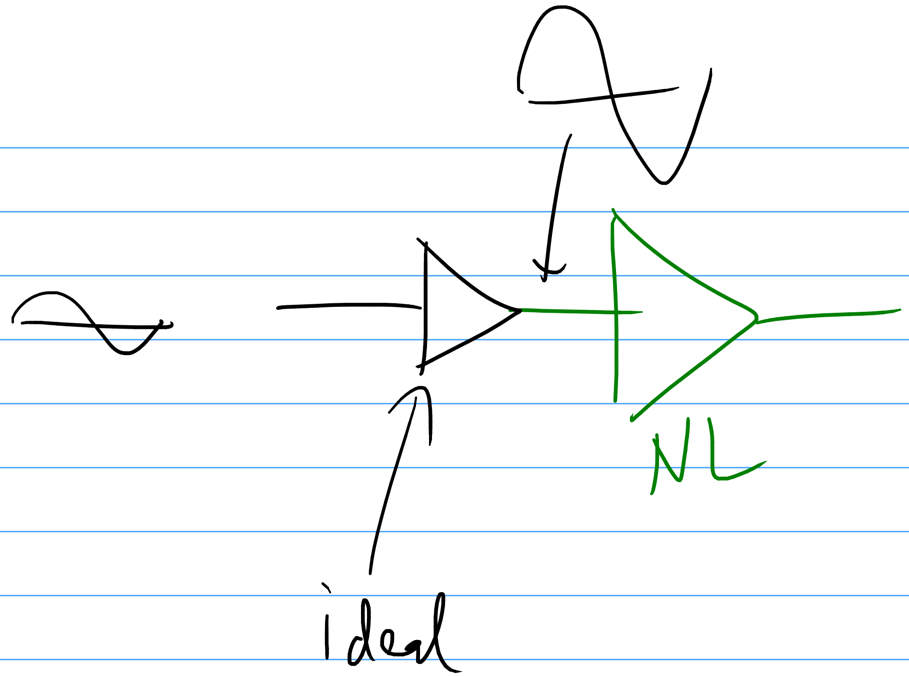
$$v_{out} = -g_{m_1} R_D \cdot Q_{in} \cdot v_s$$

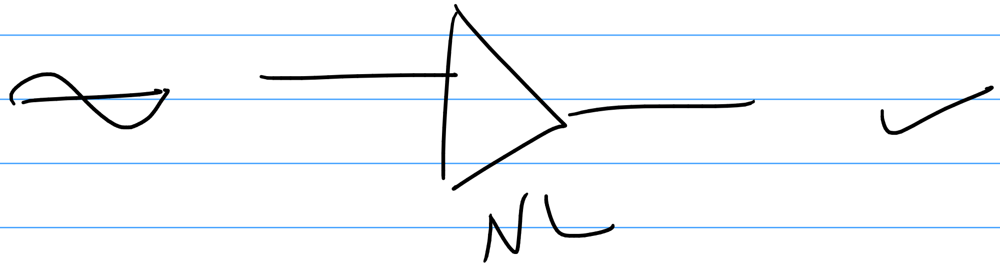




$$i_{d_1} = g_{m_1} \cdot v_{gs} = g_{m_1} \cdot Q_{in} \cdot v_s$$

$$v_{d_1} = -i_{d_1} \cdot \frac{1}{g_{m_2}} \approx -\frac{g_{m_1}}{g_{m_2}} \cdot Q_{in} \cdot v_s$$

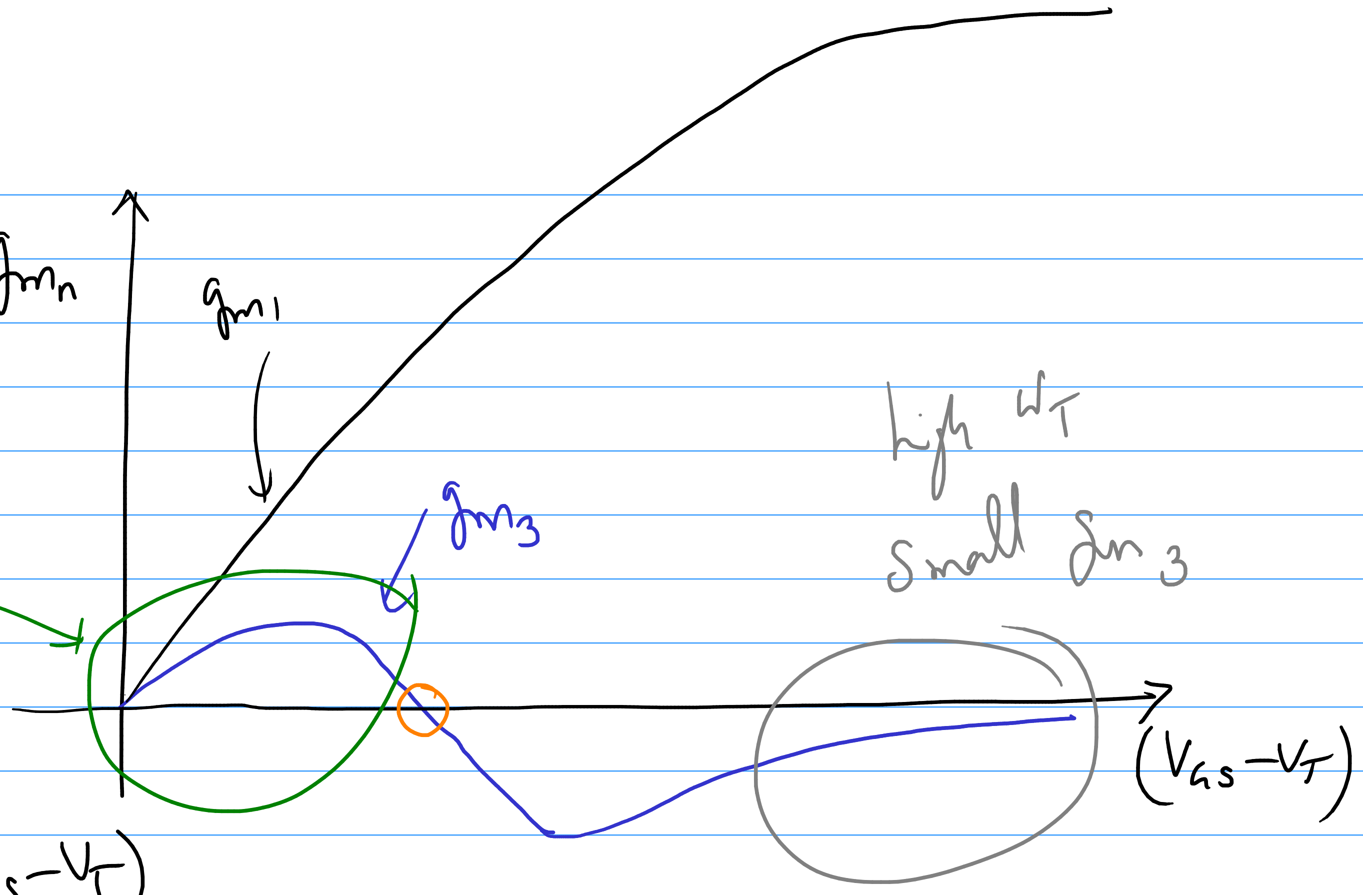

ideal
NL


NL

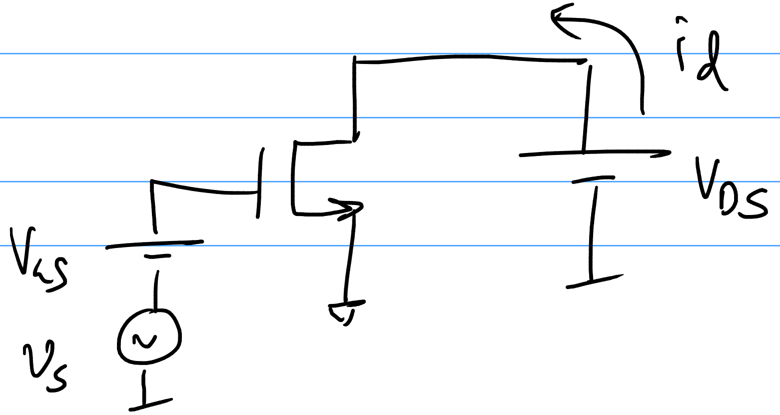
g_m NL

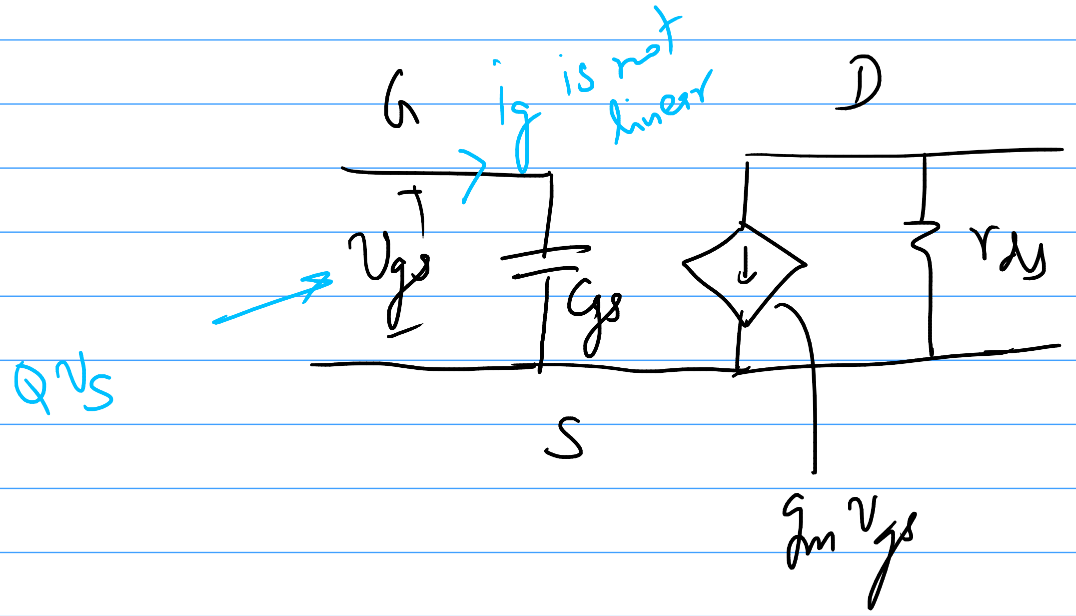
ω_T is small

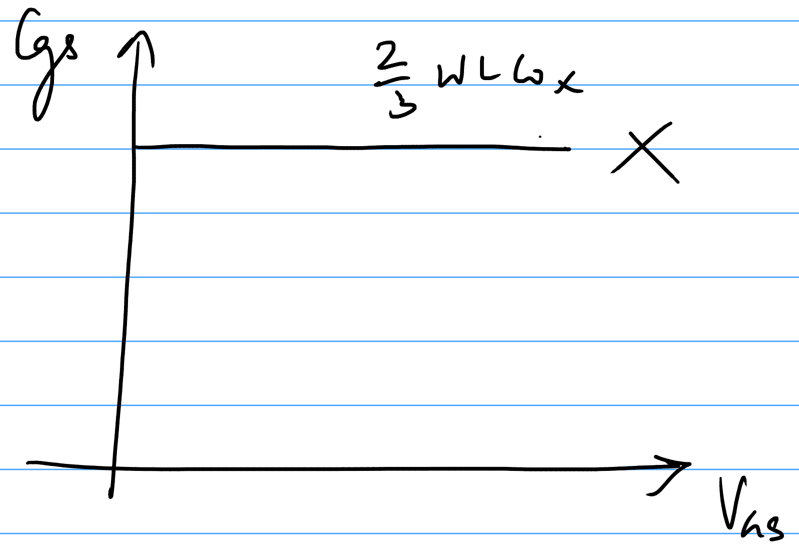
$\omega_T = \frac{g_m}{C_{gs}}$ — g_m is small, C_{gs} is large



$$g_{m_1} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)$$

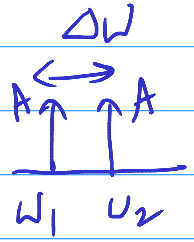


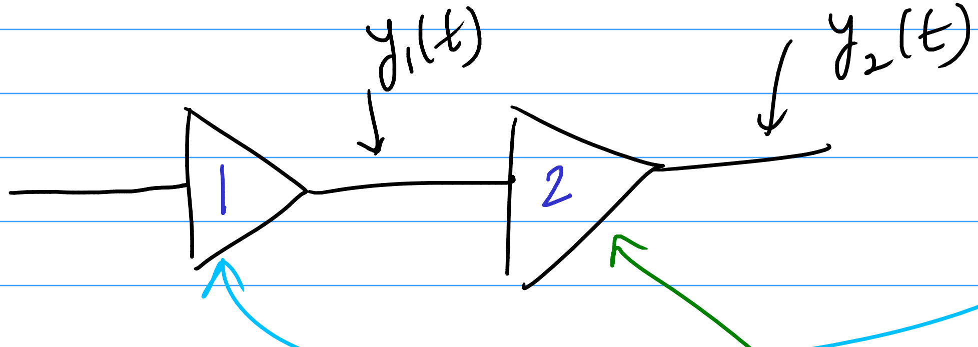


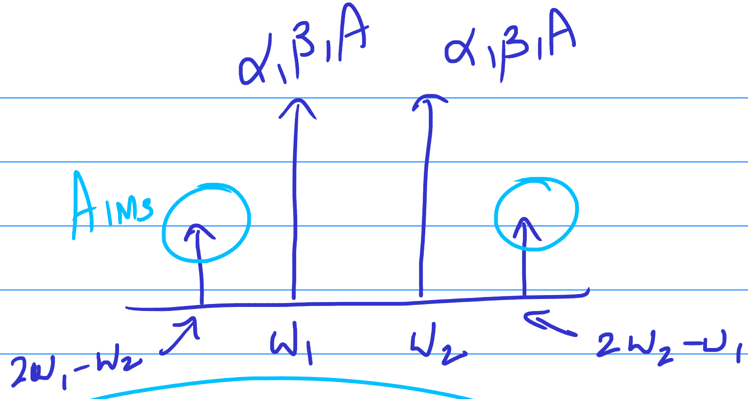


$$C_{gs} = \frac{2}{3} WLC_{ox}$$

IIP_3 of cascaded NL Systems







$$y_1(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_0 + \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$A_{IM_3} \rightarrow$ 1

$\rightarrow$ 2

$\rightarrow$ 3

$$A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\overbrace{(\qquad)}^{\text{Small signal gain}}}{\underbrace{(\qquad)}_{\text{3rd order term}}}\right|}$$

$$= \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}$$

Worst case $1|\beta_3$:

$$A_{1\beta_3} = \sqrt{\frac{4}{3}\frac{|\alpha_1\beta_1|}{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}}$$

$$\frac{1}{A_{1\beta_3}^2} = \frac{3}{4}\frac{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|}$$

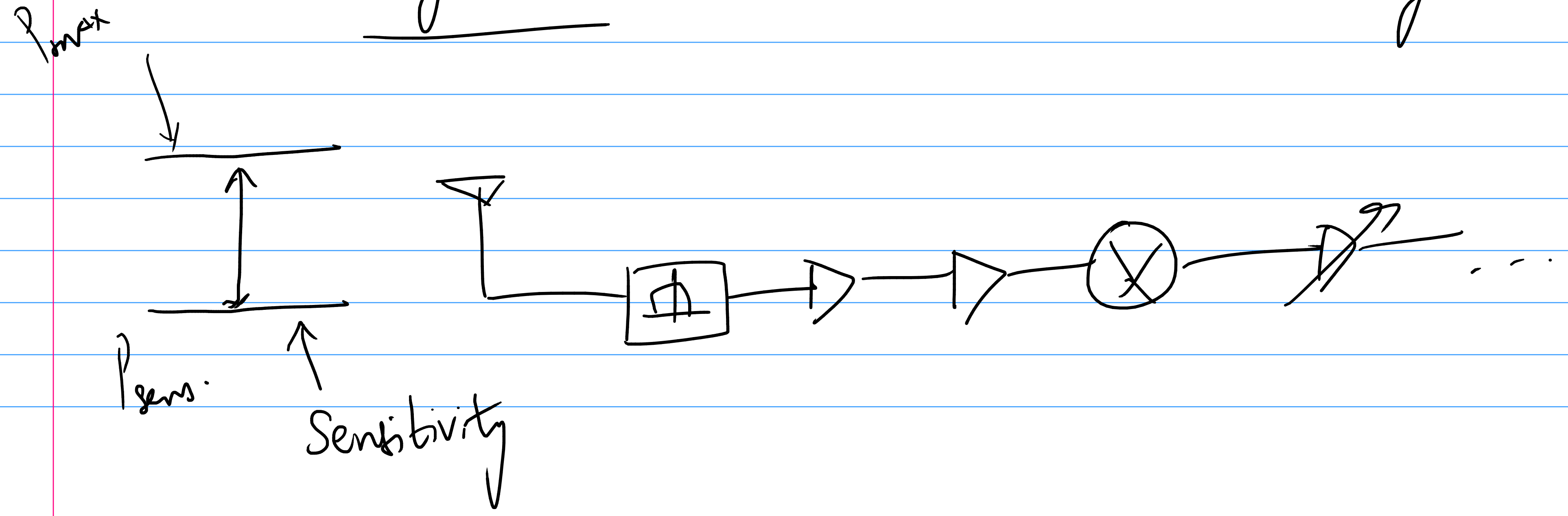
$$\frac{1}{A_{1P_3}^2} = \frac{1}{A_{1P_{3_1}}^2} + \frac{3}{4}\frac{|2\alpha_2\beta_2|}{|\beta_1|} + \frac{\alpha_1^2}{A_{1P_{3_2}}^2}$$

≈ 0 for narrowband ampl.

$$\frac{1}{A_{1P_{3\,tot.}}^2} = \frac{1}{A_{1P_{3_1}}^2} + \frac{G_1^2}{A_{1P_{3_2}}^2} + \frac{G_1^2 G_2^2}{A_{1P_{3,3}}^2} + \cdots$$

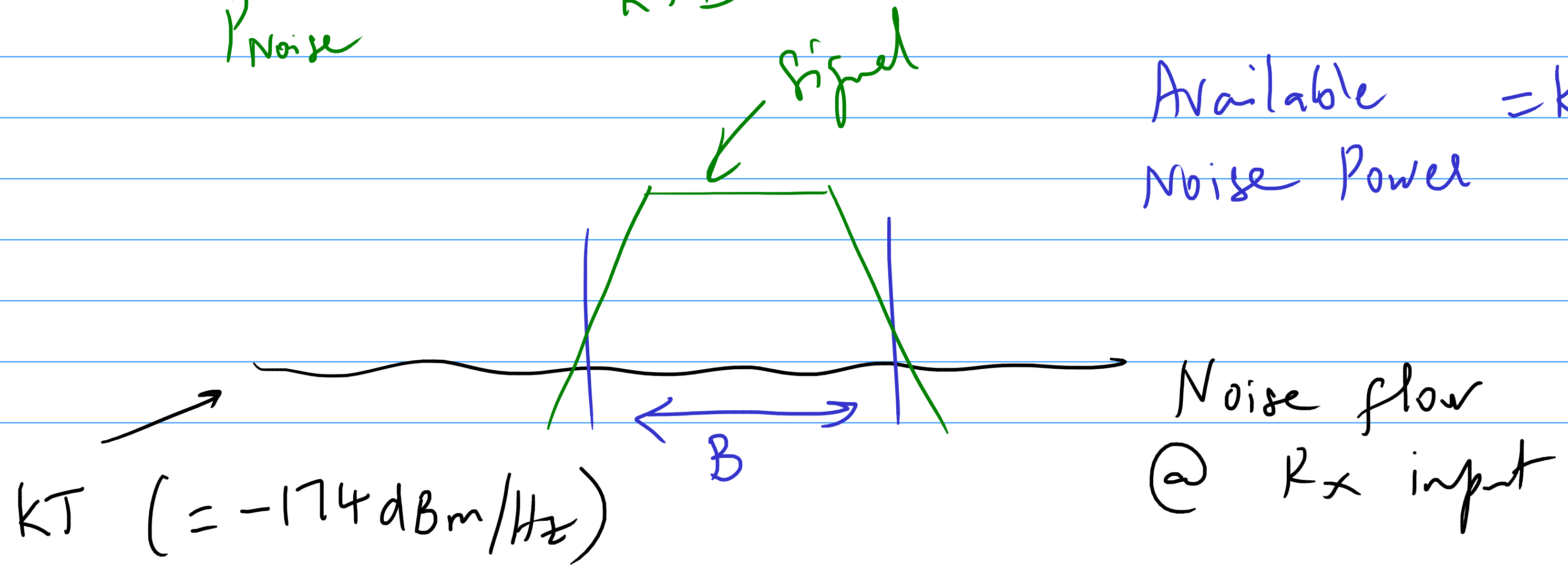
Low Noise Figure $\rightarrow$ Have high gain up front in the chain

High IIP$_3$ $\rightarrow$ Not too much gain up front



$P_{sensitivity}$ = Minimum power that the receiver can
($P_{min.}$) receive while meeting a required SNR
(BER)

$$SNR = \frac{P_{sig}}{P_{Noise}} = \frac{P_{sig}}{KTB \cdot F}$$

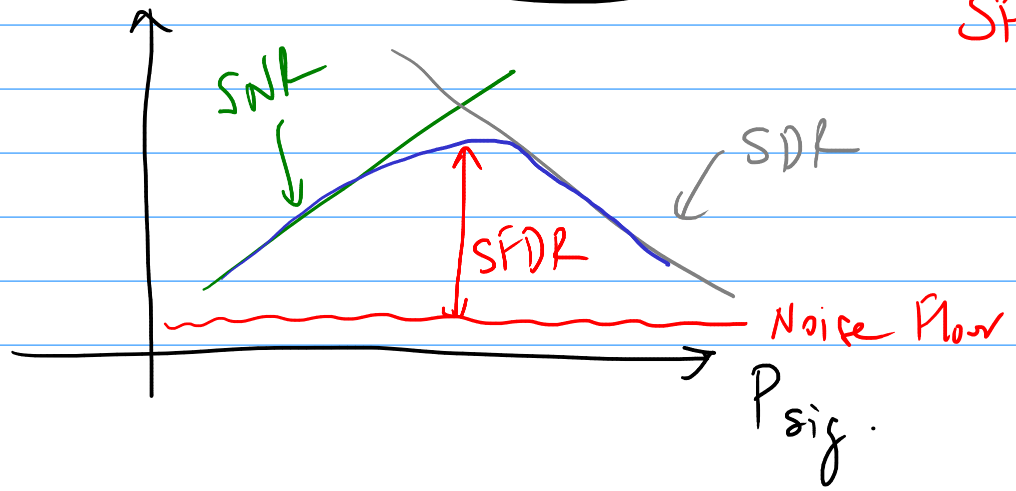


Available Noise Power = KTB

$$SNR_{min.} = \frac{P_{sens.}}{KTB \cdot F}$$

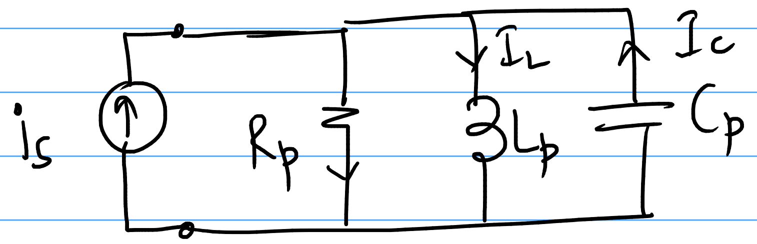
$$\underset{(dB)}{P_{sens.}} = -174\ dBm/Hz + 10\log(B) + NF + SNR_{min.}$$

SNR
SDR
SNDR



SFDR = spurious free dynamic Range

Parallel Resonance

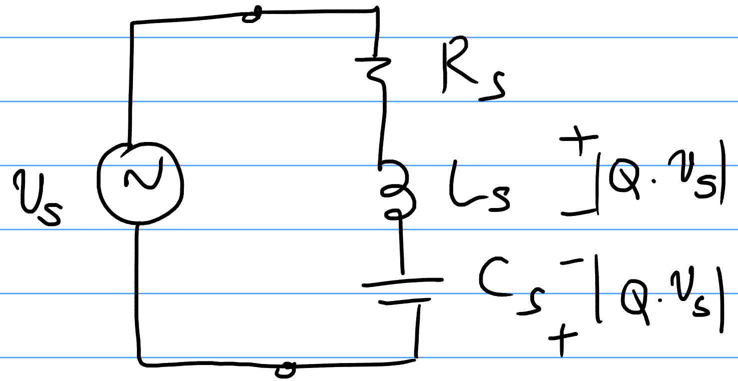


$$\omega_0 = \frac{1}{\sqrt{L_p C_p}}$$

$$Z_{resonance} = R_p$$

$$|I_L| = |I_c| = Q \cdot i_s \text{ @ } \omega_0$$

Series Resonance



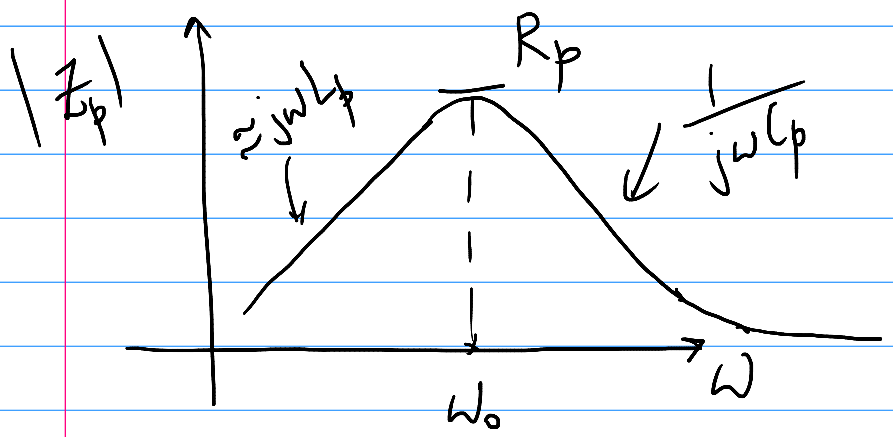
$$\omega_0 = \frac{1}{\sqrt{L_s C_s}}$$

$$Z_{resonance} = R_s$$

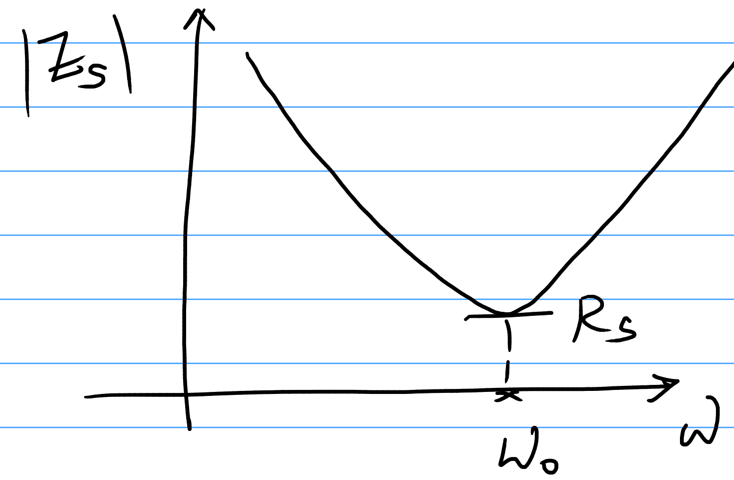
$$|V_L| = |V_c| = Q \cdot v_s$$

@ ω_0

Parallel

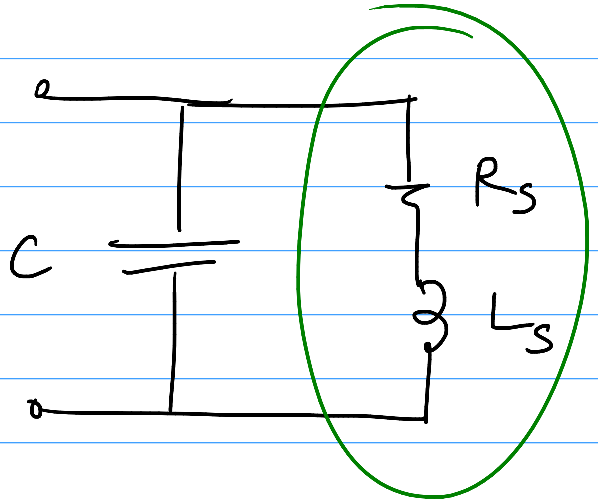


Series



$$Q_p = \frac{R_p}{\omega_0 L_p} = \frac{R_p}{\sqrt{\frac{L_p}{C_p}}}$$

$$Q_s = \frac{\omega_0 L_s}{R_s} = \frac{\sqrt{\frac{L_s}{C_s}}}{R_s}$$



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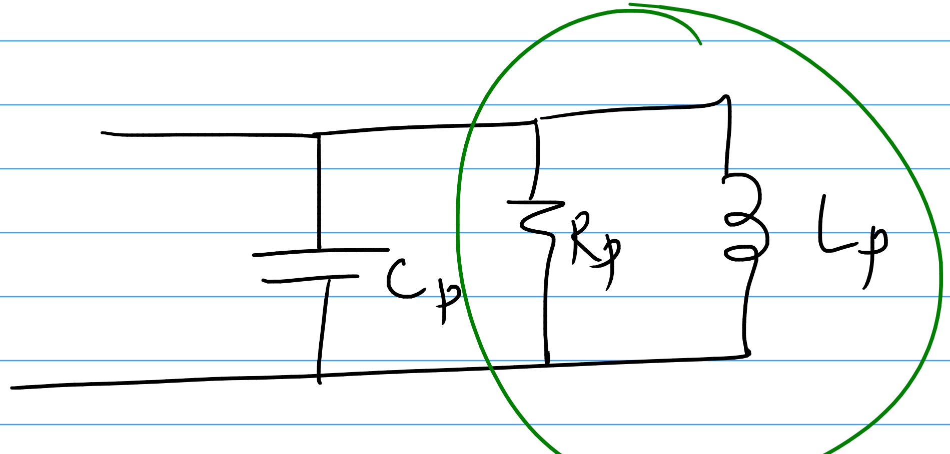
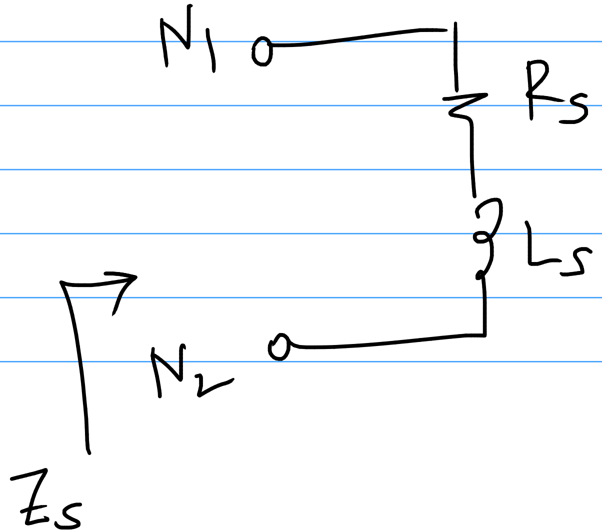
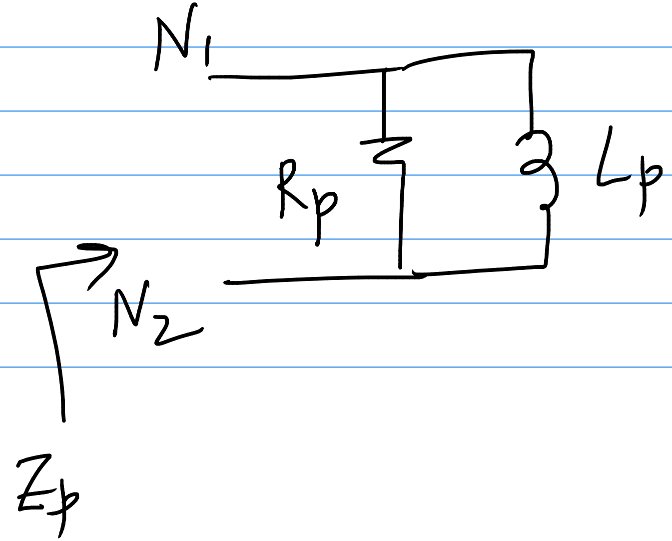


$C_p = C$



≡



$$Z_s = R_s + j\omega L_s$$

$$Z_p = \frac{R_p \cdot j\omega L_p}{R_p + j\omega L_p}$$

$$= \frac{(R_p - j\omega L_p) \cdot j\omega L_p R_p}{R_p^2 + \omega^2 L_p^2}$$

$$= \frac{\omega^2 L_p^2 R_p}{R_p^2 + \omega^2 L_p^2} + \frac{j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

$$Y_s(j\omega) = \frac{1}{R_s + j\omega L_s} = \frac{R_s - j\omega L_s}{R_s^2 + \omega^2 L_s^2}$$

$$Y_p(j\omega) = \frac{1}{R_p} + \frac{1}{j\omega L_p} = \frac{1}{R_p} - \frac{j}{\omega L_p}$$

$$\frac{1}{R_p} = \frac{R_s}{R_s^2 + \omega^2 L_s^2} = \frac{1}{R_s} \cdot \frac{R_s^2}{R_s^2 + \omega^2 L_s^2}$$

$$= \frac{1}{R_s} \cdot \frac{1}{1 + \omega^2 L_s^2 / R_s^2}$$

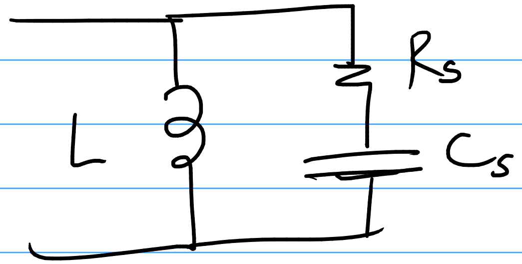
$$R_p = R_s(1+Q^2) \quad @\ \omega_o \qquad Q = \frac{\omega_o L_s}{R_s}$$

$$\omega_o \to \frac{1}{\sqrt{L_p C_p}}$$

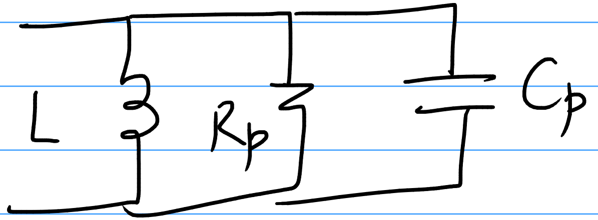
$$\frac{-j}{\omega L_p} = \frac{-j\omega L_s}{R^2+\omega^2 L_s^2}$$

$$\frac{1}{\omega L_p} = \frac{1}{\omega L_s} \cdot \frac{\omega^2 L_s^2}{R^2+\omega^2 L_s^2}$$

$$L_p = L_s \frac{(1+Q^2)}{Q^2}$$



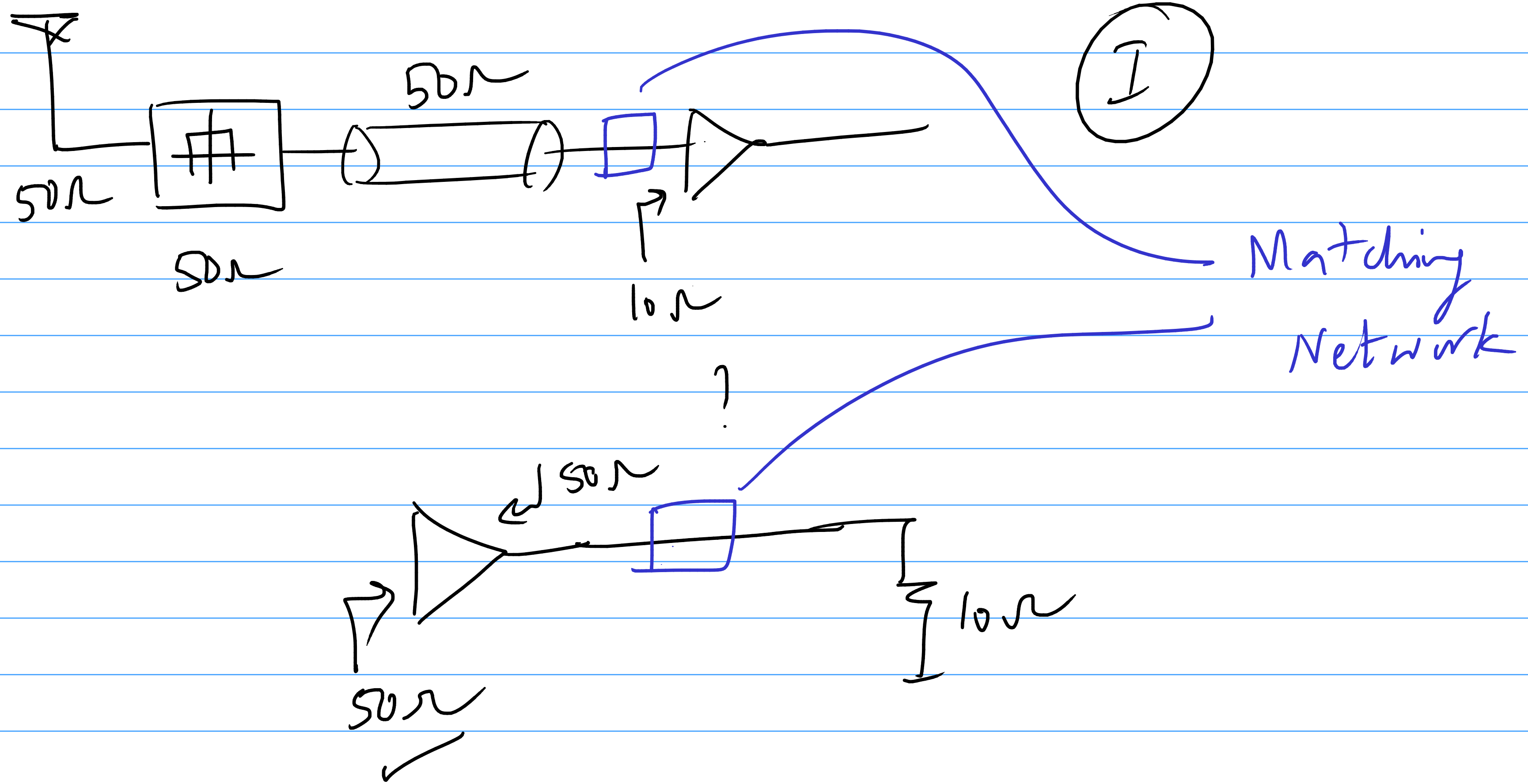
$\equiv$

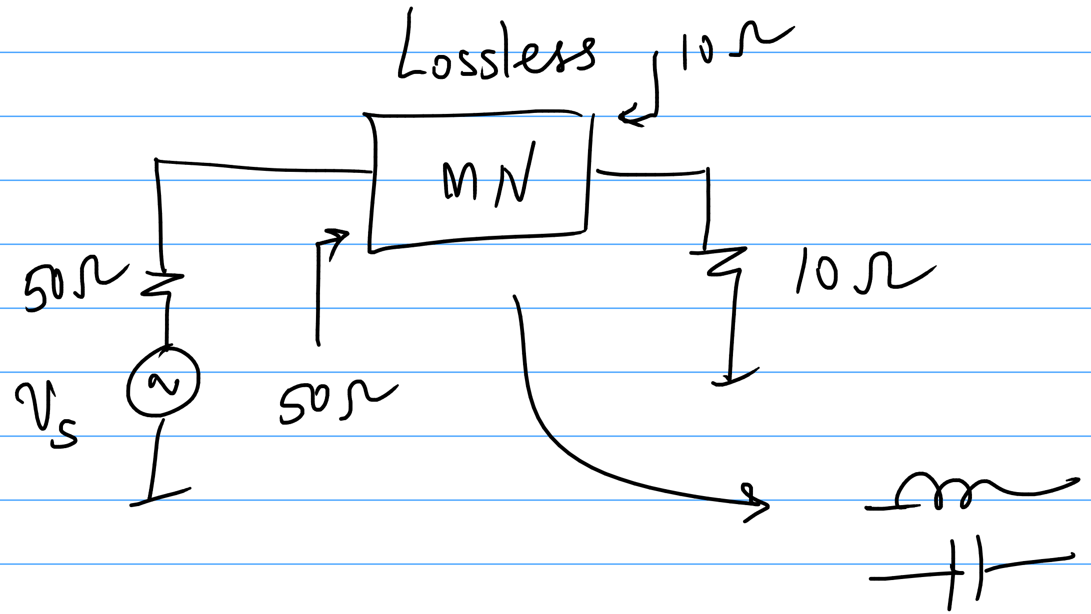


$$Q = \frac{1}{w R_s C_s}$$

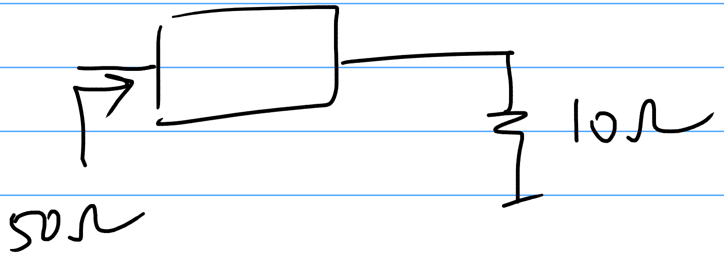
$$R_p = R_s(1+Q^2)$$

$$C_p = C_s \cdot \frac{Q^2}{1+Q^2}$$


50Ω
50Ω
50Ω
10Ω
I
Matching
Network
?
50Ω
10Ω
50Ω

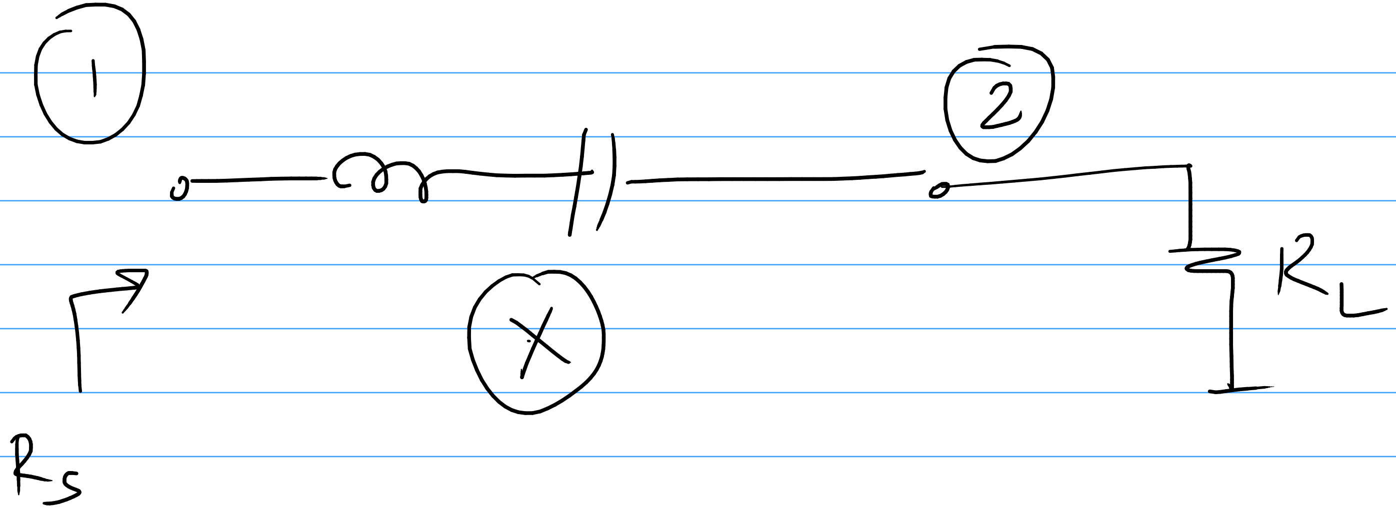


1)



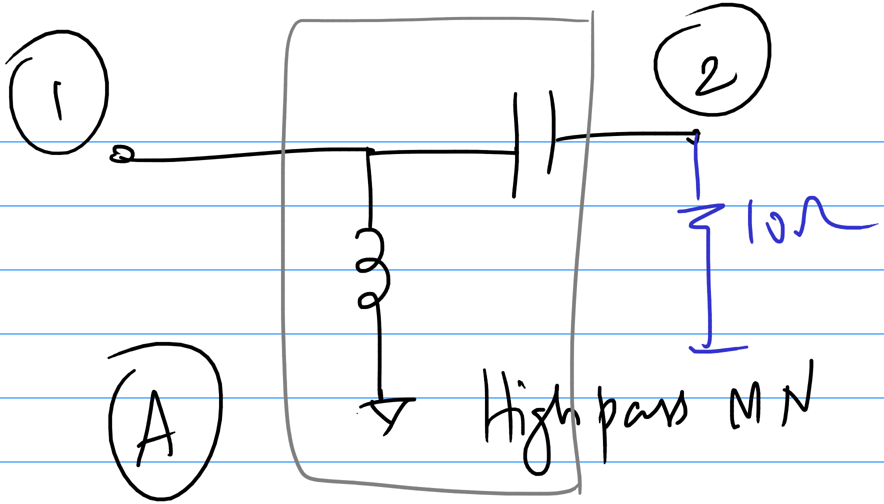
2) ω_0 = desired freq.

$\frac{R_L}{R_s}$ i.e. Q

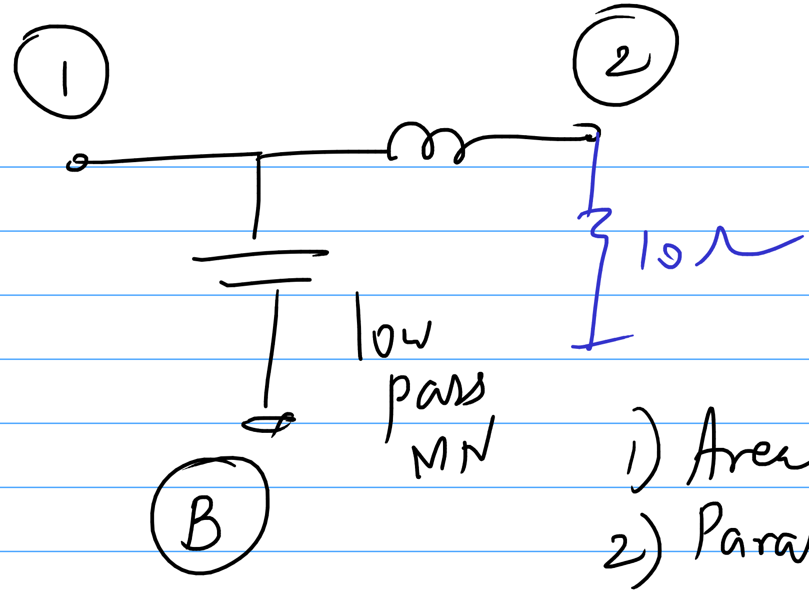


X

A & B
Upward matching network



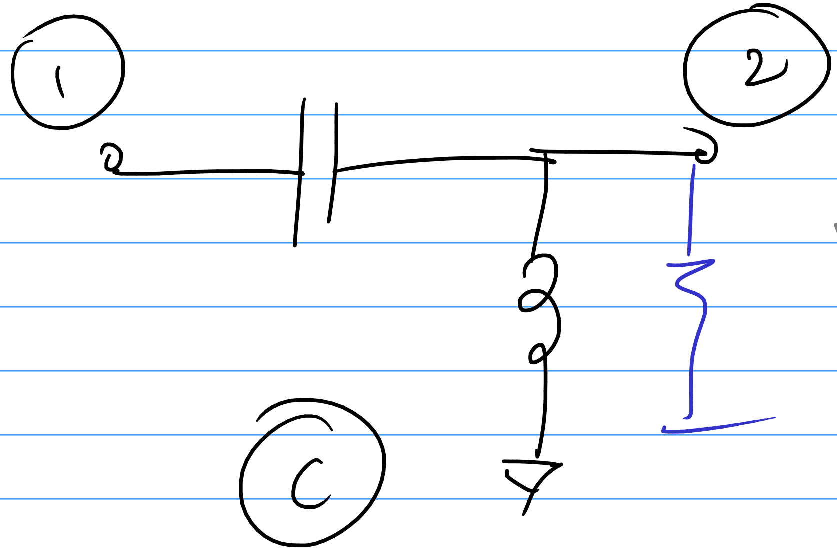
High pass MN



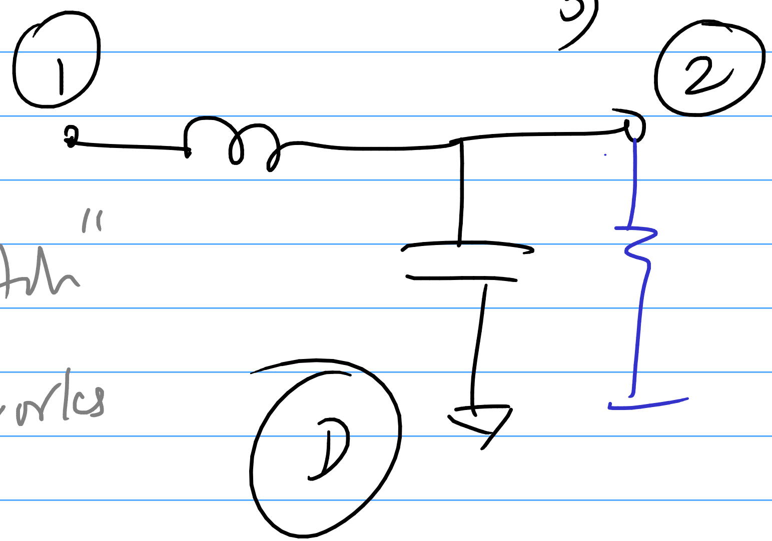
low pass MN

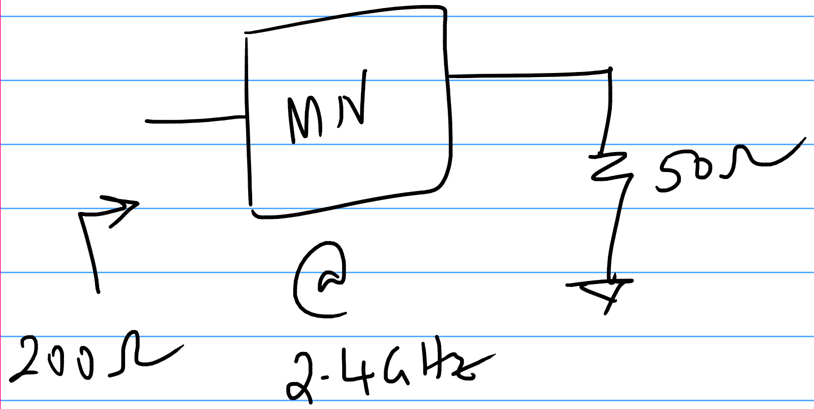
1) Area
2) Parasitics
3)

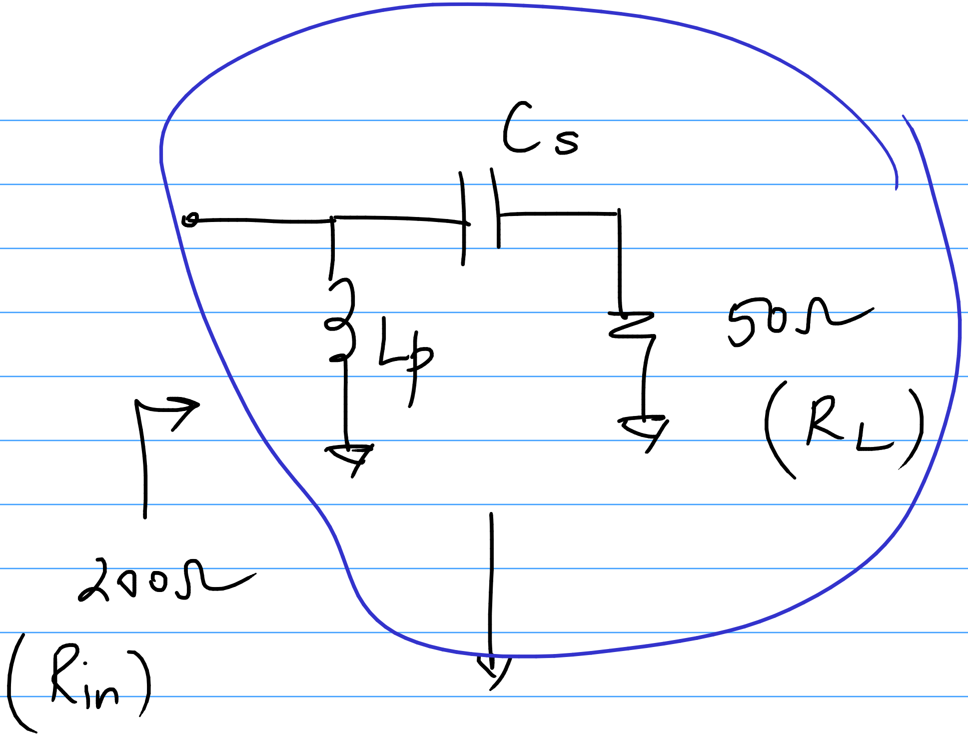
C & D
downward MN.



"L-match" Networks



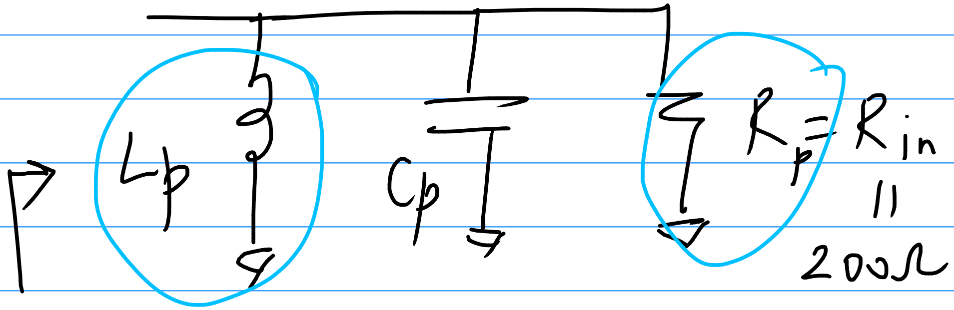




$$R_p = R_{in}$$

$$\omega_0 = \frac{1}{\sqrt{L_p C_p}}$$

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{3} = 1.73$$

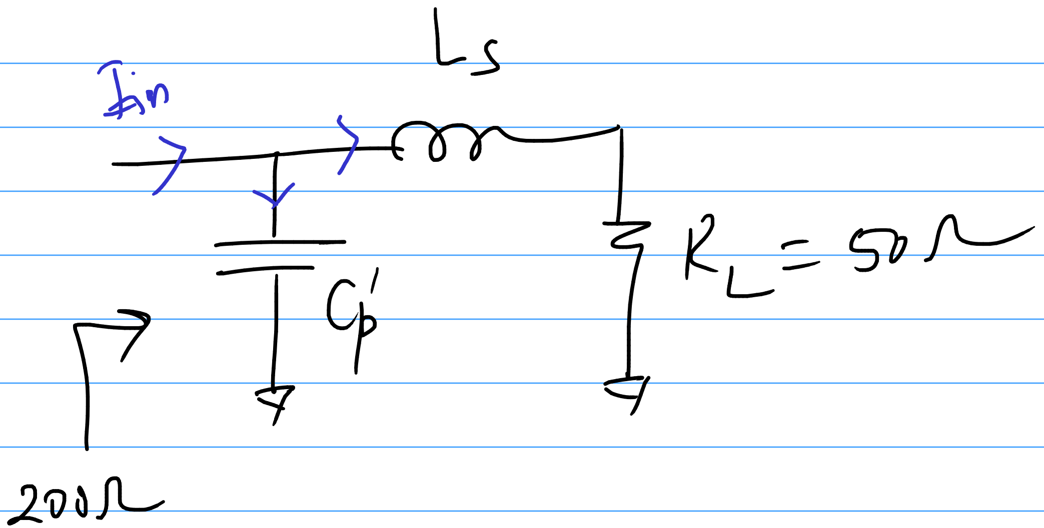


$$\frac{R_p}{\omega_0 L_p} = Q = \sqrt{3} \Rightarrow L_p = 7.66\,nH$$

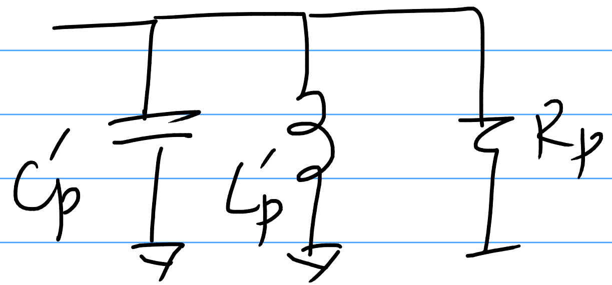
$\omega_0 = 2\pi \times 2.4\,GHz$

$$\omega_0 = \frac{1}{\sqrt{L_p C_p}} \Rightarrow C_p = 575\,fH$$

$$C_s = C_p \cdot \frac{(1+Q^2)}{Q^2} = 766\,fF$$



$\Rightarrow$



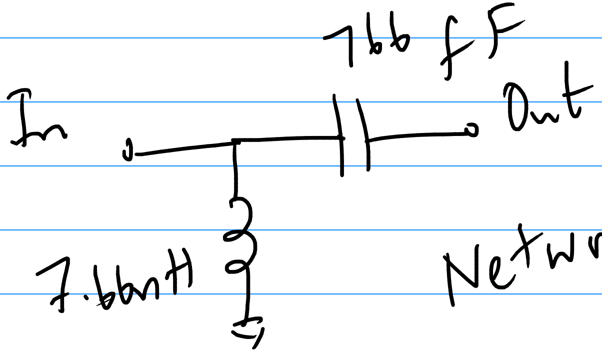
$R_p = 200\,\Omega$

$Q = \sqrt{3}$

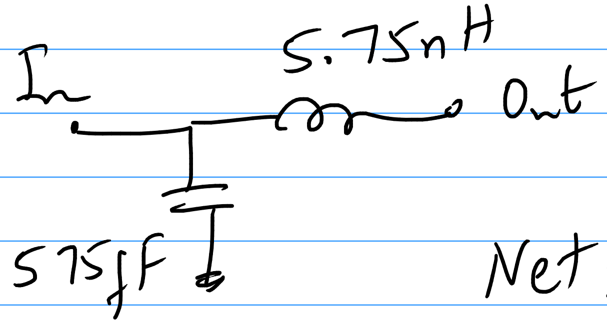
Sets the BW of the Match

$Q = \omega_0 C_p' R_p \Rightarrow 1.732 = 2\pi \times 2.4\,GHz \times C_p' \times 200$

$C_p' = 575\,fF \Rightarrow L_p' = 7.66\,nH \Rightarrow L_s = 5.75\,nH$



Network 1



Network 2

Choice

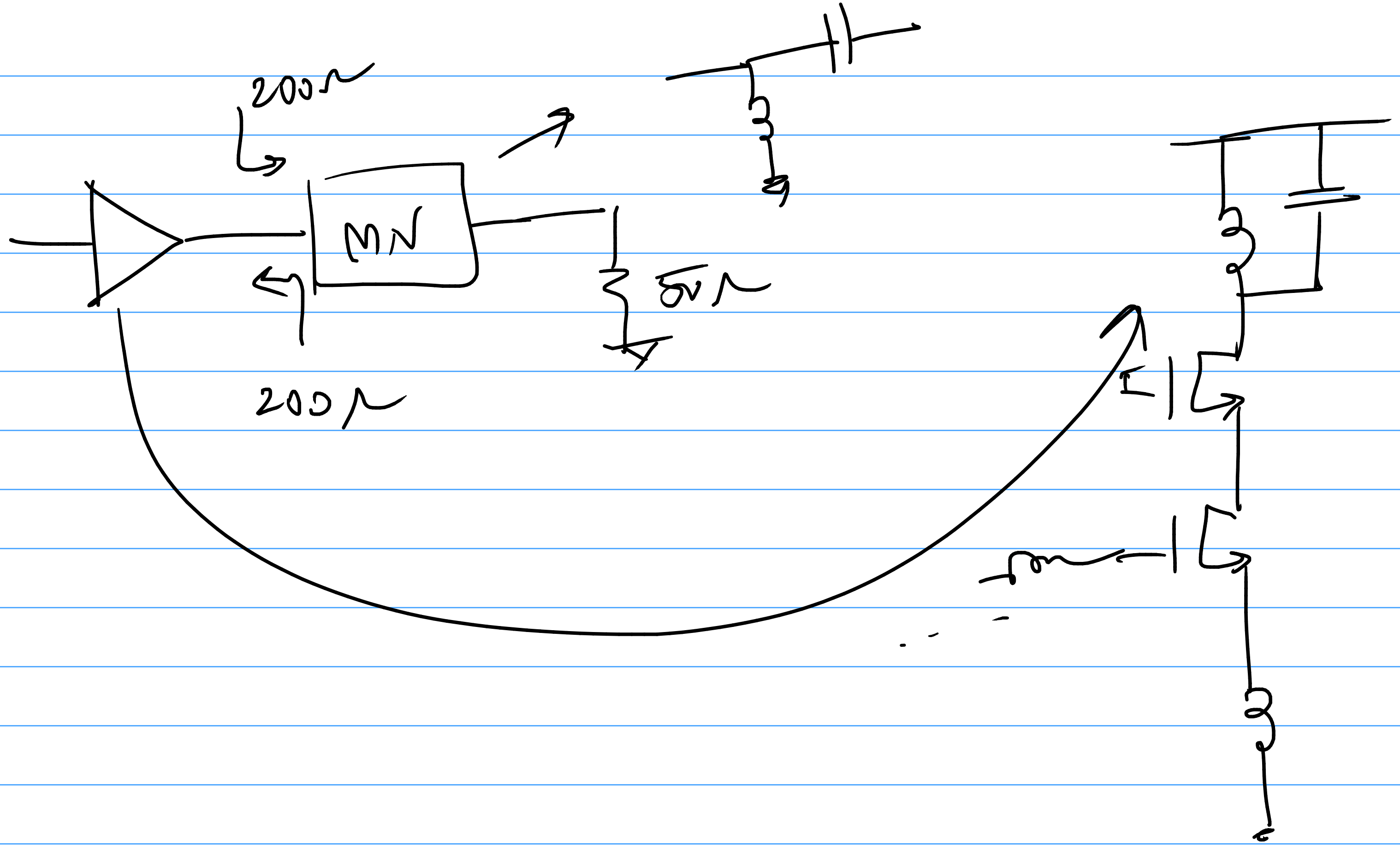
1) 2 has lower area ✓

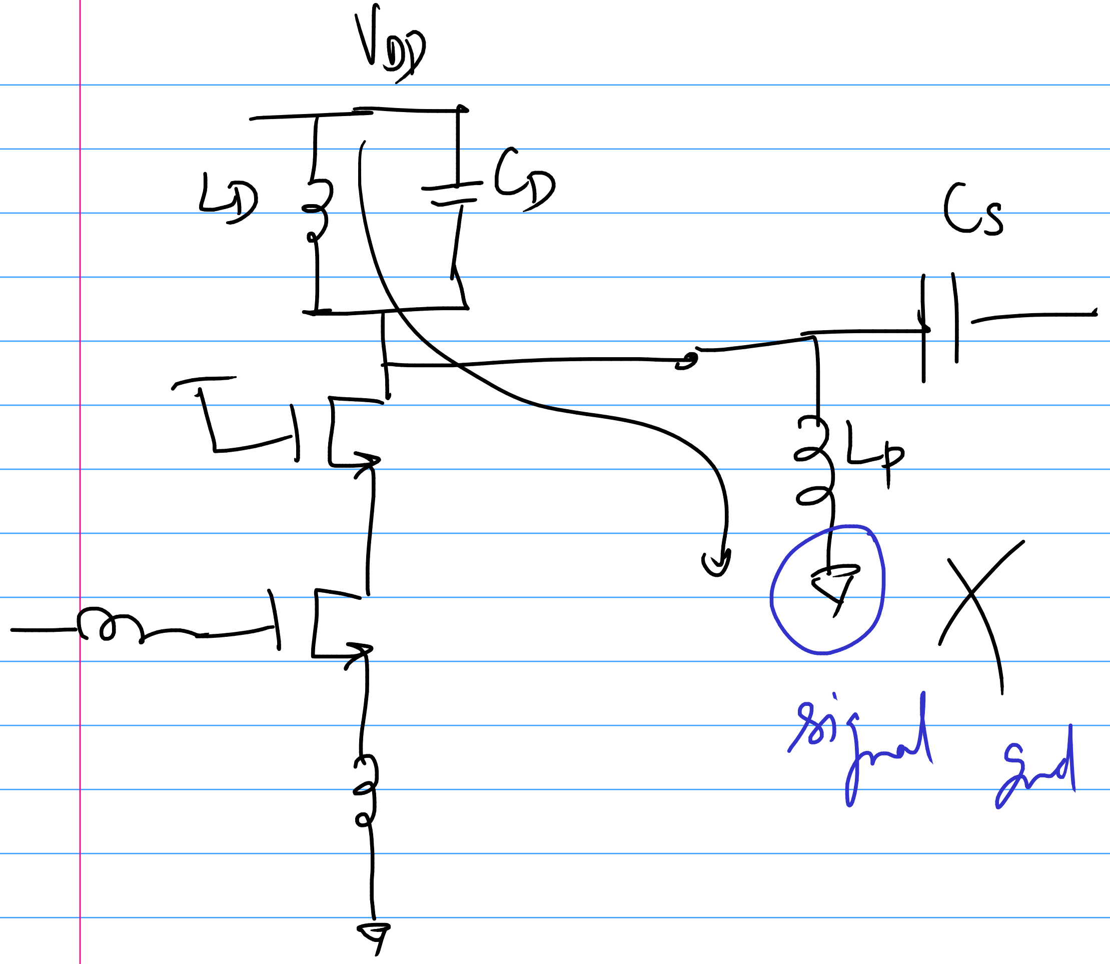
2) 1 → In is DC shorted to gnd

Out is ac coupled to In

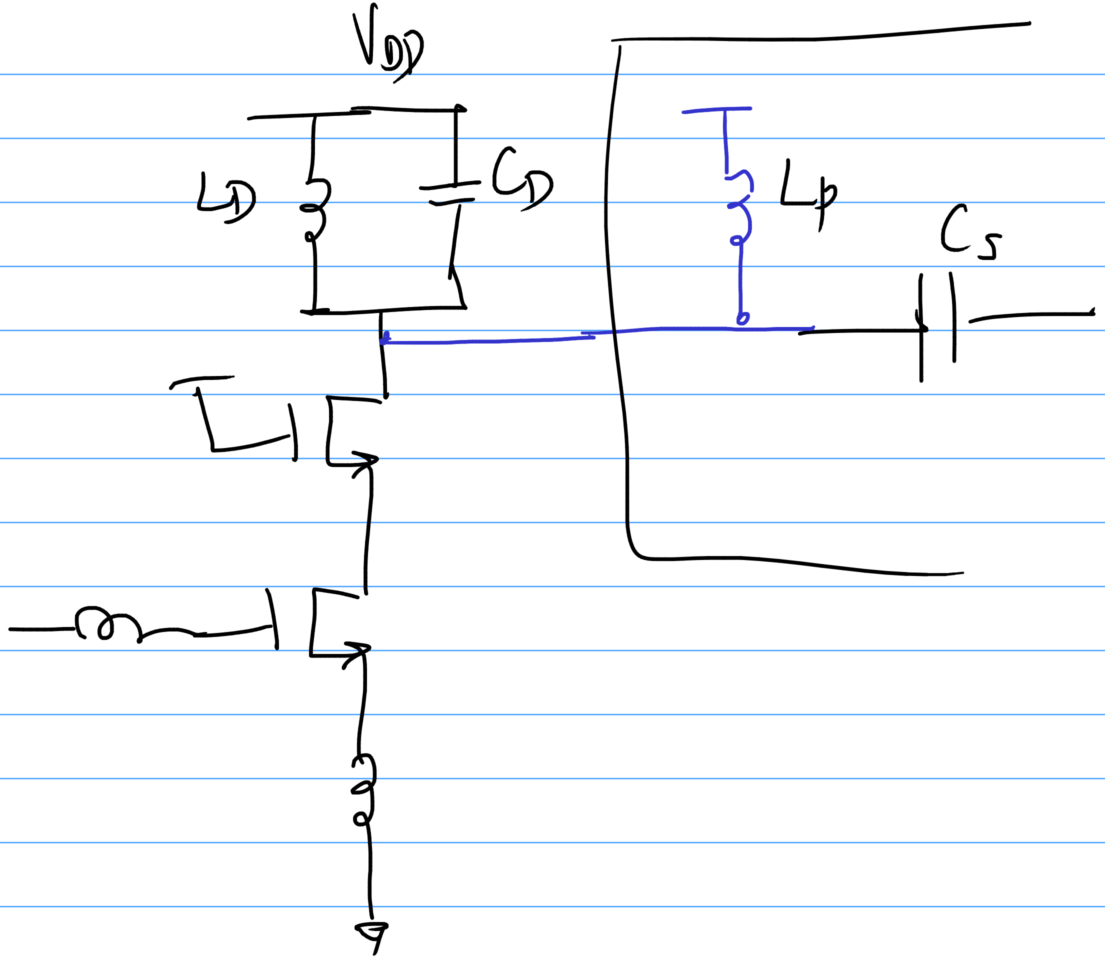
2 → Out is dc coupled to In

3) Absorb (portion of) MN into circuit

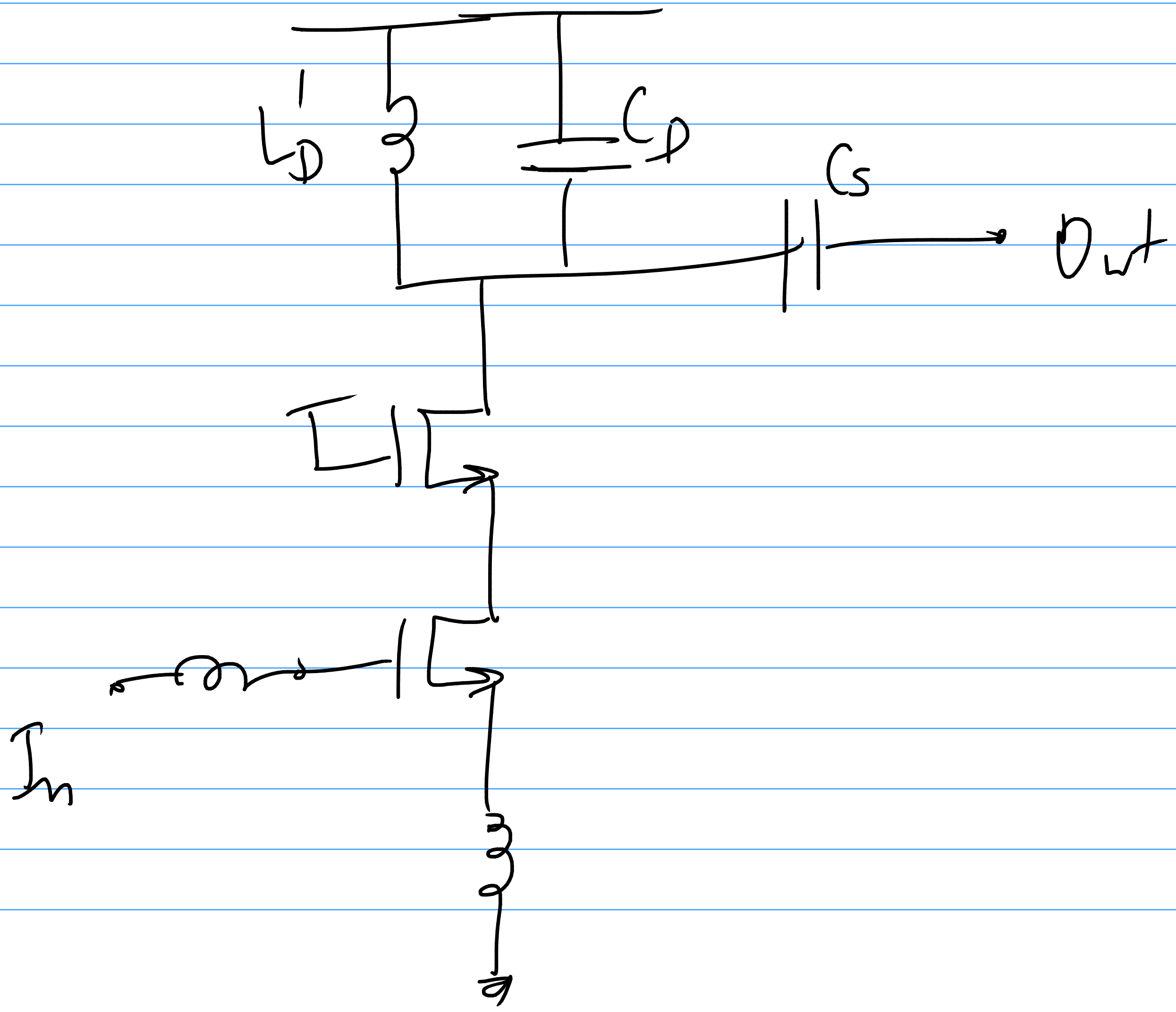

200Ω
MN
50Ω
200Ω



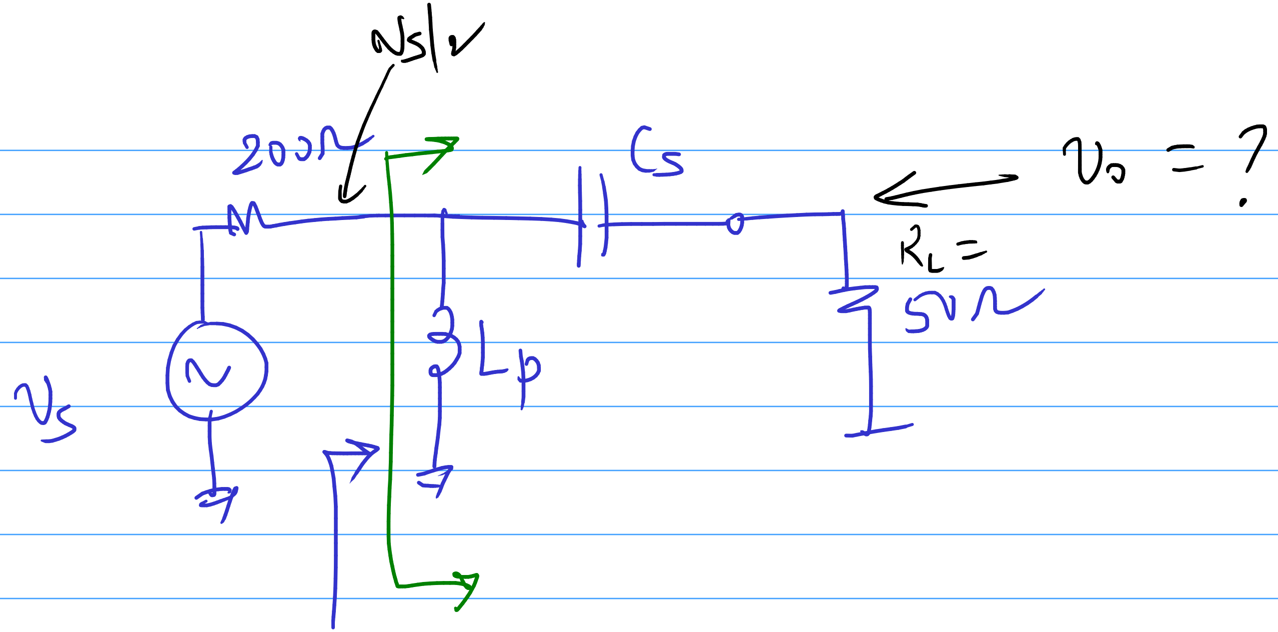
V_{DD}
L_D
C_D
C_S
L_p
signal
gnd



V_{DD}
L_D
C_D
L_p
C_S

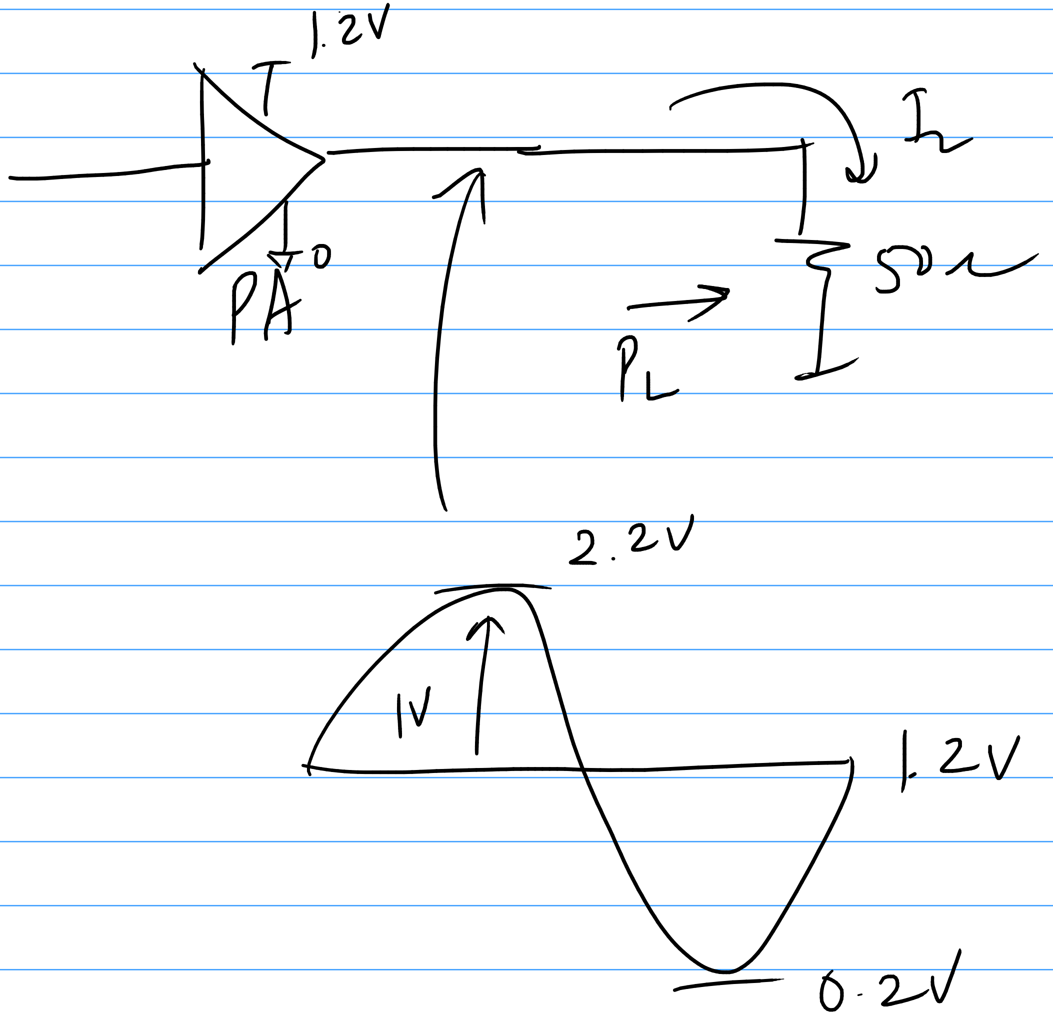


$$L_D' = L_D \| L_p$$



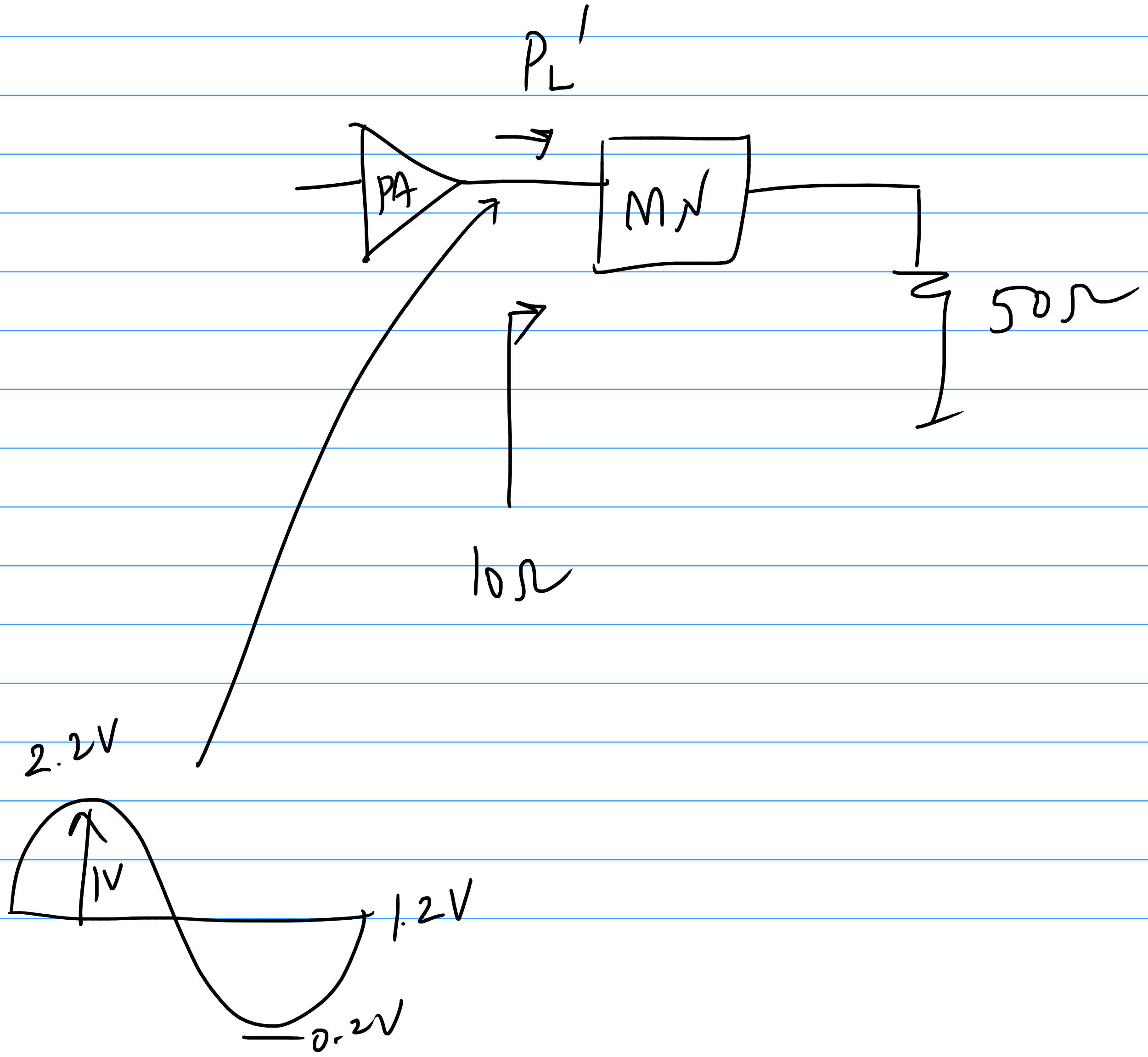
$R_{in} = 200\Omega$

$$\frac{(v_s/2)^2}{2} \times \frac{1}{R_{in}} = \frac{v_o^2}{2R_L} \Rightarrow v_o = \frac{v_s}{4}$$



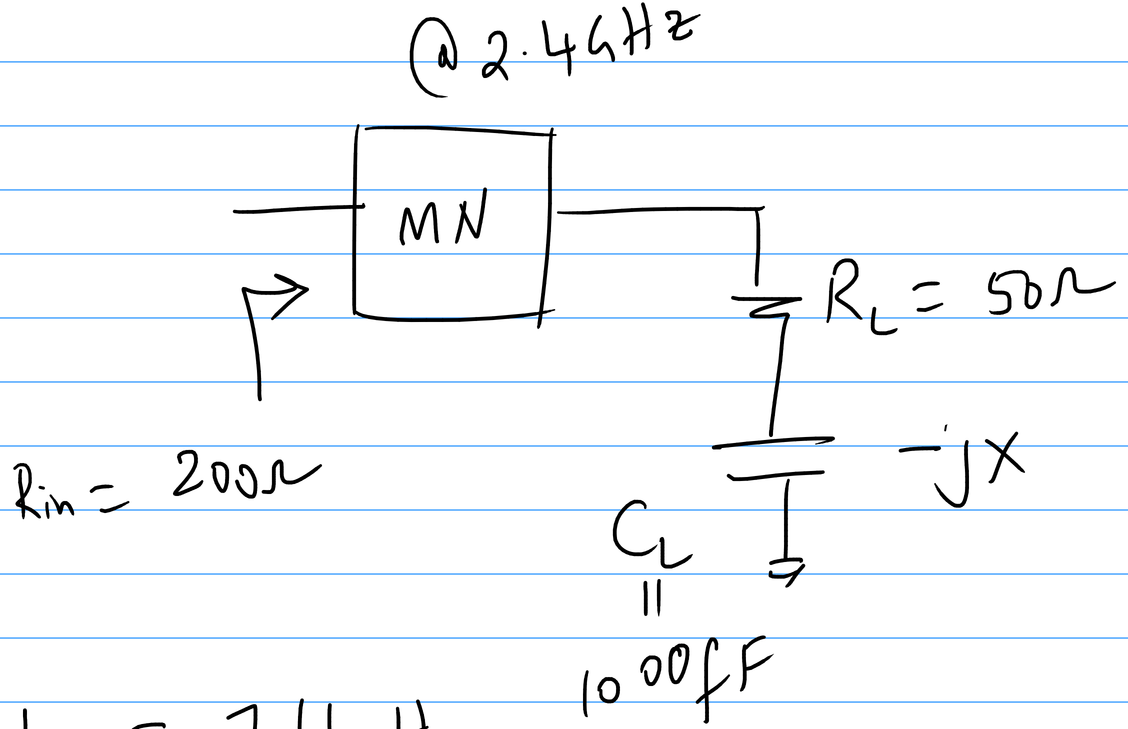
$$P_L = \frac{1^2}{2} \times \frac{1}{50\Omega}$$

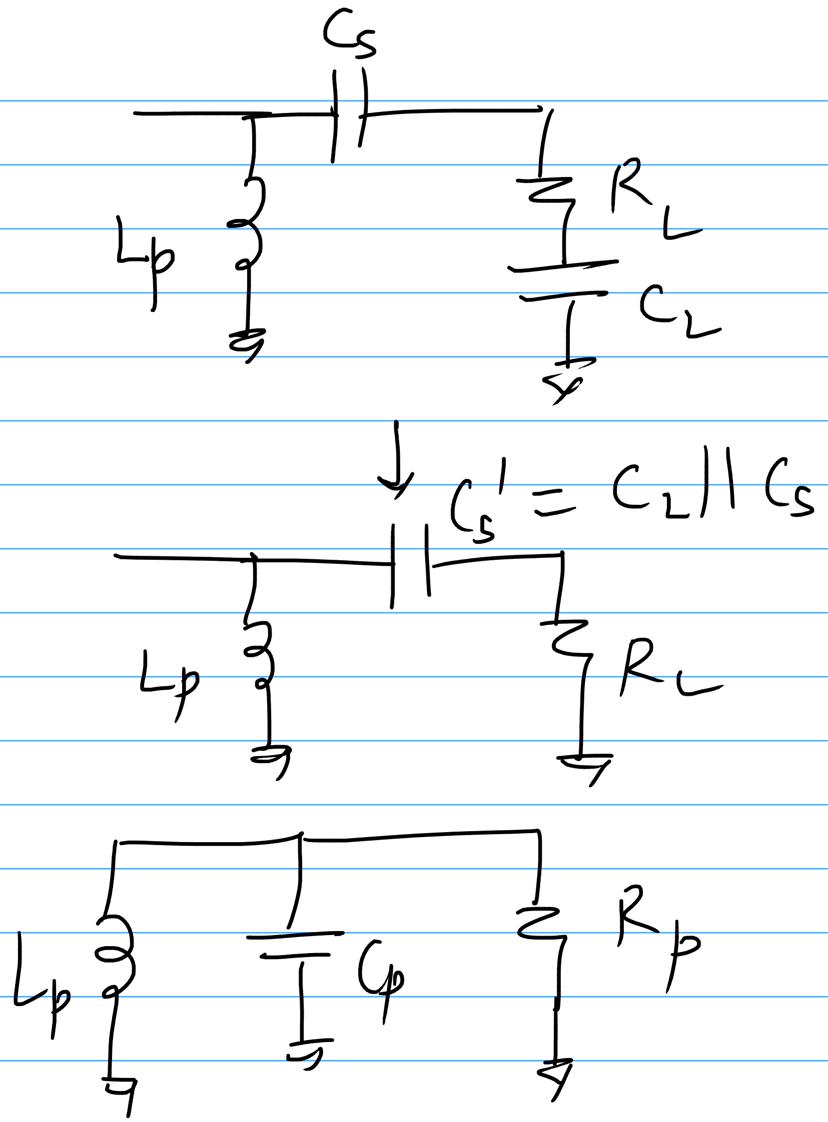
$$= 10mV$$



$$P_L' = \frac{1^2}{2} \times \frac{1}{10\Omega}$$

$$= 50mW$$



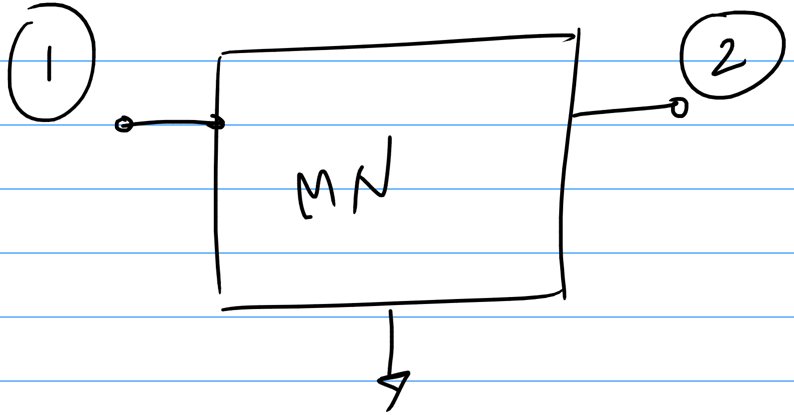


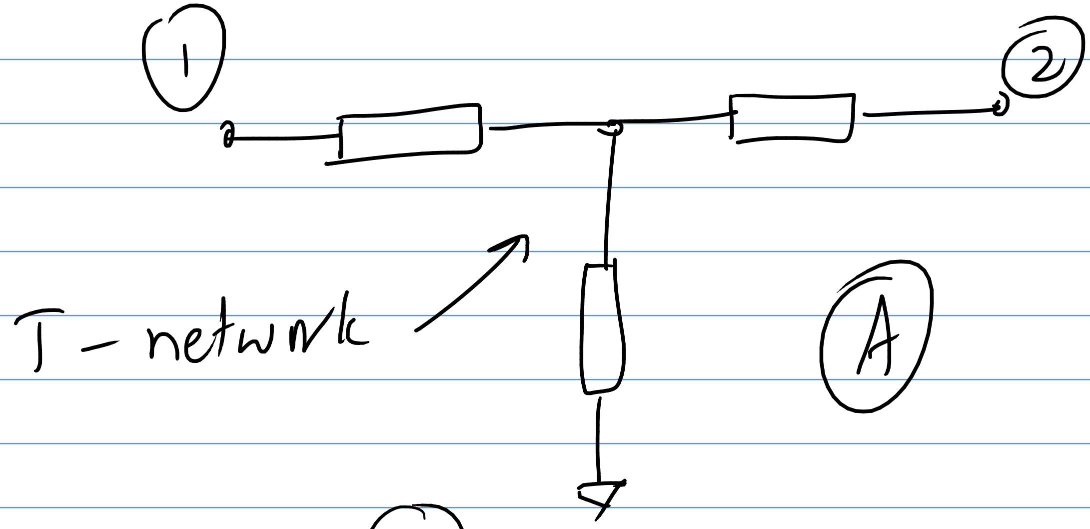
$$L_p = 7.66nH$$

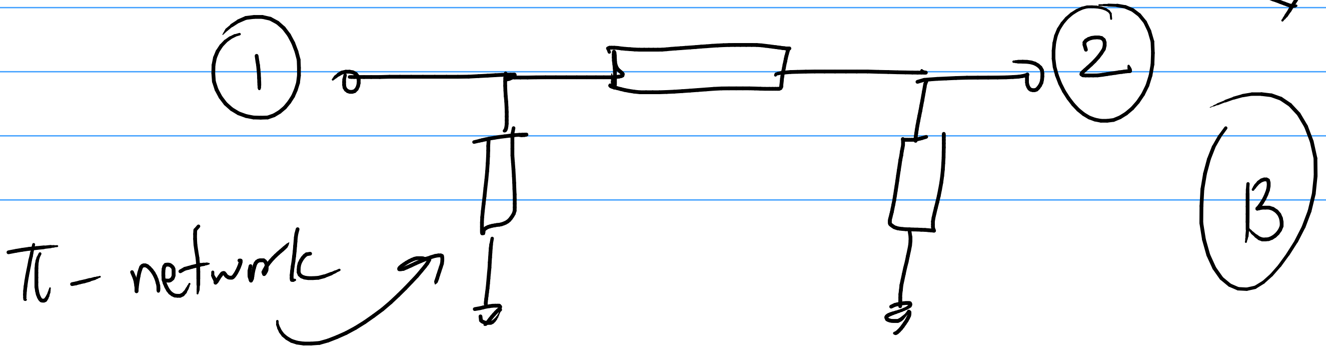
$$C_s' = 766fF = C_s || C_L$$

$$\frac{1}{766fF} = \frac{1}{C_s} + \frac{1}{C_L (= 1000fF)}$$

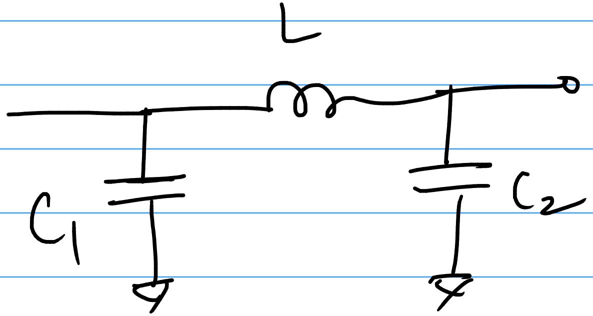
Want to set Q, $\frac{R_p}{R_s}$, f_o $\rightarrow$ 3 element match





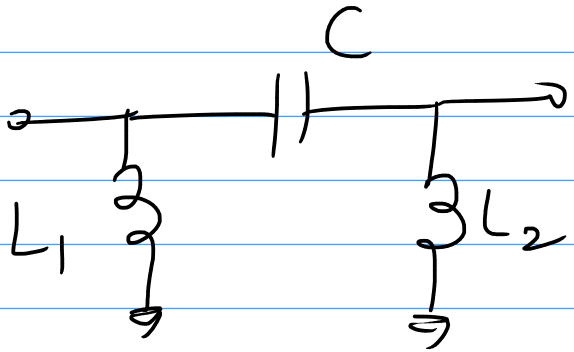


1)



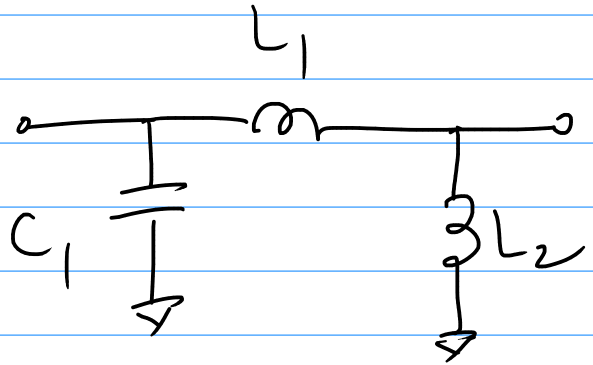
Low pass

2)



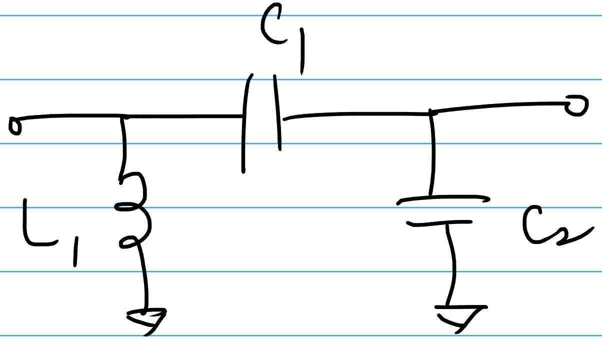
High pass

3)



tapped -ind. MN

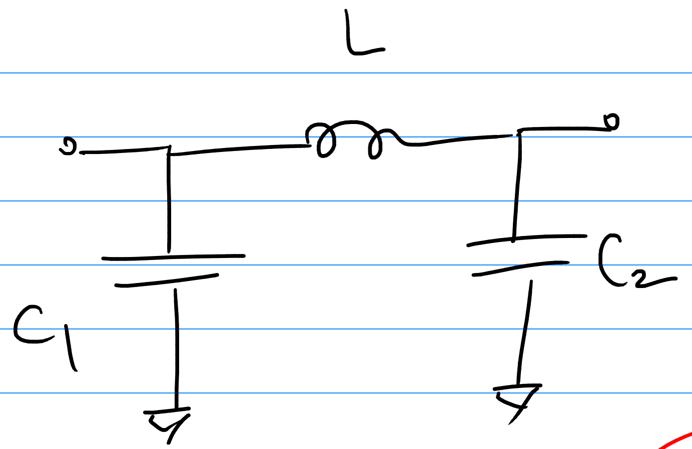
4)

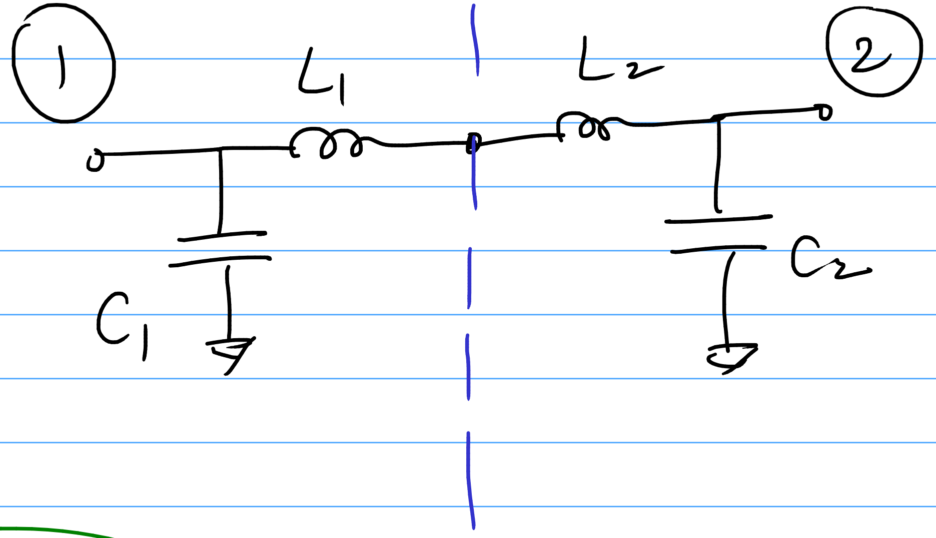


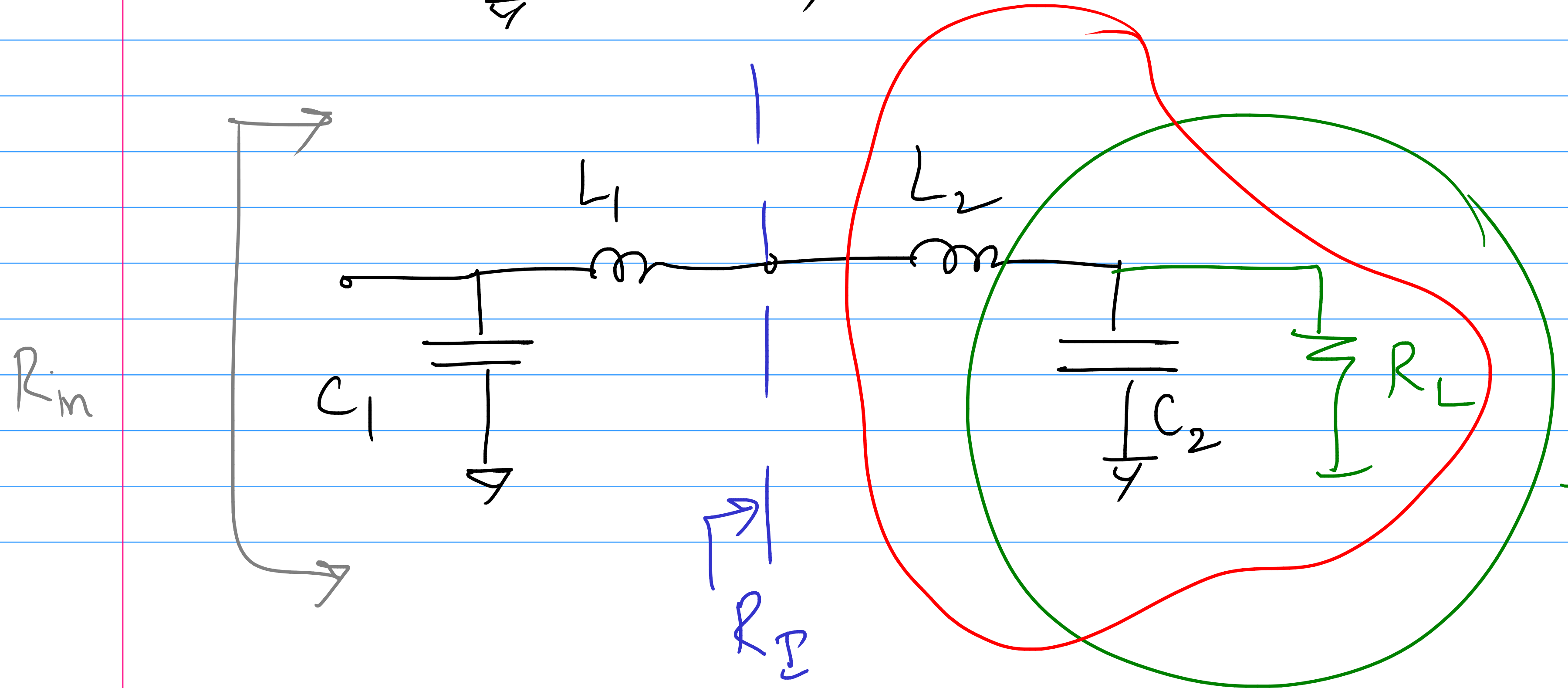
tapped cap. MN

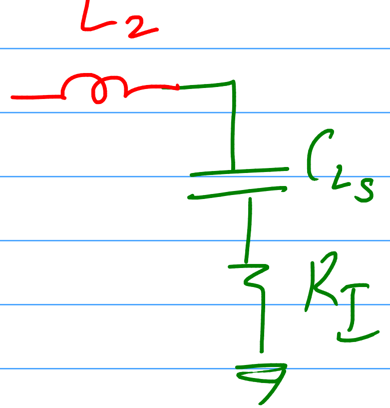
π-match

$R_I < R_{in}, R_L$

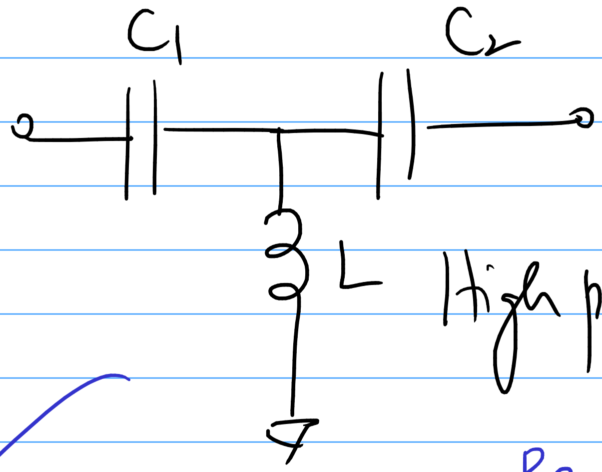




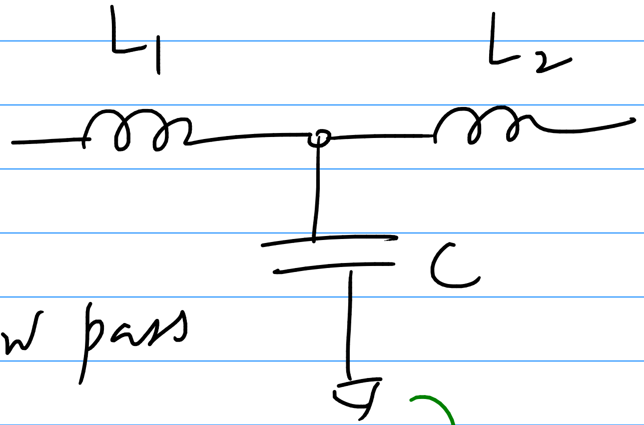




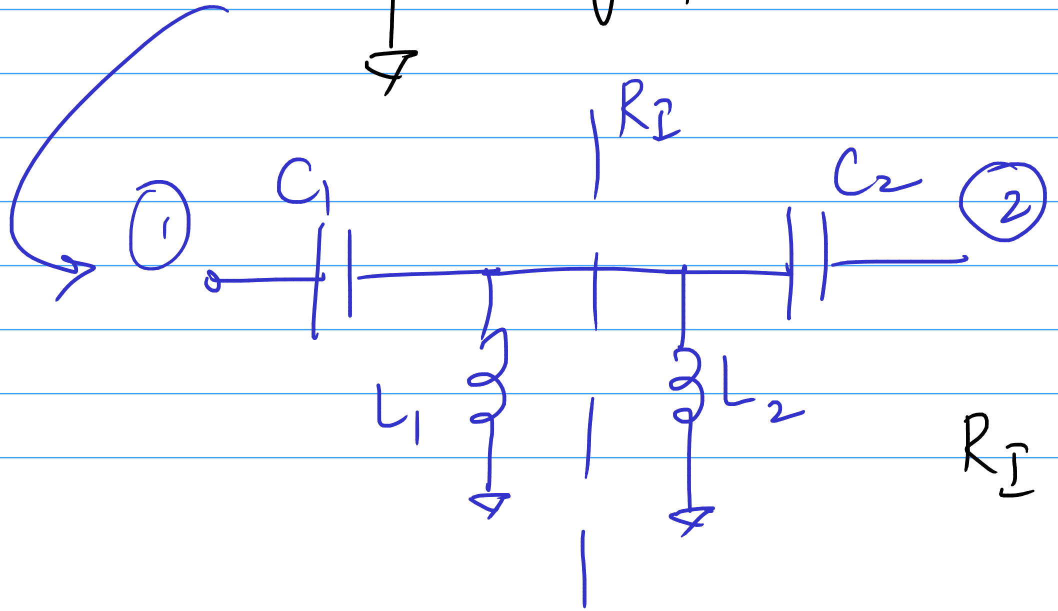
T-match Networks

1) 

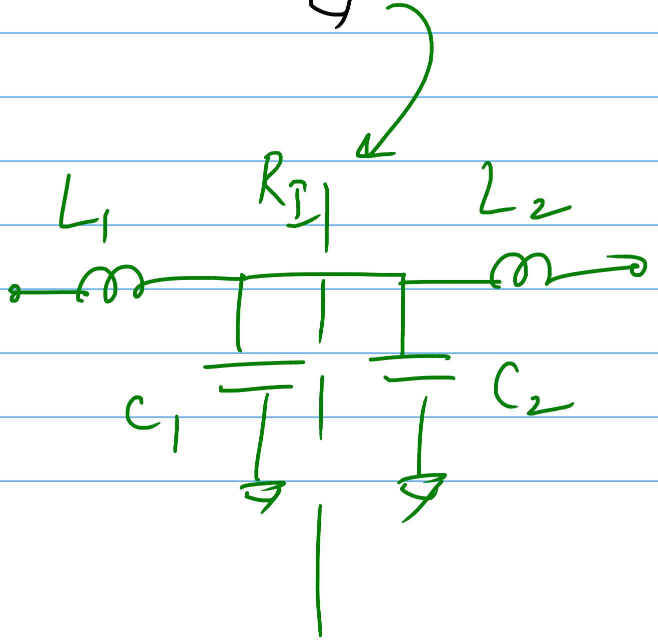
High pass

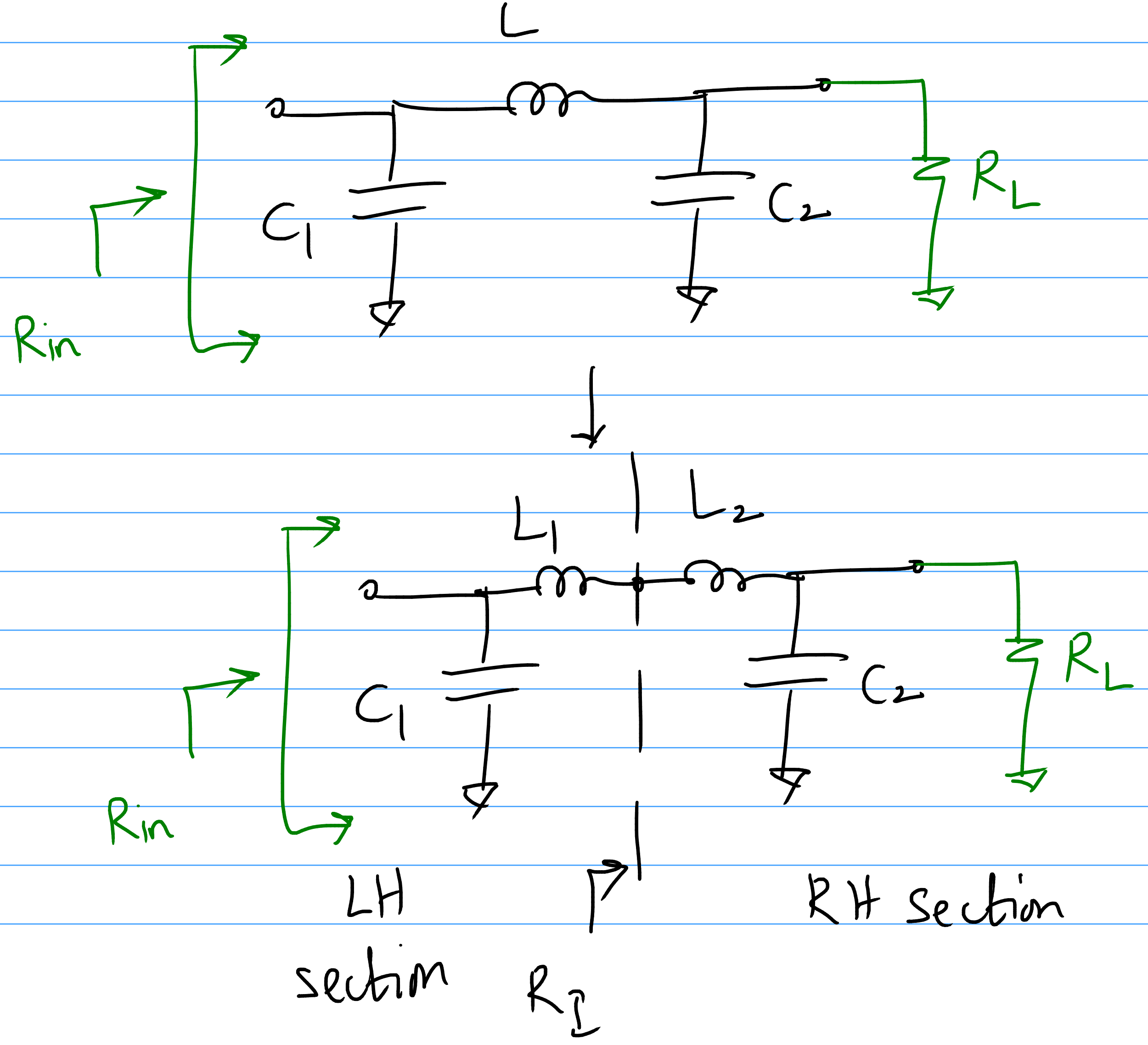
2) 

Low pass

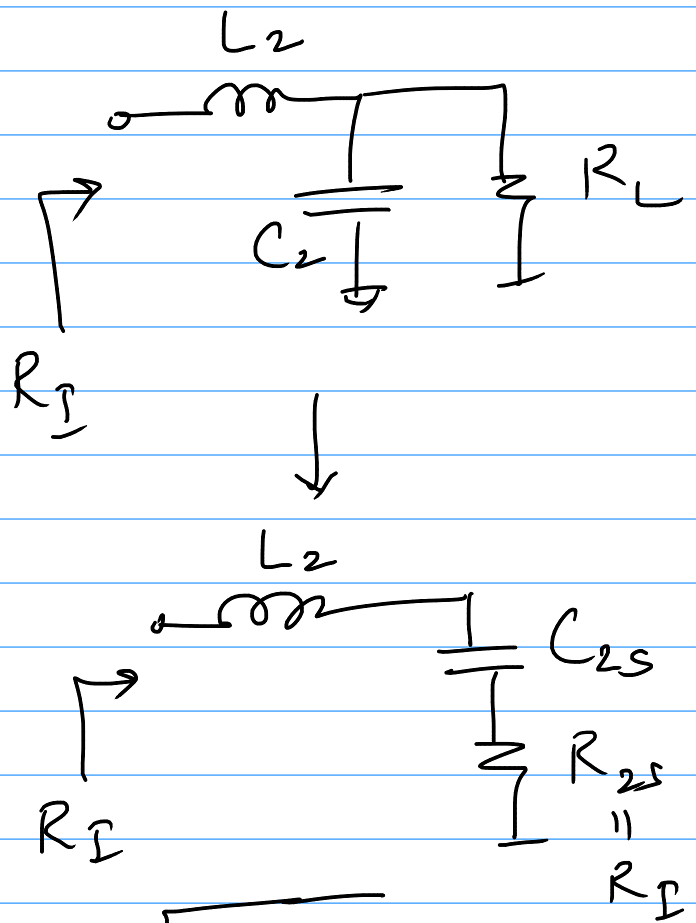


$R_I > R_{in}, R_L$



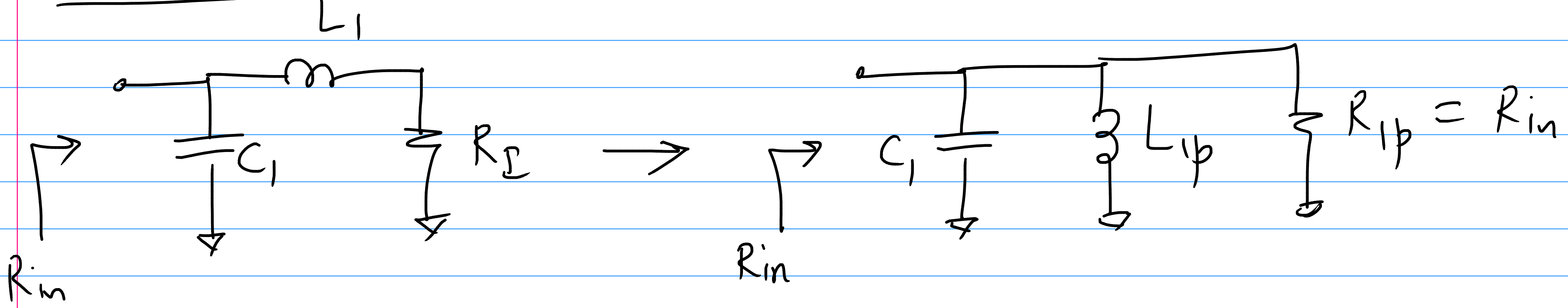


RH Section



$$Q_R = \sqrt{\frac{R_L}{R_I} - 1} = \frac{\omega_0 L_2}{R_I}$$

LH Section



$$Q_L = \sqrt{\frac{R_{in}}{R_I} - 1} = \omega_0 C_1 R_{in} = \frac{\omega_0 L_1}{R_I}$$

$$\boxed{Q_{total} = Q_L + Q_R}$$

$$\omega_0 = \frac{1}{\sqrt{L_2 C_{2s}}} \quad ; \quad \omega_0 = \frac{1}{\sqrt{C_1 L_{1p}}}$$

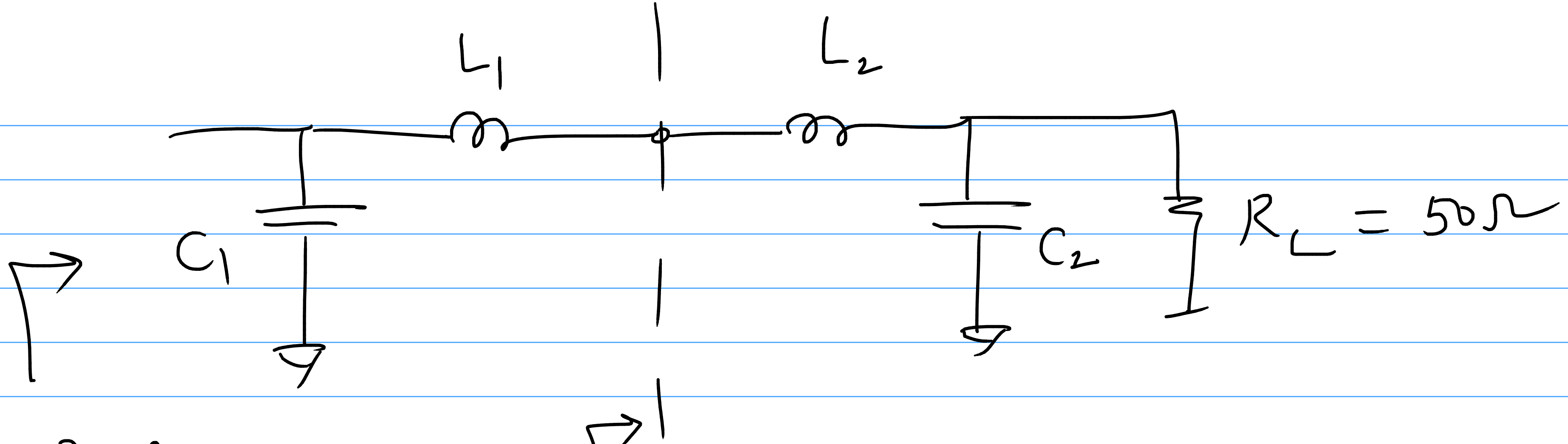
$$Q_{total} = \frac{\omega_0 (L_1 + L_2)}{R_I}$$

$$Q_{tot.} = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_L}{R_I} - 1}$$

Example! $R_L = 50\Omega$, $R_{in} = 200\Omega$, $f_0 = 2.4\,GHz$, $Q = 10$

$$Q = Q_L + Q_R$$

$$10 = \sqrt{\frac{200}{R_I} - 1} + \sqrt{\frac{50}{R_I} - 1} \Rightarrow \underline{\underline{R_I = 4.3\Omega}}$$



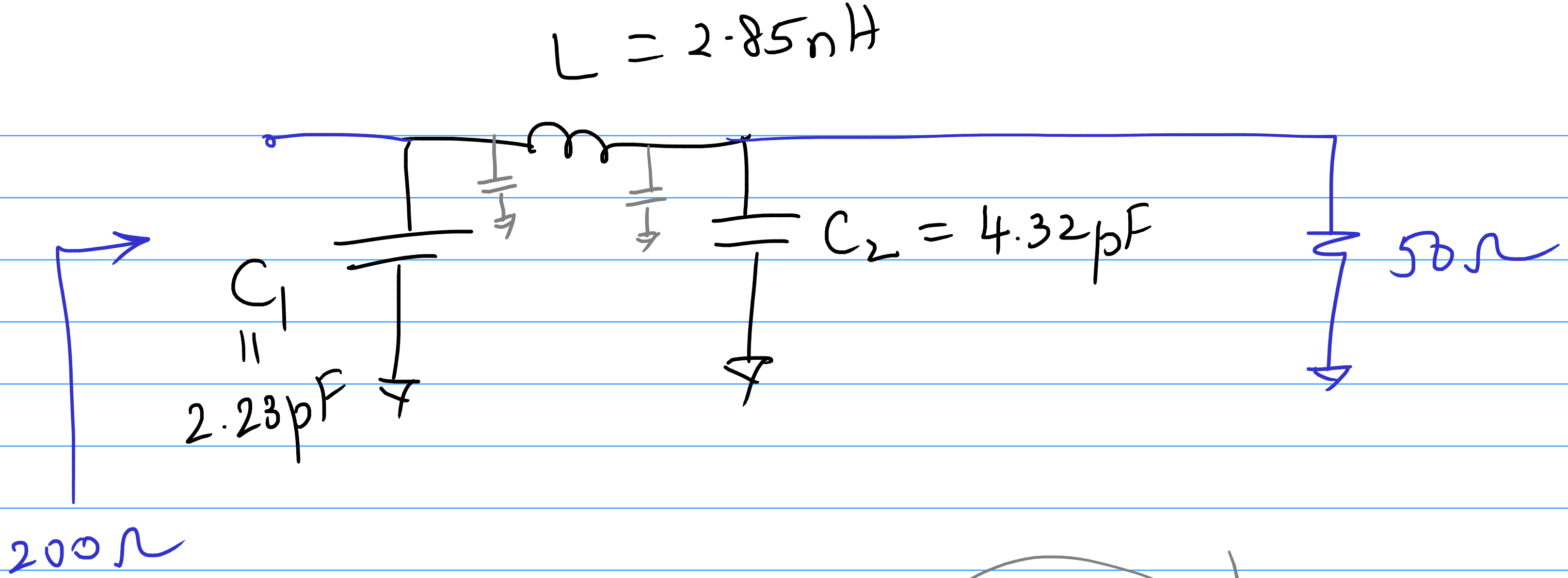
$R_{in} = 200\,\Omega$ $Q_L = 6.74$ $R_I = 4.3\,\Omega$ $Q_R = 3.26$

$$C_1 = \frac{Q_L}{\omega_0 R_{in}} = 2.23\,\text{pF}$$

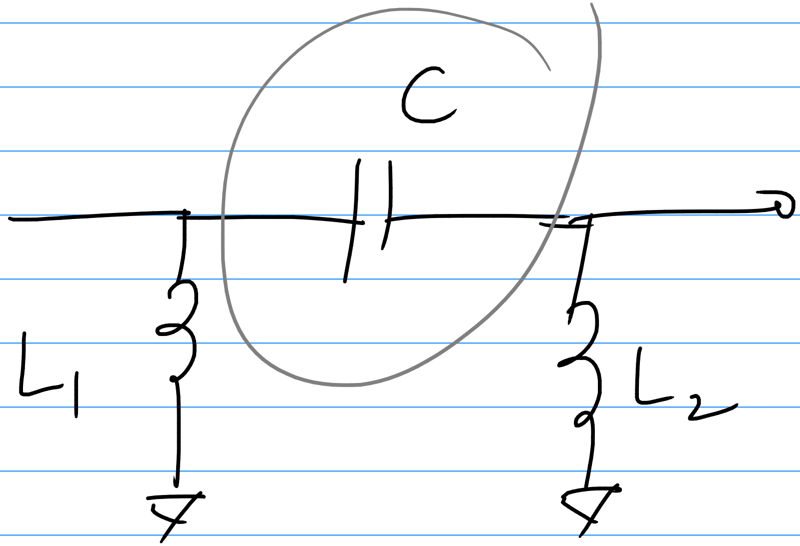
$$C_2 = Q_R / \omega_0 R_L = 4.32\,\text{pF}$$

$$L_1 = \frac{R_I\, Q_L}{\omega_0} = 1.92\,\text{nH}$$

$$L_2 = \frac{Q_R \cdot R_I}{\omega_0} = 0.93\,\text{nH}$$

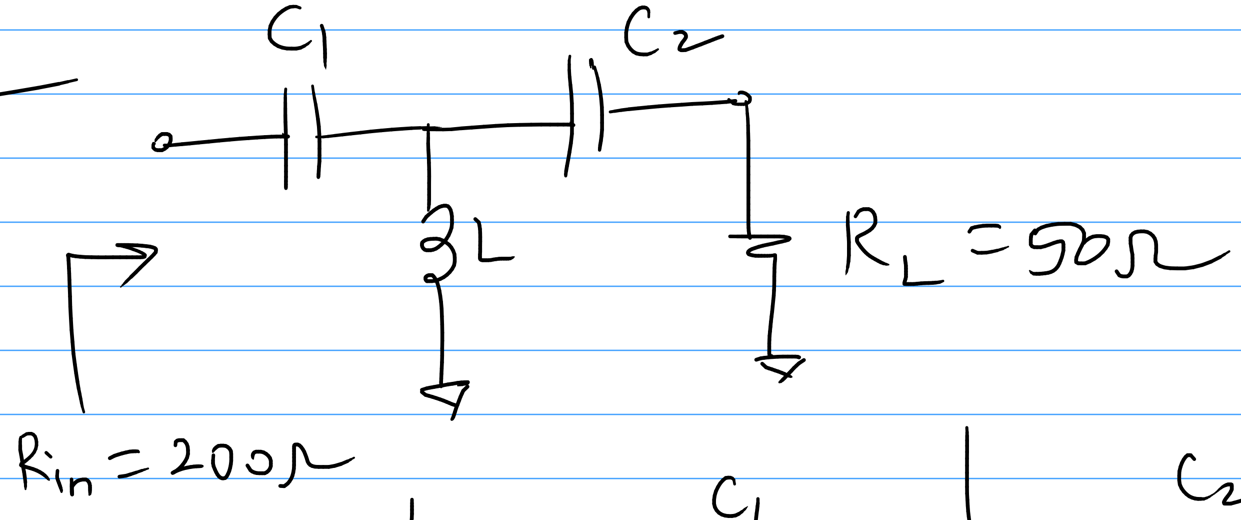


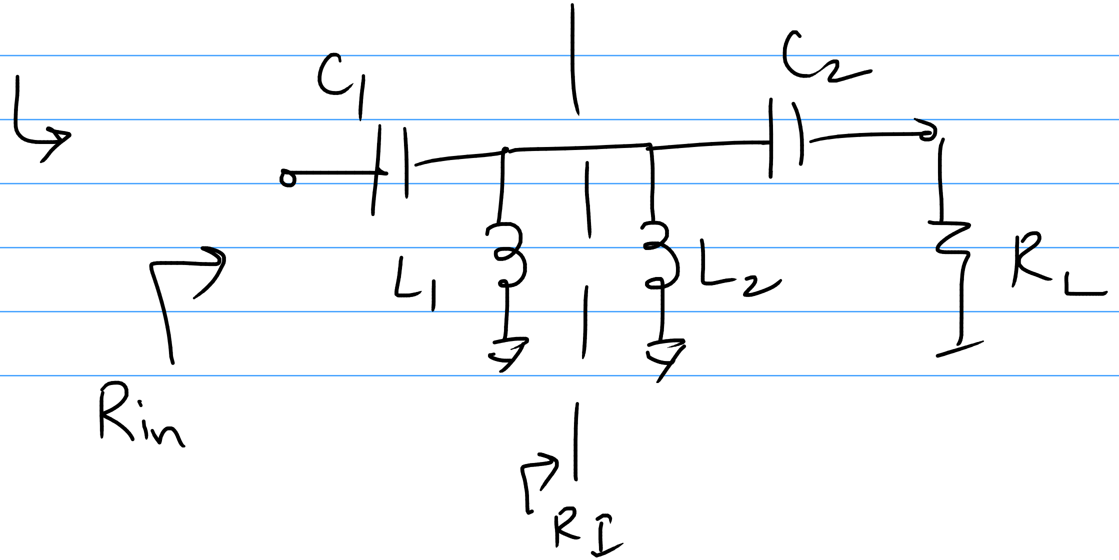
HW :



Example 2 $R_L = 50\Omega$, $R_{in} = 200\Omega$, $f_0 = 2.4\,GHz$, $Q = 10$

T-match





$$Q = \underbrace{\sqrt{\frac{R_I}{R_{in}} - 1}}_{Q_L} + \underbrace{\sqrt{\frac{R_I}{R_L} - 1}}_{Q_R}$$

$$10 = \sqrt{\frac{R_I}{200} - 1} + \sqrt{\frac{R_I}{50} - 1}$$

$R_I = 2.32\ k\Omega$

$Q_L = 3.26$ $\quad$ $Q_R = 6.74$

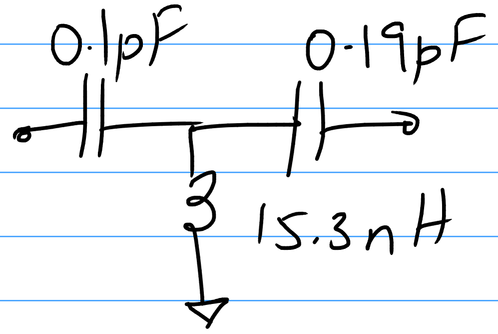
$C_1 = 0.1\ pF$

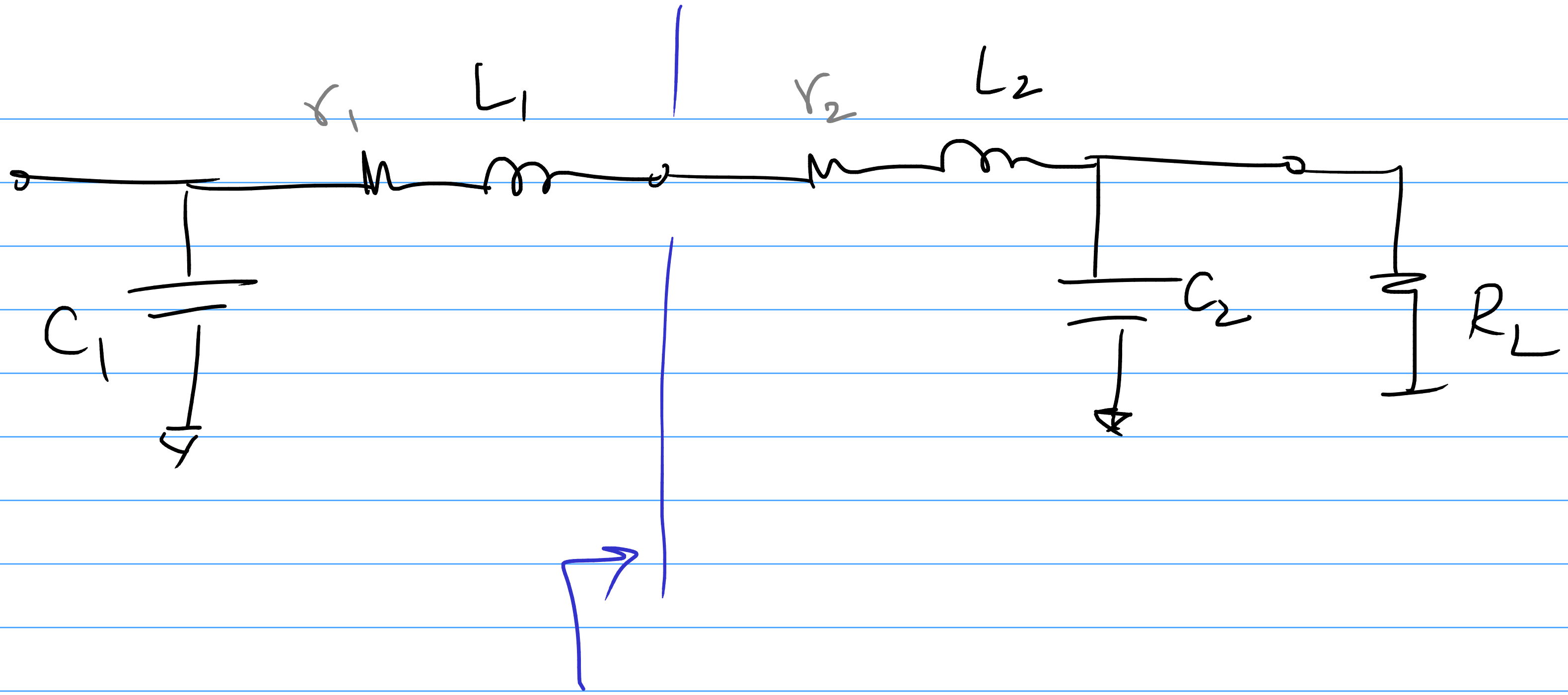
$L_1 = 47.2\ nH$

$C_2 = 0.19\ pF$

$L_2 = 22.8\ nH$

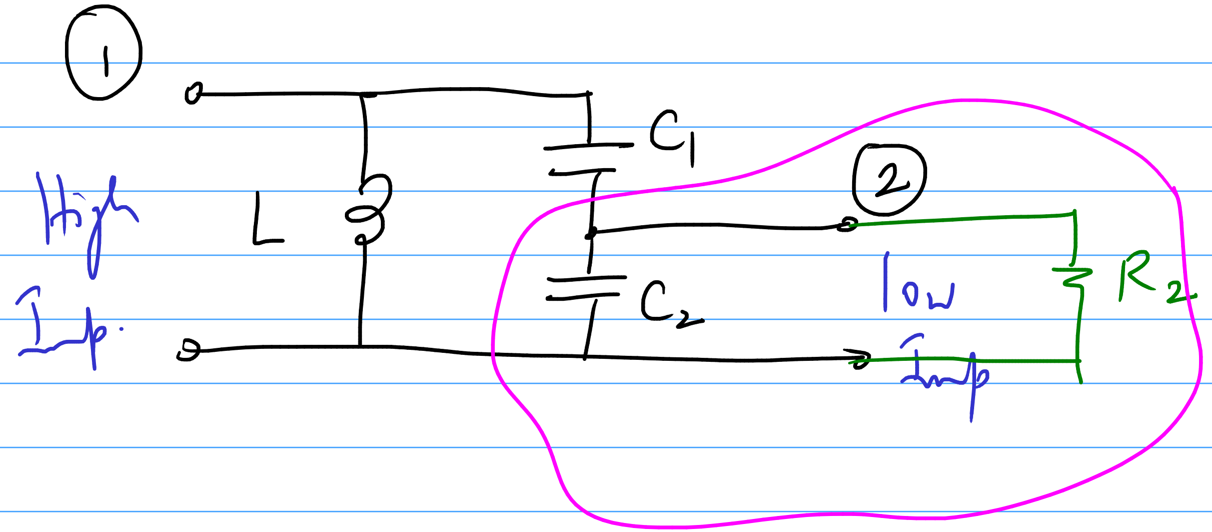
$L = 15.3\ nH$

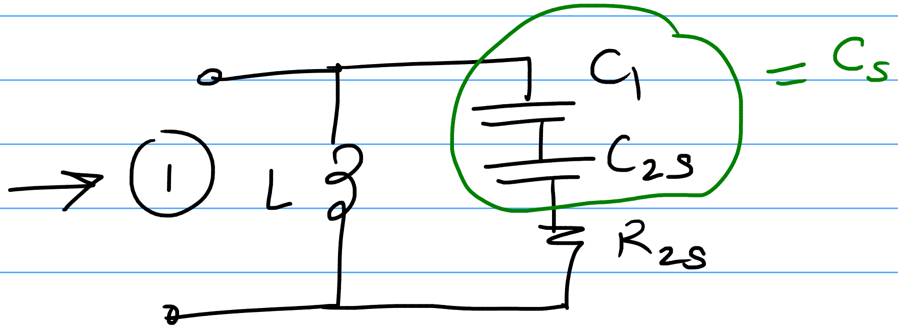


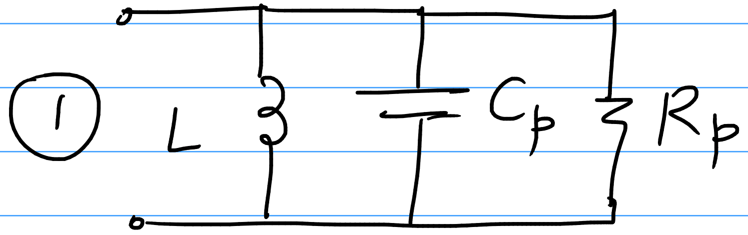


$R_I \gg r_2, r_1$

Tapped Cap. MN





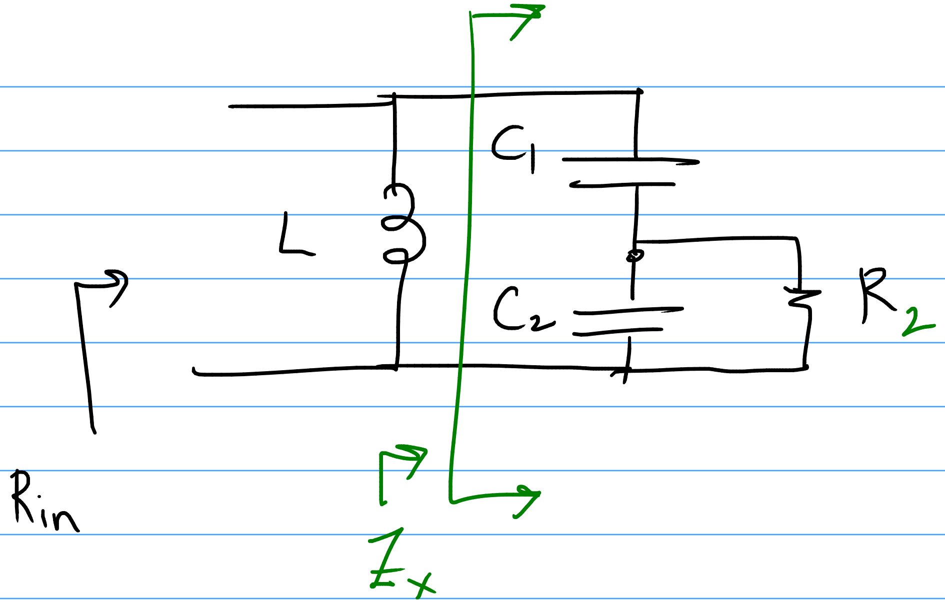


$$R_{in} = R_p$$

$$Q = w_0 C_p R_p$$

$$w_0 = \frac{1}{\sqrt{L C_p}}$$

$$Q = \frac{R_p}{w_0 L}$$



$$Z_x(s) = \frac{1}{sC_1} + \frac{R_2}{1 + sC_2R_2}$$

$$= \frac{1 + sC_2R_2 + sC_1R_2}{sC_1 + s^2C_1C_2R_2}$$

$$Y_x(j\omega) = \frac{1}{Z_x(j\omega)} = \frac{j\omega C_1 - \omega^2 C_1C_2R_2}{1 + j\omega R_2(C_1 + C_2)}$$

$$= \frac{j\omega C_1 + \omega^2 C_1R_2(C_1 + C_2) - \omega^2 C_1C_2R_2 - j\omega^3 C_1C_2R_2^2(C_1+C_2)}{1 + \omega^2 R_2^2 (C_1 + C_2)^2}$$

$$\frac{1}{R_p} = G_p = \frac{\omega^2 C_1^2 R_2}{1 + \underbrace{\omega^2 R_2^2 (C_1+C_2)^2}_{\gg 1}} \approx \frac{\omega^2 C_1^2 R_2}{\omega^2 R_2^2 (C_1+C_2)^2}$$

$$G_p \approx G_2 \cdot \left(\frac{C_1}{C_1+C_2}\right)^2 = \frac{G_2}{n^2}$$

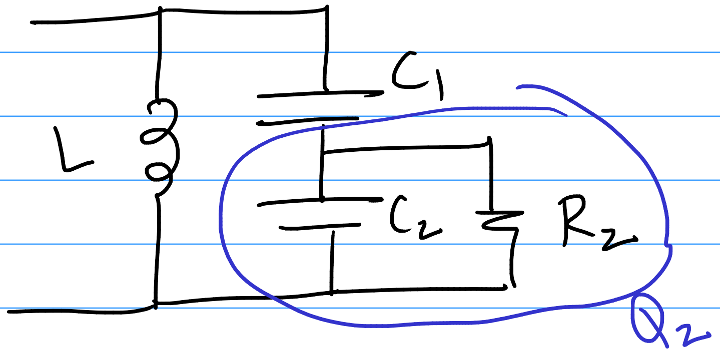
$R_p > R_2$

$$n = \text{tap ratio} = \frac{C_1+C_2}{C_1}$$

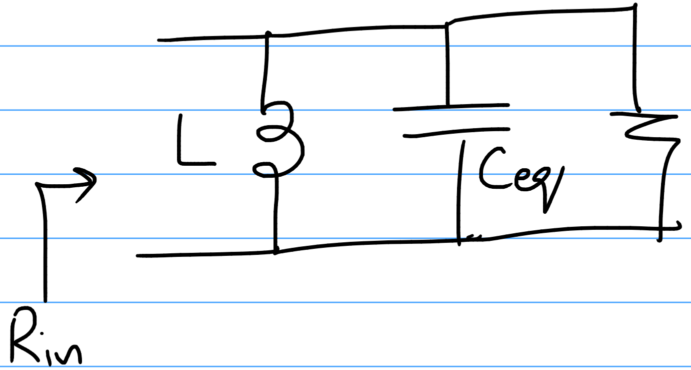
$$B_p = \frac{\omega C_1 - \omega^3 C_1 C_2 R_2^2 (C_1+C_2)}{1+\omega^2 R_2^2 (C_1+C_2)^2}$$

$$\approx \frac{-\omega^3 C_1 C_2 R_2^2 (C_1+C_2)}{\omega^2 R_2^2 (C_1+C_2)^2} \quad \text{@ high freq.}$$

$$\approx -\omega \cdot \frac{C_1 C_2}{C_1+C_2} \approx -\omega C_{eq}$$



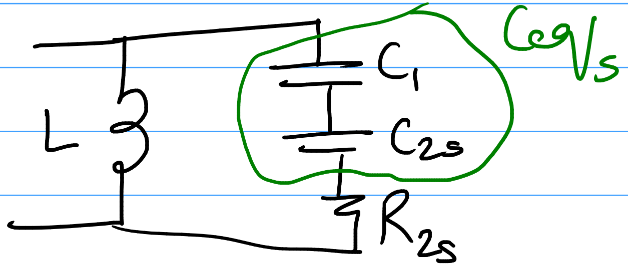
$\longrightarrow$



$R_p = n^2 \cdot R_2$

Design equations

1) $Q = \dfrac{R_{in}}{w_0 L} \Rightarrow \boxed{L = \dfrac{R_{in}}{w_0 Q}}$



$$C_{2s} = C_2 \times \left(\frac{1+Q_2^2}{Q_2^2}\right)$$

$$Q_2 = W_0 C_2 R_2$$

$$R_{in} = R_{2s}(1+Q^2)$$

$$\frac{R_2}{1+Q_2^2} = \frac{R_{in}}{1+Q^2}$$

2) $$Q_2 = \sqrt{\frac{R_2}{R_{in}}(1+Q^2) - 1}$$

3) $$C_2 = \frac{Q_2}{w_0 R_2}$$

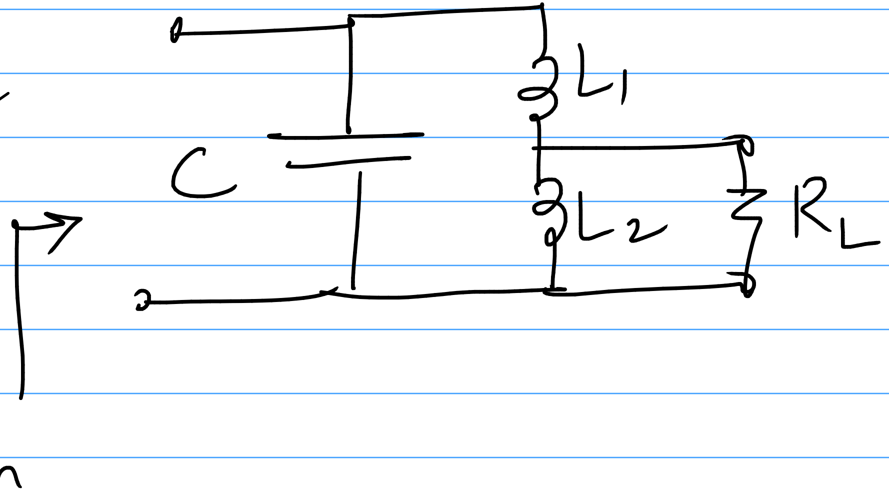
$$C_{eq_{Vs}} = \frac{C_1 \cdot C_{2s}}{C_1 + C_{2s}}$$

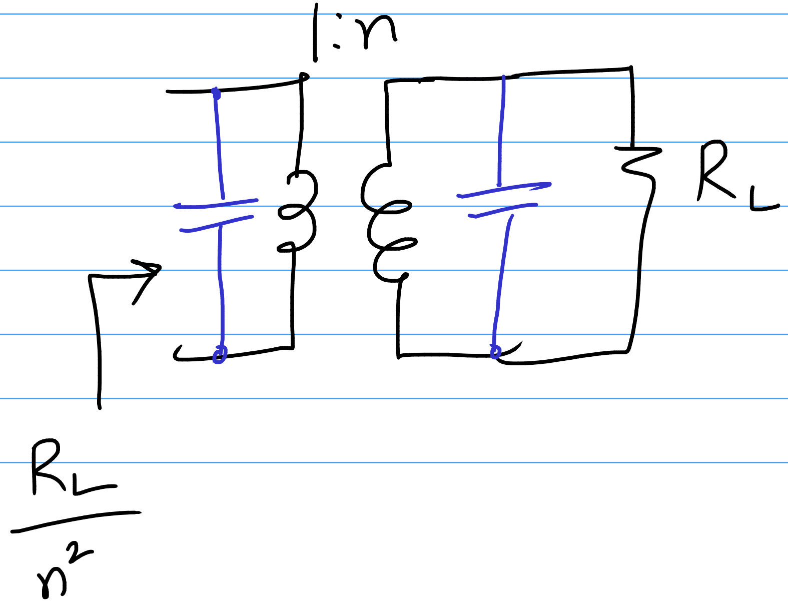
$$Q = \frac{1}{W_0 \, C_{eq_{Vs}} \cdot R_{2s}} = \frac{C_1 + C_{2s}}{W_0 R_{2s} \cdot C_1 C_{2s}}$$

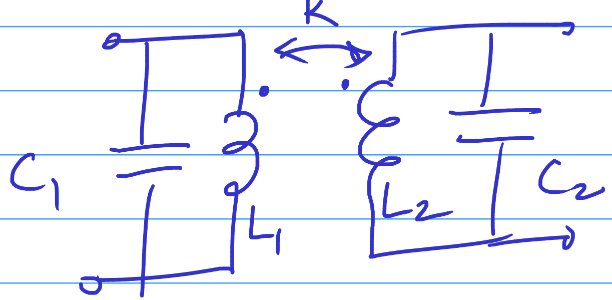
4) $$C_1 = \frac{C_2 \left(1 + Q_2^2\right)}{Q Q_2 - Q_2^2}$$

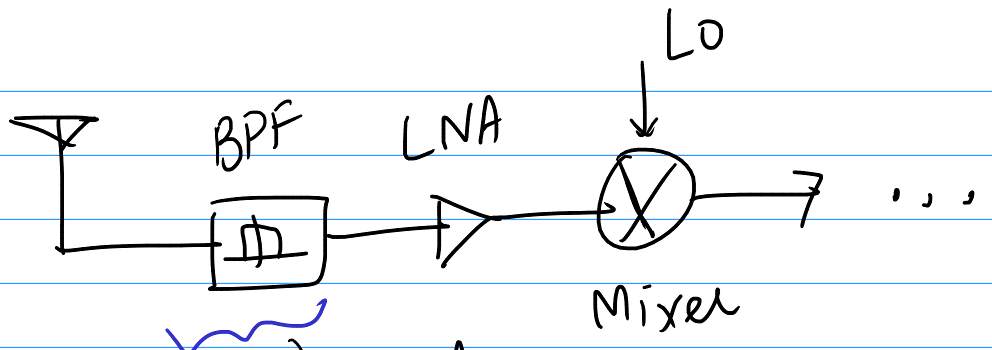
HW

Tapped-L match









IIP_3 (passive) $= \infty$ (ideal)

$= +50\,dBm$ (100W)

L_o = loss

$F_o = ?$

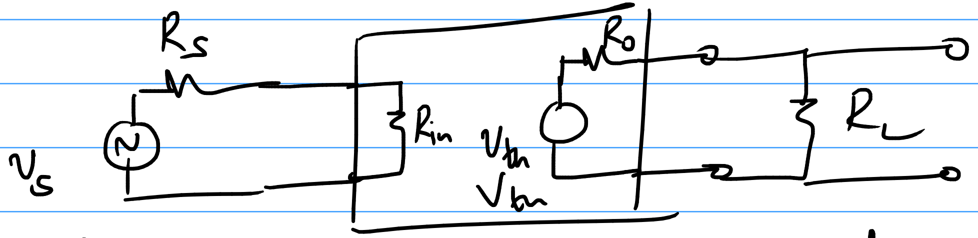
$IIP_3 = ?$

A_1 $\quad A_2$

F_1 $\quad F_2$

$IIP_{3,1}$ $\quad IIP_{3,2}$

$F_o = ?$



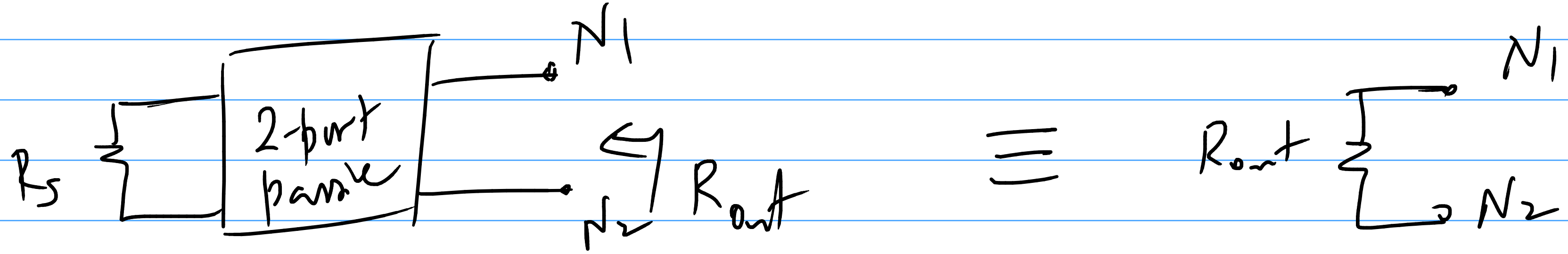
v_o

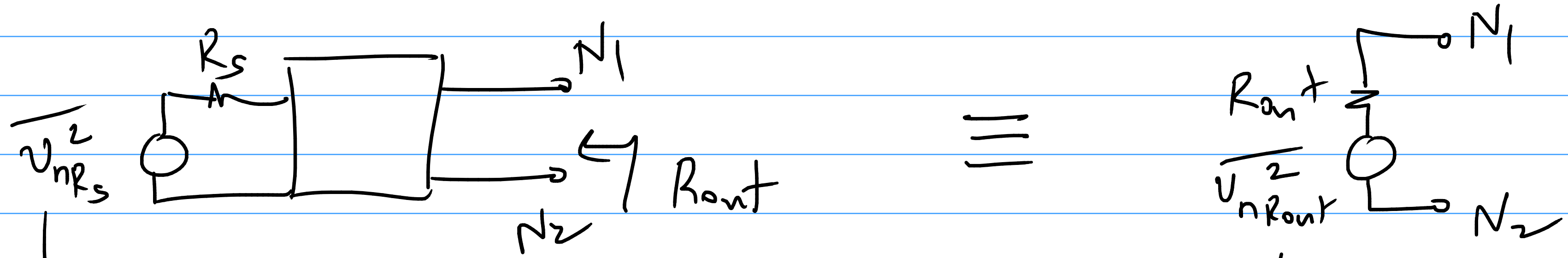
$R_{in} = R_s$

$R_o = R_L$

loss $\quad L = \dfrac{P_{in}}{P_{out}} = \dfrac{V_s^2/4R_{in}}{V_{th}^2/4R_L}$

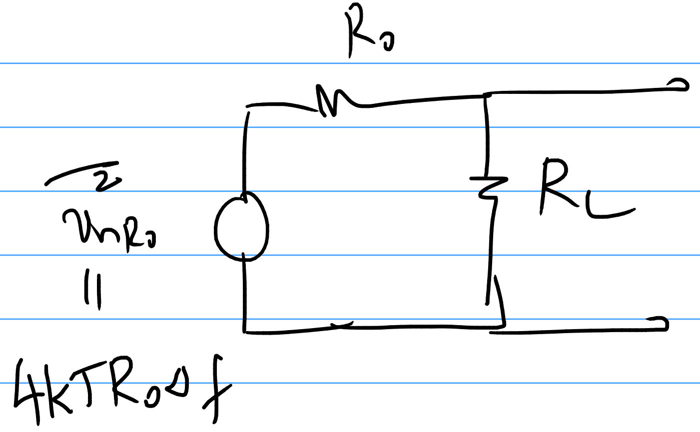
$$L = \frac{V_s^2}{V_{in}^2} \cdot \frac{R_L}{R_{in}}$$





$4kTR_s \Delta f$

$4kTR_{out} \Delta f$



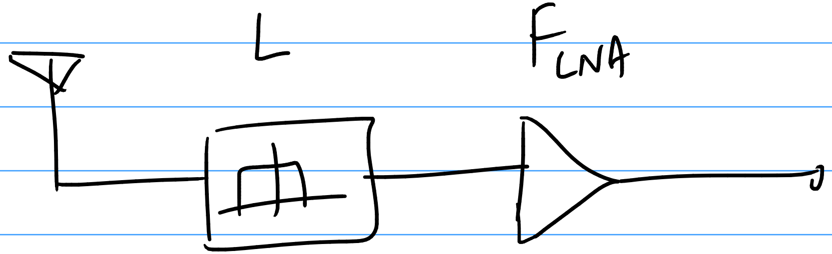
$$\overline{v_{n_L}^2} = 4kTR_o\Delta f \times \left(\frac{R_L}{R_L+R_o}\right)^2$$

$$A_v = \frac{V_{th}}{V_s} \cdot \left(\frac{R_L}{R_L+R_o}\right)$$

$$F = \frac{\overline{v_{n_L}^2}}{\overline{v_{nR_s}^2} \cdot A_v^2} = \frac{4kTR_o\Delta f \left(\frac{R_L}{R_L+R_o}\right)^2}{4kTR_s\Delta f \cdot \left(\frac{V_{th}}{V_s}\right)^2 \left(\frac{R_L}{R_L+R_o}\right)^2}$$

$$F = \left(\frac{V_s}{V_{th}}\right)^2 \cdot \frac{R_o}{R_s} = L$$

Noise Factor of a Passive Block = Loss of the block



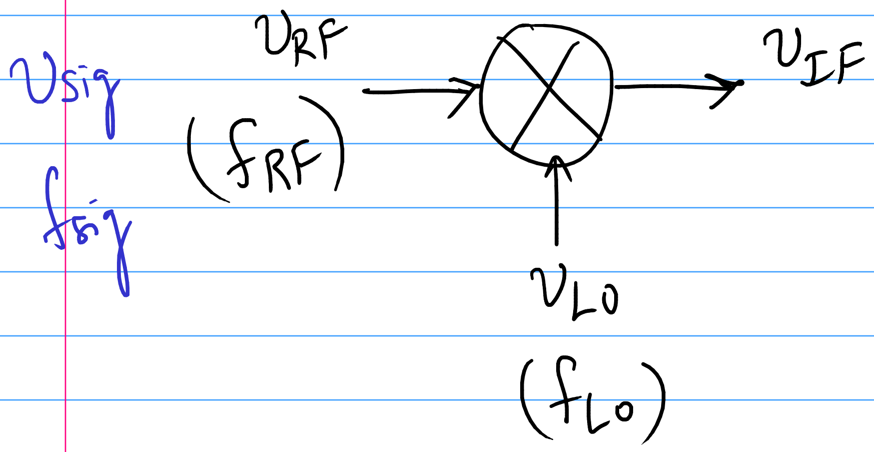
$$F = L + \frac{F_{LNA} - 1}{1/L}$$

$$= L + L(F_{LNA} - 1)$$

$$F = L \cdot F_{LNA}$$

$$\boxed{NF_{tot.} = L_{dB} + NF_{LNA}}$$

Mixers

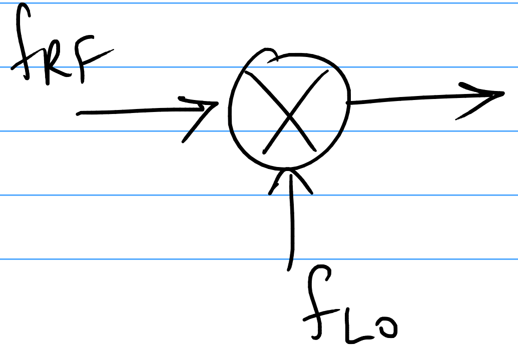


$f_{LO} + f_{RF}$

$f_{LO} - f_{RF}$

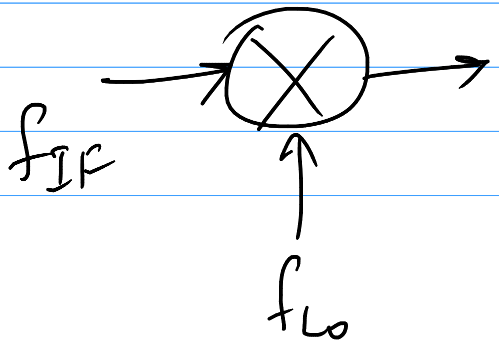
$f_{LO} + f_{sig}$ → up-conversion (Tx)

$f_{LO} - f_{sig}$ → down-conversion (Rx)

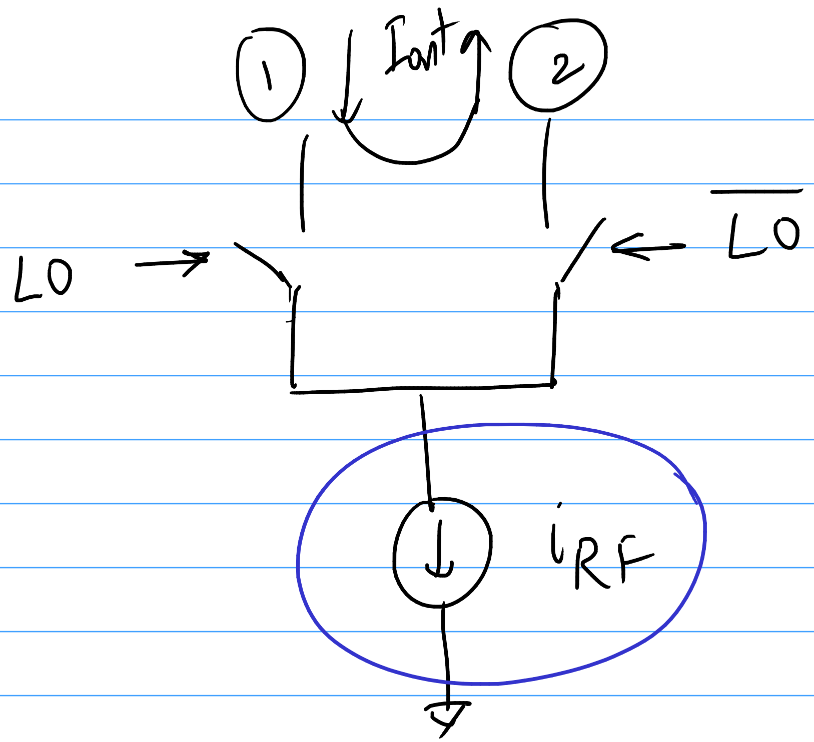


$f_{RF} = f_{LO} - f_{RF}$

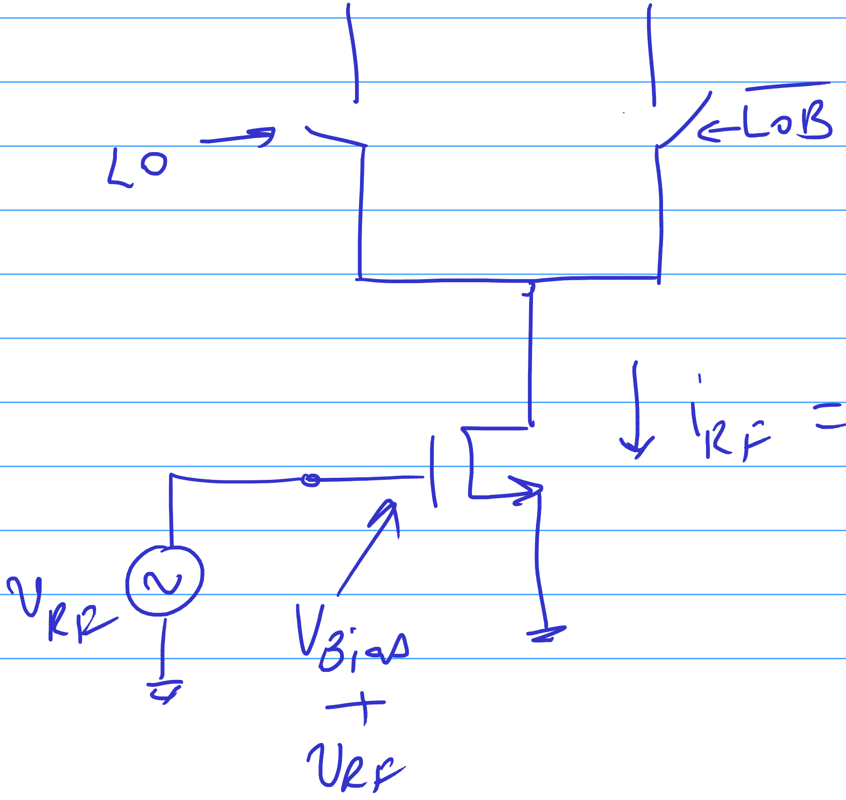
Rx



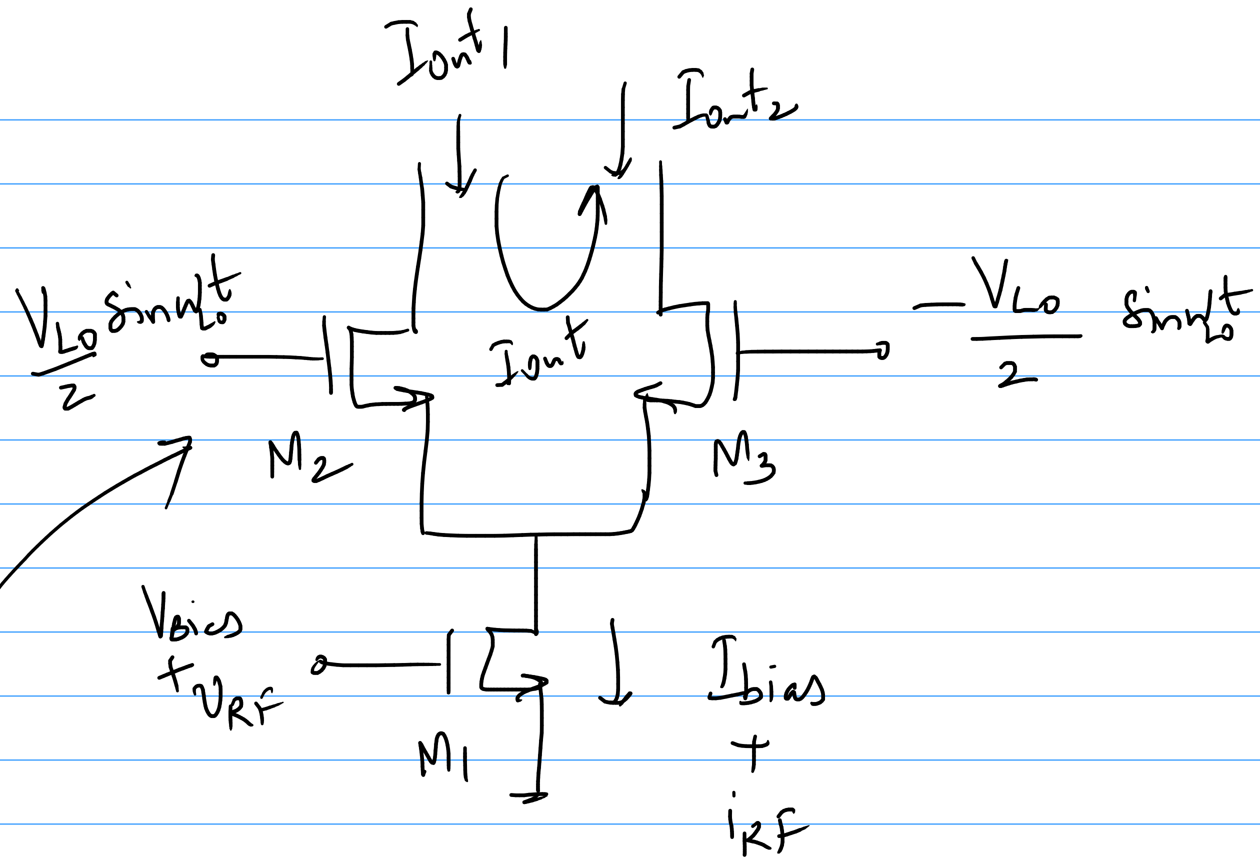
$f_{RF} = f_{LO} + f_{IF}$

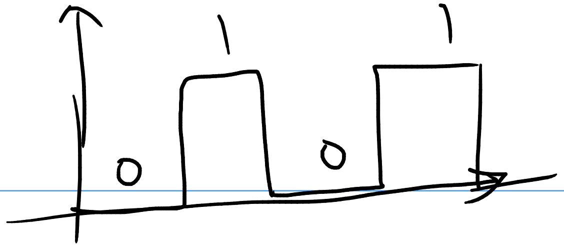


$$I_{out} = i_{RF} \times \mathrm{sgn}(\sin \omega_{LO} t)$$



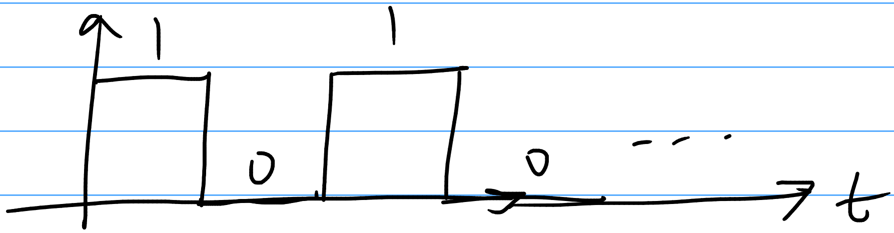
$$i_{RF} = g_m \cdot v_{RF} + I_{bias}$$



$s_2(t) =$ 

V_{LO} is large enough for

1) full switching
2) fast switching

$s_1(t) \Rightarrow$ 

$$S_1(t) = 0.5 + 0.5\, s(t)$$

$$I_{out_1} = \left(I_{bias} + I_{RF} \cdot \cos \omega_{RF} t \right) \times \left(\underline{0.5 + 0.5}\, s(t) \right)$$

fourier series

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_{Lo} t) + b_n \sin(n\omega_{Lo} t) \right]$$

$$\frac{a_0}{2} = 0 \; ; \quad a_n = 0 \; ; \quad b_n = \frac{2}{T_{Lo}} \int_0^{T_{Lo}} sgn(t) \sin(n\omega_{Lo} t)\, dt$$

$$b_n = \frac{4}{T_{Lo}} \int_0^{T_{Lo}/2} \sin(n\omega_{Lo}t)\, dt$$

$$= \frac{4}{T_{Lo}} \cdot \frac{1}{n\omega_{Lo}} \cdot \Big[-\cos(n\omega_{Lo}t)\Big]_0^{T_{Lo}/2}$$

$$= \frac{2}{n\pi} \cdot (1 - \cos n\pi)$$

$$b_n = \begin{cases} 0 & \text{for even } n \\ \dfrac{4}{n\pi} & \text{for odd } n \end{cases}$$

~~I_{out}~~

$\underline{I_{Bias} \text{ term}}$: $\dfrac{4 I_{Bias}}{\pi}\left[\sin \omega_{LO}t + \dfrac{1}{3}\sin(3\omega_{LO}t) + \dfrac{1}{5}\sin(5\omega_{LO}t) + \ldots\right]$ $\Rightarrow$ LO feed-through

$$s(t) = \frac{4}{\pi}\left[\sin \omega_{LO}t + \frac{1}{3}\sin 3\omega_{LO}t + \ldots\right]$$

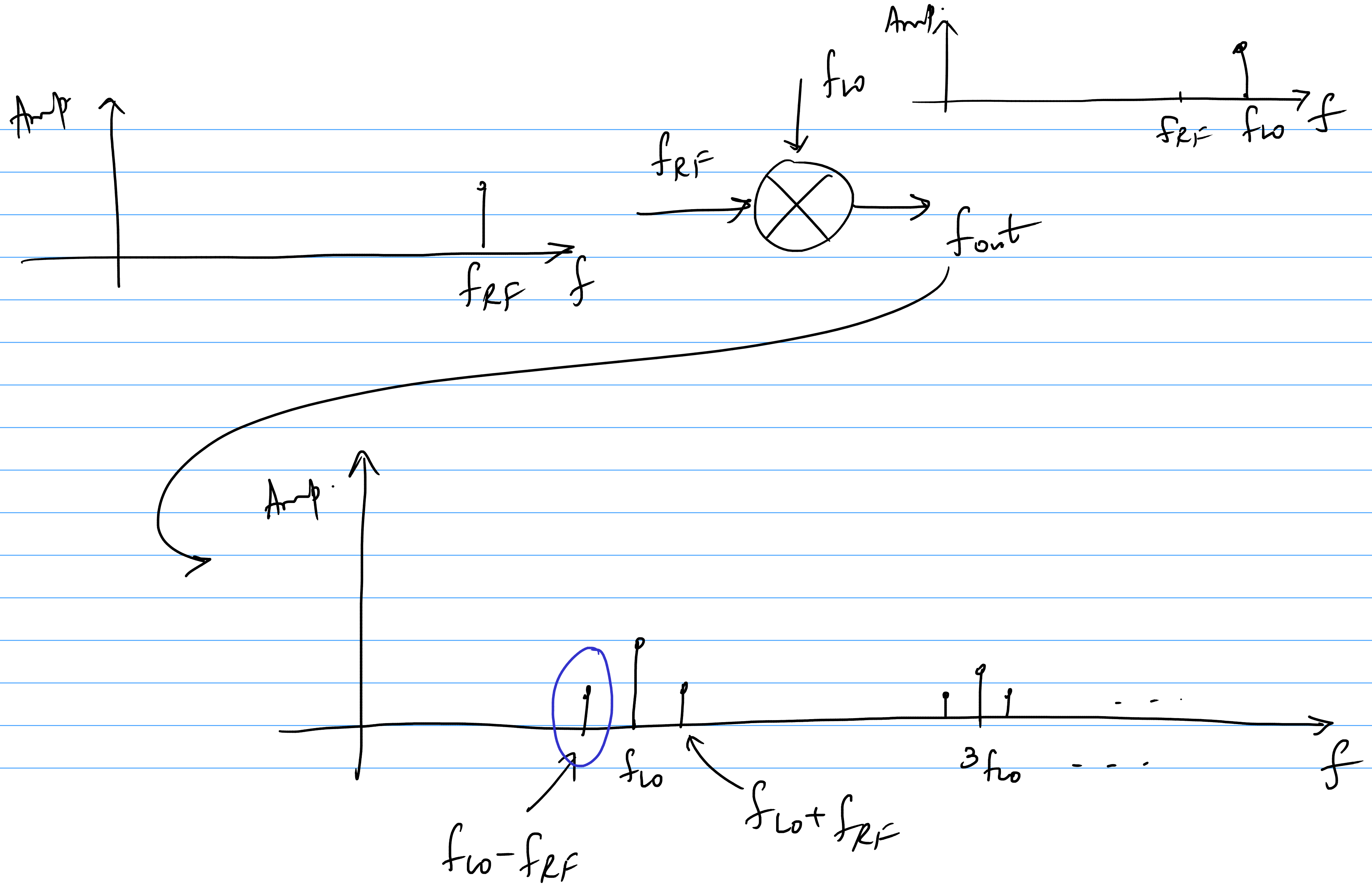
$$I_{out}(t) = \left[I_{Bias} + I_{RF}\cos \omega_{RF}t\right] \cdot s(t)$$

$\rightarrow$ I_{RF} term

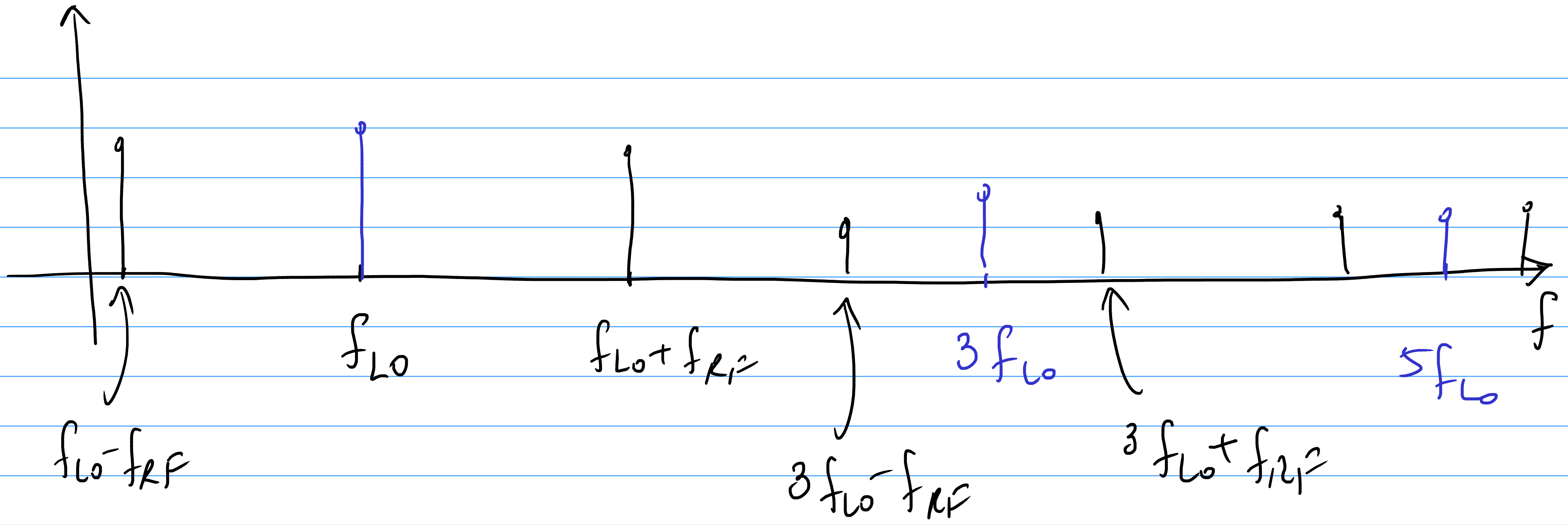
I_{RF} term

$$\frac{4I_{RF}}{\pi}\left[\underbrace{\cos(\omega_{RF}t)\cdot\sin(\omega_{LO}t)}^{\omega_{LO}\pm\omega_{RF}} + \frac{1}{3}\underbrace{\cos(\omega_{RF}t)\cdot\sin(3\omega_{LO}t)}^{3\omega_{LO}\pm\omega_{RF}} + \ldots\right]$$

$$= \frac{2I_{RF}}{\pi}\left[\sin(\omega_{LO}-\omega_{RF}) + \sin(\omega_{LO}+\omega_{RF}) + \frac{1}{3}\sin(3\omega_{LO}-\omega_{RF}) + \sin(3\omega_{LO}+\omega_{RF}) + \ldots\right]$$



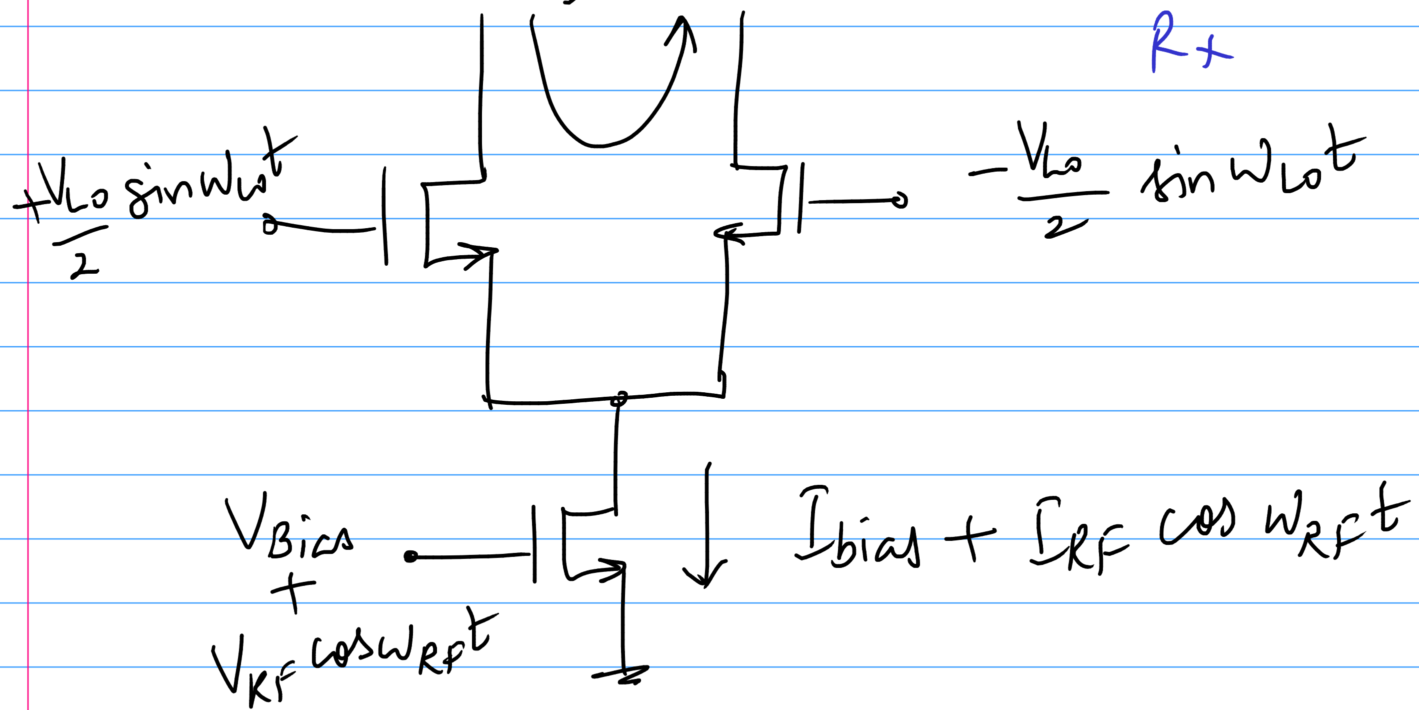
Amp
f_{RF}
f
f_{LO}
f_{RF}
f_{out}
Amp
f_{RF}
f_{LO}
f
Amp.
f_{LO}
$3f_{LO}$
f
$f_{LO}-f_{RF}$
$f_{LO}+f_{RF}$

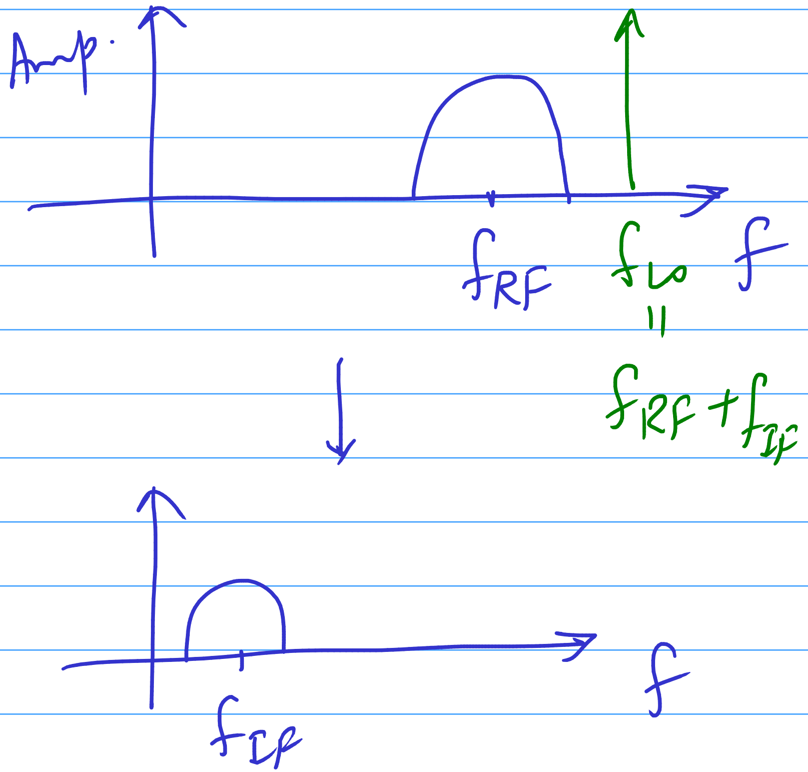


$$\text{current gain} = \frac{2}{\pi}$$

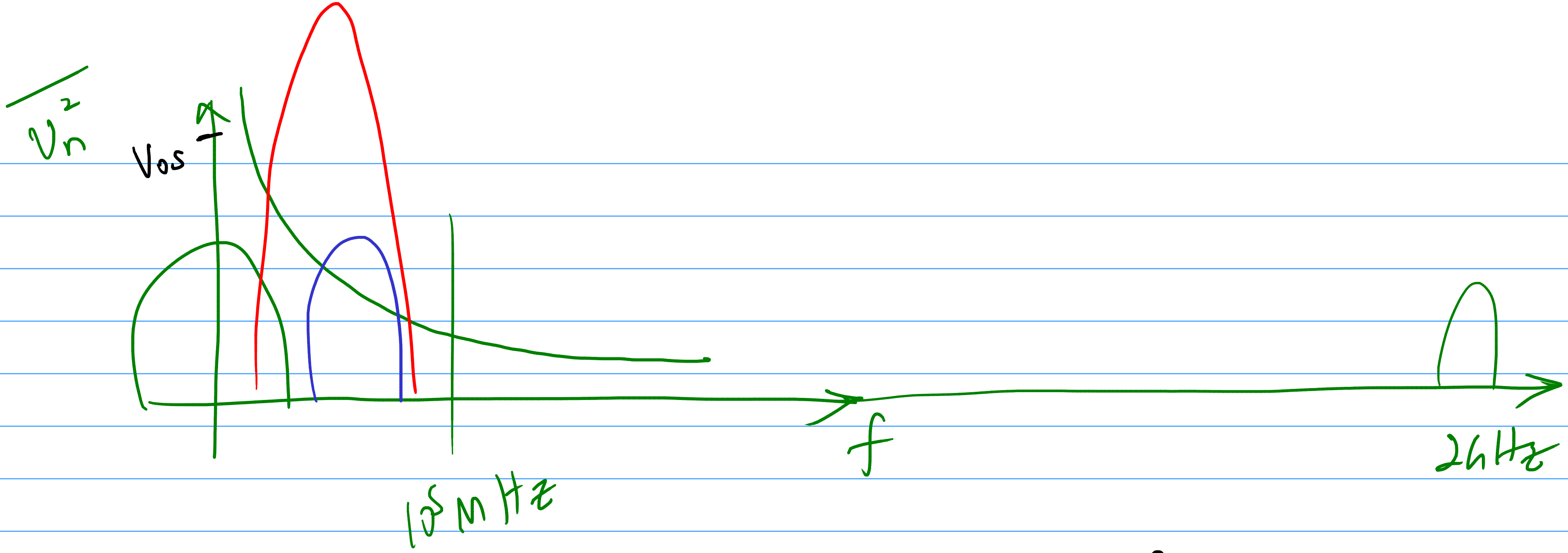
$$\left(\frac{I_{out}}{I_{RF}}\right)$$

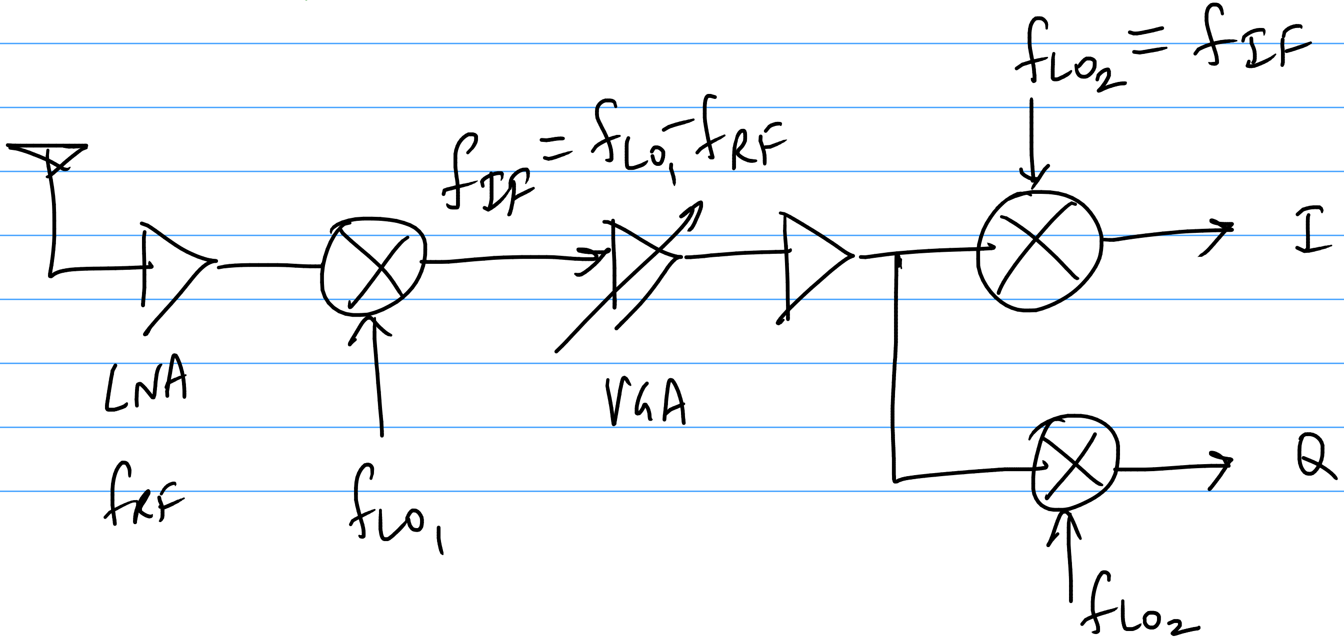
$$I_{out} = \frac{2}{\pi} I_{RF} \left[\underbrace{\cos(\omega_{LO}-\omega_{RF})t}_{Rx} + \underbrace{\cos(\omega_{LO}+\omega_{RF})t}_{Tx} \right]$$





IF = Intermediate freq.



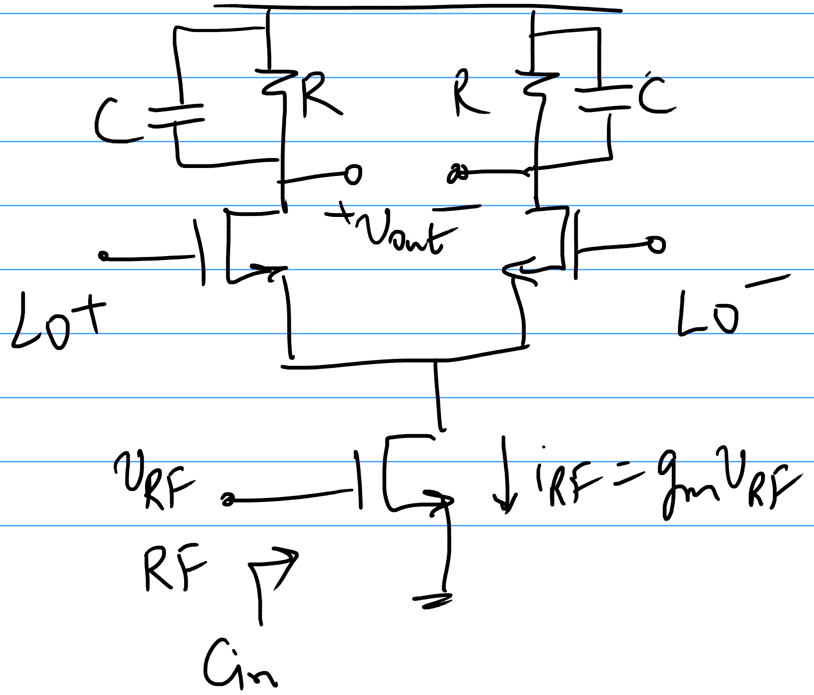


Heterodyne Receiver

$$f_{RF} = 3.5\,\text{GHz}, \quad f_{LO} = 3.6\,\text{GHz} \quad \Big\} \quad f_{IF} = 100\,\text{MHz}$$

$I_{out} \rightarrow$ 100 MHz, 3.6 GHz, 7.1 GHz, ...

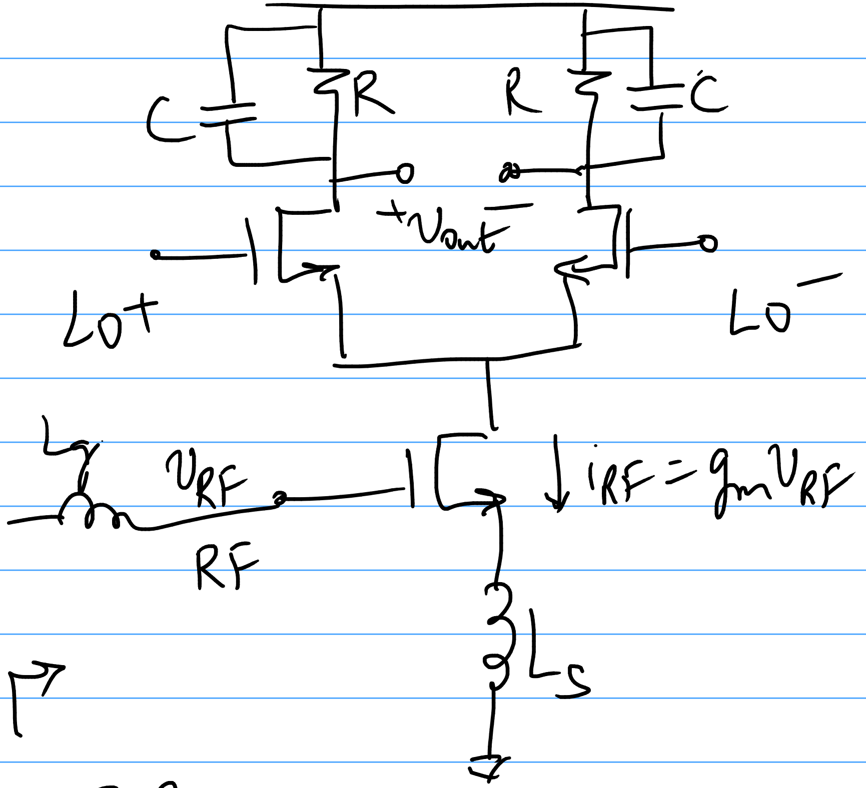
"Gilbert-cell" Mixer



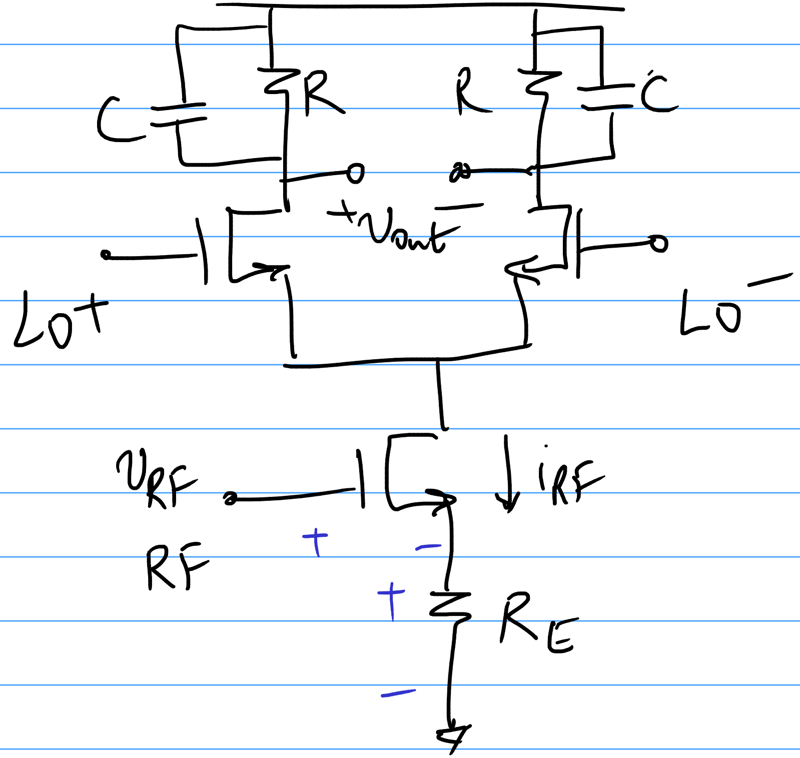
gain → decides R

filter → decides C

$$\text{gain } A_v = \frac{v_{out}}{v_{RF}} = \frac{2}{\pi} g_m R$$

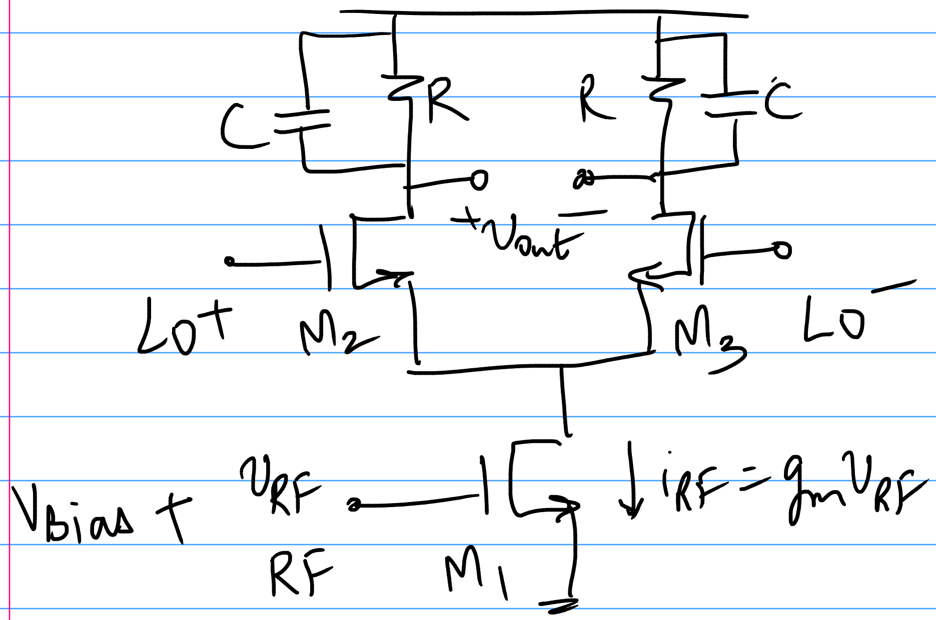


$R_{in} = 50\Omega$

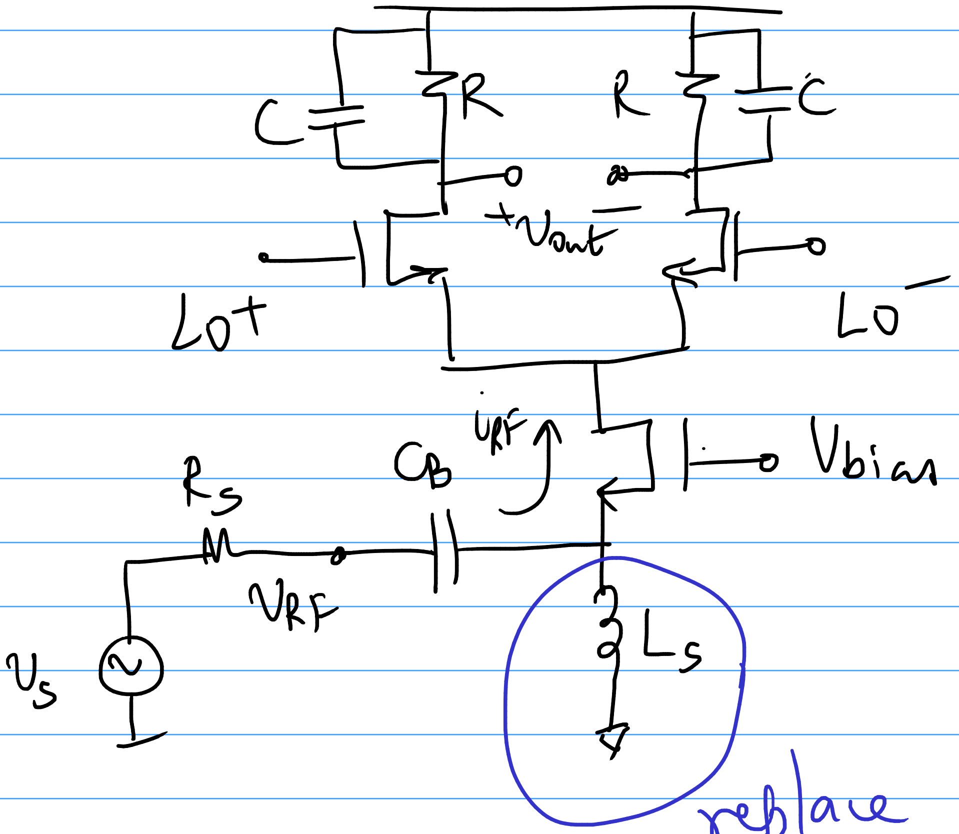


$$i_{RF} = v_{RF} \cdot \frac{g_m}{1 + g_m R_E}$$

$$g_m \gg 1/R_E \Rightarrow i_{RF} \approx \frac{v_{RF}}{R_E}$$



choose V_{Bias} so that $g_{m_3}(M_1) \approx 0$

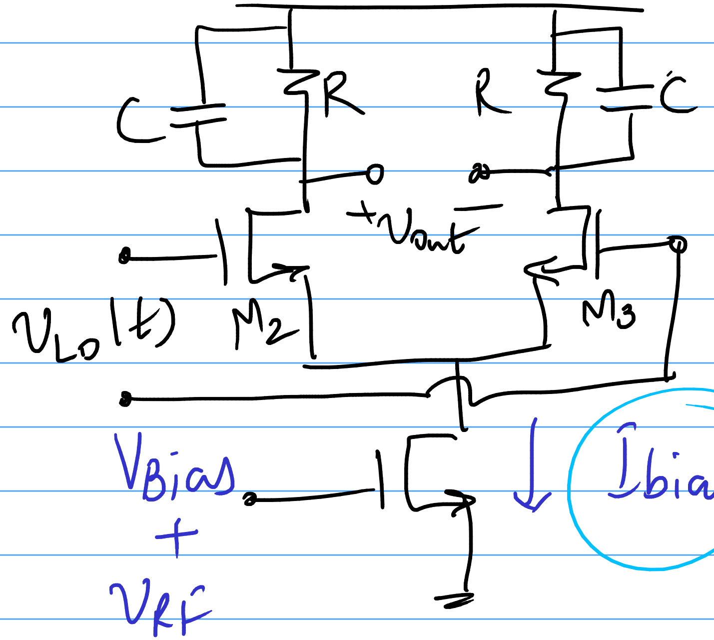


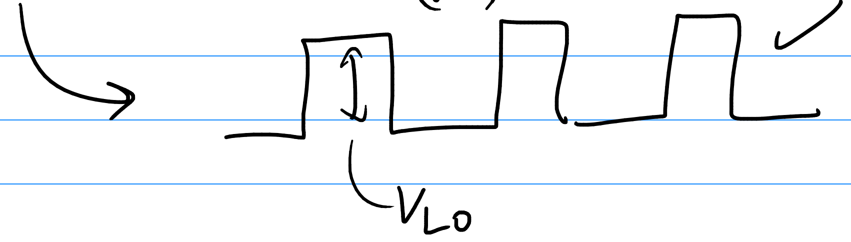
$$i_{RF} = \frac{v_s}{R_s + 1/g_m} = \frac{g_m v_s}{1 + g_m R_s} \approx \frac{v_s}{R_s} \text{ if } 1/g_m << R_s$$

replace with Ideal current source

$v_{LO}(t) = V_{LO} \sin \omega_{LO} t$

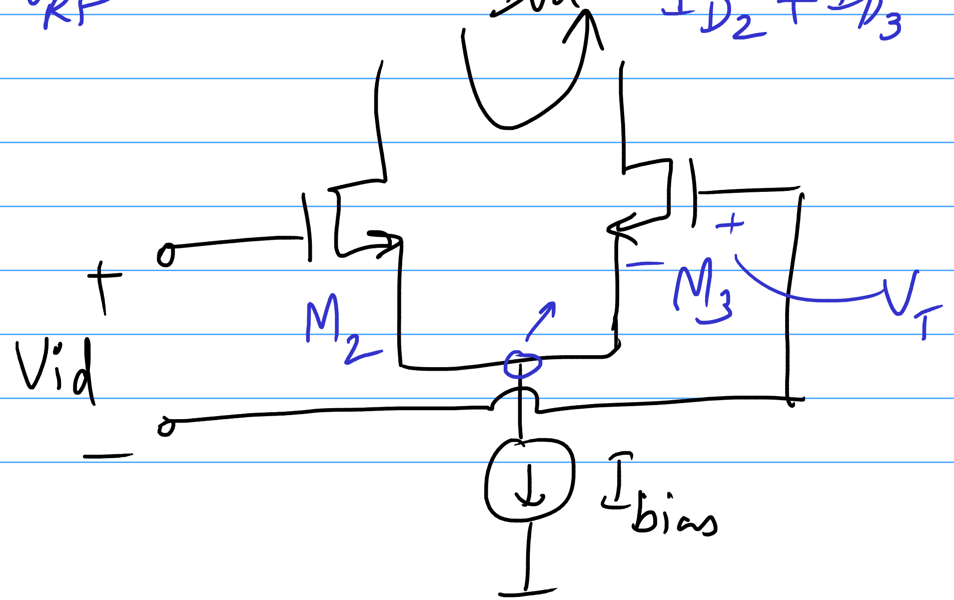
(or)





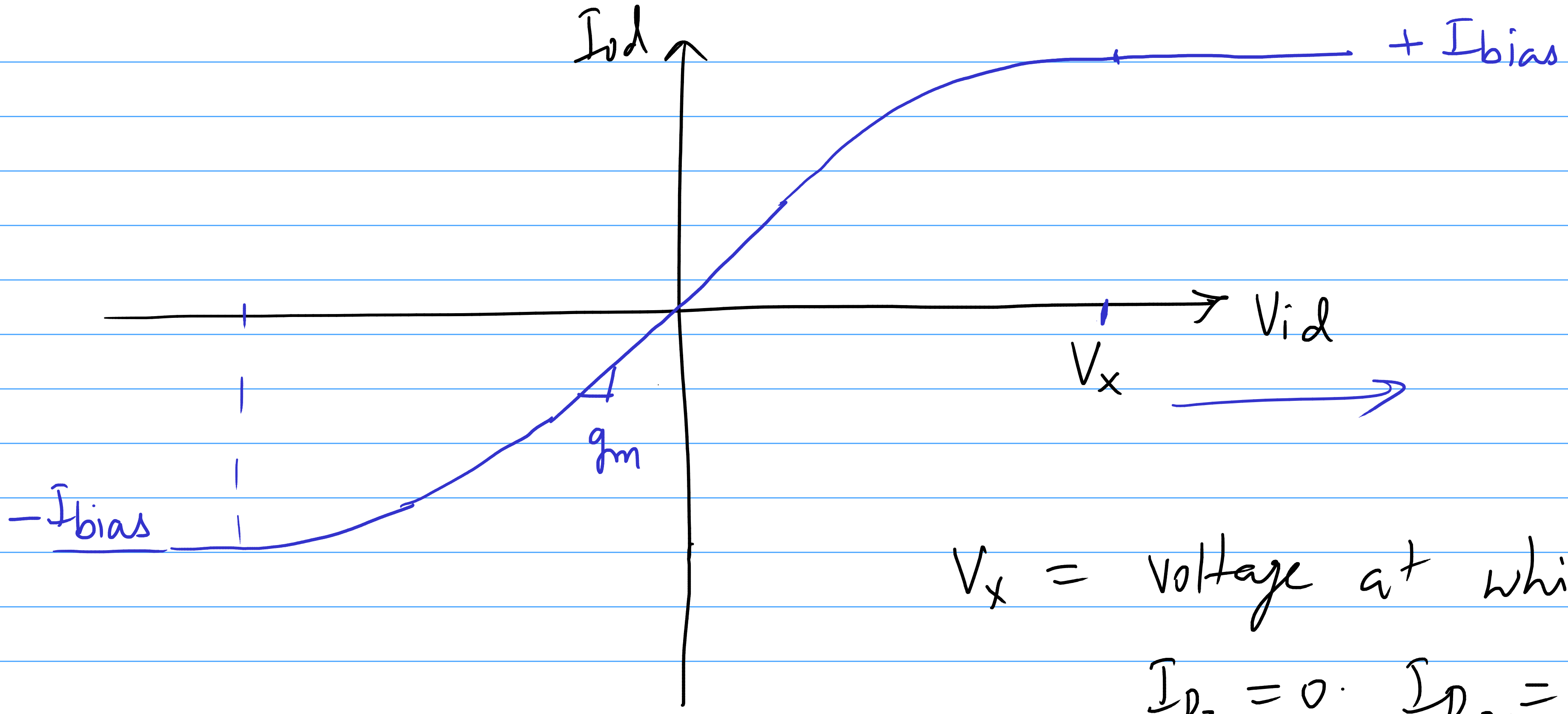
$I_{bias} + i_{RF}$

$I_{D_2} + I_{D_3} = I_{bias}$



$$I_{D_2} = I_{bias} + g_m \frac{V_{id}}{2}$$

$$I_{D_3} = I_{bias} - g_m \frac{V_{id}}{2}$$

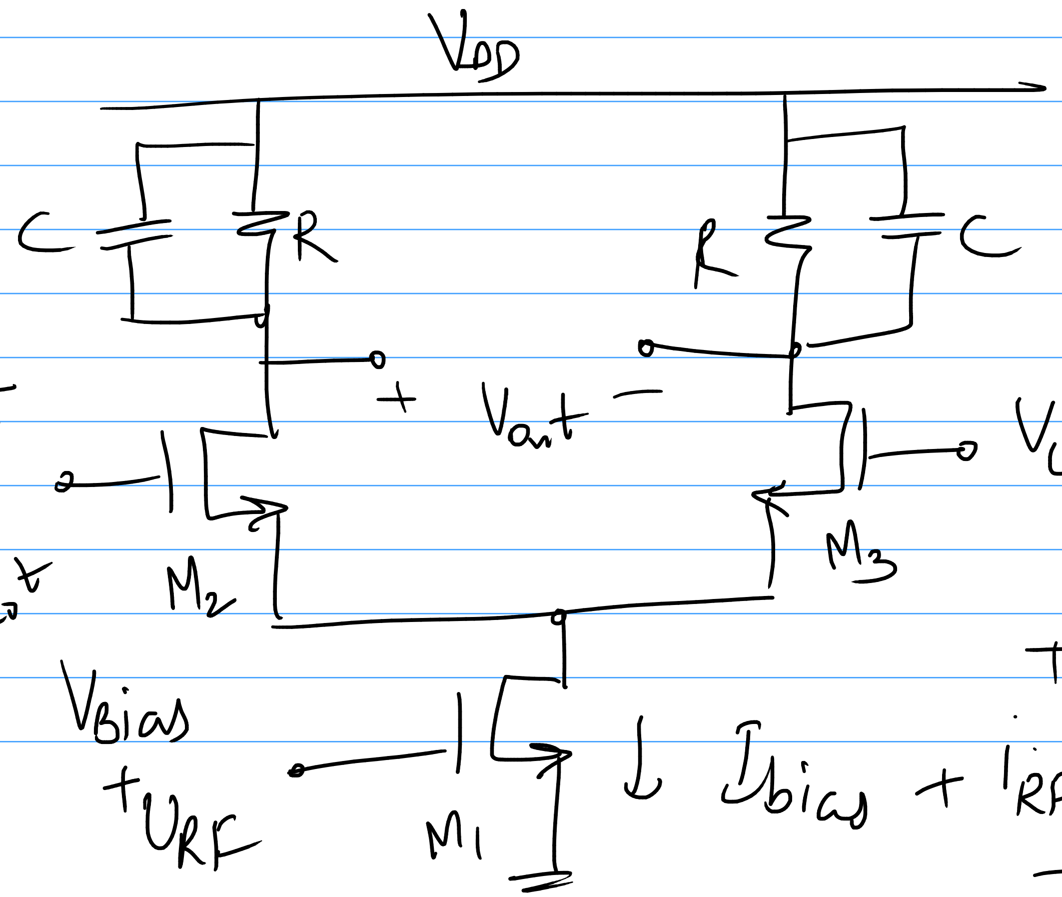


V_x = voltage at which

$I_{D_3} = 0$; $I_{D_2} = I_{bias}$

$= \sqrt{2}\,(V_{GS_{nom.}} - V_T)$

If $V_{LO} < \sqrt{2}\, V_{OV}$ → lower Mixer gain



$V_{LO}^{+} \to V_B + \frac{V_{LO}}{2}\sin\omega_{LO}t$

$V_{LO}^{-} \to V_B - \frac{V_{LO}}{2}\sin\omega_{LO}t$

$V_{Bias} + v_{RF}$

$\uparrow A_v \to \uparrow g_{m1} \to \uparrow I_{bias},\ \uparrow \frac{W_1}{L_1}$

$\uparrow A_v \to \uparrow R$

$V_{out,CM} = V_{DD} - \frac{I_{bias}}{2}R$

$V_{DS1} = V_B - V_{GS_{2,3}}\Big|_{I_{bias}/2}$

1) Choose V_B so that M_1 is in Sat. $\Rightarrow V_{DS_1} = V_B - V_{GS_{23}}$

$\geq V_{Bias} - V_{T_1}$

2) $V_{DD} - \frac{I_{Bias}}{2} R \geq V_B - V_{T_{23}}$ so that M_2, M_3 are in sat. (bias)

3) If $\frac{W_1}{L_1} \uparrow \Rightarrow C_{in} \uparrow \Rightarrow$ loads LNA

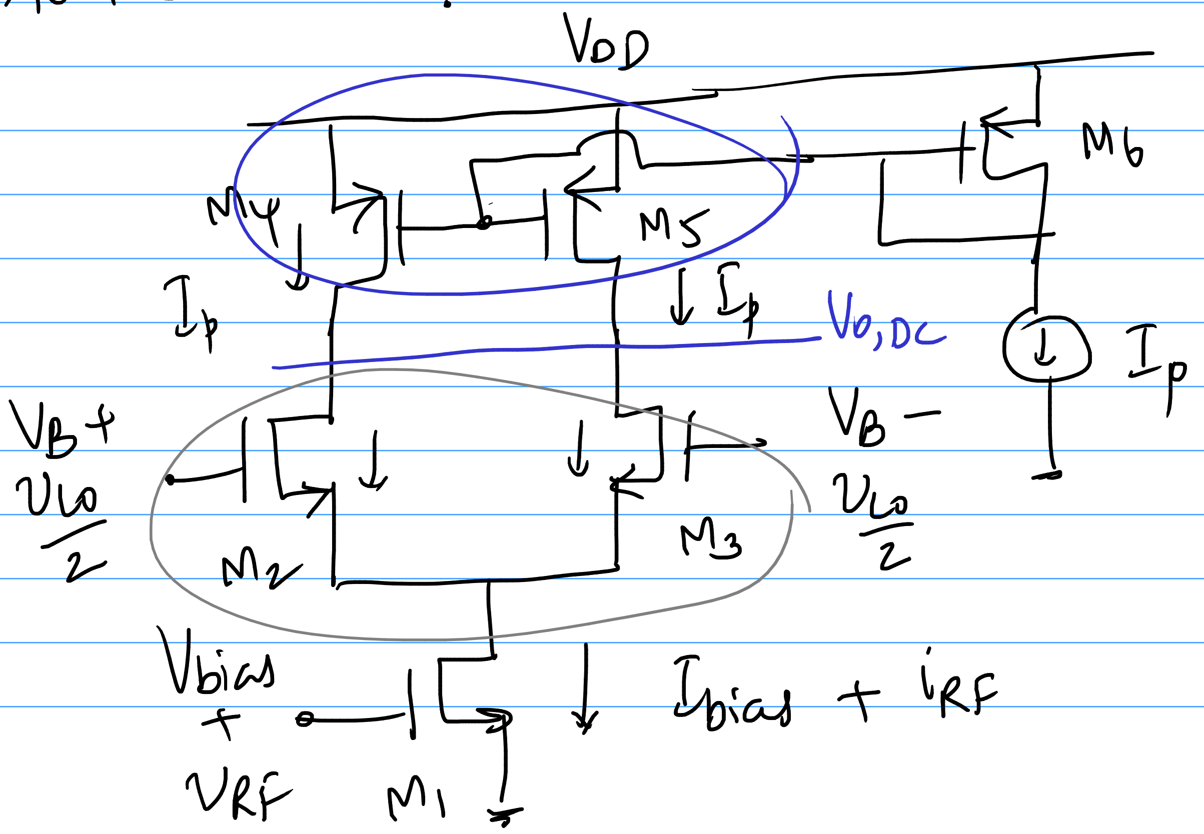
4) If M_1 goes to Moderate / Weak Inversion

↳ $\uparrow \frac{W_1}{L_1}$ does not $\uparrow g_m$

↳ device speed $\downarrow$ (f_T)

5) $\uparrow I_{bias}$ or $\uparrow R \Rightarrow V_{o,DC} \downarrow$

6) Active load ?



$r_{ds_{4,5}} \gg R$

so that A_v is larger

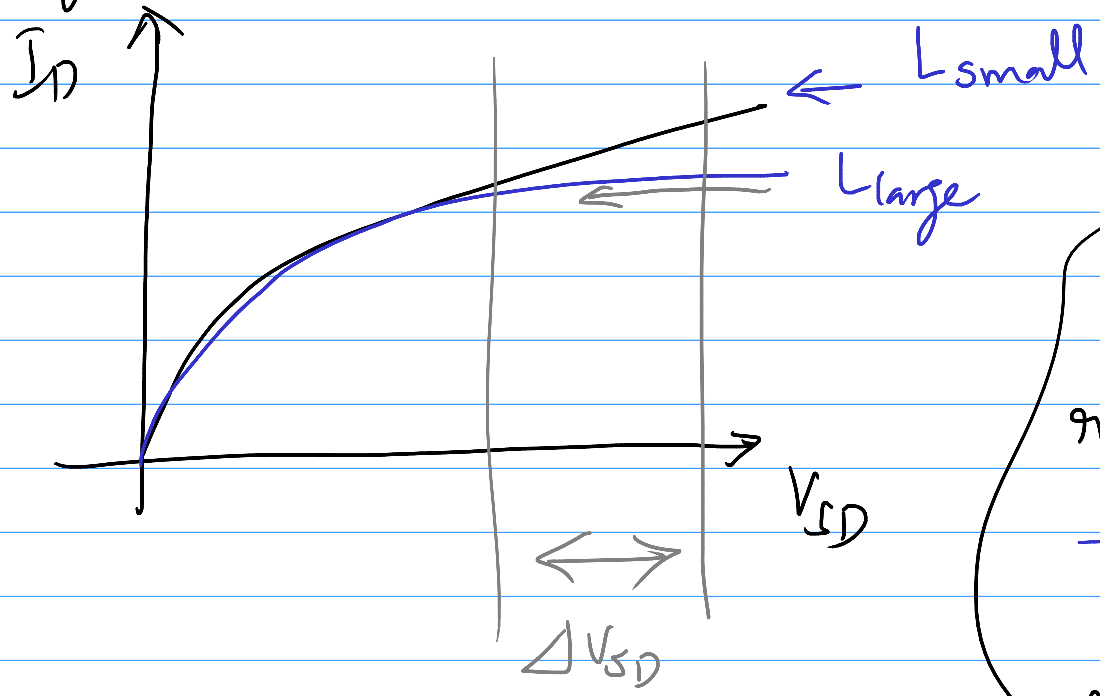
$M_6 = M_5 = M_4$

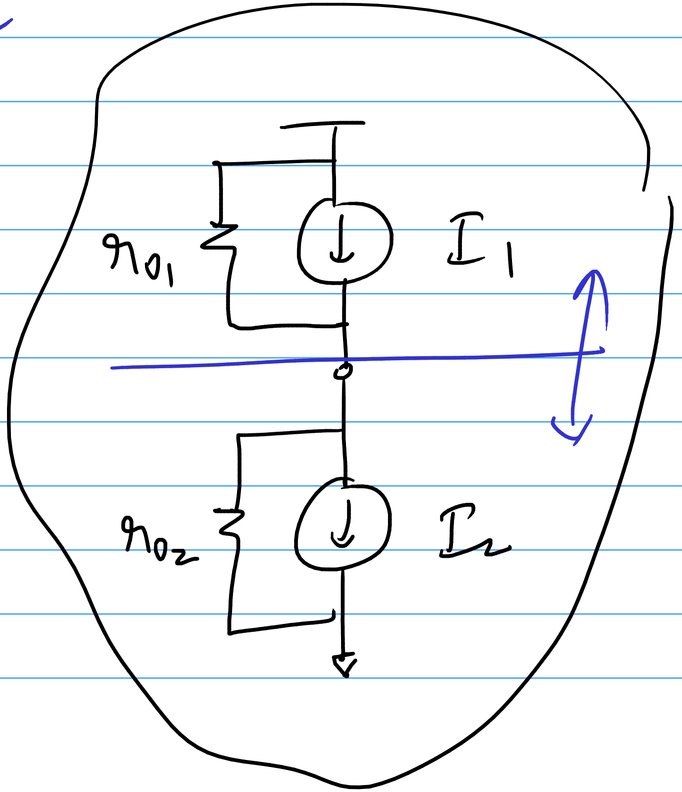
What if $\frac{I_{bias}}{2} \neq I_p$

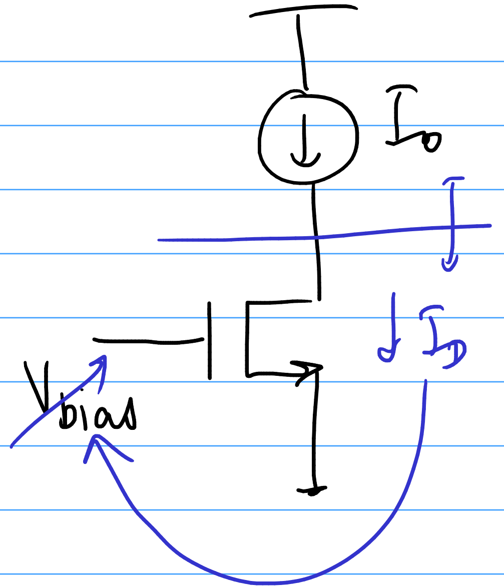
If $I_p > \frac{I_{bias}}{2} \Rightarrow V_{o,DC} \uparrow$

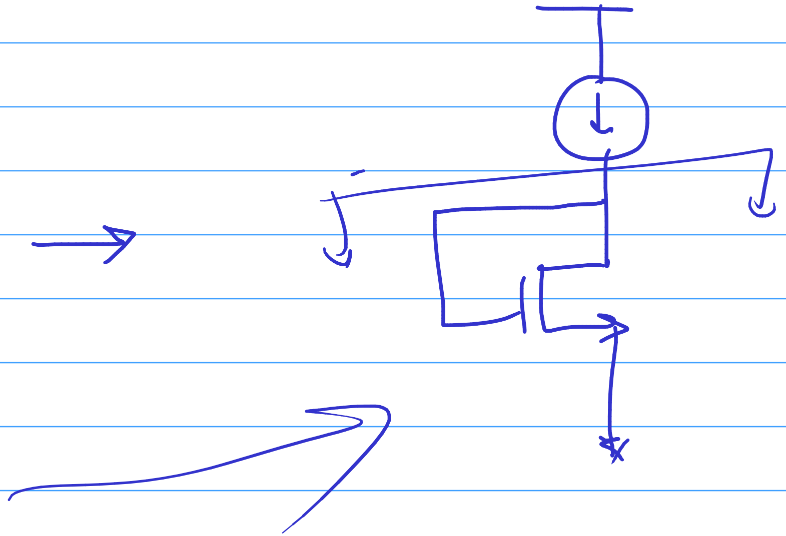
If $I_p < \frac{I_{bias}}{2} \Rightarrow V_{o,DC} \downarrow$

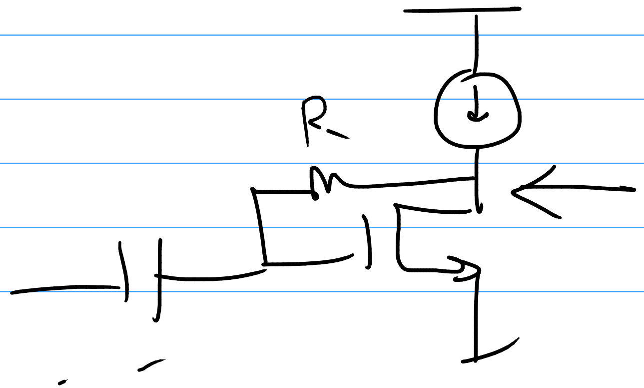
$\uparrow r_{ds_{4,5}}$ by $\uparrow L_{4,5} \Rightarrow \lambda \downarrow$

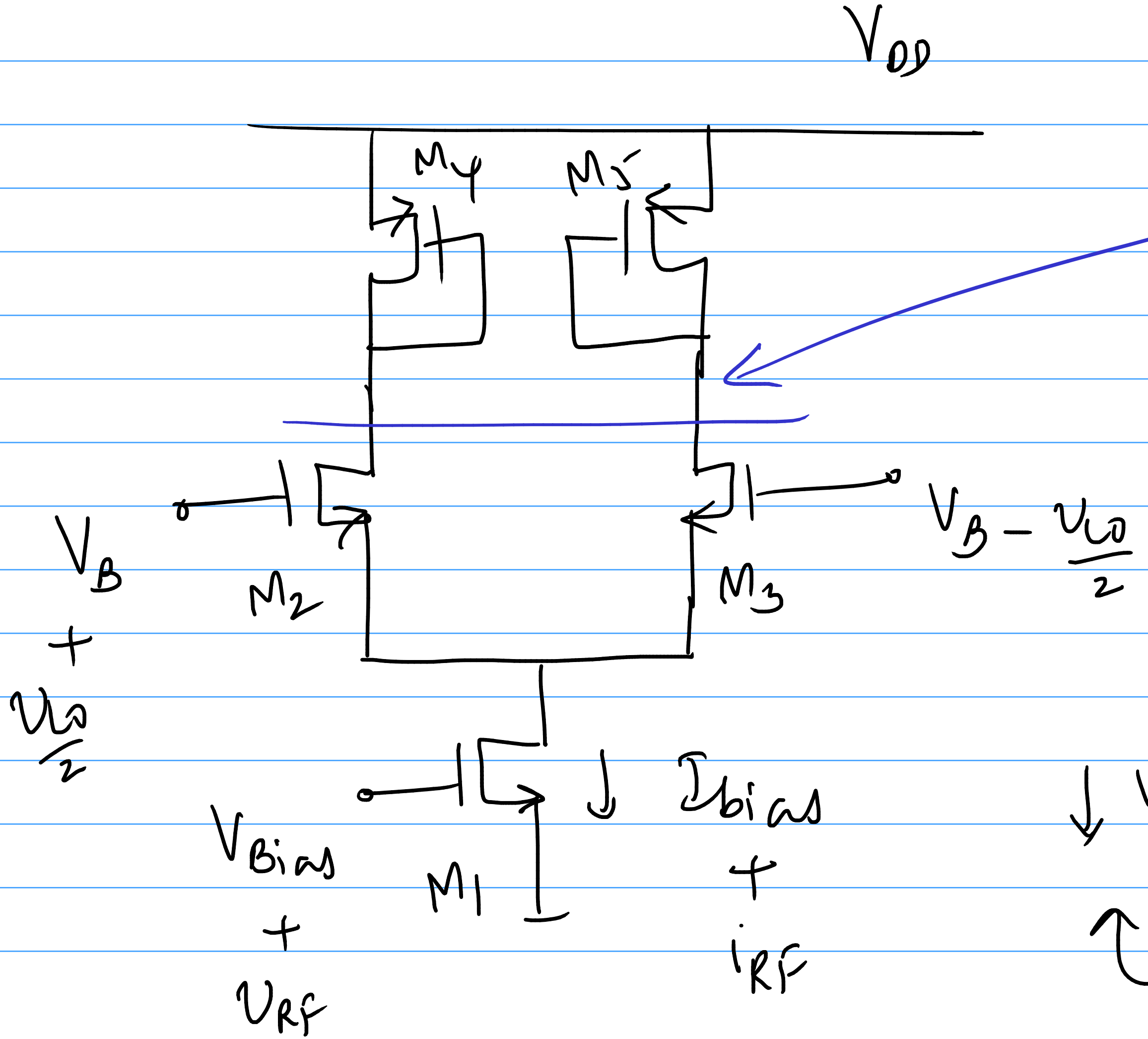





Io
ID
Vbias




R



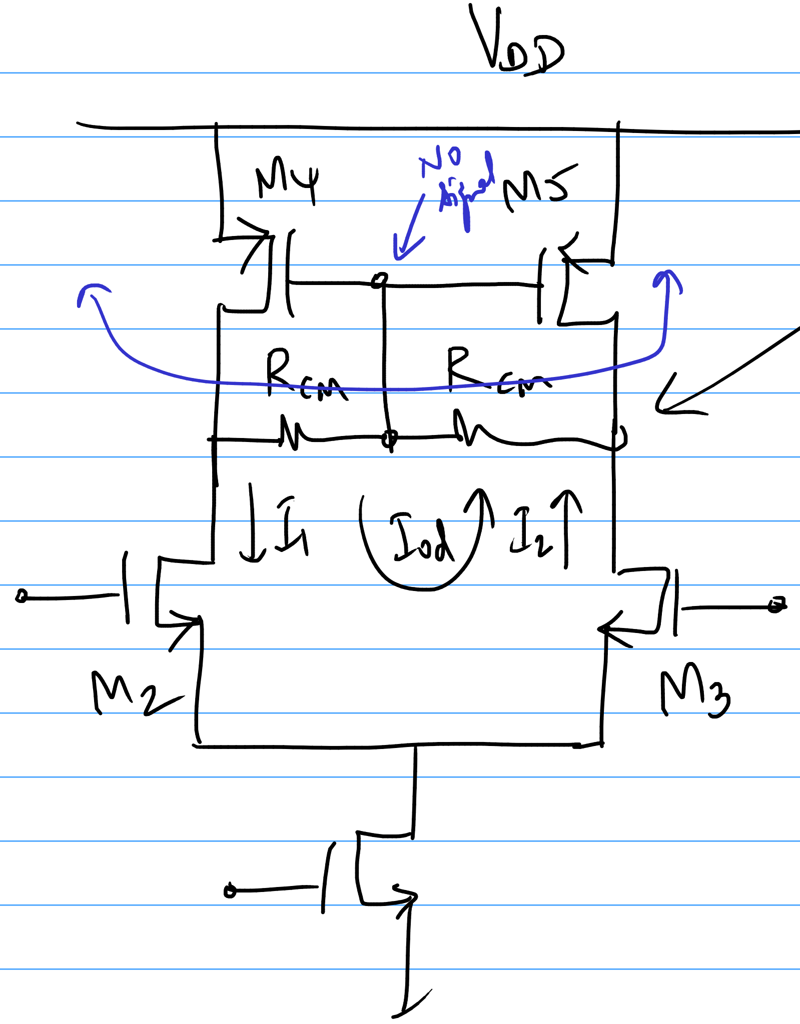
$$V_{o,DC} = V_{DD} - V_{SG_{4,5}} \Big|_{\frac{I_{bias}}{2}}$$

$$V_{o,ac} = (\ ?\ ) \cos \omega_{IF} t$$

$$A_v = \frac{2}{\pi} g_{m_1} \times \frac{1}{g_{m_{4,5}}}$$

$\downarrow V_{o,DC}$

$\uparrow \downarrow \left(\frac{W}{L}\right)_{4,5} \leftarrow$ Make this small

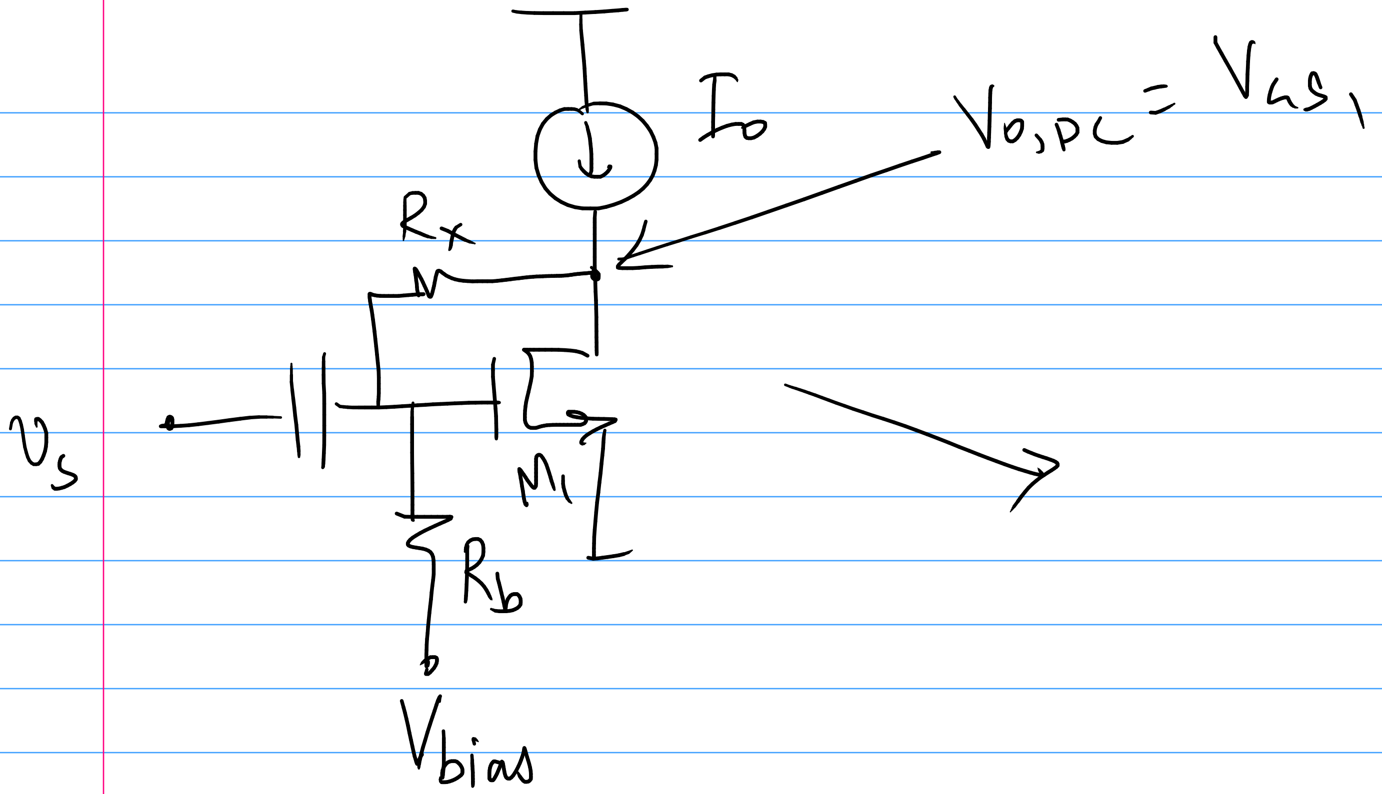


$$V_{o,DC} = V_{DD} - V_{SG_{4,5}}\Big|_{\frac{I_{bias}}{2}}$$

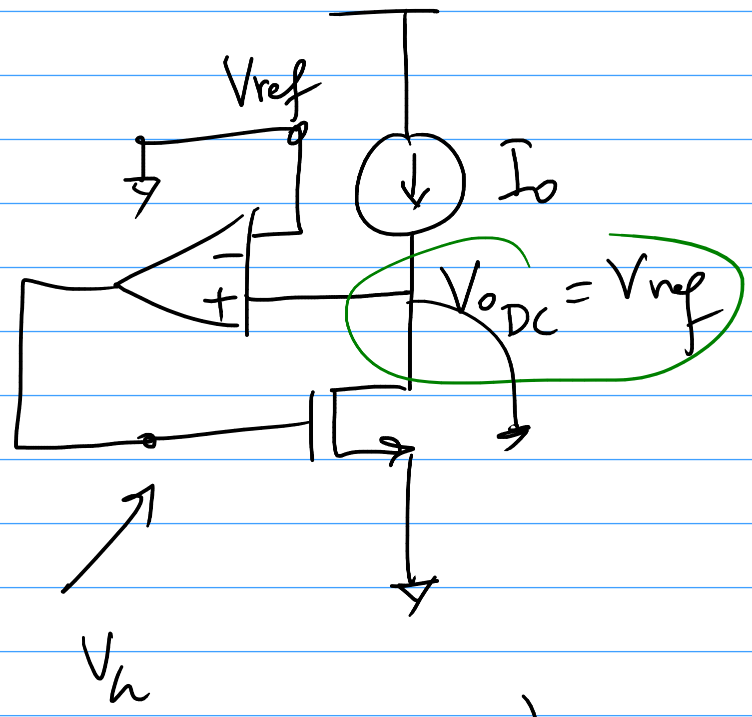
$$A_v = \frac{2}{\pi} g_m (r_{ds_{4,5}} \| R_{CM})$$

Set $V_{o,DC} \rightarrow$ set $\left(\frac{W}{L}\right)_{4,5}$

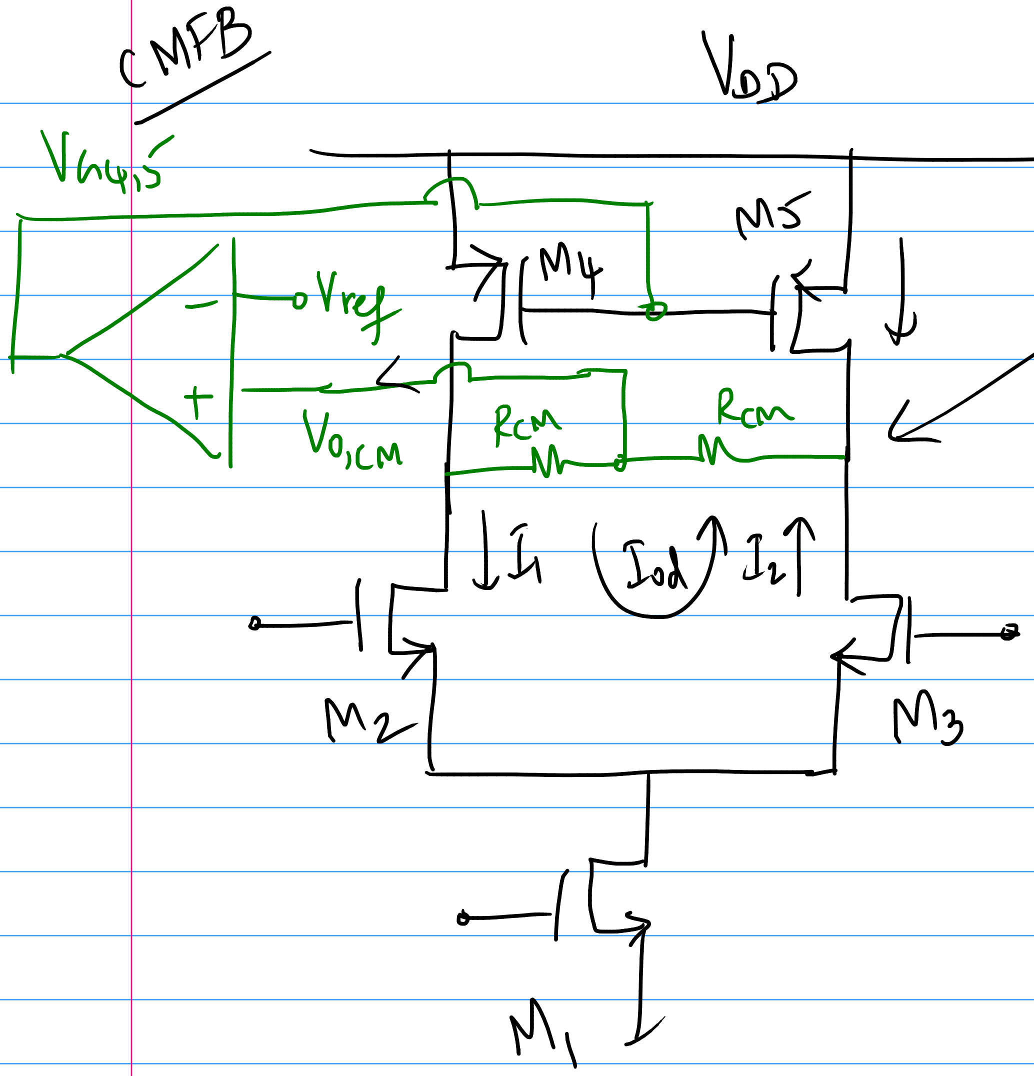
Set $A_v \rightarrow$ set $L_{4,5}$



I_o
$V_{o,DC} = V_{GS1}$
R_x
v_s
M_1
R_b
V_{bias}

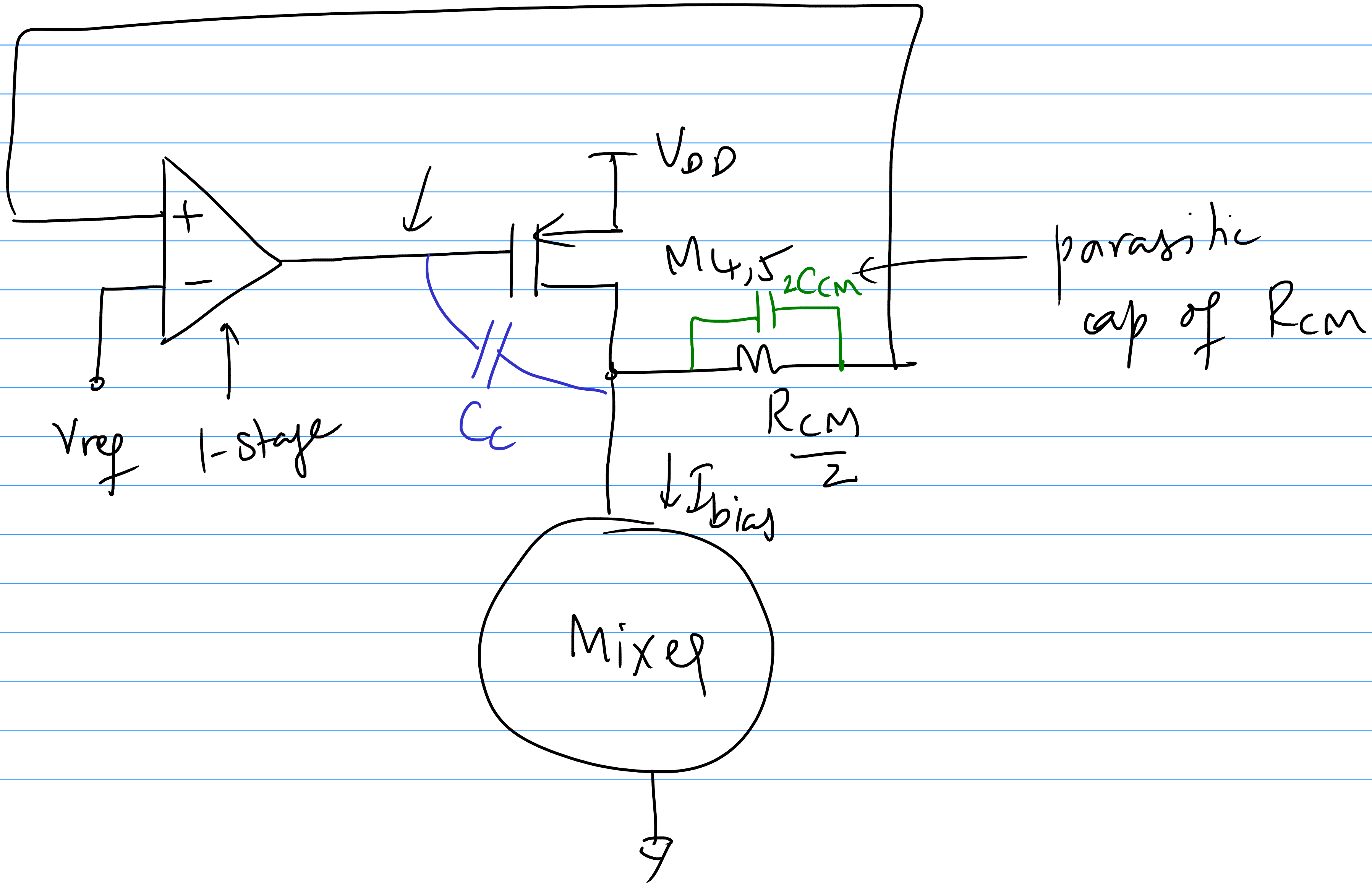


V_{ref}
I_o
$V_{o,DC} = V_{ref}$
V_b
(set by opamp)

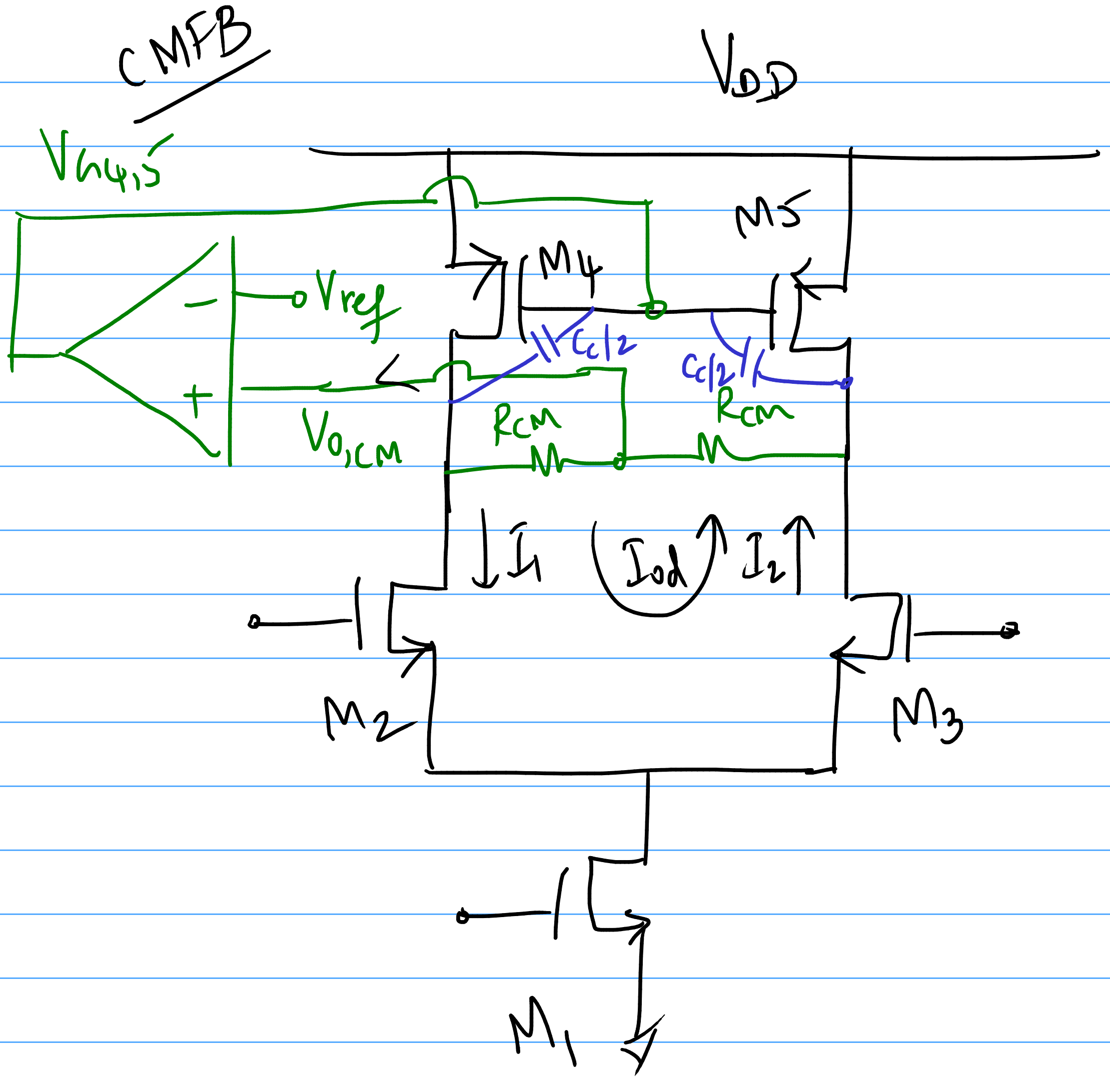


$V_{o,DC} = V_{ref}$ (set for max swing)

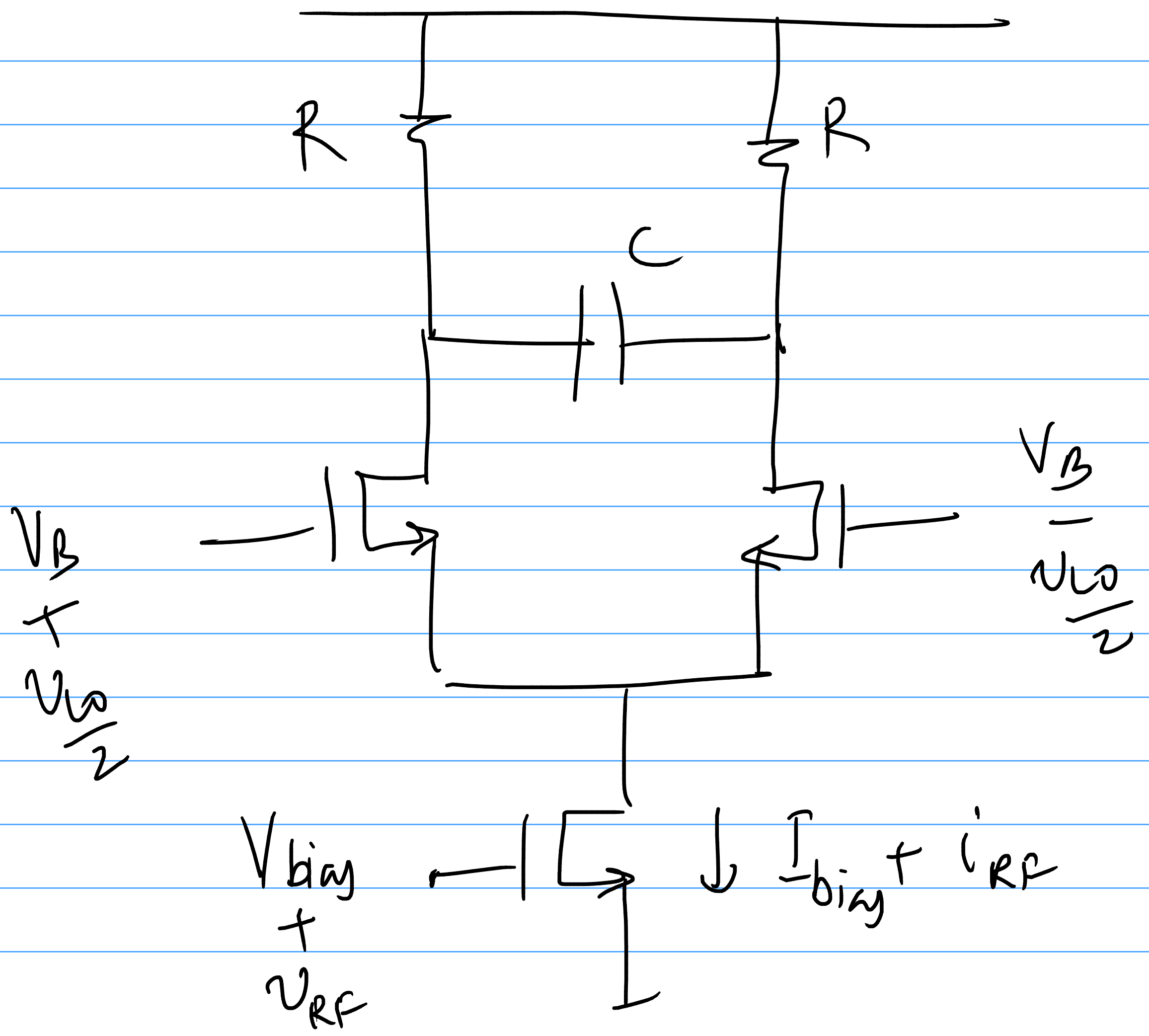
$$A_v = \frac{2}{\pi} g_m \left(r_{ds4,5} \,\|\, R_{CM} \right)$$



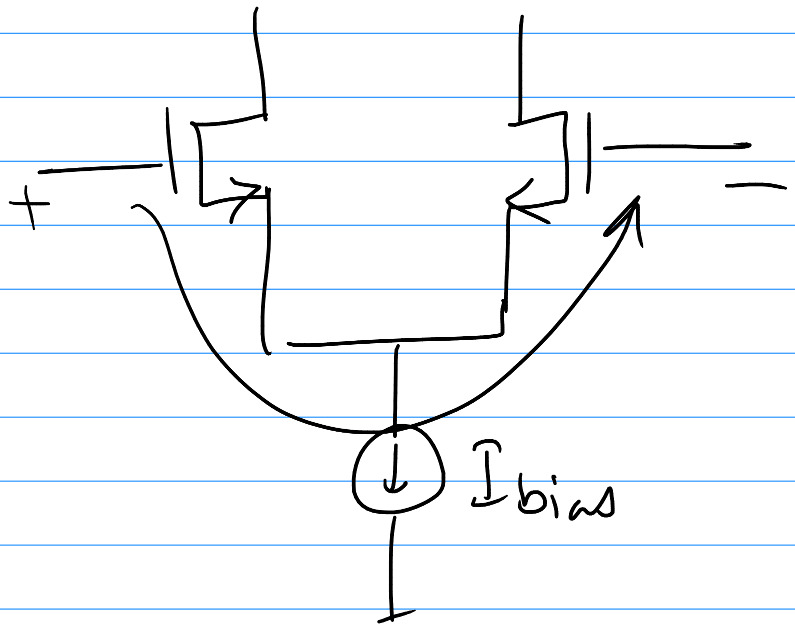
+
−
V_{DD}
$M_{4,5}$
$2C_{CM}$
parasitic cap of R_{CM}
V_{ref}
1-stage
C_c
$\frac{R_{CM}}{2}$
I_{bias}
Mixer



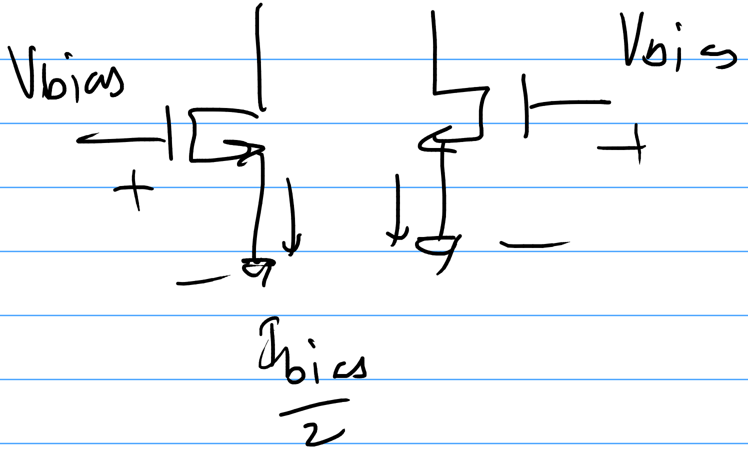
CMFB
V_{DD}
$V_{G4,5}$
M5
M4
V_{ref}
$C_c/2$
$C_c/2$
R_{CM}
R_{CM}
$V_{O,CM}$
I_1
I_{od}
I_2
M_2
M_3
M_1



"Single-balanced" Mixer



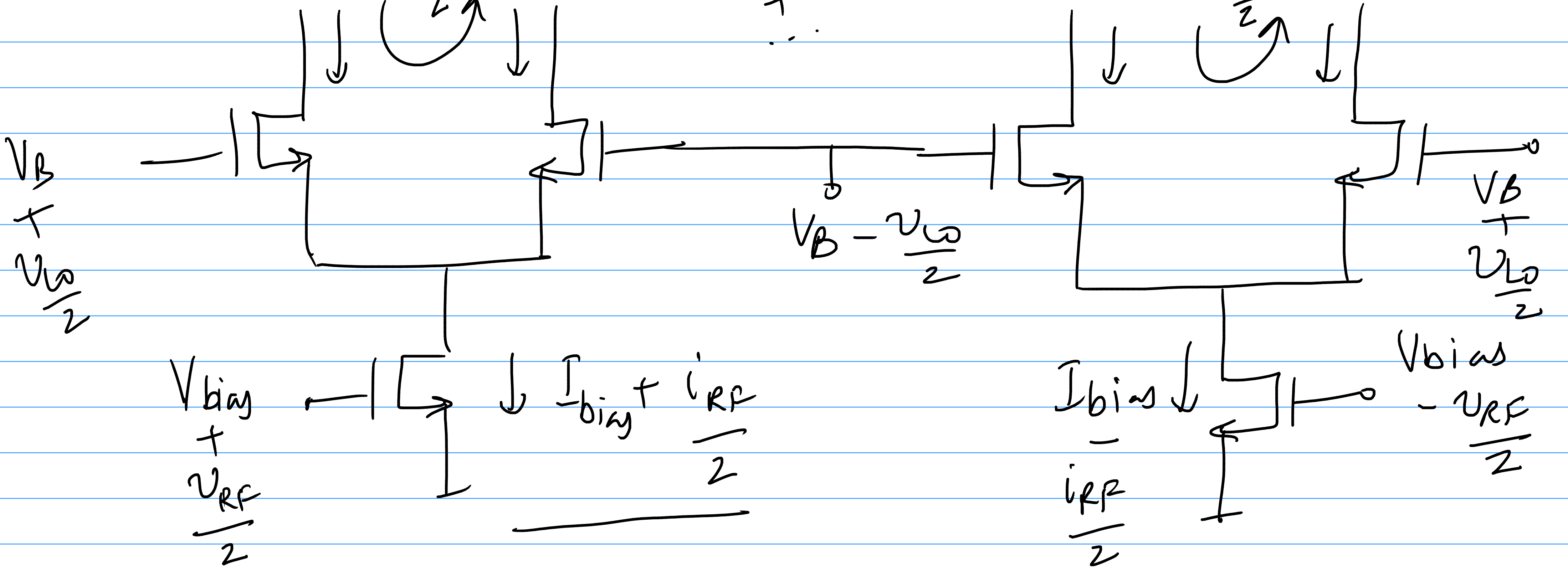
fully diff

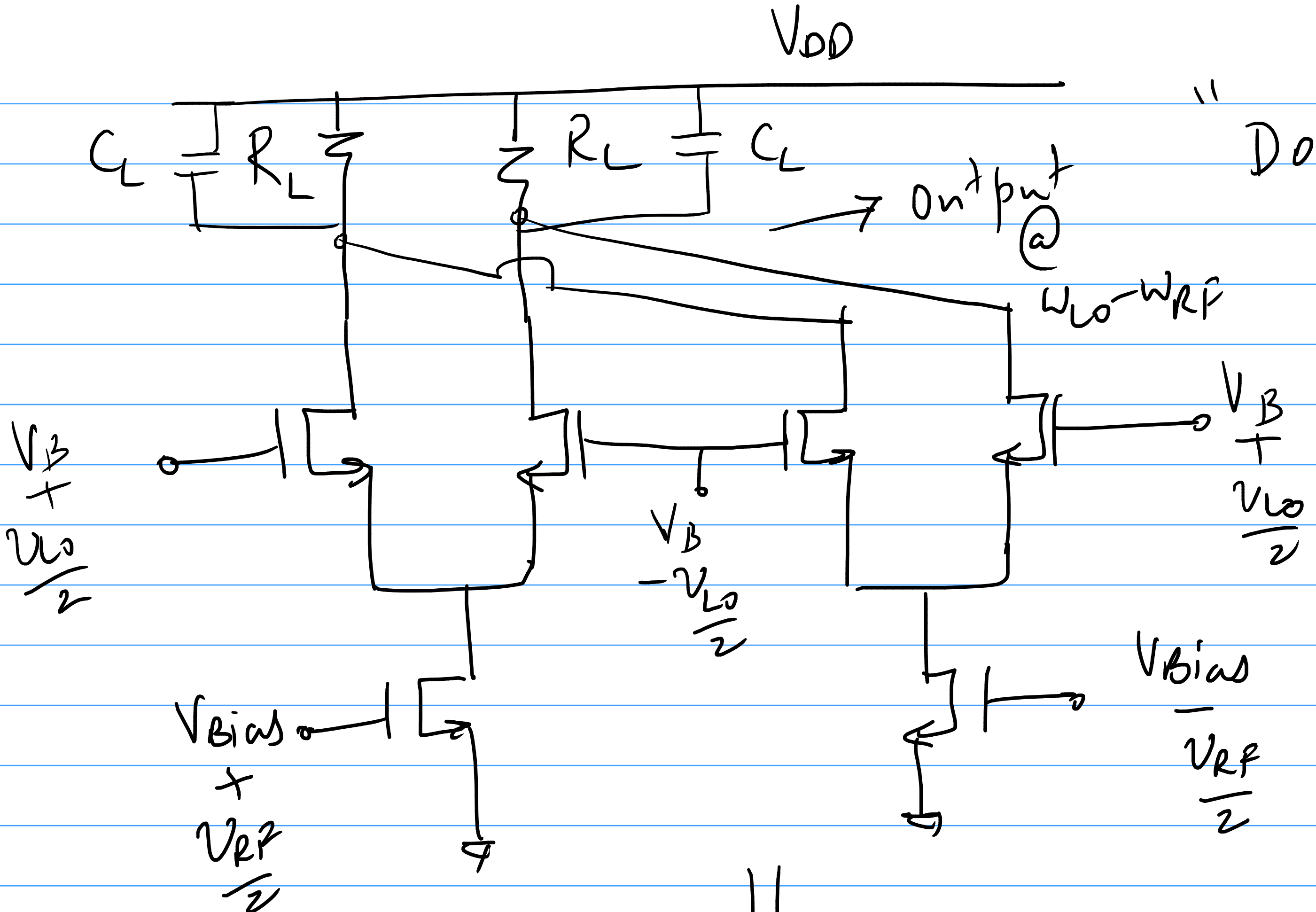


pseudo diff.

$$\frac{i_{od}}{2} = \frac{I_{RF}}{\pi}\left[\sin(\omega_{LO}-\omega_{RF})t + \sin(\omega_{LO}+\omega_{RF})t + \cdots\right]$$

$$-\frac{i_{od}}{2} = -\frac{I_{RF}}{\pi}\left[\cdots\right]$$



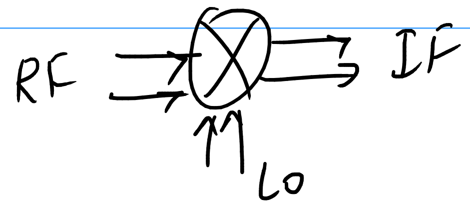


"Double-balanced" Mixer

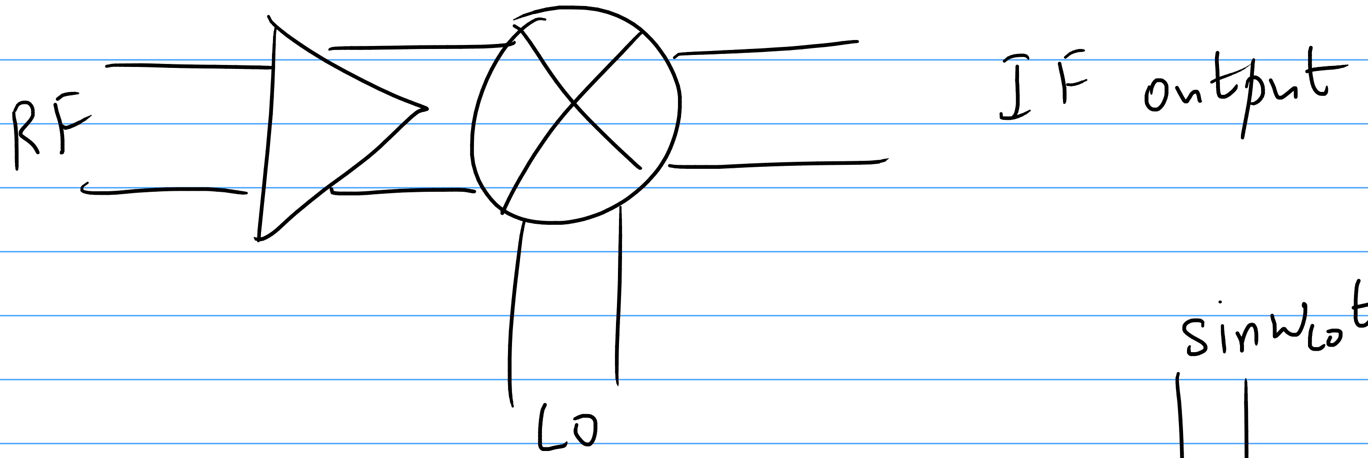
(No LO feed-through)

Same gain

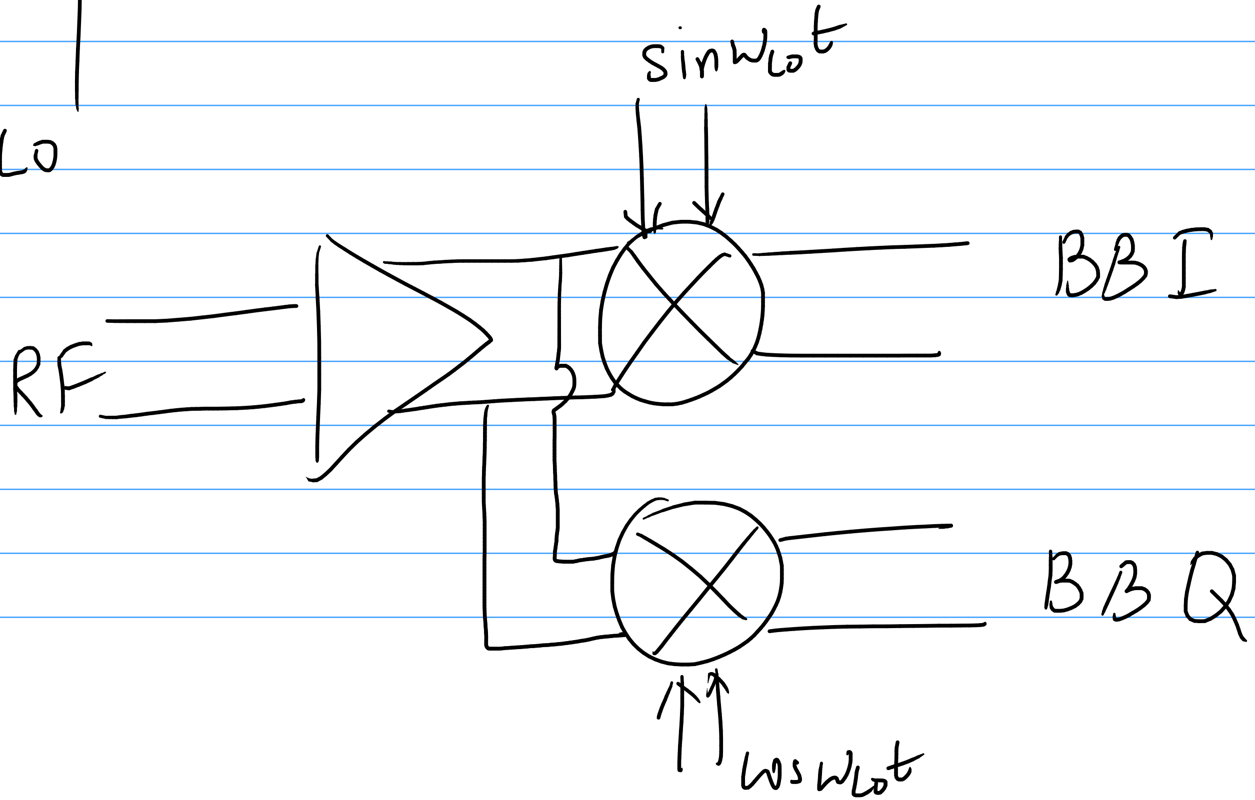
||

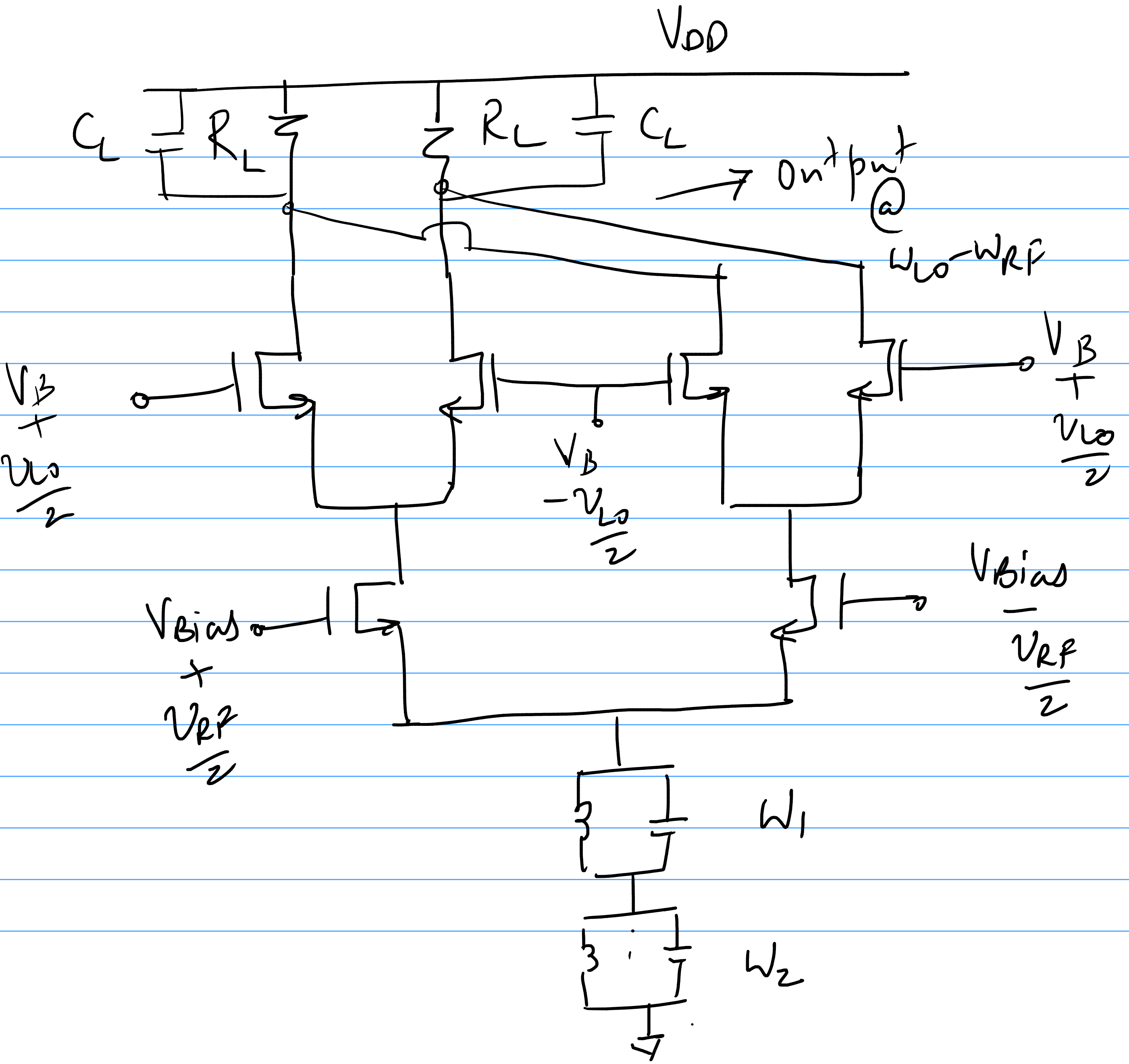


1)

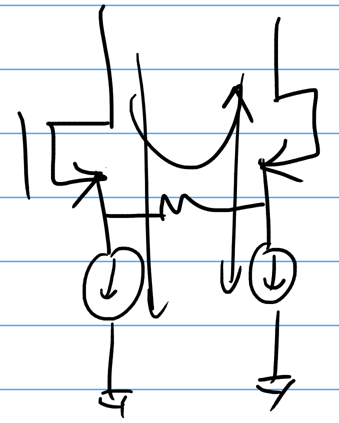


2)

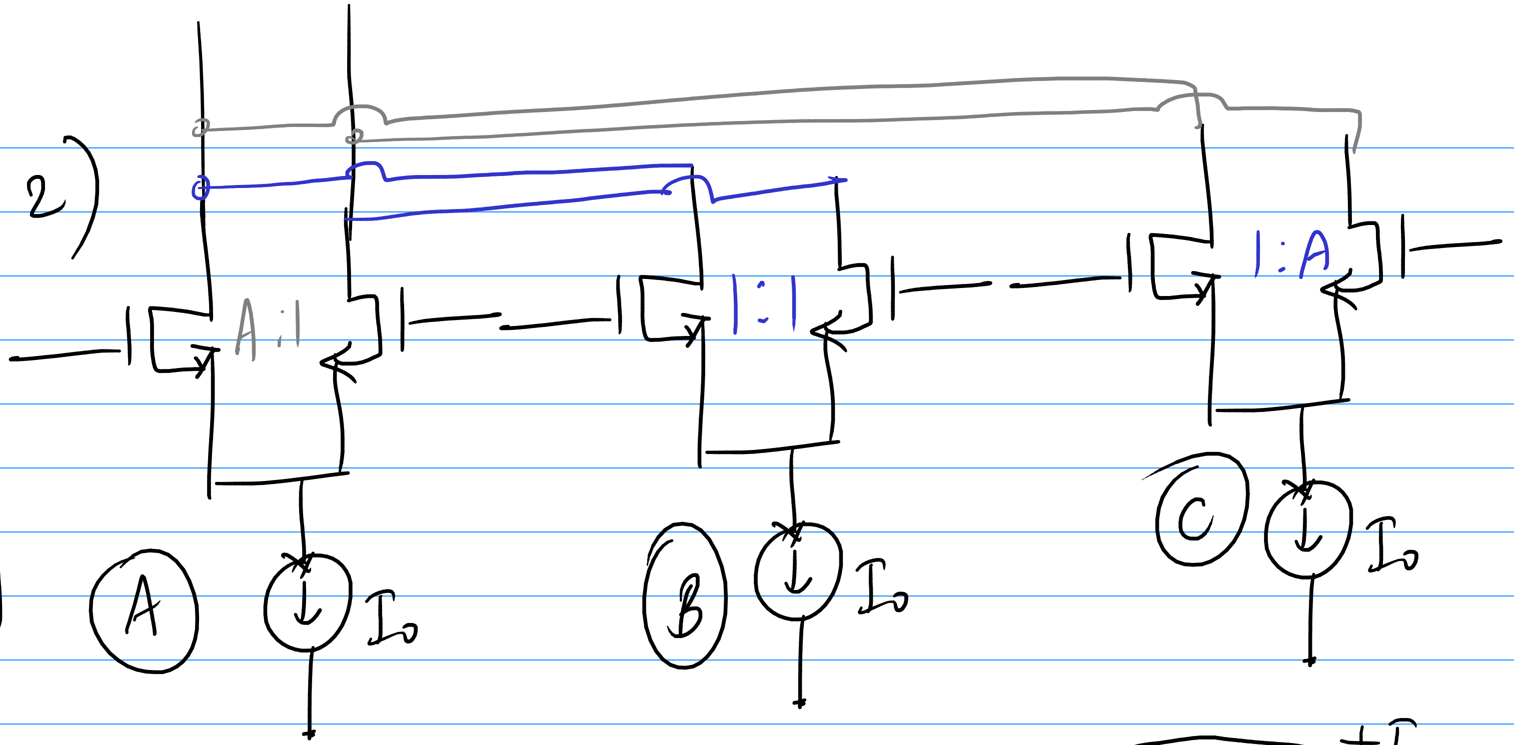


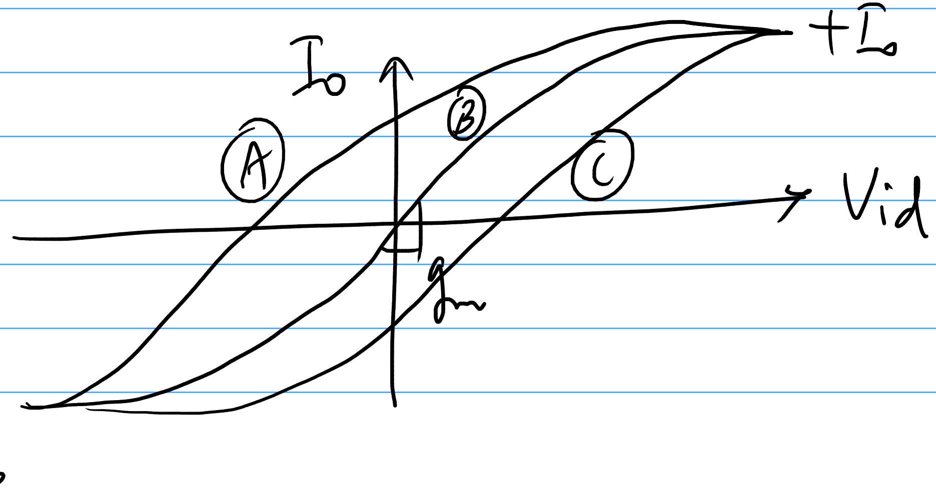

VDD
CL
RL
RL
CL
output @ ωLO-ωRF
VB + vLO/2
VB - vLO/2
VB + vLO/2
VBias + vRF/2
VBias - vRF/2
W1
W2

1)

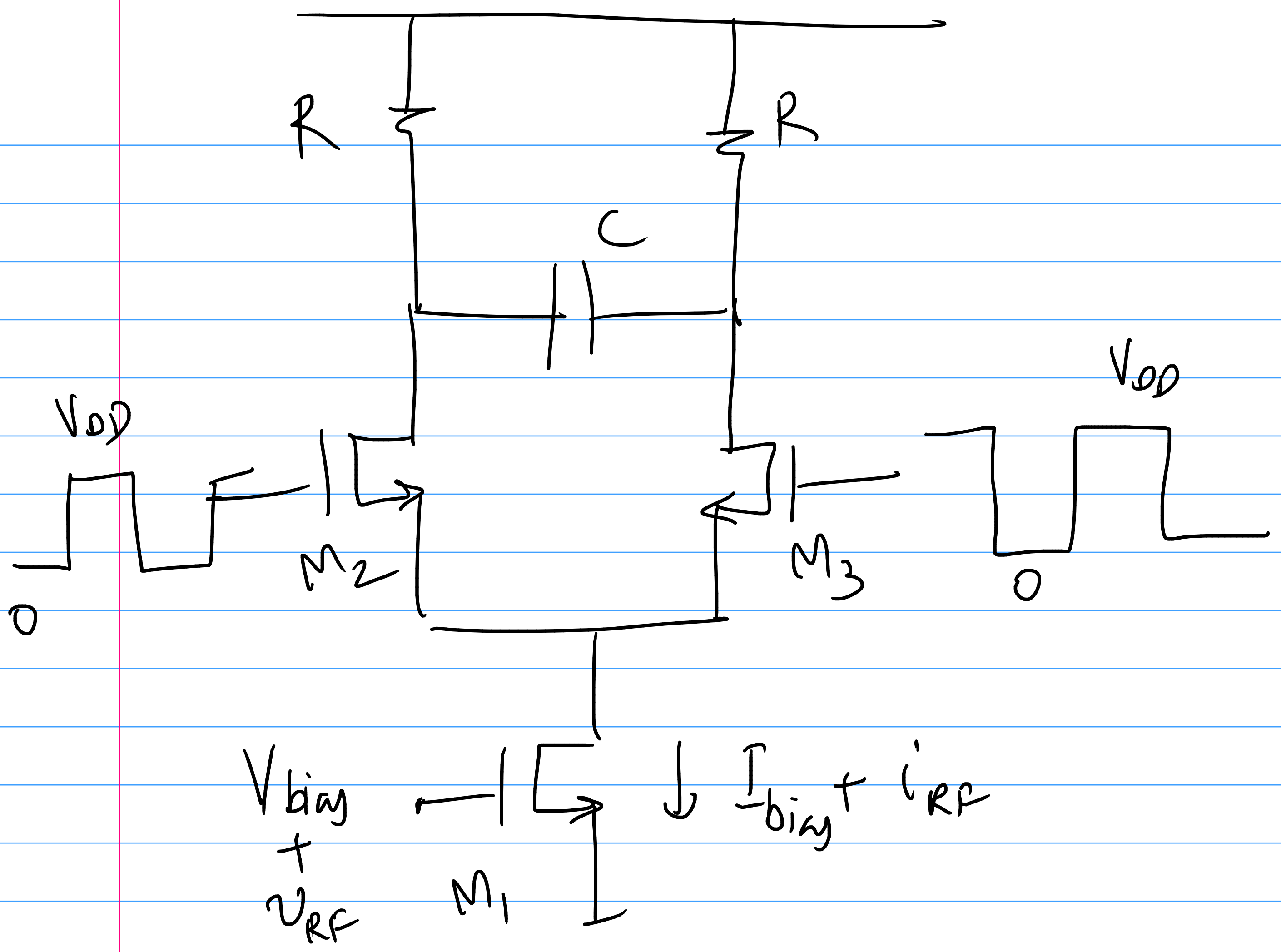


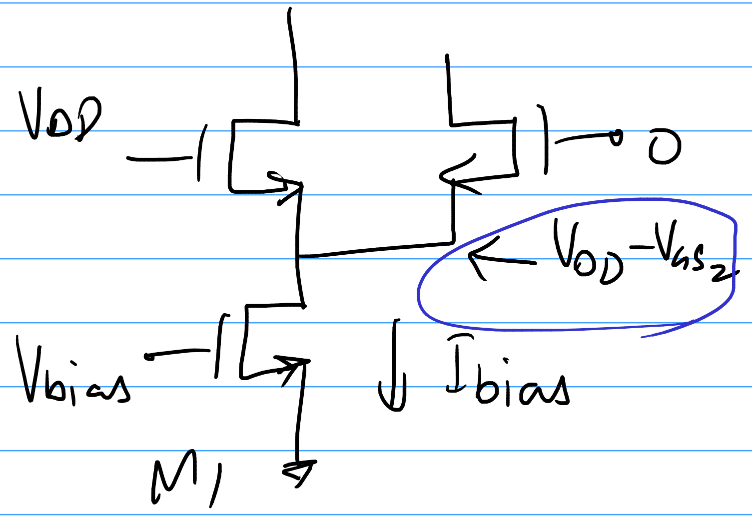
2)

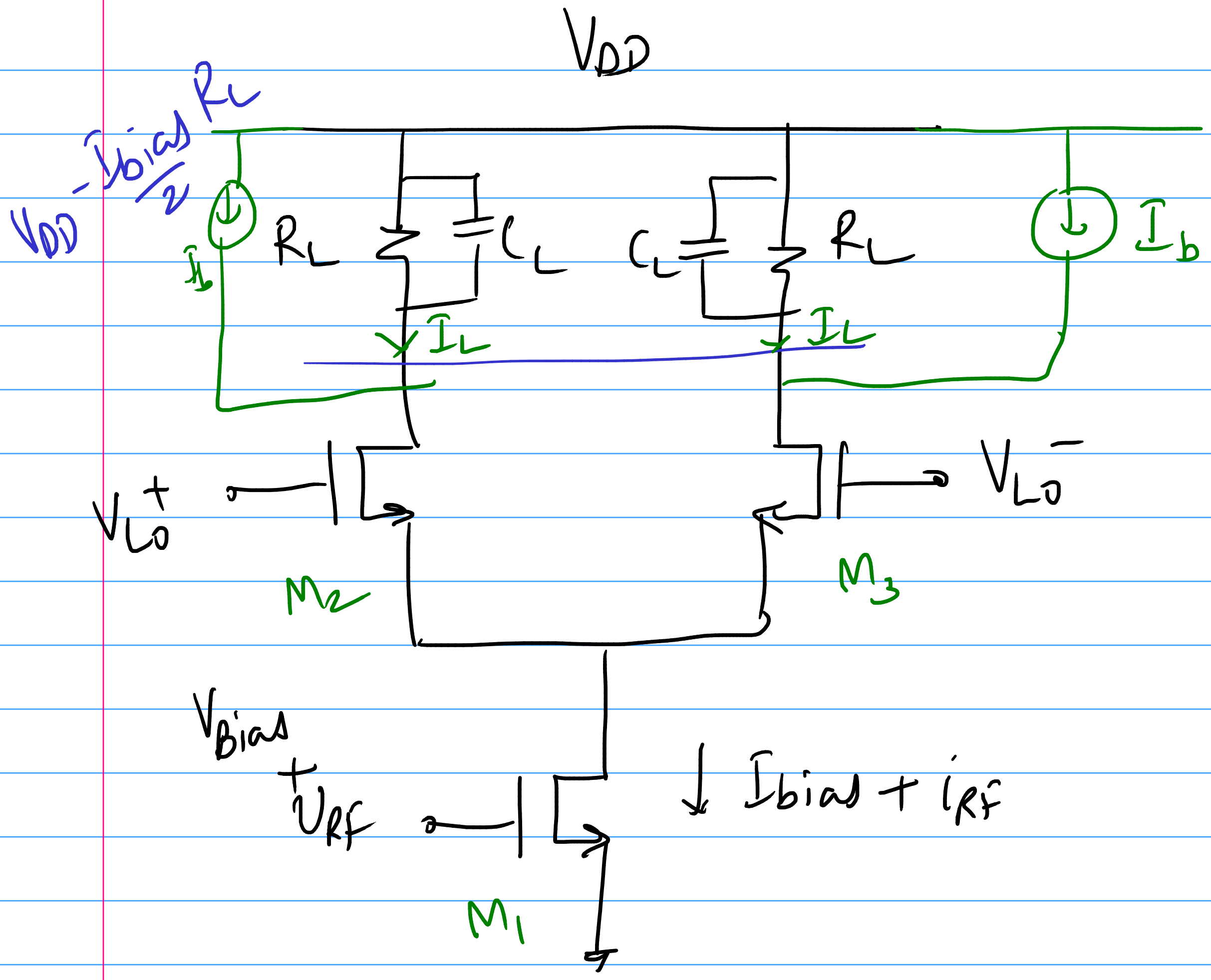




" Multi-tanh" transconductor

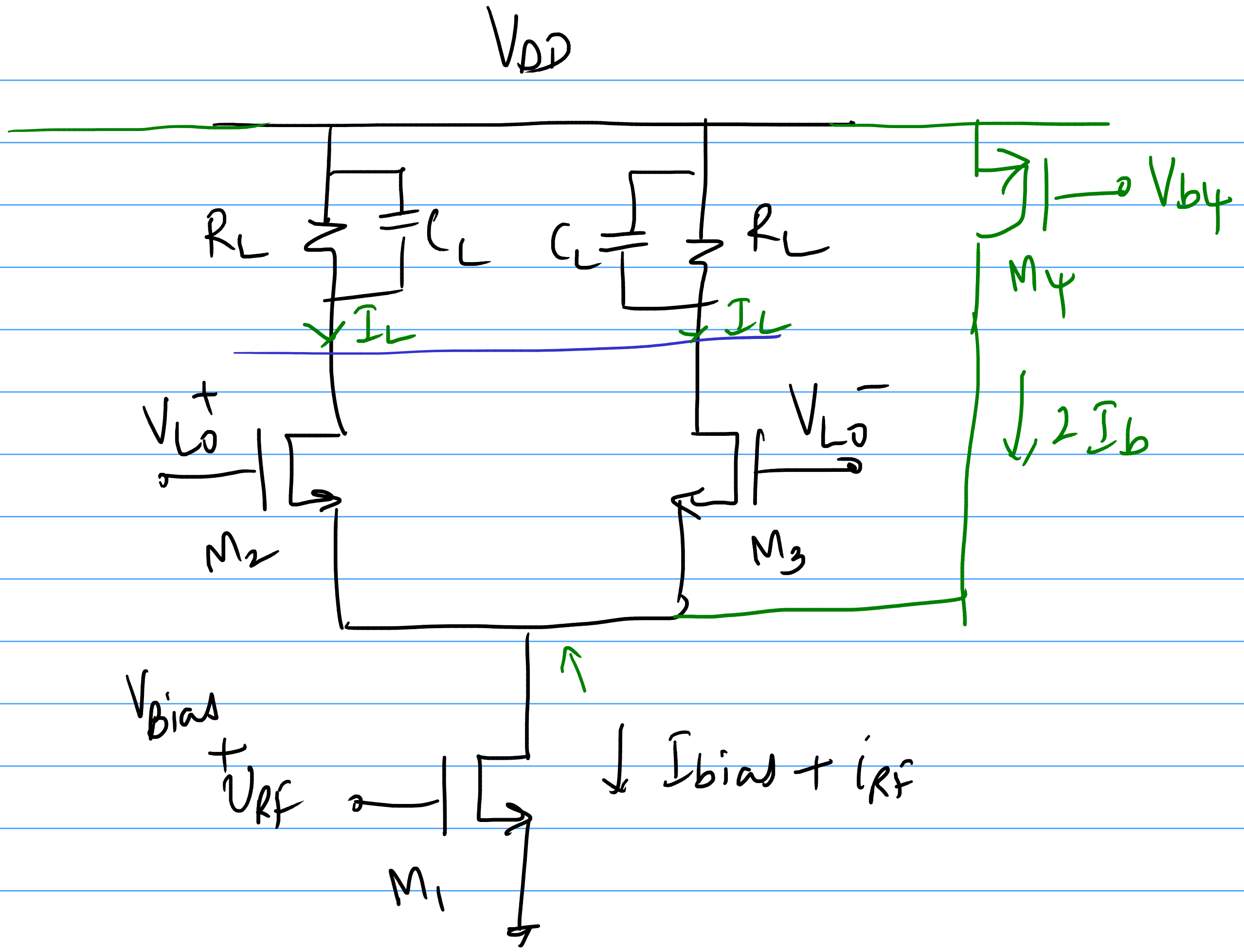

R
R
C
V_{DD}
0
M_2
M_3
V_{DD}
0
$V_{bias} + v_{RF}$
M_1
$I_{bias} + i_{RF}$


V_{DD}
0
$V_{DD} - V_{GS_2}$
V_{bias}
I_{bias}
M_1



$$I_L = \frac{I_{bias}}{2} - I_b$$

We want small $V_{Dsat_{2,3}}$



reduce $V_{Dsat_{2,3}}$

$\downarrow$

$\uparrow \left(\frac{W}{L}\right)_{2,3}$, $\downarrow I_{D_{2,3}}$

$\downarrow$ $\quad$ $\downarrow$

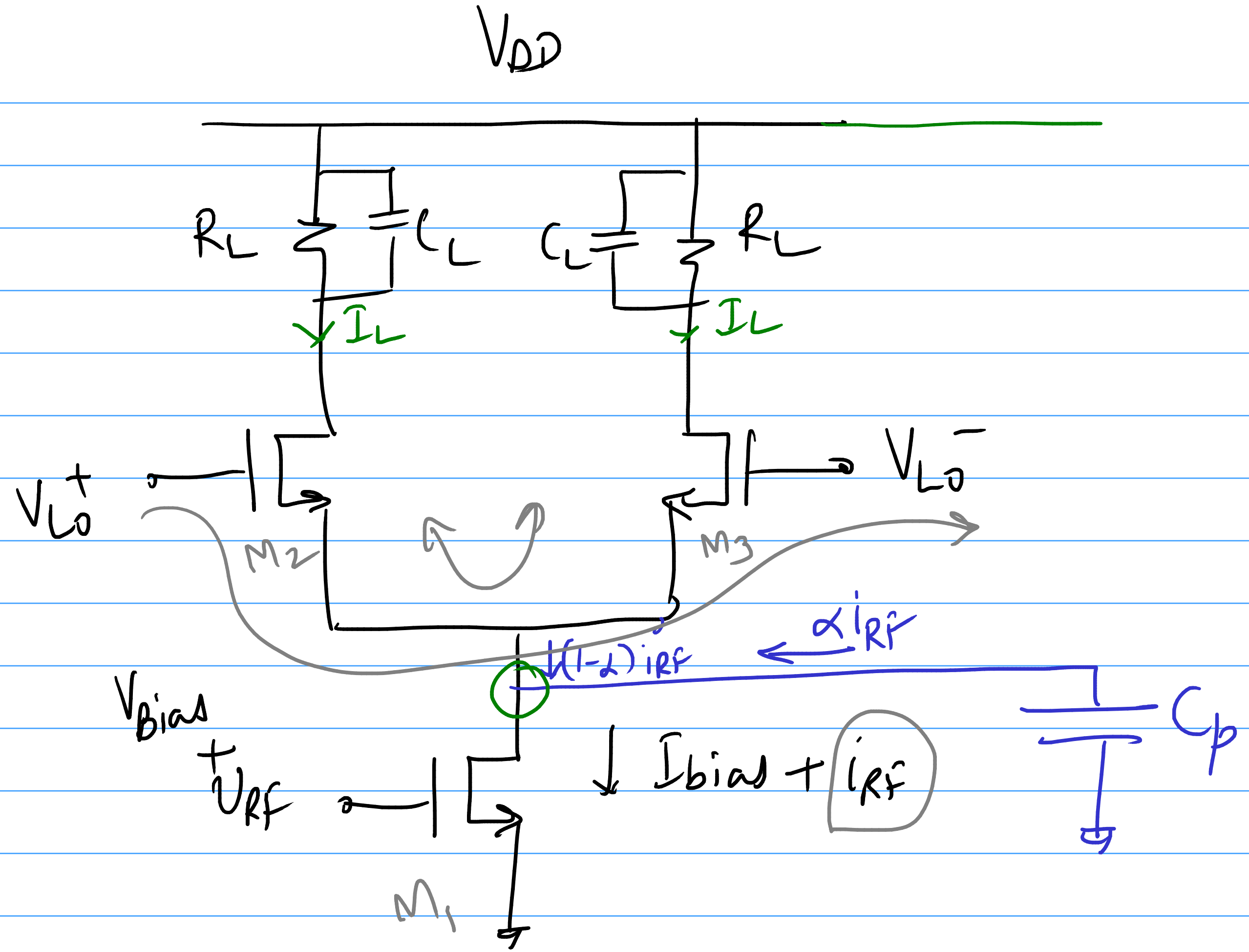
$\uparrow C_{gs_{2,3}}$ $\quad$ $\frac{I_{bias}}{2} - I_b$ ✓

$C_{db_{2,3}}$

$C_{sb_{2,3}}$

$\vdots$

X



$$V_{LO}^+ - V_{LO}^- = V_{GS_2} - V_{GS_3}$$ ✓

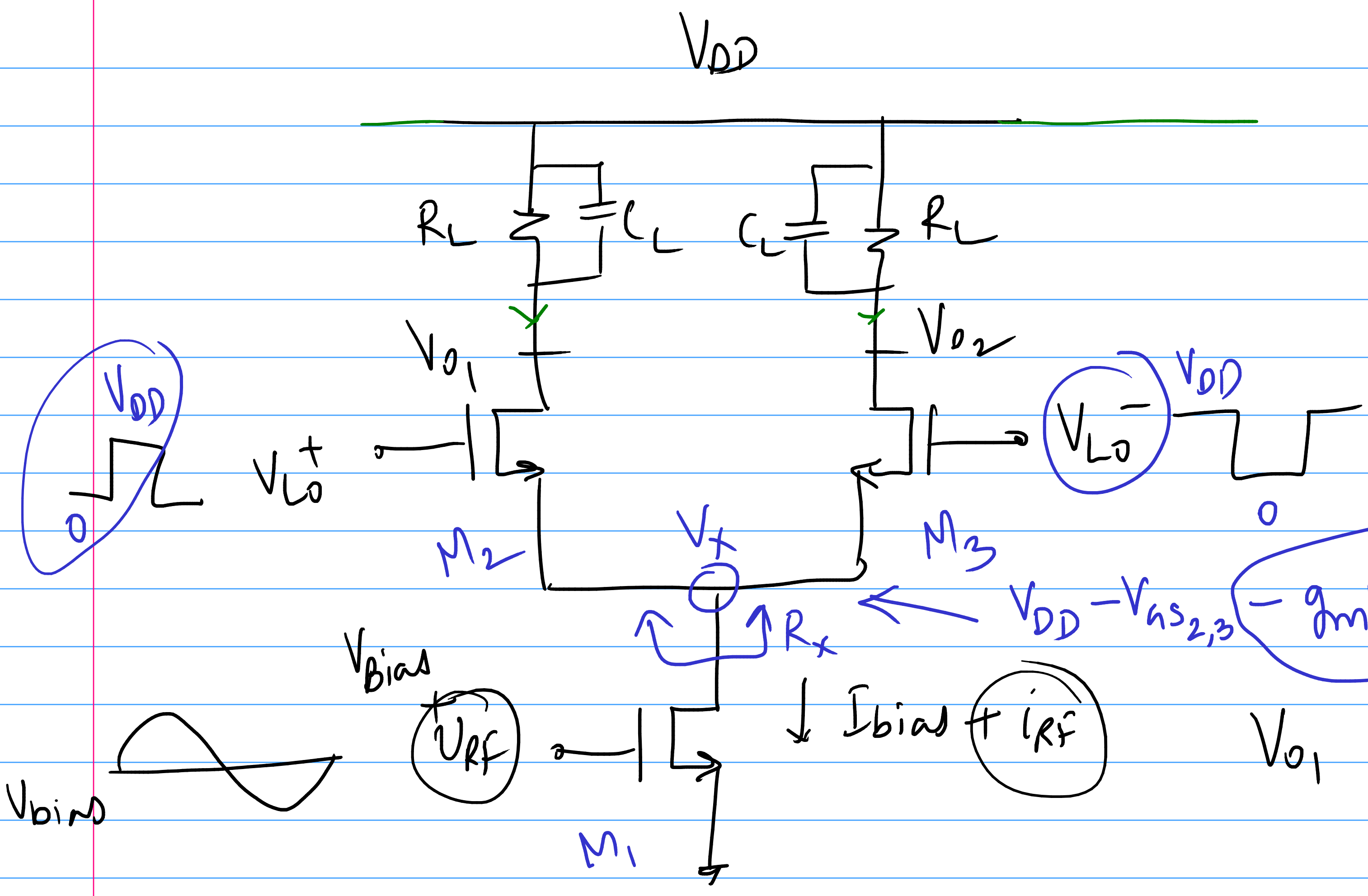
$$I_{D_k} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_k\left(V_{GS_k}-V_T\right)^2$$

$$I_{D_2} + I_{D_3} = I_{bias} + I_{C_p}$$

$$\downarrow$$

$$C_p\frac{dV_{C_p}}{dt}$$

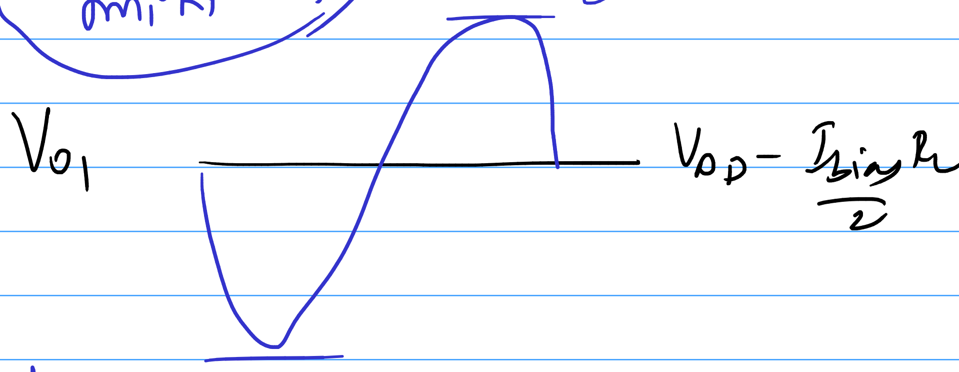
?



Linearity of Mixer

P_{1dB}, IIP_3 ← linear g_m stage

$V_{DD} - V_{GS_{2,3}} - g_{m_1} v_{RF} \cdot R_x$

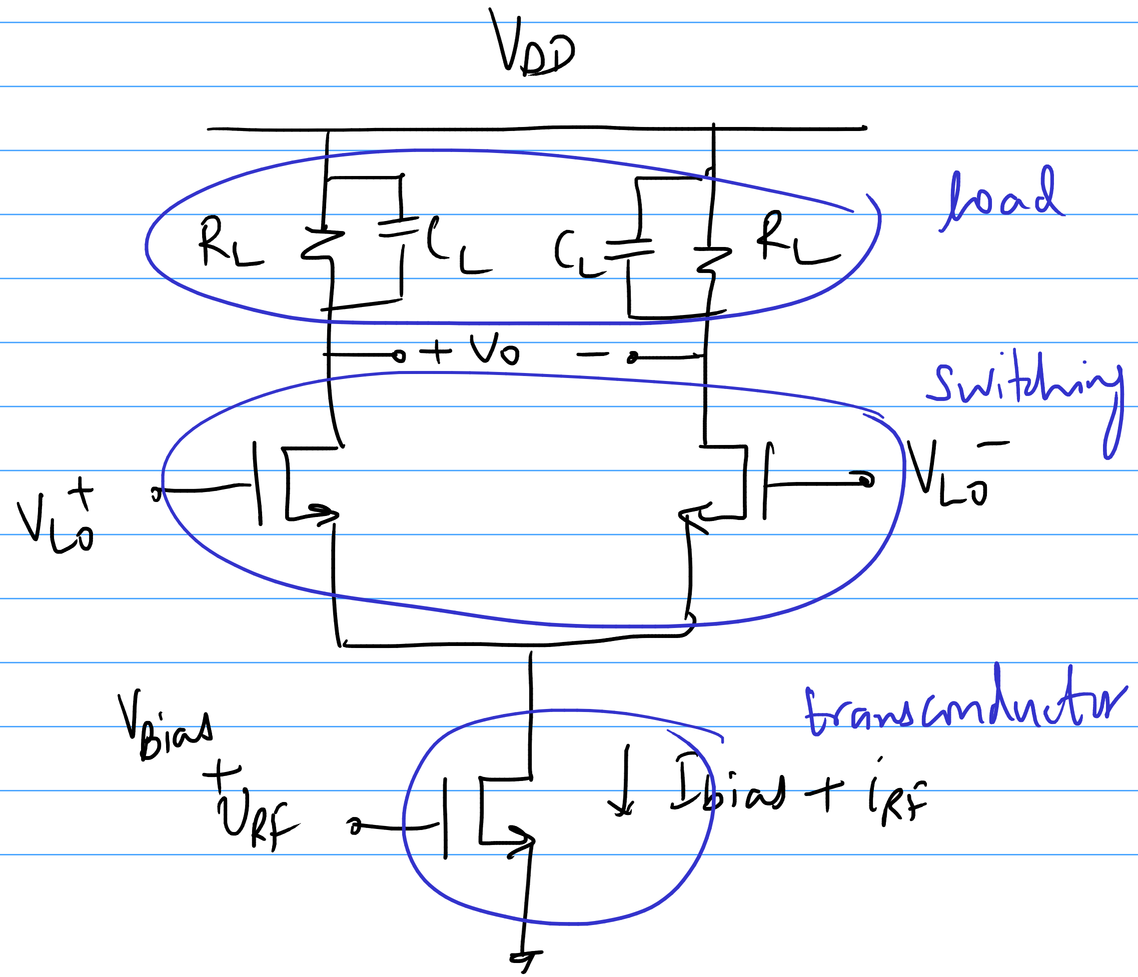


Output swing limited →

↖ triode limit of M_2

$V_{DD} - V_{T_2}$

Noise in active mixers $\longrightarrow NF, F$



1) Load noise

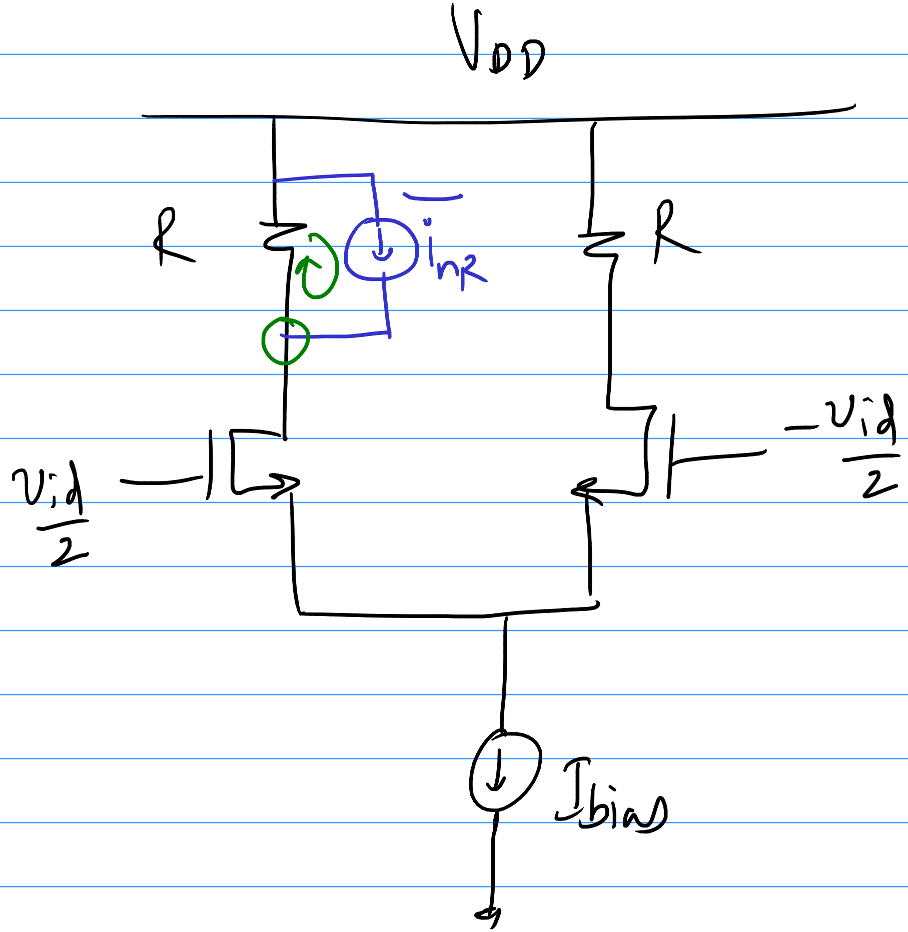
(a) $\frac{\overline{V_{on_{R_L}}^2}}{\Delta f} = 8kTR_L$

or ($\uparrow R_L$ for lower NF)

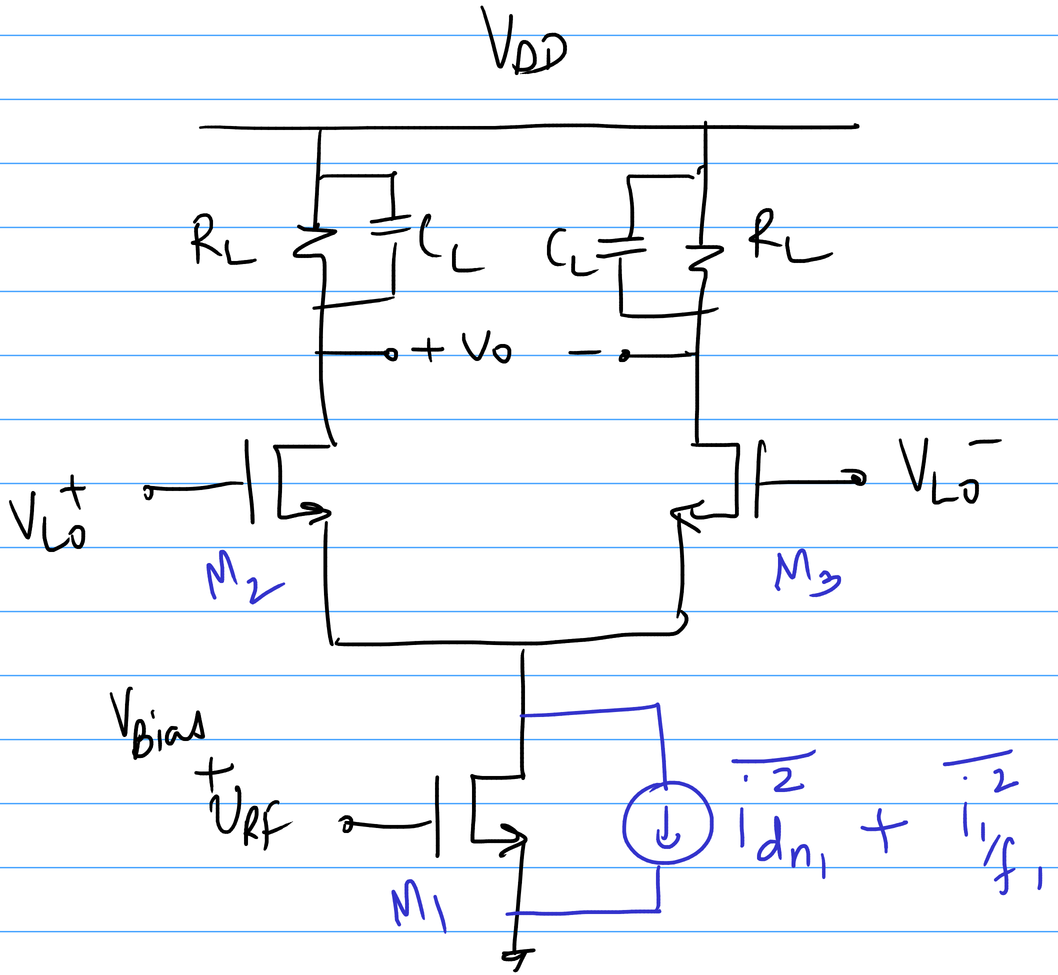
(b) PMOS w/ CMFB

(x) $4kT\gamma g_{m_{PMOS}}$

$\downarrow g_{m_{PMOS}} \rightarrow \downarrow \left(\frac{W}{L}\right)_{PMOS}$

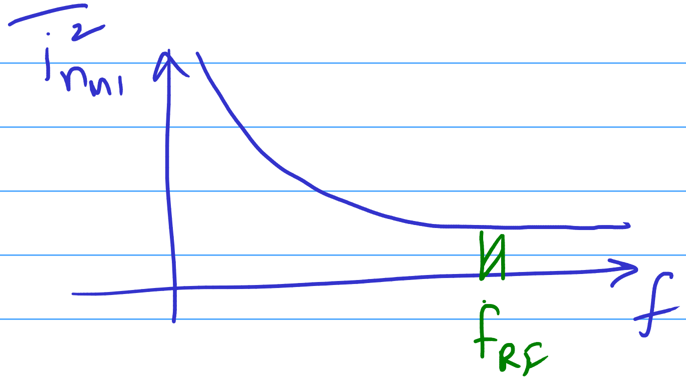


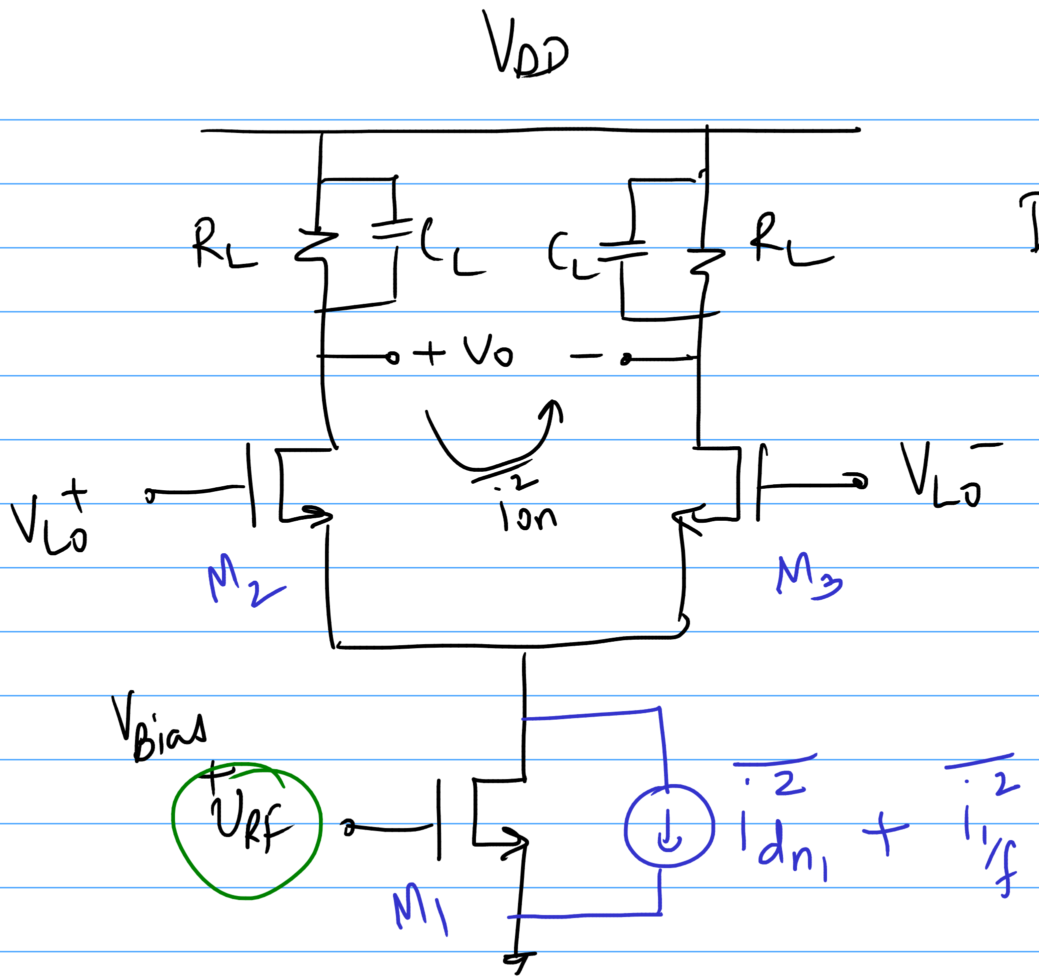
HW

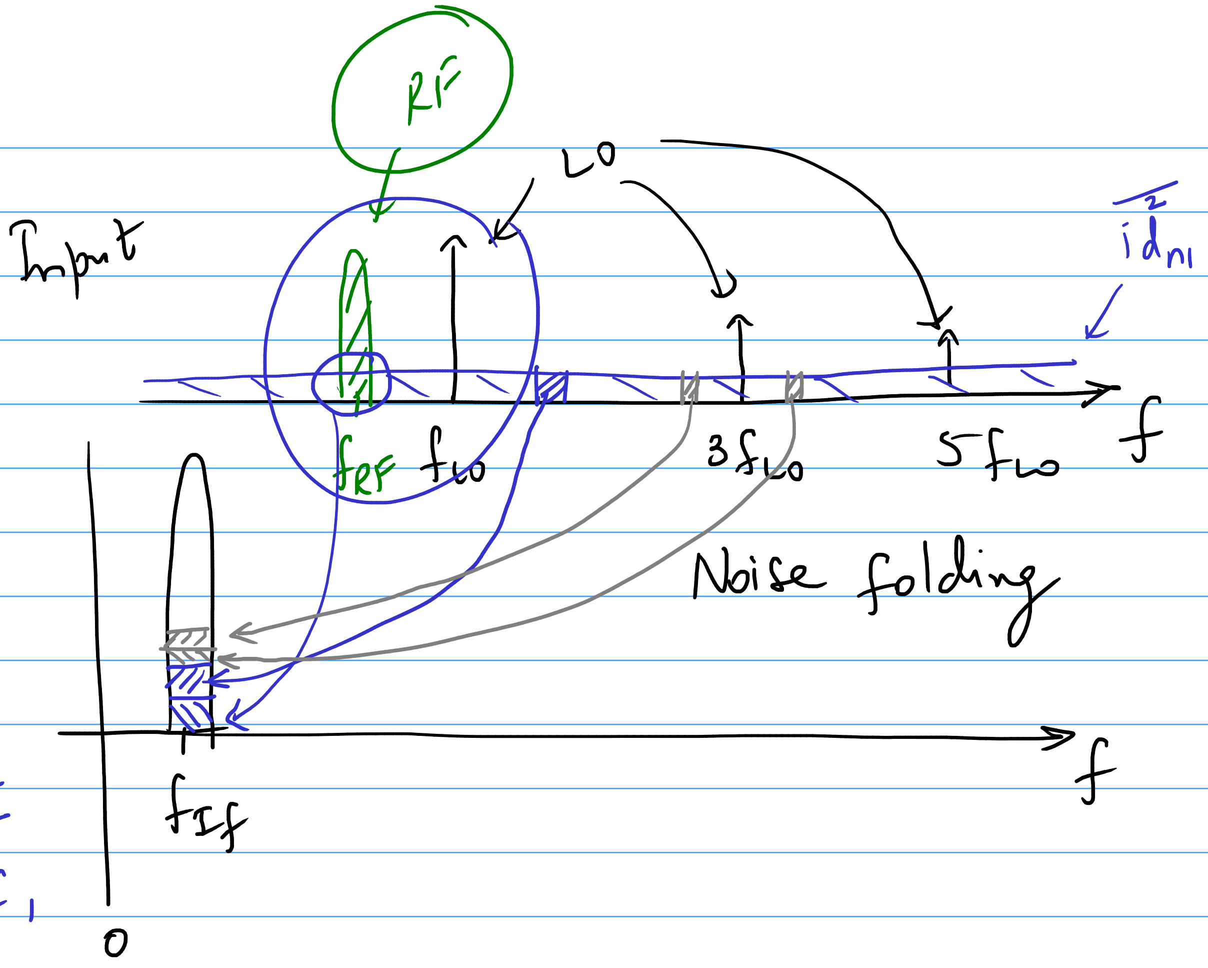


<u>LPTV</u> analysis

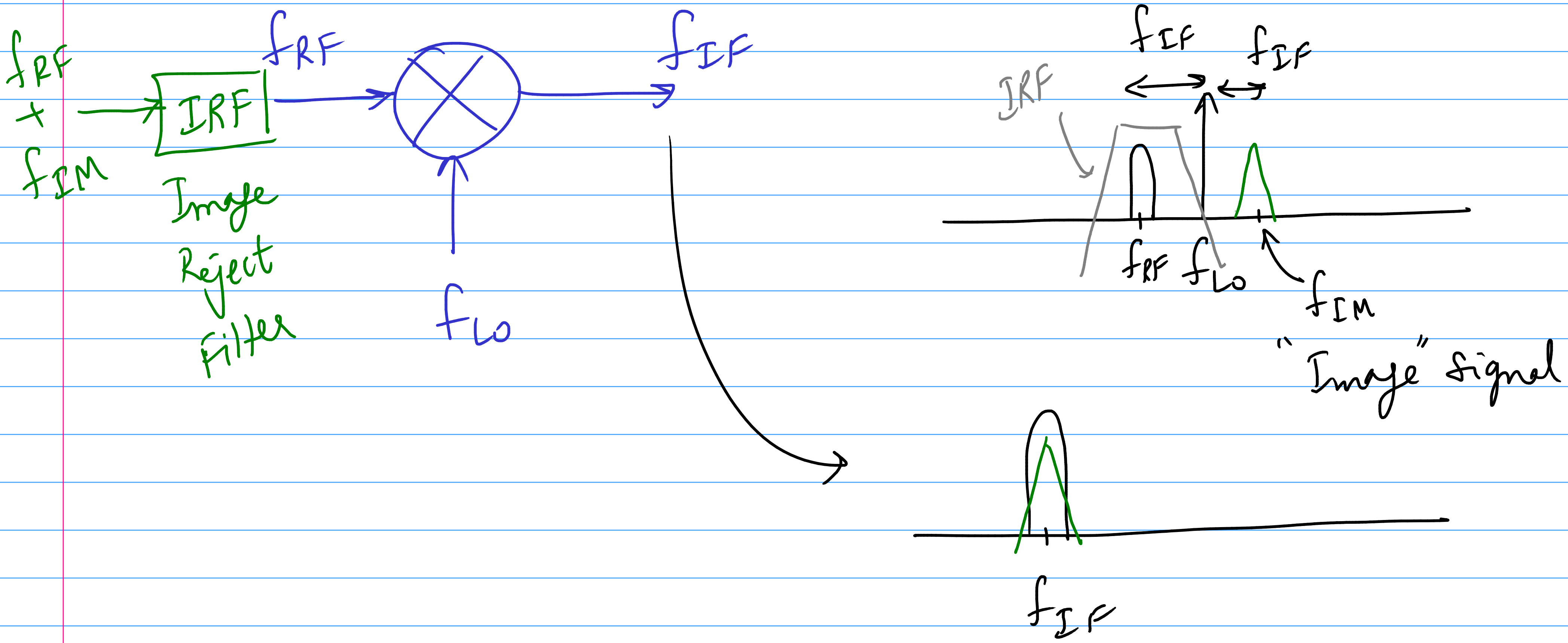
Effect on $\overline{i_{nM_1}}$ is same as that for i_{RF}





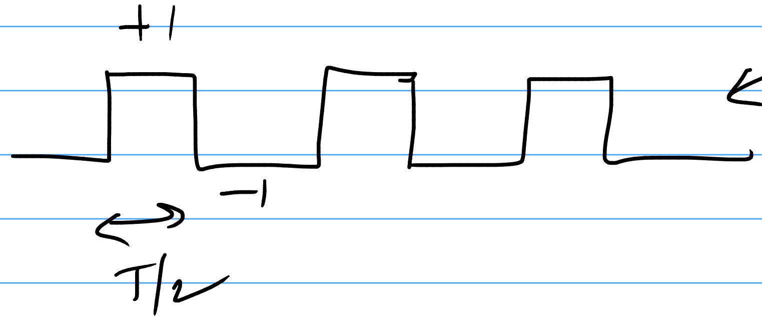


$$\frac{\overline{i_{on_1}^2}}{\Delta f} = 4kT\gamma g_m \times \left(\frac{2}{\pi}\right)^2 \underbrace{\left[1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]}_{\pi^2/8} \times 2$$


f_{RF} + f_{IM}
IRF
Image Reject Filter
f_{RF}
f_{IF}
f_{LO}
f_{IF}
f_{IF}
IRF
f_{RF} f_{LO}
f_{IM}
"Image" signal
f_{IF}

$$\frac{\overline{i_{on_1}^2}}{\Delta f} = 4kT\gamma g_m$$

$s(t) =$



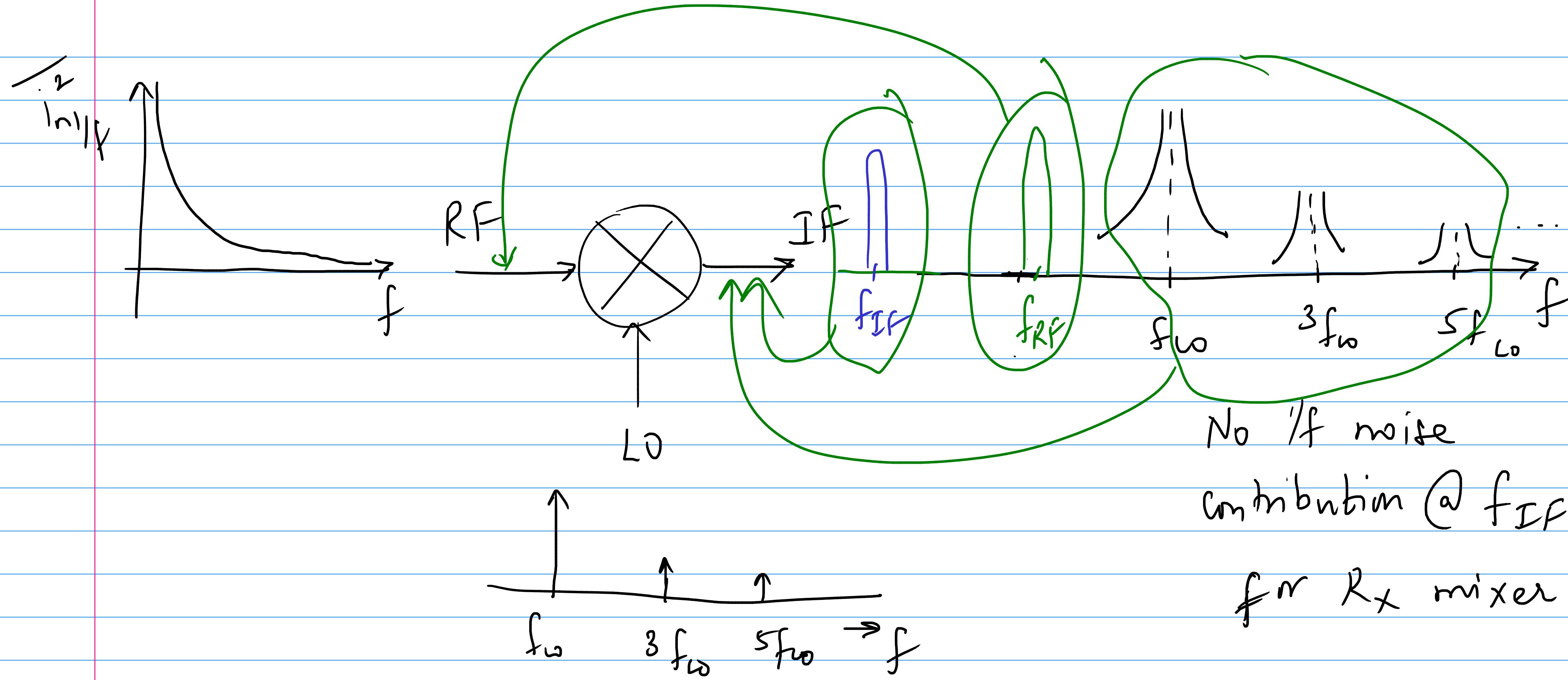
$\sum |c_k|^2 = 1$

$$s(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_{LO}t}$$

drain thermal noise

$$S_i(f - kf_{LO}) = S_i$$

$$S_o(f) = \sum_{-\infty}^{\infty} |c_k|^2 \cdot S_i(f - kf_{LO}) = S_i \sum_{-\infty}^{\infty} |c_k|^2$$

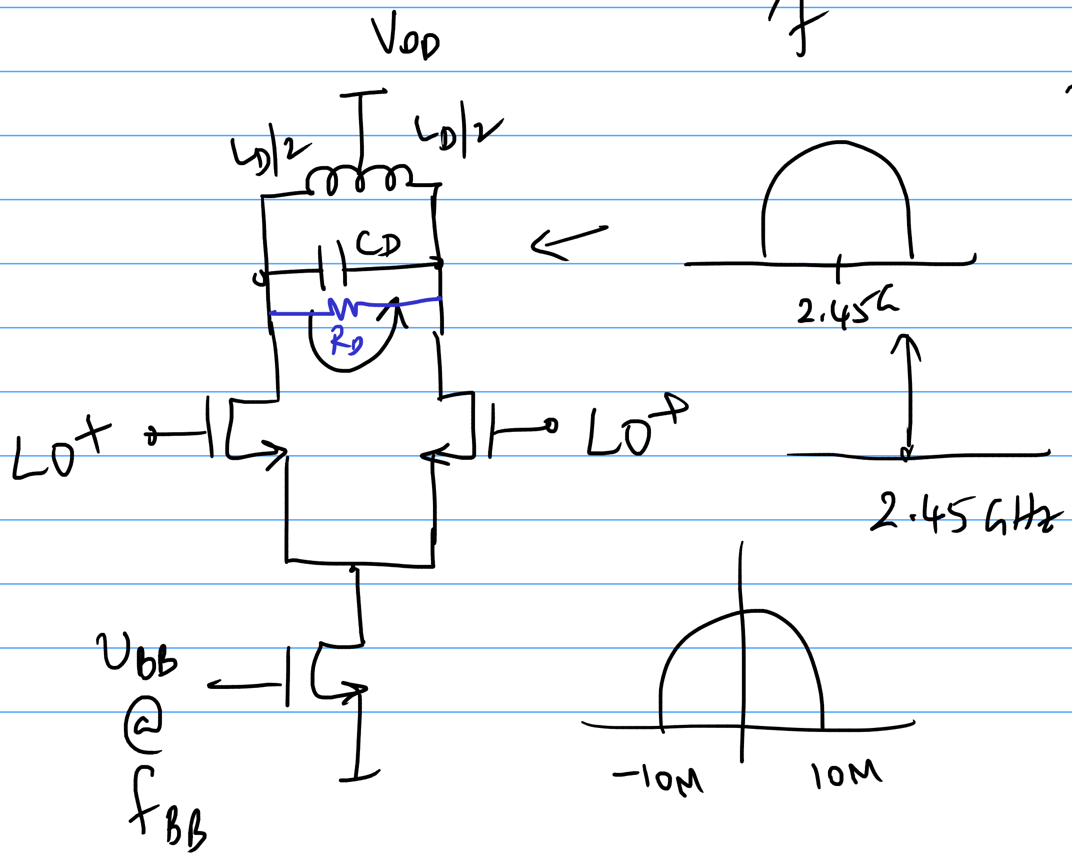


No 1/f noise
contribution @ f_{IF}
for R_x mixer

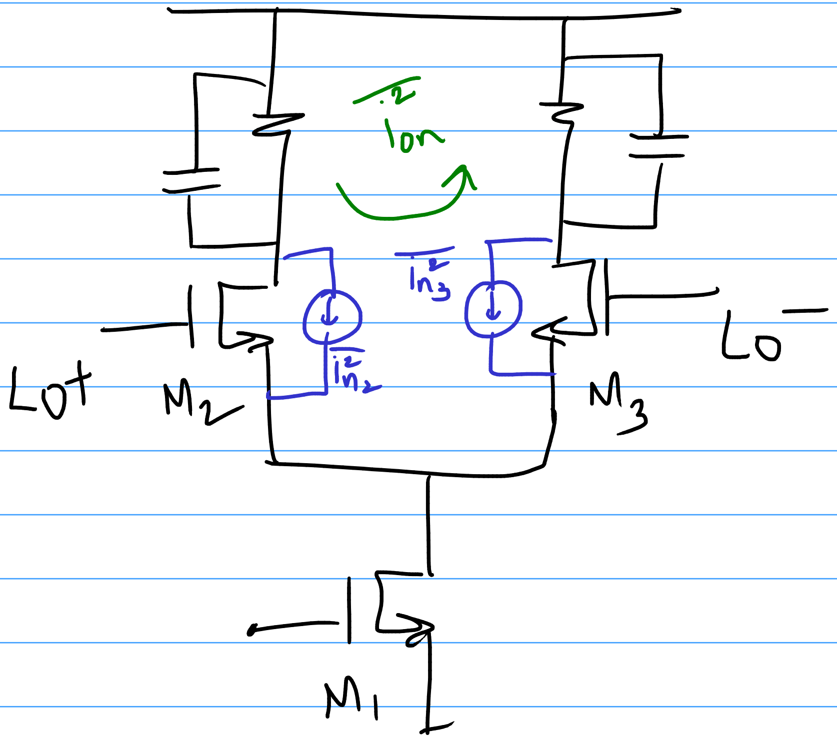
Tx mixer

f_{BB} + $1/f$ noise → ⊗ ← f_{LO}

$$f_{RF} = f_{LO} + f_{BB} + 1/f \text{ noise}$$



Noise from Switches



$$\overline{i_{o_2}(t)} = \overline{i_{n_2}(t)} \cdot a_2(t)$$

$$\overline{i_{o_3}(t)} = \overline{i_{n_3}(t)} \cdot a_3(t)$$

$$\left.\begin{aligned} a_2(t) &= a_2(t - kT_{LO}) \\ a_3(t) &= a_3(t - kT_{LO}) \end{aligned}\right\} \text{LPTV}$$

$$a_m(t) = \sum_{k=-\infty}^{\infty} a_{m_k} \cdot e^{jk\omega_{LO}t}$$

$$i_{o_m}(t) = \overline{i_{n_m}}(t) \cdot a_m(t)$$

$$= \left[\sum_{-\infty}^{\infty} a_{mk}\, e^{-jk\omega_{LO}t}\right] \cdot \overline{i_{n_m}}(t)$$

$$= \sum_{-\infty}^{\infty} a_{mk} \left(e^{-jk\omega_{LO}t} \cdot \overline{i_{n_m}}(t)\right)$$

$$S_{o_n}^{tot}(f) = \sum_m \sum_{k=-\infty}^{\infty} |a_{m_k}|^2 \cdot S_{n_m}(f - k f_{LO})$$

$$S_{n_m}(f) = 4kT\gamma g_{m_{SW}} = S_{n_m}(f - k f_{LO})$$

$$S_{on}^{tot}(f) = \sum_{m} \sum_{k=-\infty}^{\infty} |a_{mk}|^2 \left(4kT\gamma g_{m_{sw}}\right)$$

$$= \sum_{m} 4kT\gamma g_{m_{sw}} \cdot \underbrace{\sum_{k=-\infty}^{\infty} |a_{mk}|^2}$$

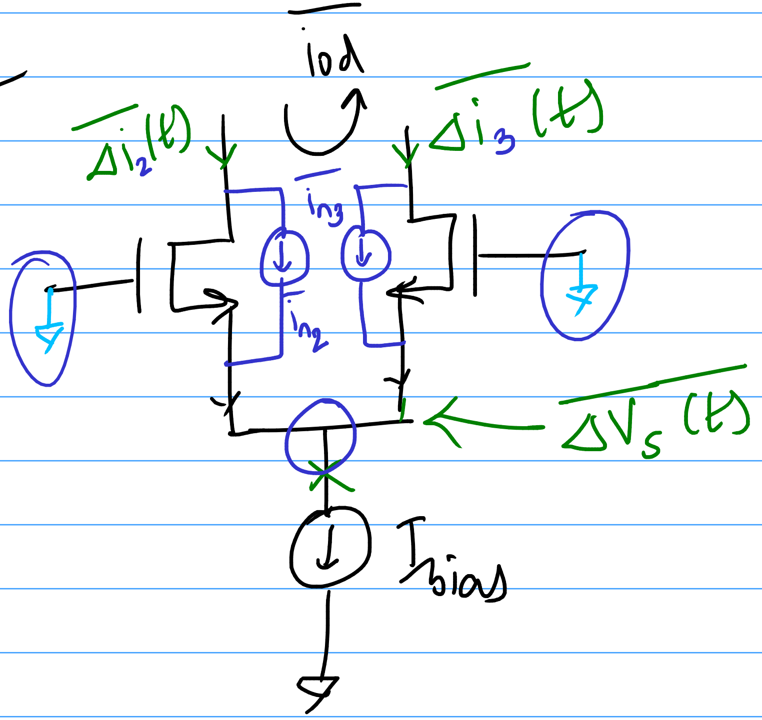
$$= \frac{1}{T_{LO}} \int_{0}^{T_{LO}} |a_m(t)|^2 \, dt$$

$$= A$$

$$S_{on}^{tot.}(f) = 2 \times 4kT\gamma g_{m_{sw}} \cdot A$$

$$a_m(t) = \left\{ \begin{array}{ll} 0 & \text{OFF} \\ 1 & \text{ON} \end{array} \right\} \qquad A = \sum_{-\infty}^{\infty} |a_{mk}|^2 = \frac{1}{2}$$

In reality



$$\overline{\Delta i_2(t)} + \overline{\Delta i_3(t)} = 0$$

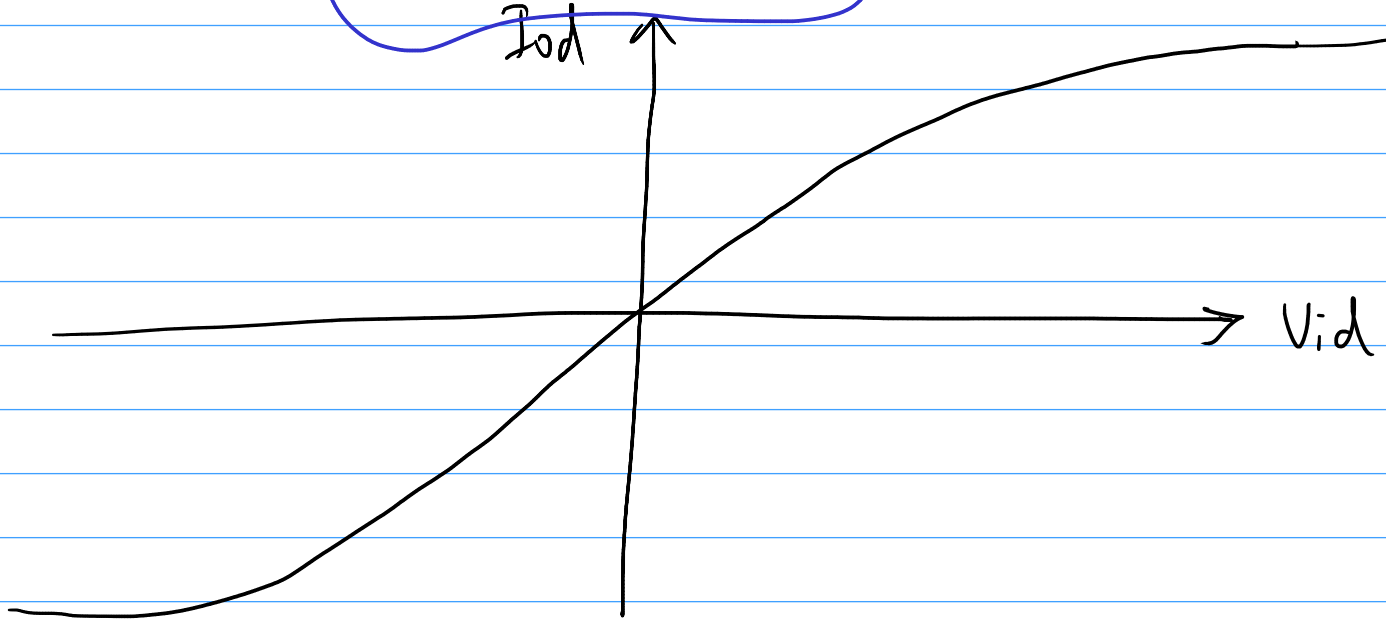
$$\overline{\Delta i_2(t)} = \overline{i_{n_2}(t)} - g_{m_2}(t)\Delta V_s(t)$$

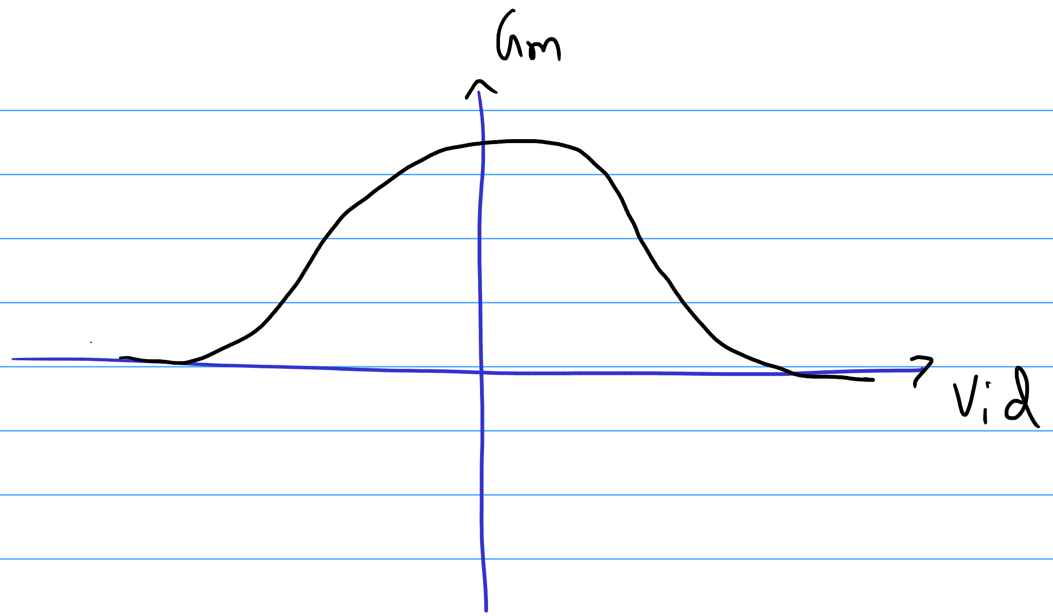
$$\overline{\Delta i_3(t)} = \overline{i_{n_3}(t)} - g_{m_3}(t)\cdot\overline{\Delta V_s(t)}$$

$$\overline{\Delta V_s(t)} = \frac{\overline{i_{n_2}(t)} + \overline{i_{n_3}(t)}}{g_{m_2}(t) + g_{m_3}(t)}$$

$$\overline{i_{od}} = \Delta i_2(t) - \Delta i_3(t)$$

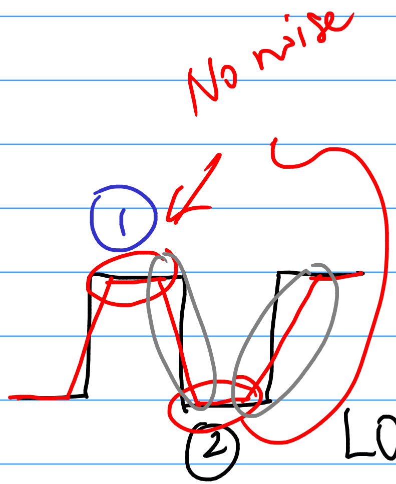
$$= \overbrace{\left(\frac{2g_{m_3}(t)}{g_{m_2}(t)+g_{m_3}(t)}\right)}^{a_2(t)} \overline{i_{n_2}(t)} - \overbrace{\left(\frac{2g_{m_2}(t)}{g_{m_2}(t)+g_{m_3}(t)}\right)}^{a_3(t)} \overline{i_{n_3}(t)}$$

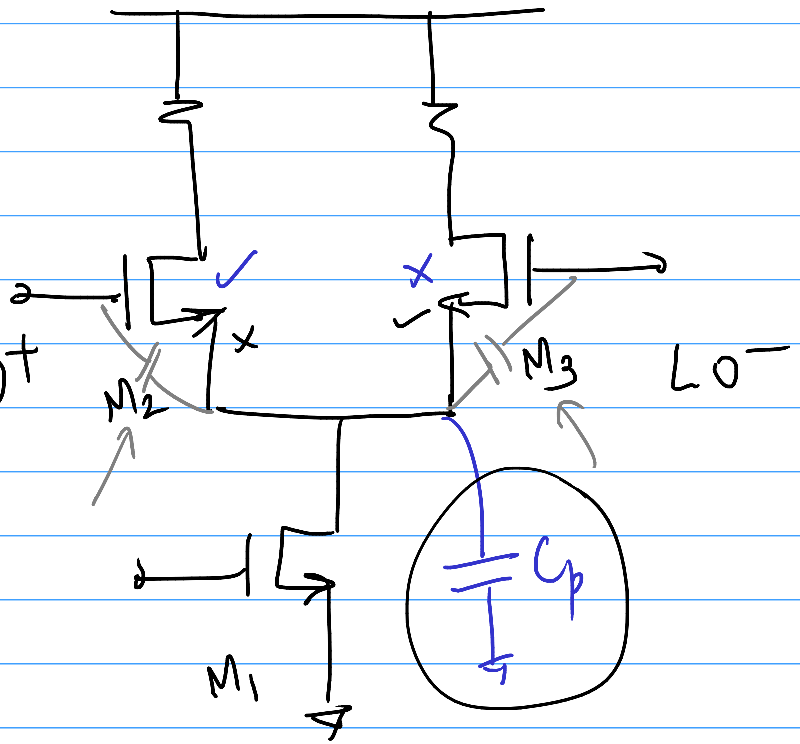


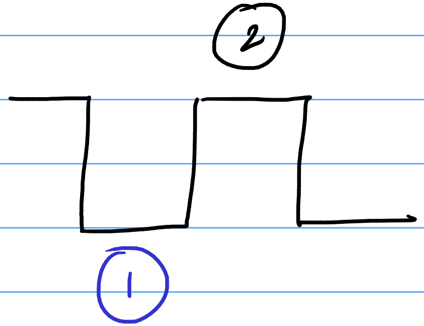


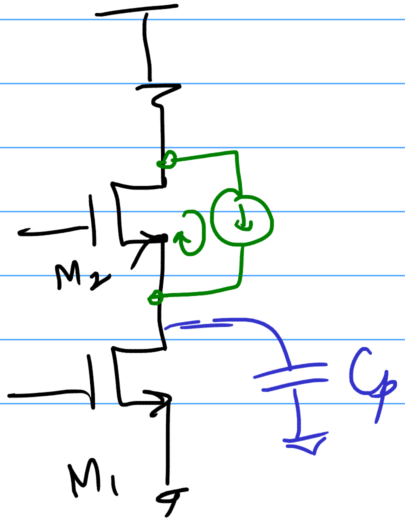
F = ?

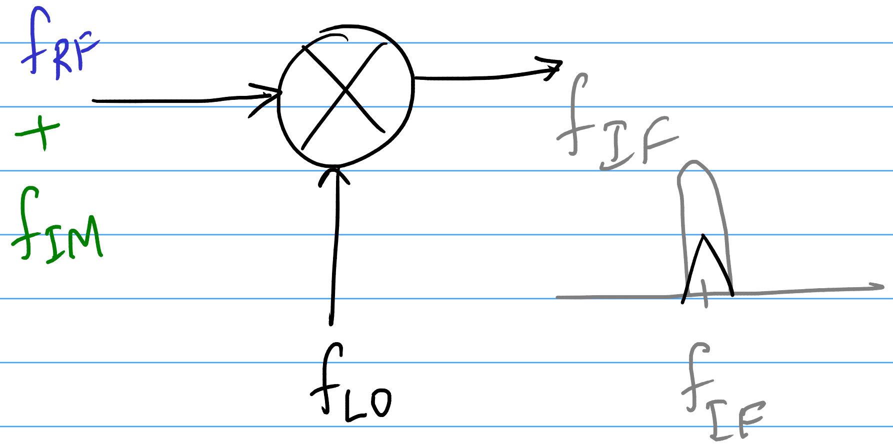
HW

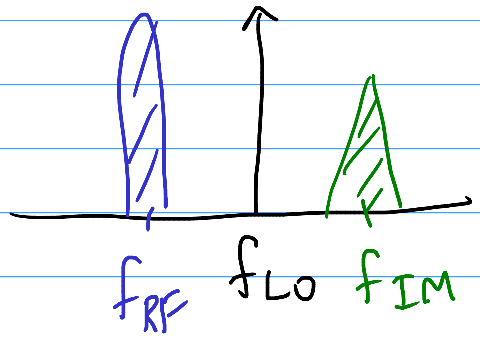

No noise
1
2


LO+
LO−
M2
M3
M1
Cp


2
1

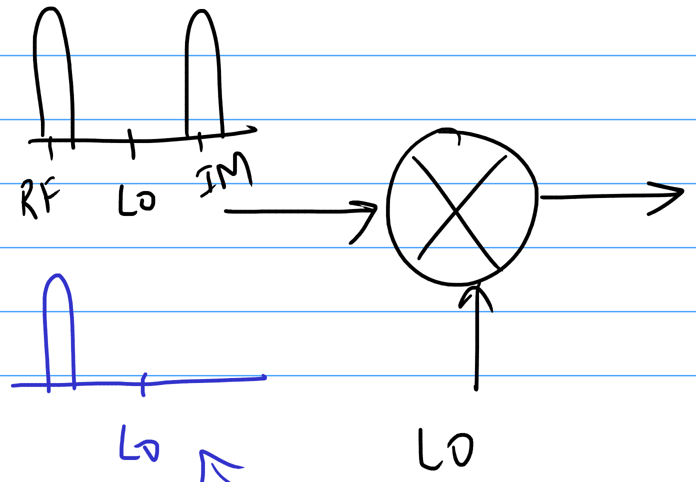

M2
M1
Cp





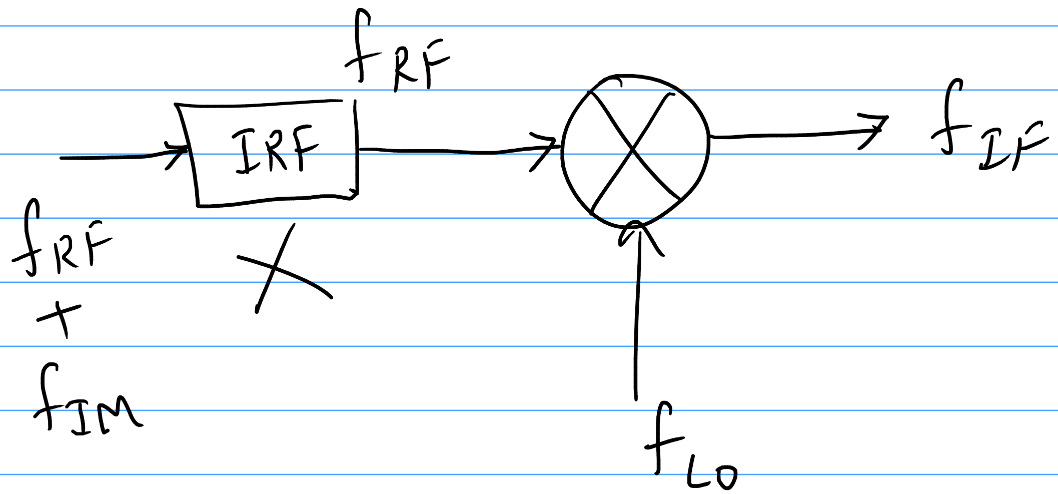
(Double Side Band)

DSB noise figure

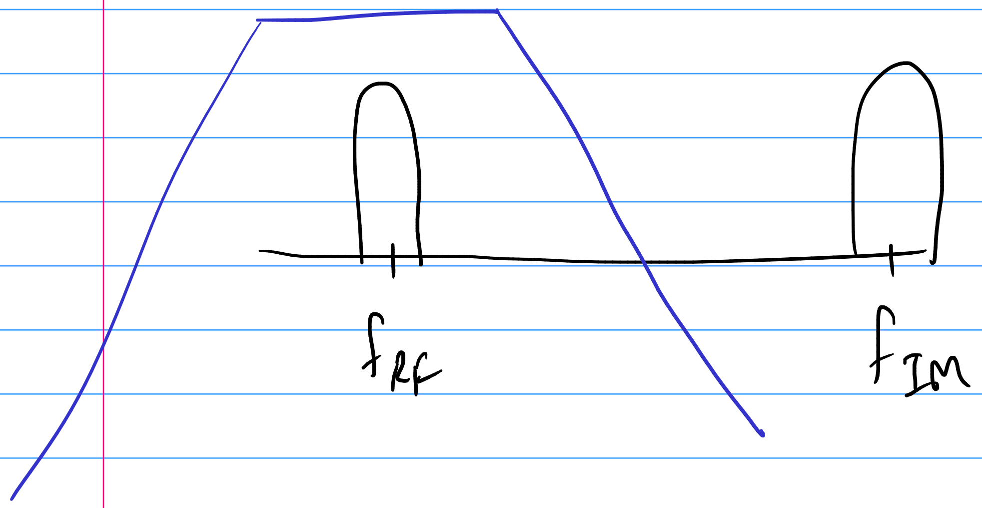


$$\frac{SNR_i}{SNR_o} = NF$$

SSB noise figure = DSB NF + 3 dB

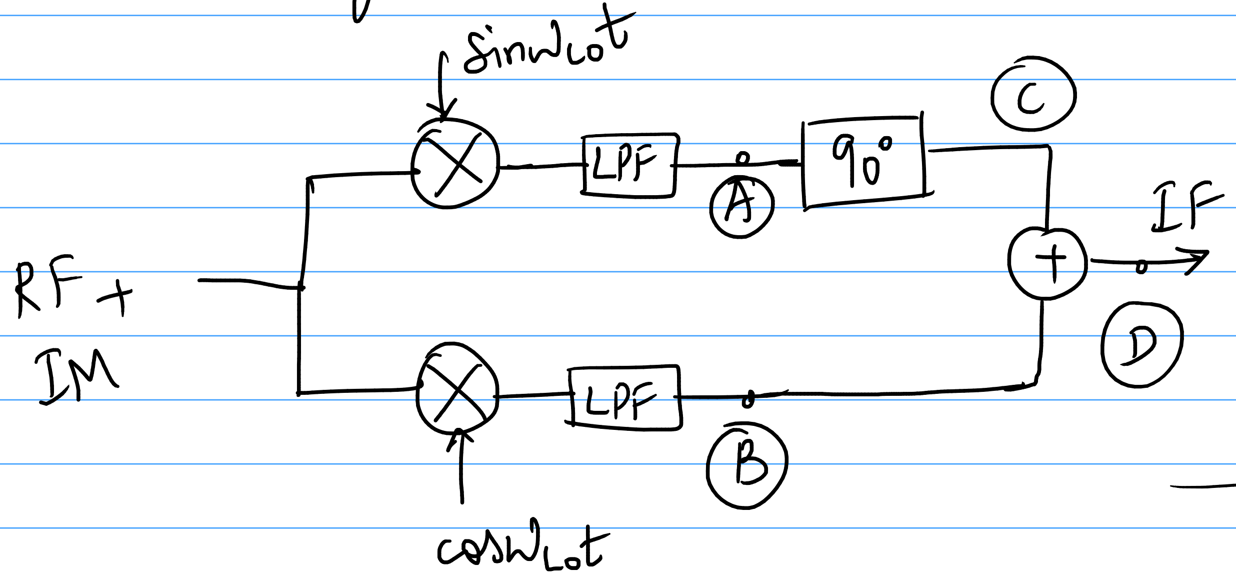


* No IRF
* Process RF & IM differently
* f_{RF} & f_{IM} are on opposite sides of f_{LO}



Hartley Image Reject architecture

$v_{RF} + v_{IM}$



$$v_{RF} = A_{RF} \cos \omega_{RF} t$$

$$v_{IM} = A_{IM} \cos \omega_{IM} t$$

$$\omega_{IF} = \omega_{RF} - \omega_{LO} = \omega_{LO} - \omega_{IM}$$

$$x_A(t) = \frac{A_{RF}}{2} \cdot \sin(\omega_{LO} - \omega_{RF})t + \frac{A_{IM}}{2} \sin(\omega_{LO} - \omega_{IM})t$$

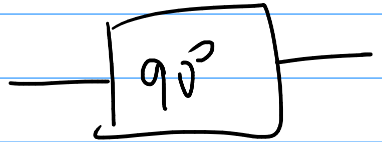
$$x_B(t) = \frac{A_{RF}}{2} \cdot \cos(\omega_{LO} - \omega_{RF})t + \frac{A_{IM}}{2} \cos(\omega_{LO} - \omega_{IM})t$$

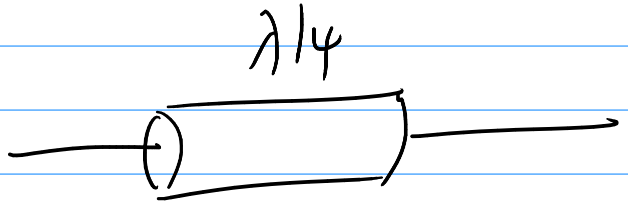
$$x_A(t) = -\frac{A_{RF}}{2}\sin\underbrace{(\omega_{RF}-\omega_{LO})}_{\omega_{IF}}t + \frac{A_{IM}}{2}\sin\underbrace{(\omega_{LO}-\omega_{IM})}t$$

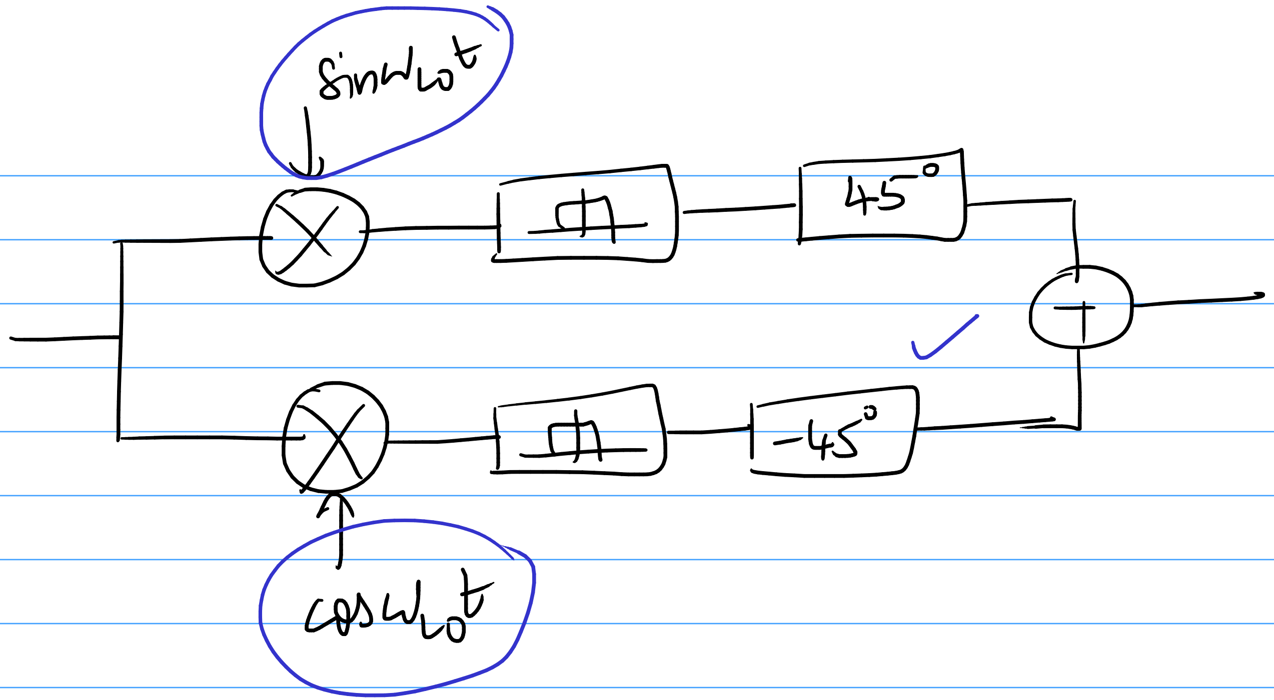
$$x_C(t) = +\frac{A_{RF}}{2}\cos(\omega_{RF}-\omega_{LO})t - \frac{A_{IM}}{2}\cos(\omega_{LO}-\omega_{IM})t$$

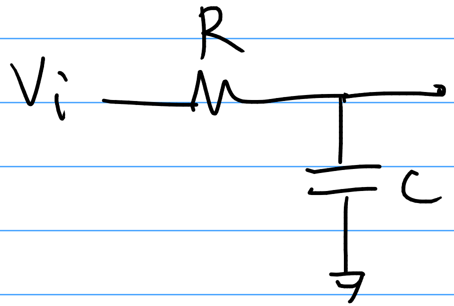
$$x_D = A_{RF}\cos(\omega_{RF}-\omega_{LO})t$$

1)



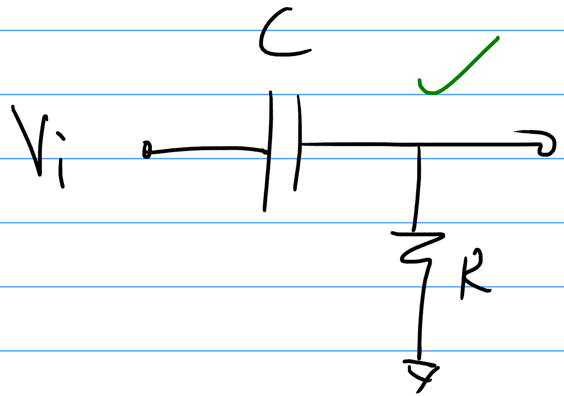






$$\frac{V_o}{V_i}(s) = \frac{1}{1 + sCR}$$

$$@\ \omega = \frac{1}{RC} \rightarrow -45^\circ$$



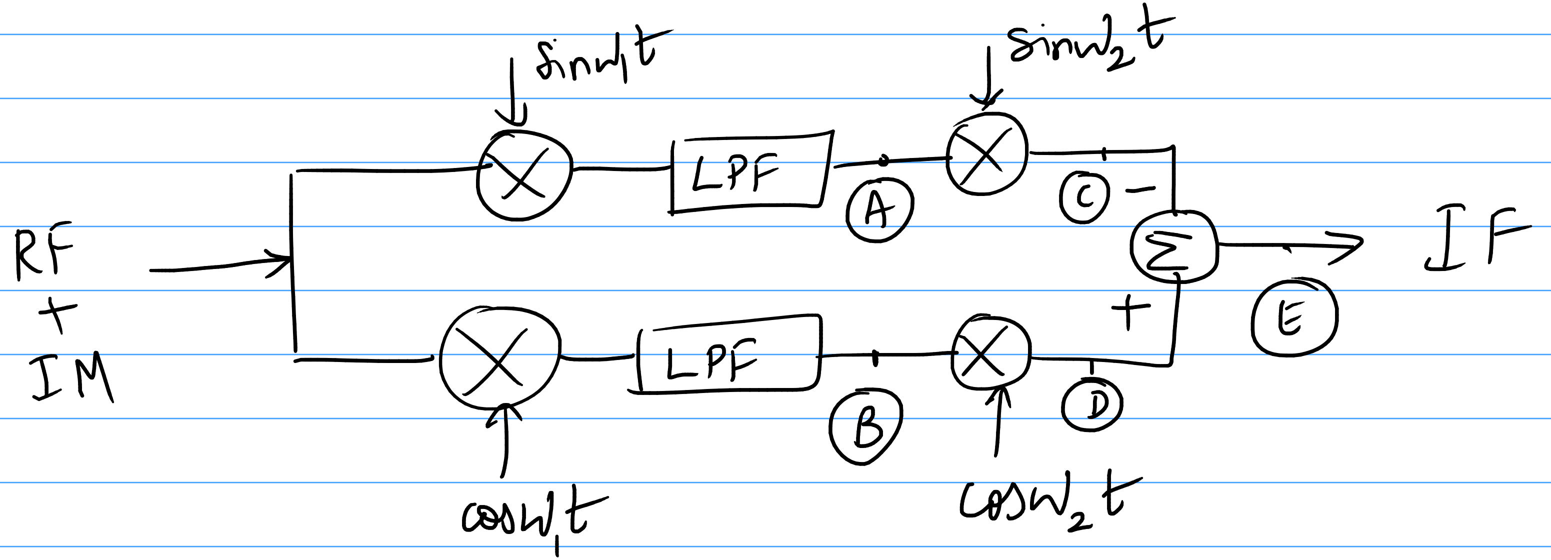
$$\frac{V_o}{V_i} = \frac{sCR}{1+sCR}$$

$$@\ \omega = \frac{1}{RC} \Rightarrow +45^\circ$$

* Amplitude mismatch
* φ mismatch

Weaver IR architecture

$\omega_{RF} > \omega_1 > \omega_{IM}$



$x_A(t)$, $x_B(t)$	same as before	$x_C(t) =$ $x_D(t) =$ $x_E(t) =$	HW

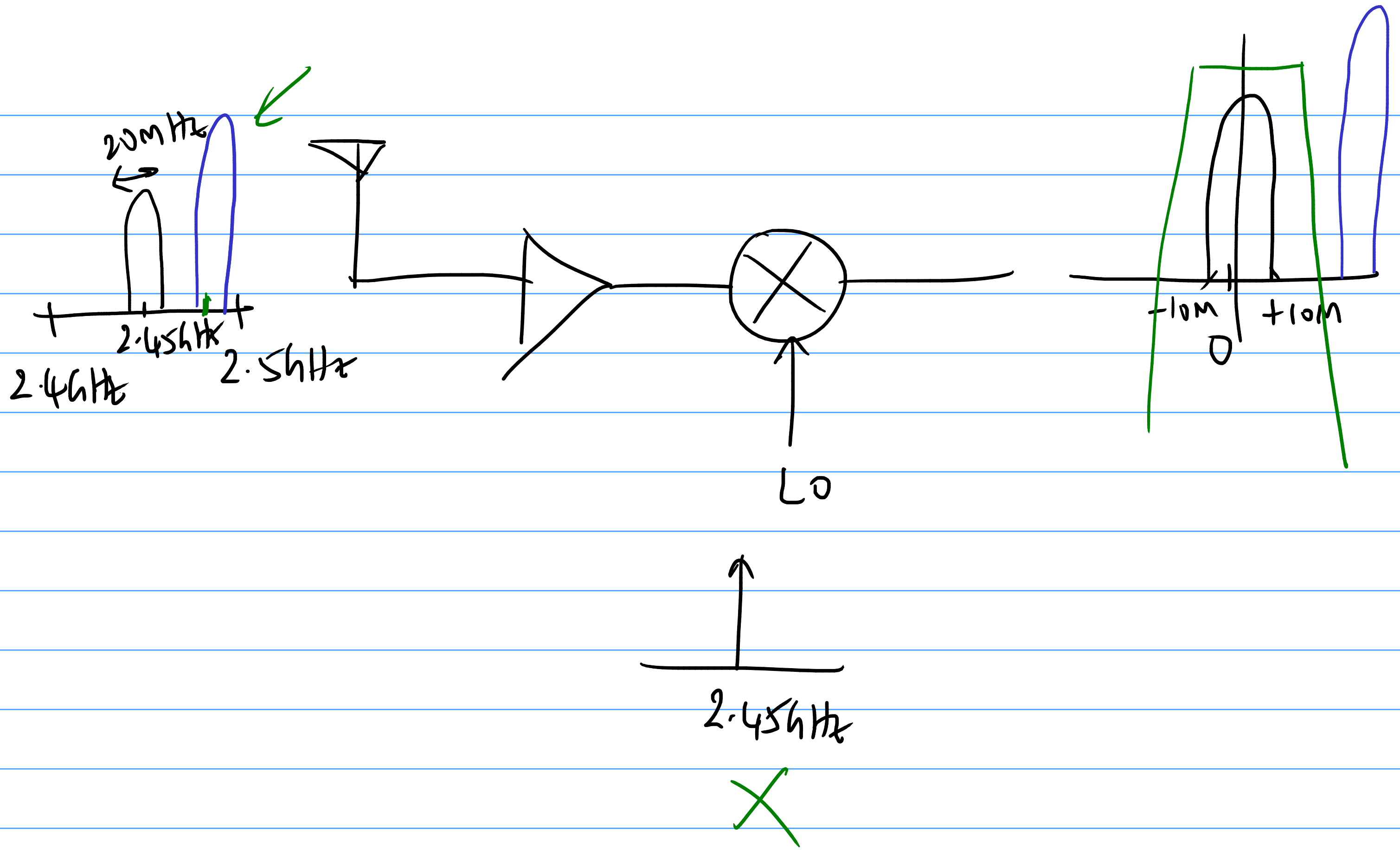
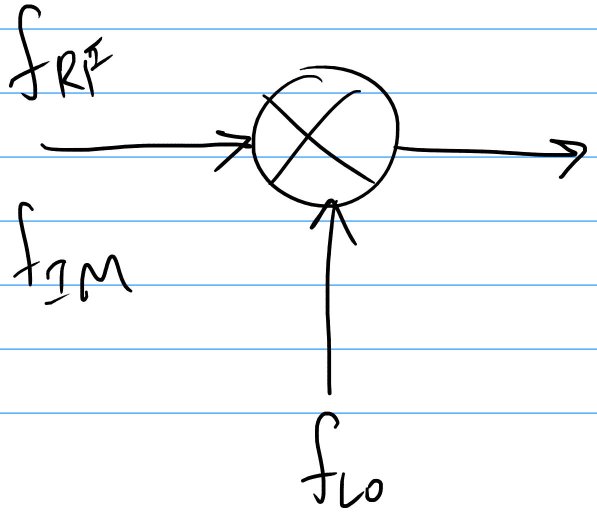

20MHz
2.4GHz
2.45GHz
2.5GHz
LO
-10M
0
+10M
2.45GHz

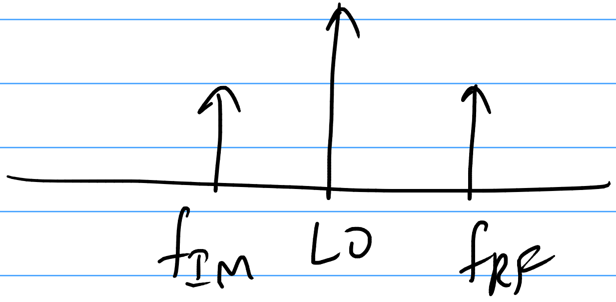
Image Reject Ratio

$$IRR = \frac{\text{Image to signal ratio @ output}}{\text{Image to signal ratio @ input}}$$

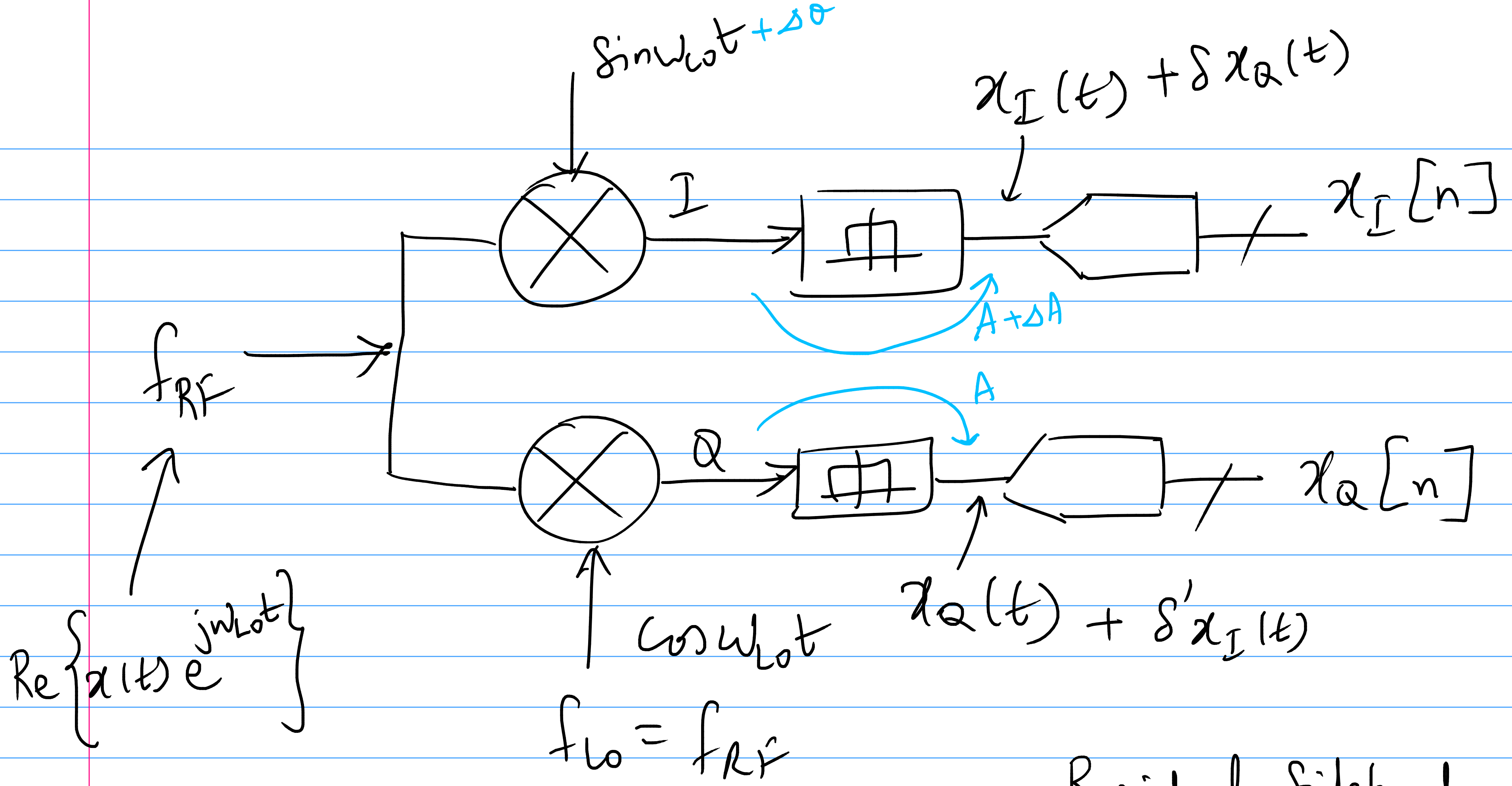


$$f_{IF} = f_{RF} - f_{LO}$$

$$= f_{LO} - f_{IM}$$



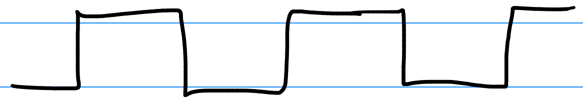
DCR $\Rightarrow$ Signal & Image are the same



Residual Sideband (RSB)

Oscillators

$\sim \longrightarrow V_0 \cos(\omega_0 t + \varphi)$ — (1)

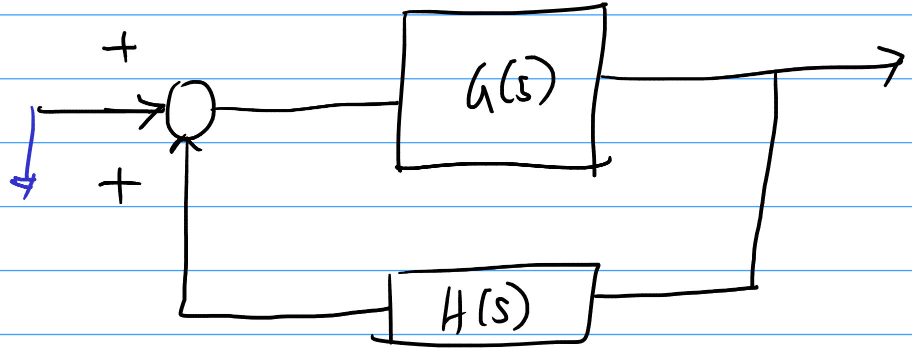
$\sim \longrightarrow$  — (2)

⋮

1) clock for digital
2) LO signal for mixer
3) clock for ADC, DAC

* Hartley Osc.
* Wien Bridge
* Ring Osc.
* Relaxation Osc.

* Colpitts Osc.
* RC phase shift Osc.
* LC osc. ← Most common in RF Systems
* Schmitt Trigger Osc.



Barkhausen Criteria

$$|G(j\omega)\cdot H(j\omega)| = 1$$

and

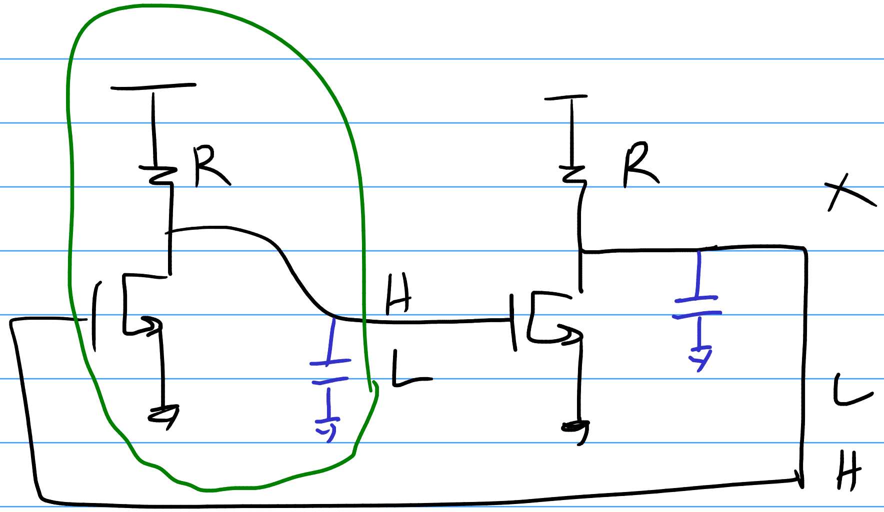
$$\angle G(j\omega)\cdot H(j\omega) = 360^\circ$$

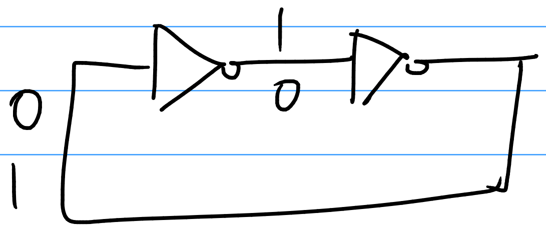
Necessary but not sufficient conditions

$$\frac{-A_0}{1+s/\omega_p}$$

$$A_0 = g_m R$$

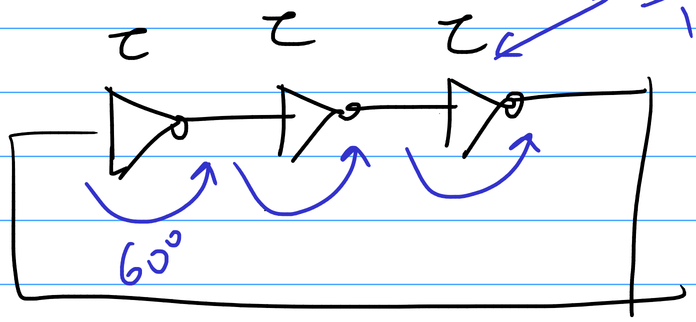
$$\omega_p = \frac{1}{RC}$$

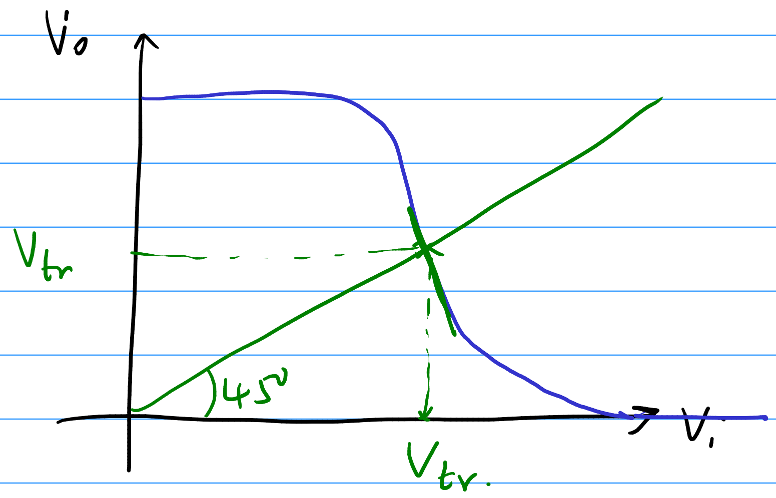




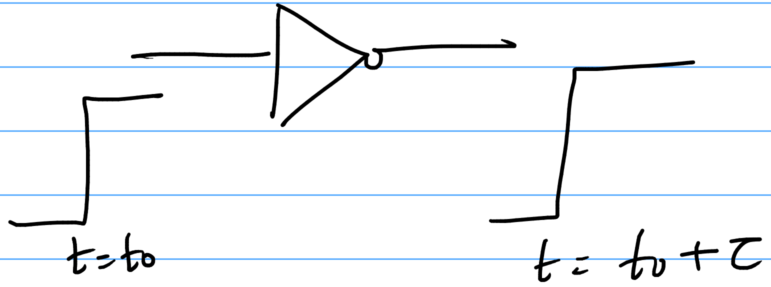
Latch

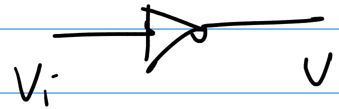
$$A(s) = \frac{-A_0}{1+s/\omega_p}$$

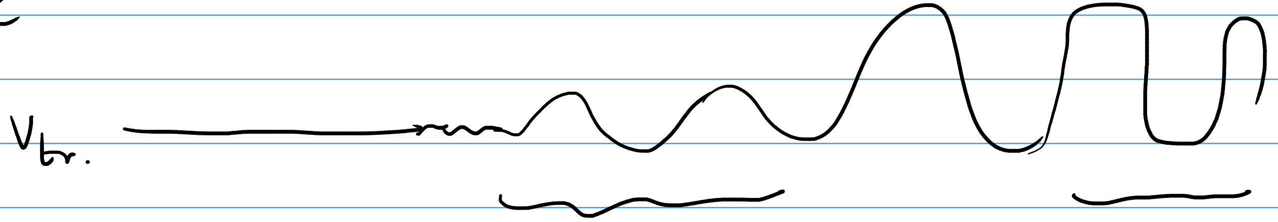




each inv gives 60° phase shift @ ω_0



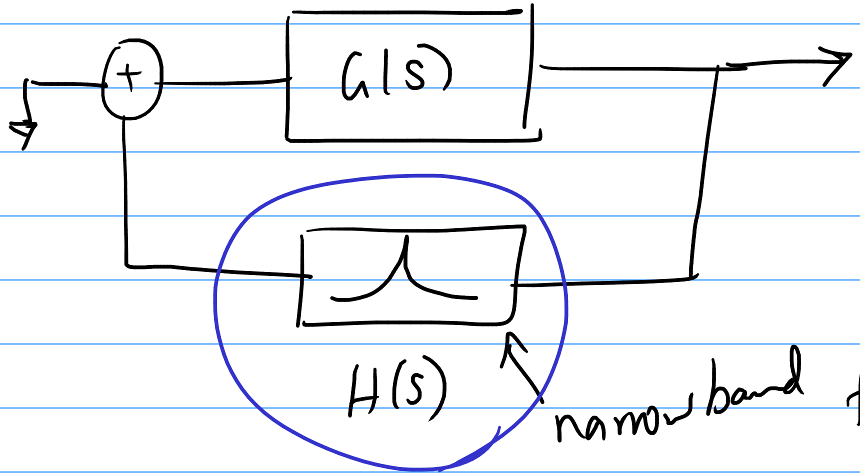




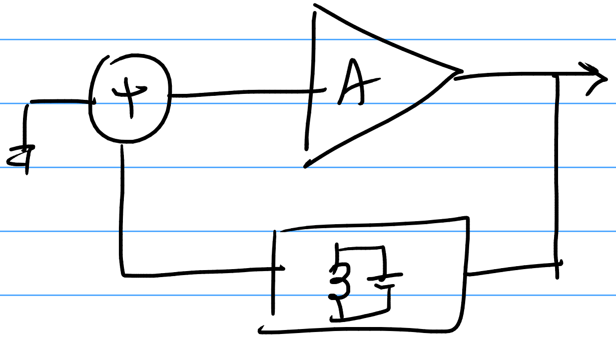
$$LG(s) = \frac{-A_0^3}{(1+s/\omega_p)^3}$$

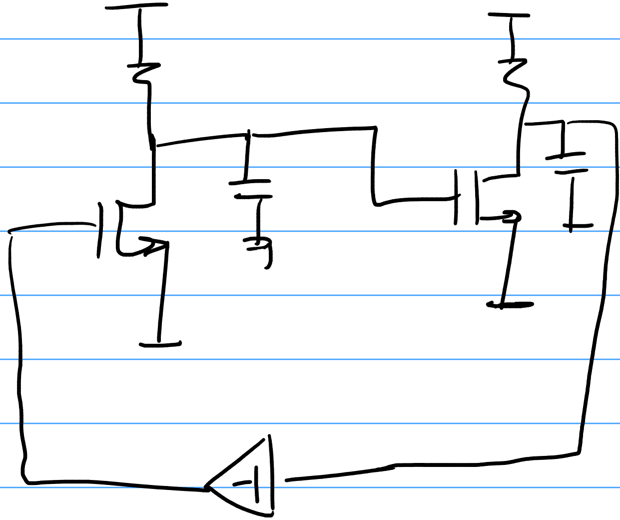
$f = ?$

$f_{ss} = \frac{1}{6\tau}$



narrowband feedback

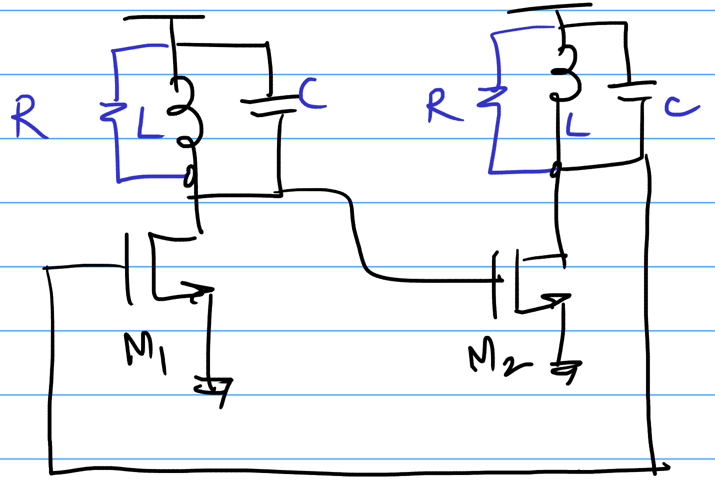




Ideal inv.

$$LG = \frac{-A_0^2}{\left(1+s/\omega_p\right)^2}$$

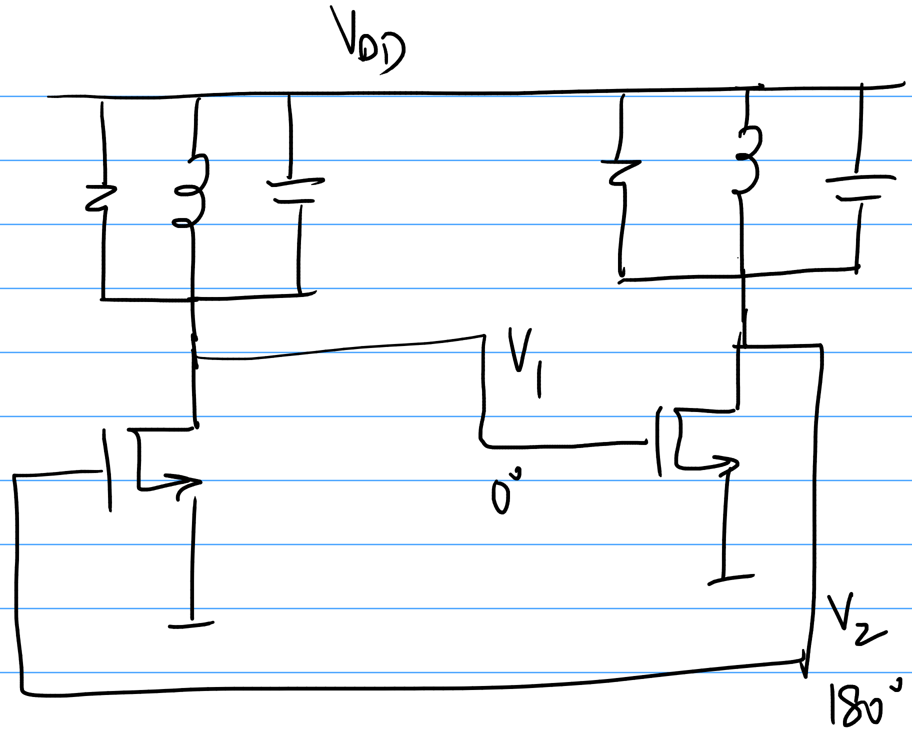
$\varphi = 360^\circ$ only @ $\omega = \infty$



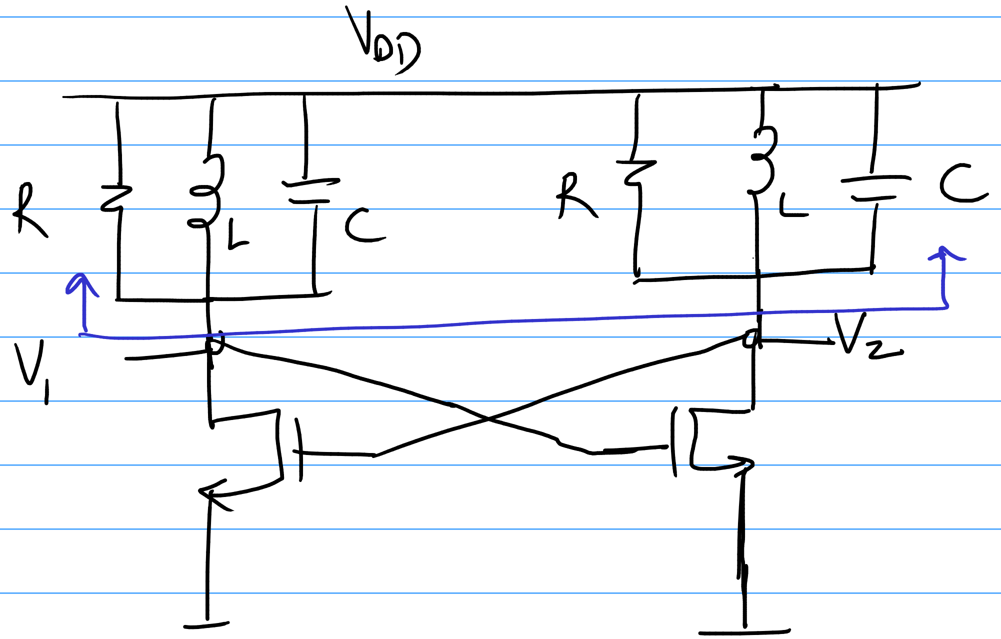
$$LG(s) = \frac{(g_m sL)^2}{\left[1 + \frac{sL}{R} + s^2 LC\right]^2}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ ← phase = $-180°$ per block

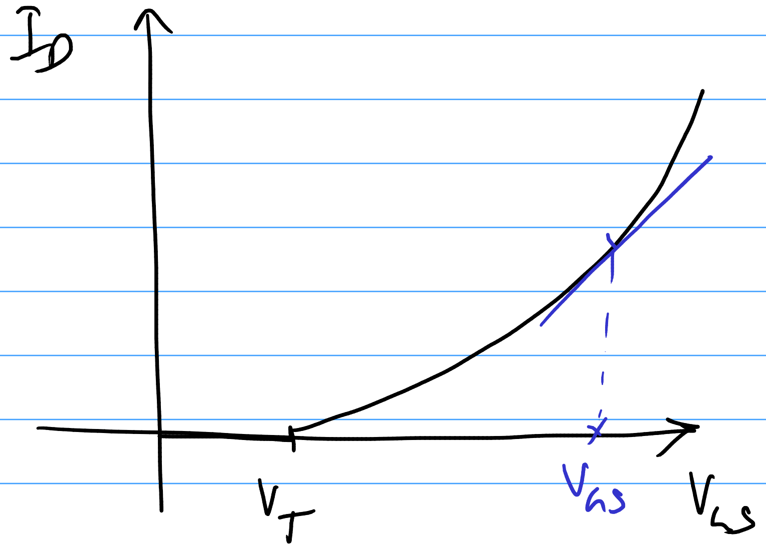
$g_m R = 1$ ⇒ Loop gain = 1 mag.

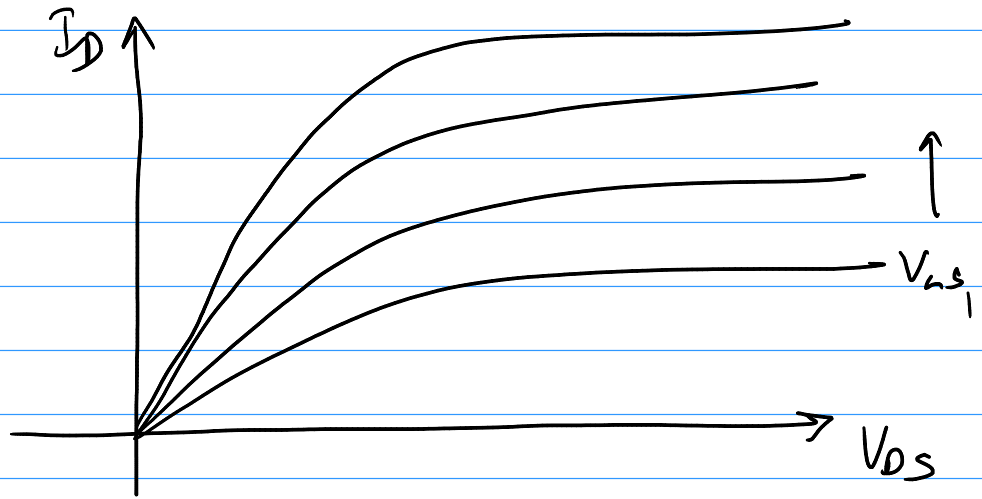


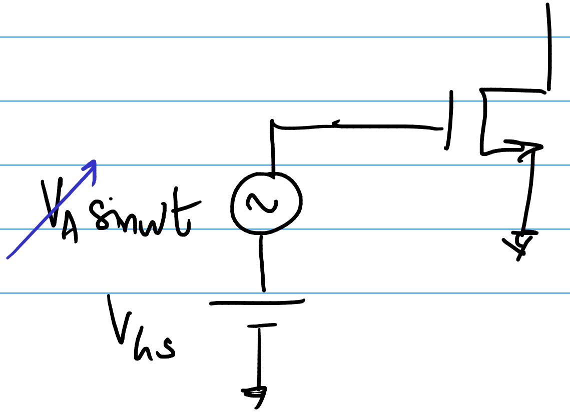
Cross-coupled LC oscillator



$$g_m = \frac{1}{R}$$

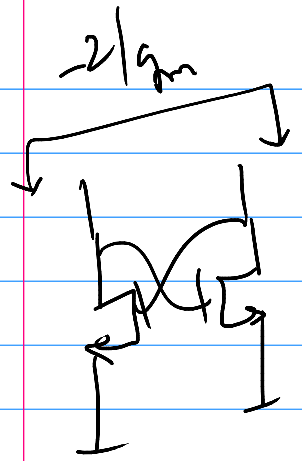


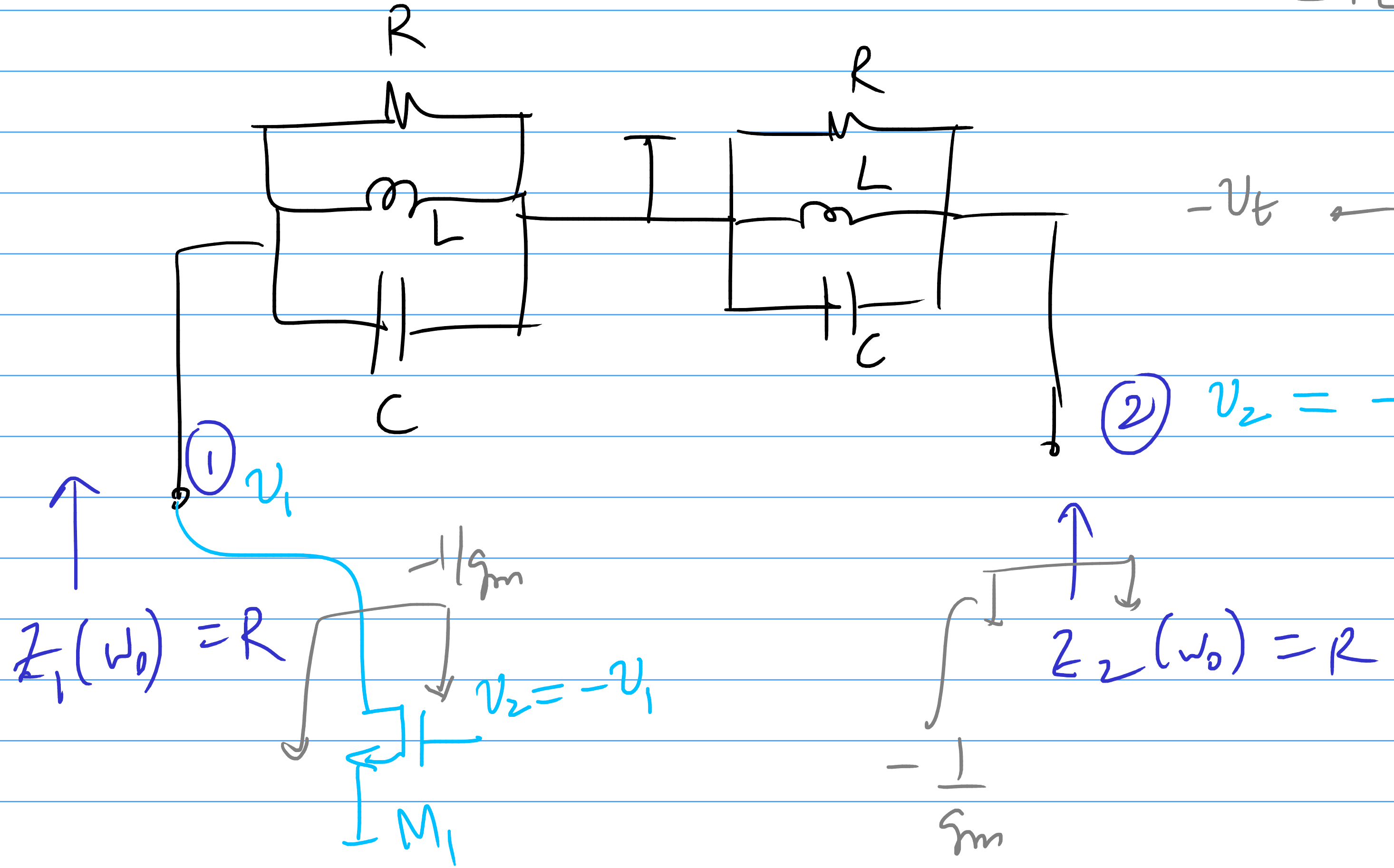


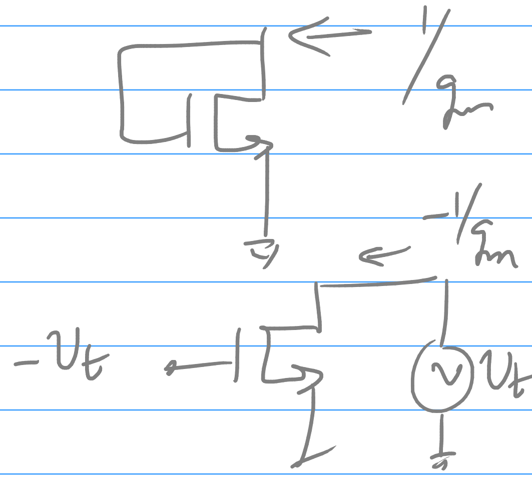


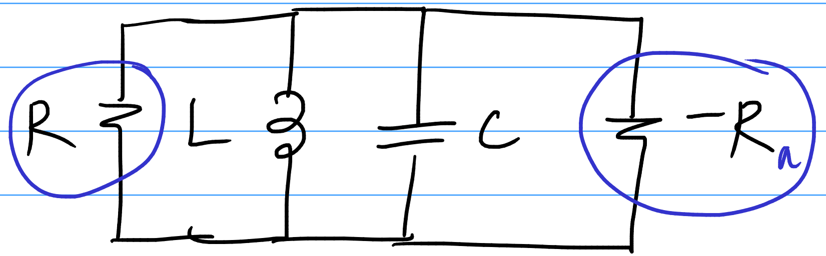
1) In sat. → does G_m stay constant?

2) If device goes to triode/cutoff → G_m → ?


-2/gm


R
L
C
R
L
C
① v1
② v2 = -v1
-1/gm
v2 = -v1
M1
Z1(w0) = R
Z2(w0) = R
-1/gm


1/gm
-1/gm
-vt
vt



$= -\frac{1}{g_m}$

positive f.b.

$$\underbrace{|R_a| < |R|}_{\text{RHP poles}}$$

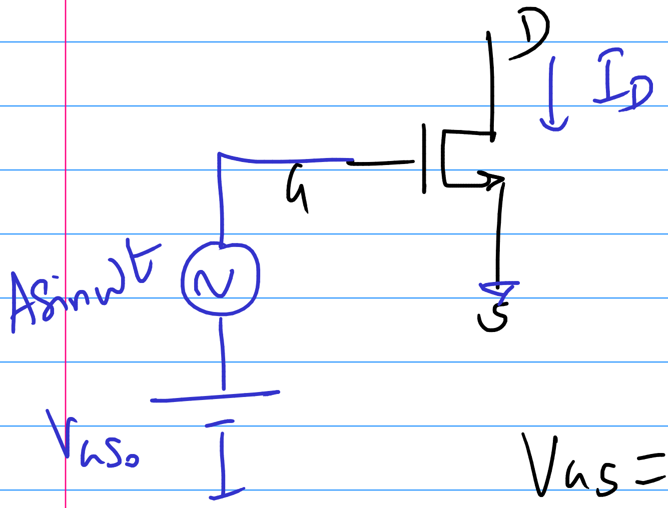
$$|R_a| = |R|$$

↳ poles on $j\omega$ axis

$$|R_a| > |R|$$

↳ decaying osc.

MOSFET G_m and $g_m(t)$



$$I_D = \frac{1}{2}\beta(V_{GS}-V_T)^2$$

$$\beta = \mu C_{ox}\left(\frac{W}{L}\right)$$

$$G_m = \beta(V_{GS_0}-V_T)$$

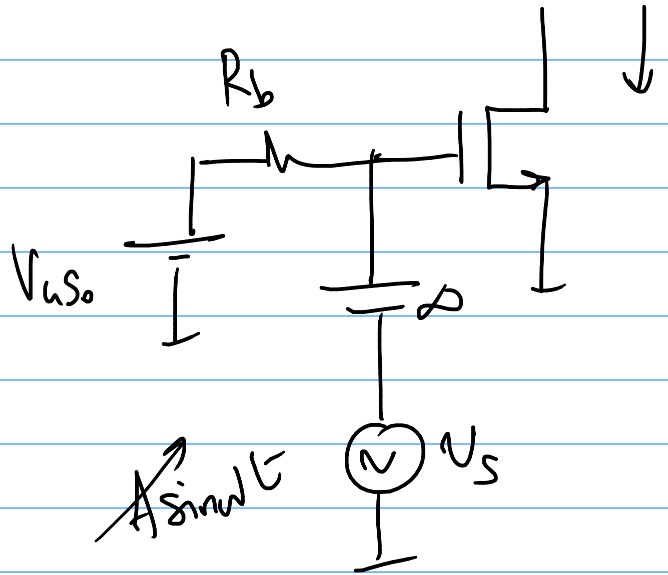
$$= g_m$$

only for a square law device

$$V_{GS} = V_{GS_0} + A\sin\omega t$$

$$I_D = \frac{1}{2}\beta(V_{GS_0} + A\sin\omega t - V_T)^2$$

$$= \underbrace{\frac{1}{2}\beta(V_{GS_0}-V_T)^2}_{I_{D_0}} + \frac{1}{2}\beta A^2\sin^2\omega t + \underbrace{\beta(V_{GS_0}-V_T)A\sin\omega t}_{G_m \cdot V_{GS}(t)}$$



$I_{DC} = \frac{1}{2}\beta(V_{GS_0} - V_T)^2$ X

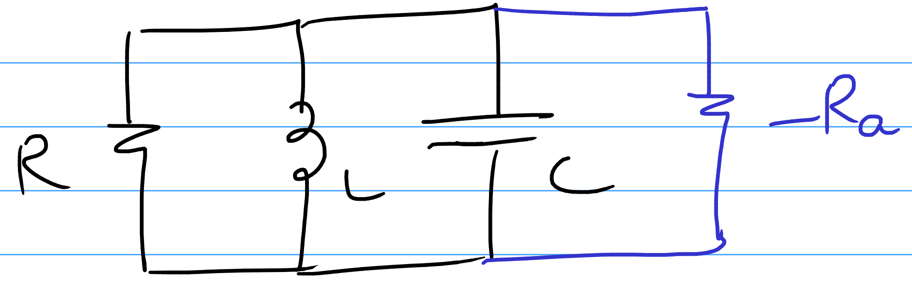
$I_{DC} = I_{average}$ over 1 period

$$g_m(t) = \frac{\partial I_D(t)}{\partial V_{GS}(t)} = \beta(V_{GS_0} - V_T) + \beta A\sin\omega t$$

$$(G_m R) = 1$$

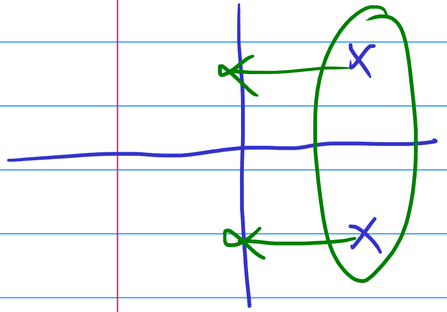
$R_x < 0$

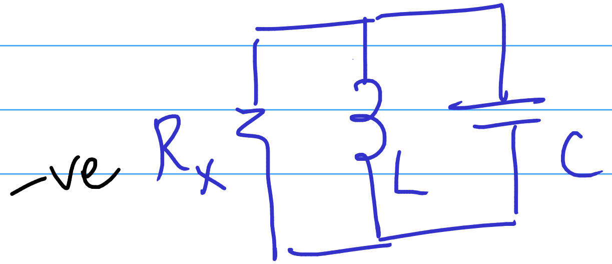
$|-R_a| < |R|$



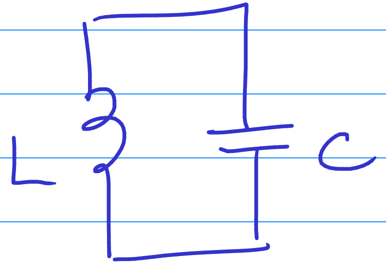
$$-R_a = -\frac{2}{g_m}$$

$$R_x = \frac{R \times -R_a}{R - R_a} = \frac{R \cdot R_a}{R_a - R}$$

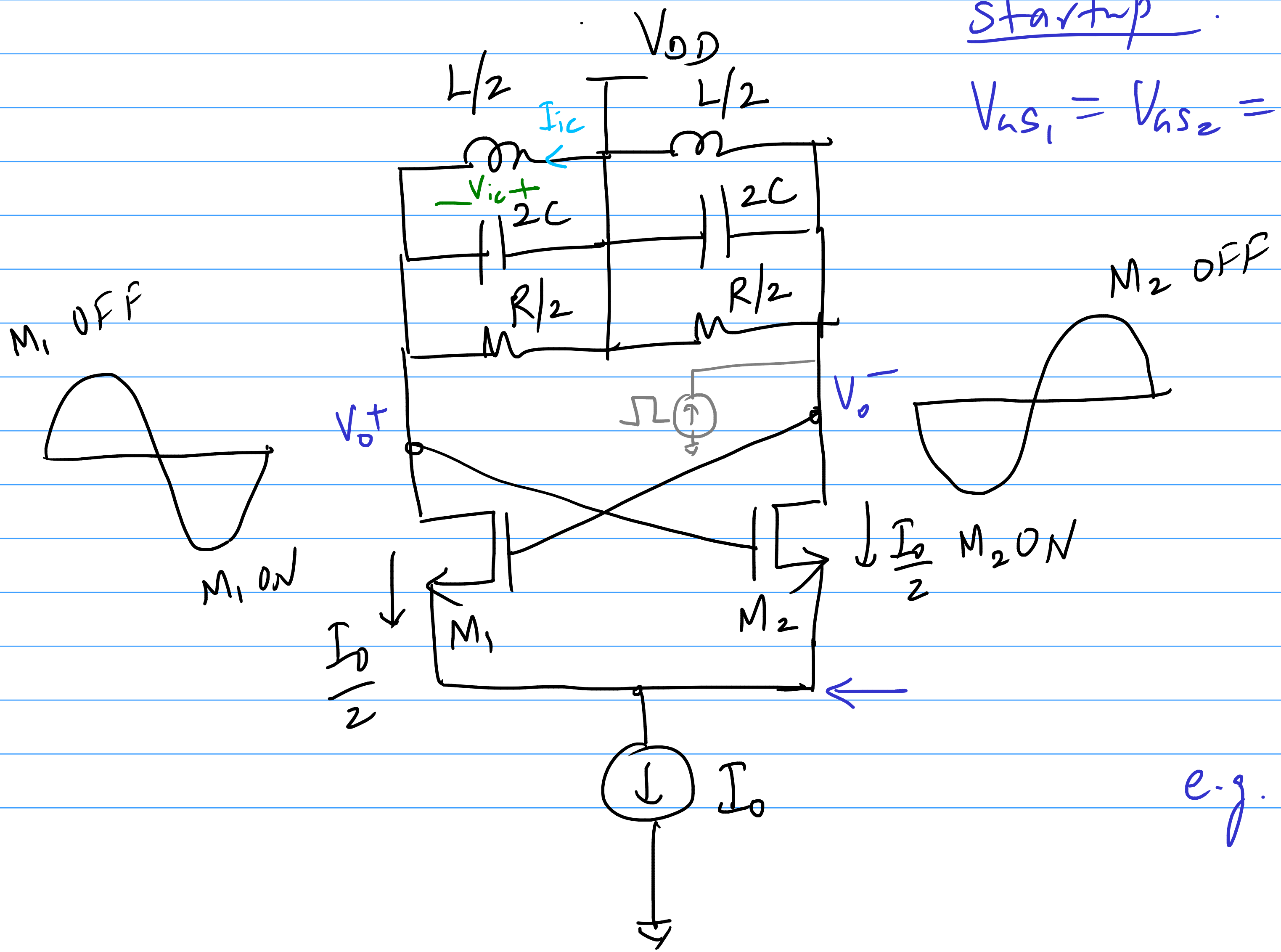




@ startup



@ steady state



startup :

$$V_{GS_1} = V_{GS_2} = V_T + \sqrt{\frac{I_o}{\beta}}$$

$$g_m = \sqrt{\beta I_o}$$

We want

$$g_m \gg \frac{2}{R}$$

$$g_{min} = \frac{2}{R}$$

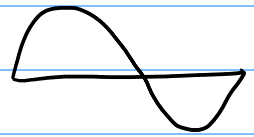
$$g_m \gg g_{min}$$

e.g. $\underline{g_m = 3 \times g_{min}}$

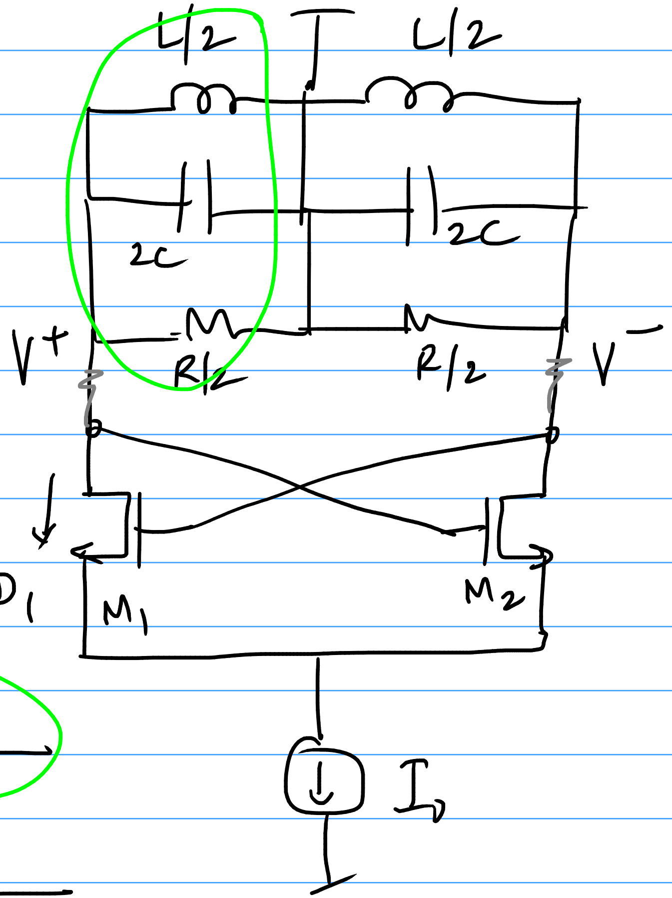
20 $Q = \dfrac{R}{\omega_0 L}$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$

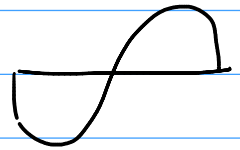
M_1 ✗



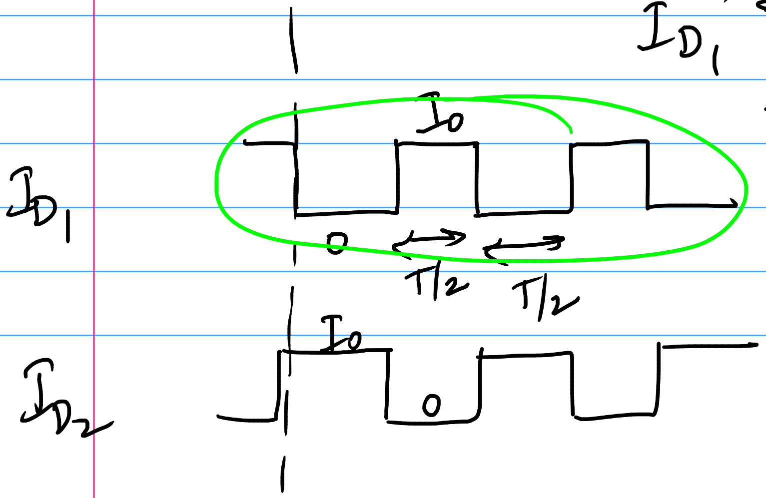
M_1 ✓



M_2 ✗



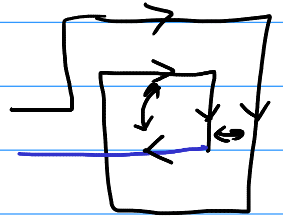
M_2 ✓

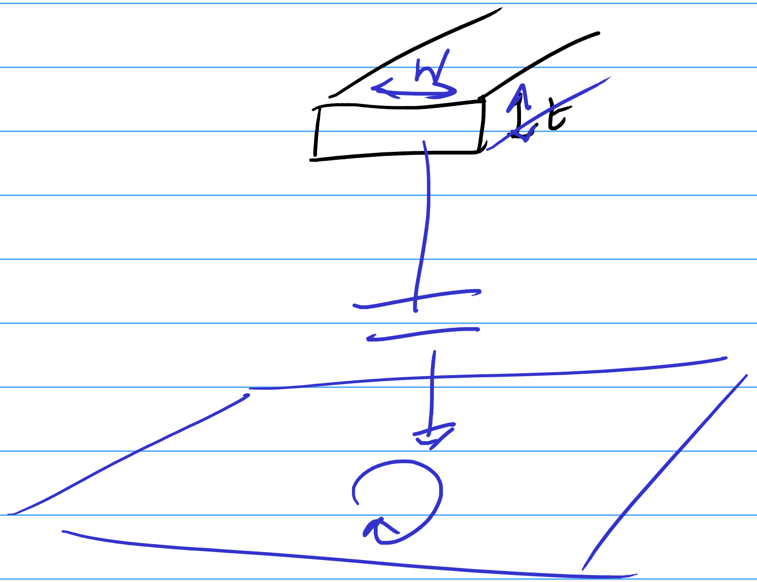


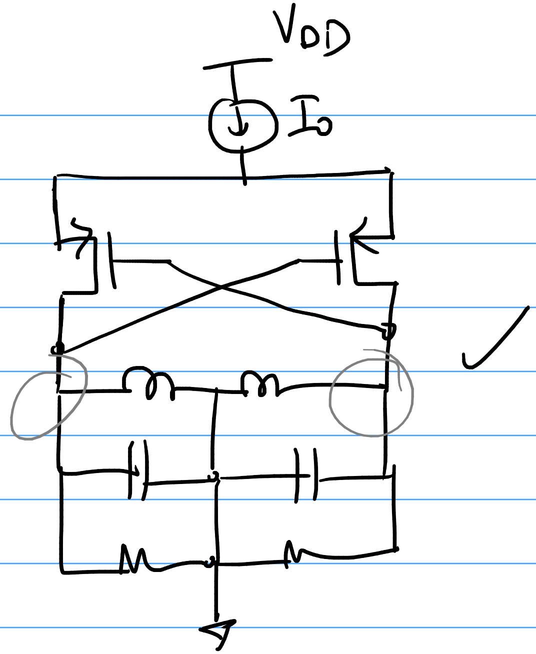
$V^+ = \dfrac{R}{2} \times \dfrac{2}{\pi} \cdot I_0$
(amplitude)

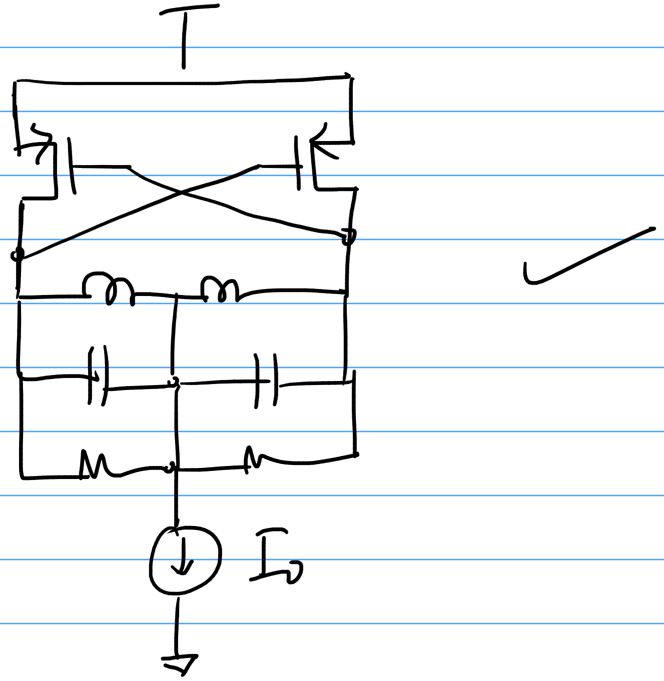
$V^- = \dfrac{R}{2} \times \dfrac{2}{\pi} I_0$

$V_{od} = V^+ - V^- = \dfrac{2 I_0 R}{\pi}$ ✓

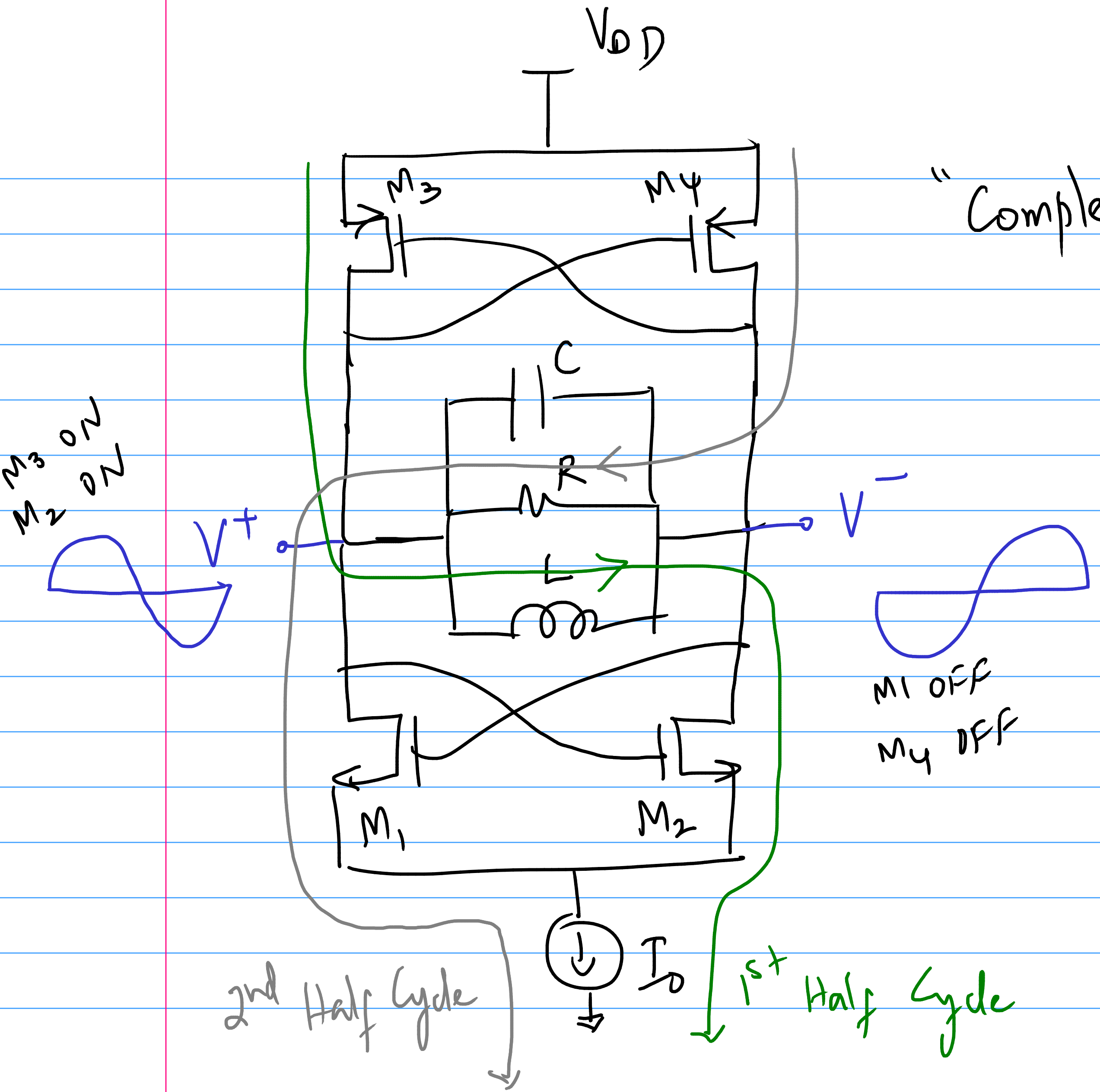



w
t


VDD
I0

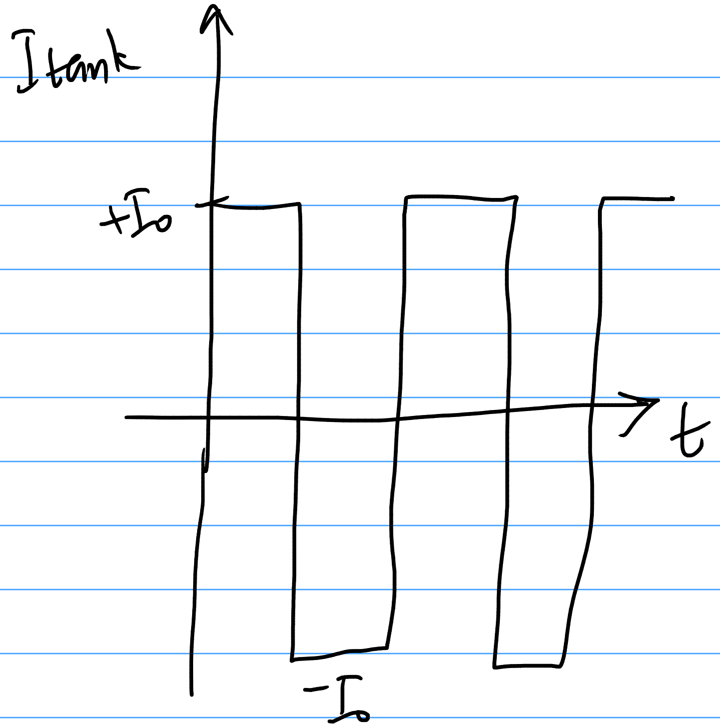

I0

"Complementary" cross-coupled LC Oscillator



$$-R_a = \frac{-2}{g_{m_n} + g_{m_p}}$$

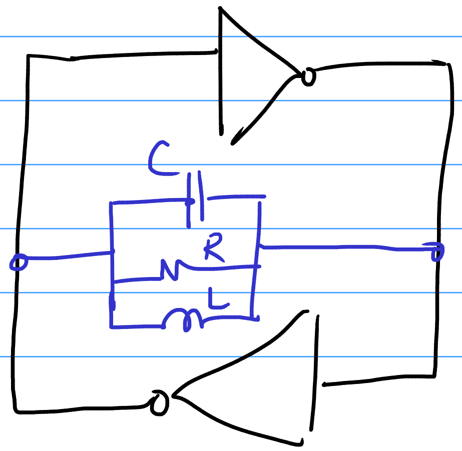
* Better startup
* V^+ / V^- "biased" at $V_{DD} - V_{SG_p}$
* Current flows in LC tank in both half cycles



$$V_{od} = V^{+} - V^{-}$$

$$= \frac{4}{\pi} I_o \cdot R \cdot \sin \omega_o t$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$



$R, L, C, \mu, t_{ox} \ldots \rightarrow$ "PVT" variations

Process → P

Voltage → V

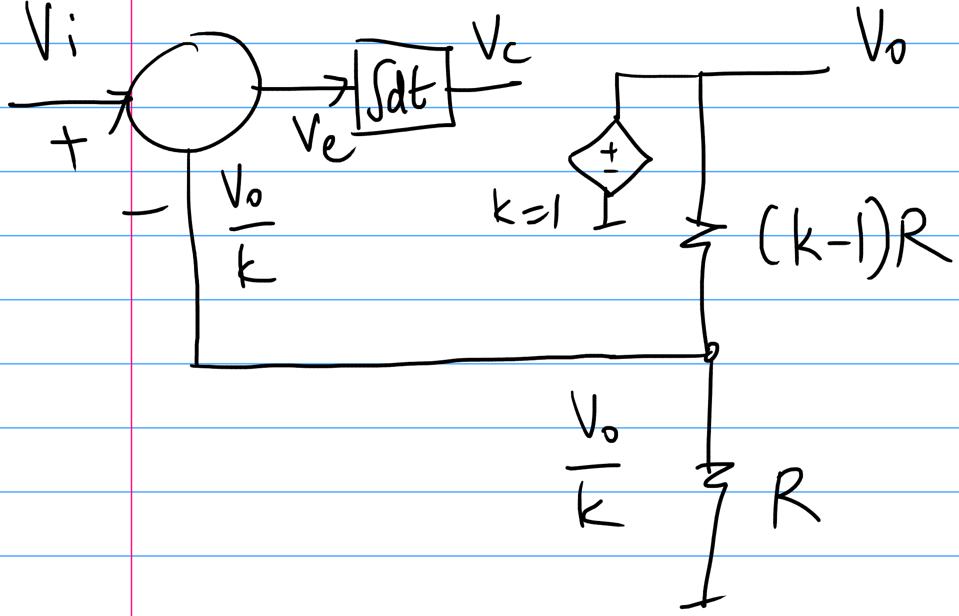
temperature → T

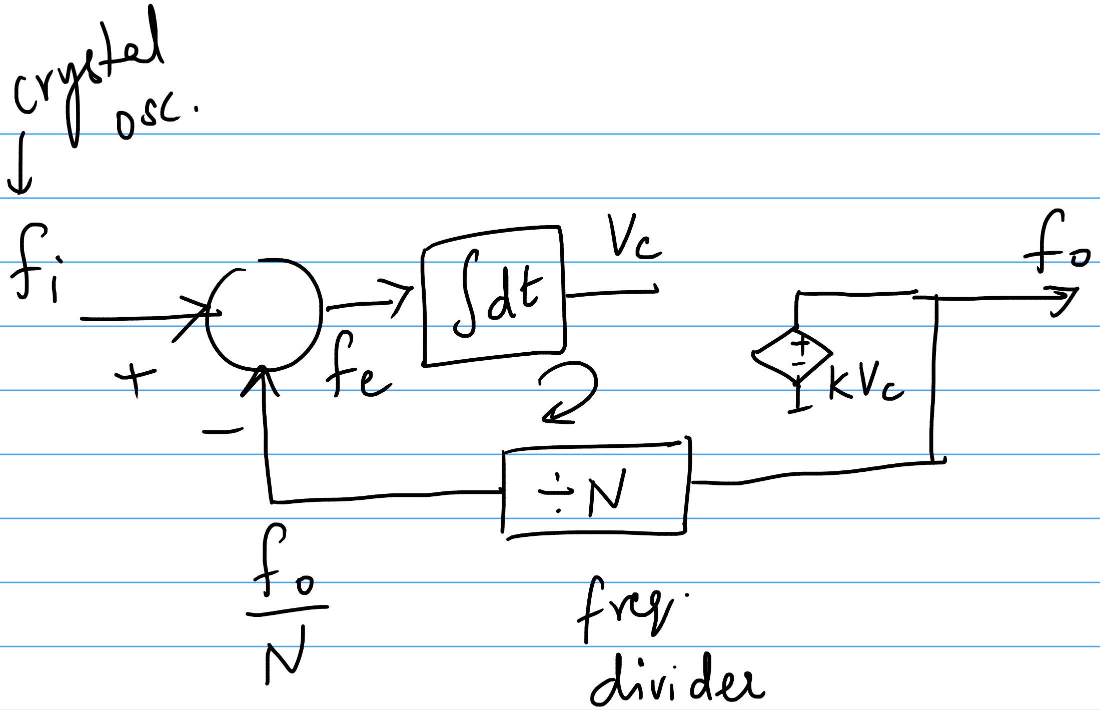
$\omega_0 = \frac{1}{\sqrt{LC}}$ is not constant for every die & over temp.

Fix using Negative feedback → "Voltage Controlled Oscillator" VCO

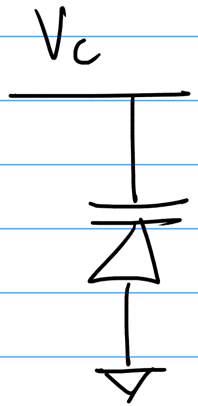
$C = f(V_c)$

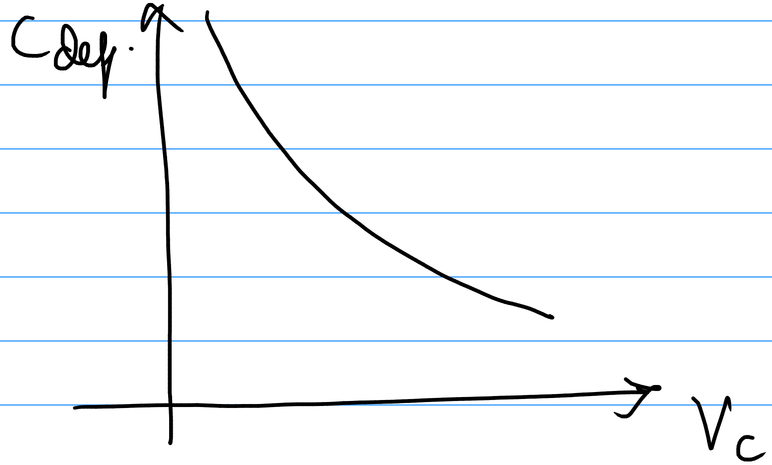
Voltage Amplifier





$$\underbrace{f_o = k V_c}_{\text{"VCO"}}$$



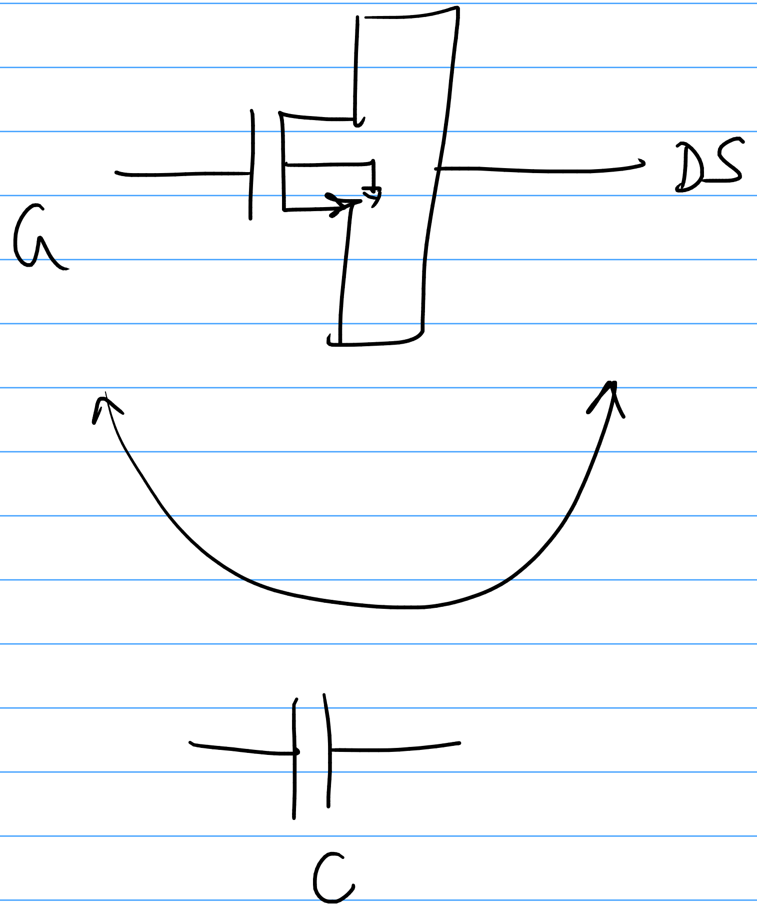


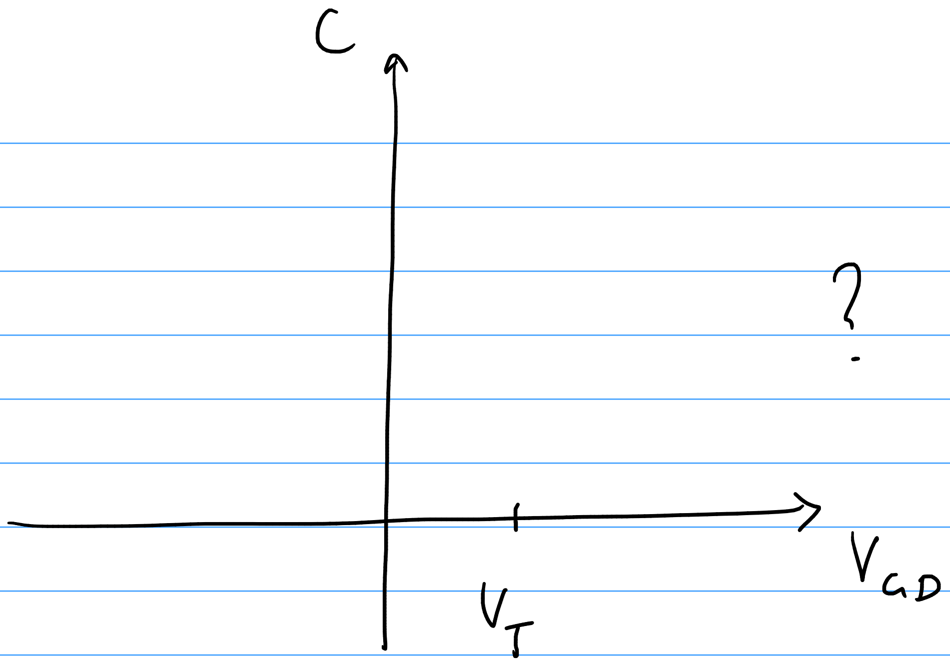
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

free running
freq. of VCO

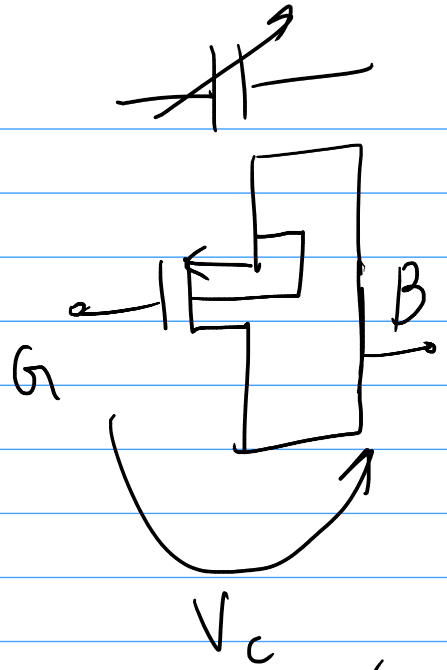
$$K_{vco} = \_\_ \ MHz$$

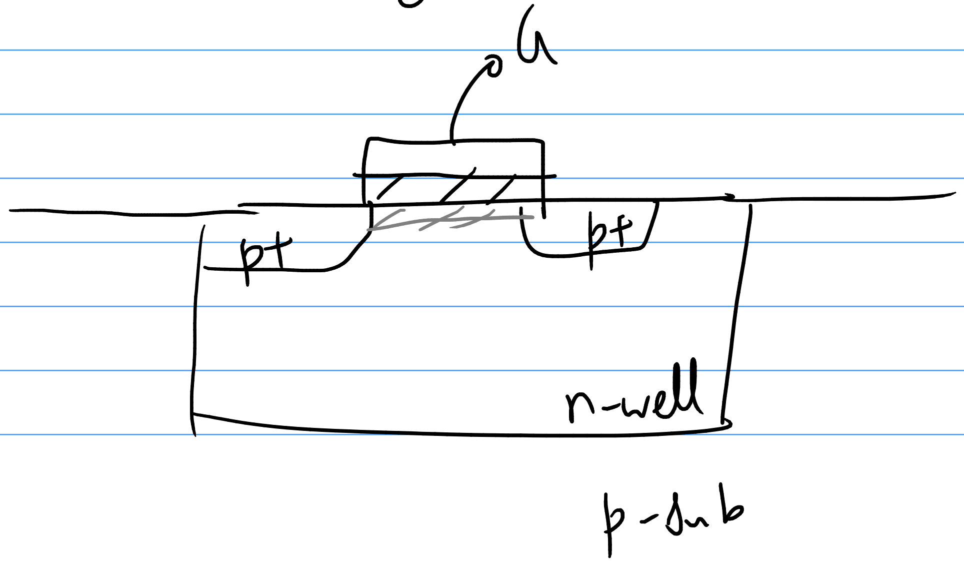
$$f_{vco} = f_{free} + K_{vco} \cdot V_c$$

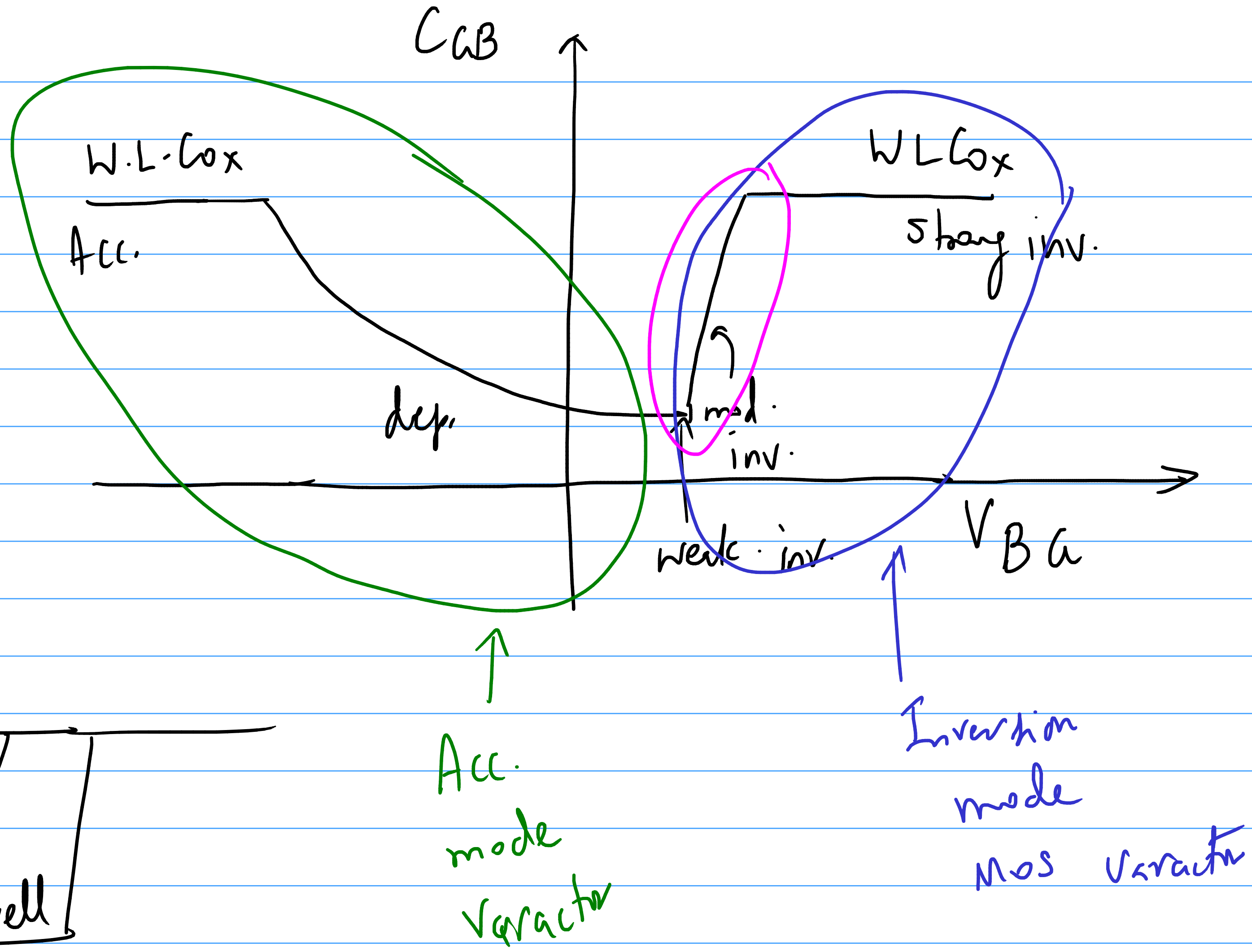


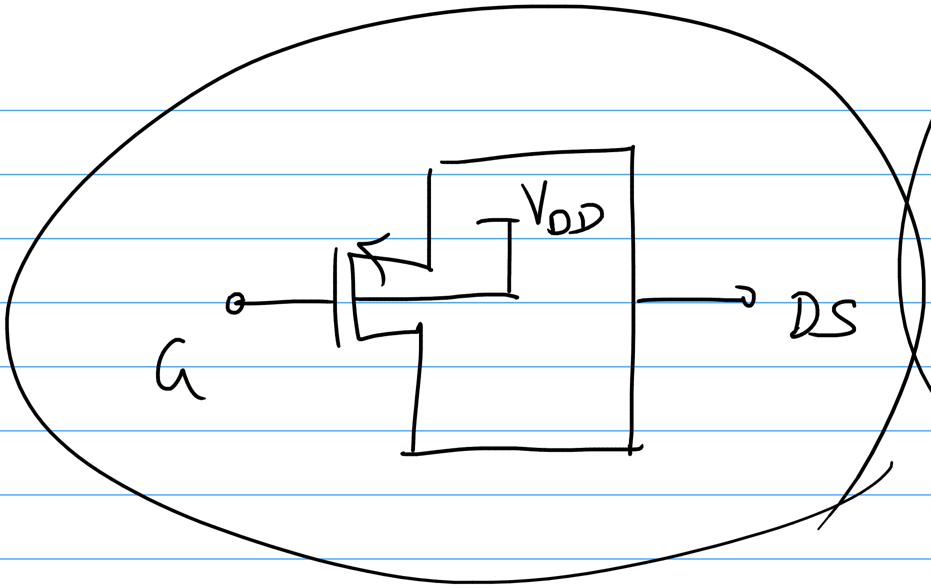


HW

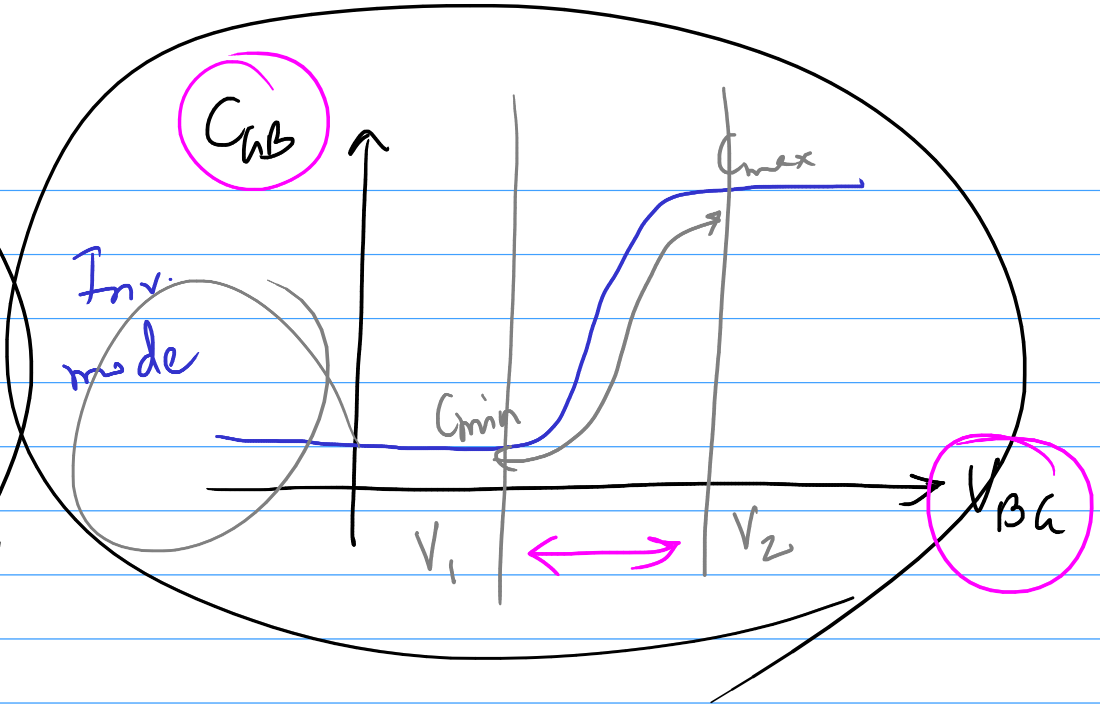

G
B
V_c


G
p+
p+
n-well
p-sub


C_{GB}
W.L.Cox
Acc.
dep.
mod. inv.
weak. inv.
WLCox
strong inv.
V_{BG}
Acc. mode Varactor
Inversion mode MOS Varactor

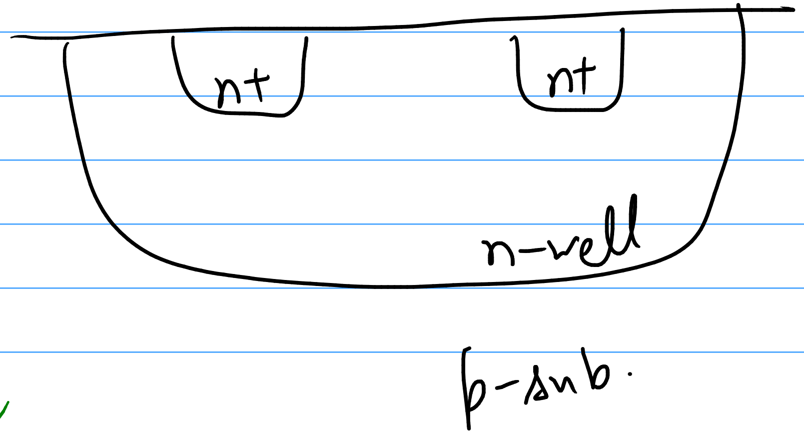


V_{DD}
G
DS

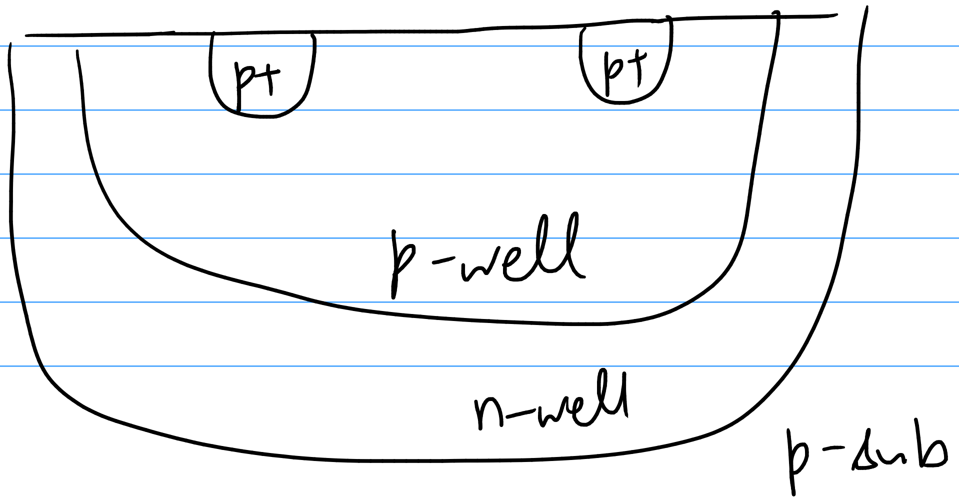


C_{GB}
Inv. mode
C_{min}
C_{max}
V_1
V_2
V_{BG}

acc. mode

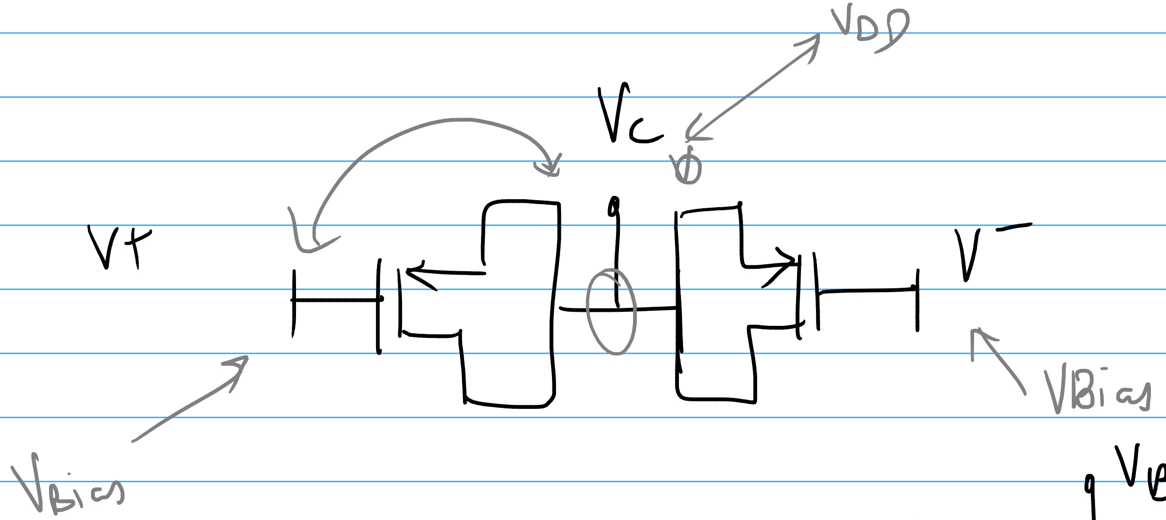


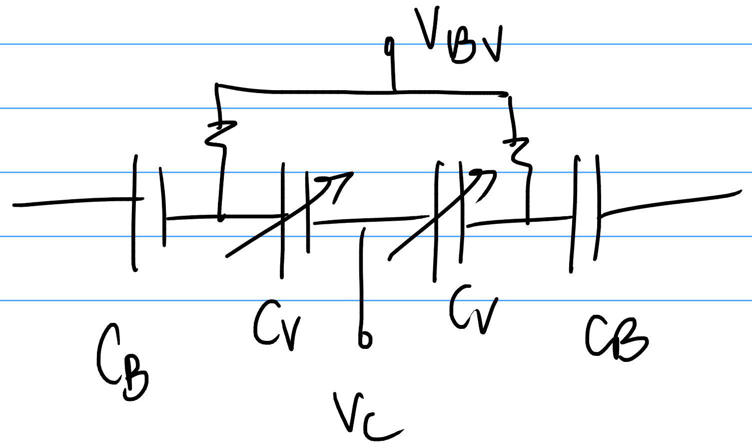
n+
n+
n-well
p-sub.

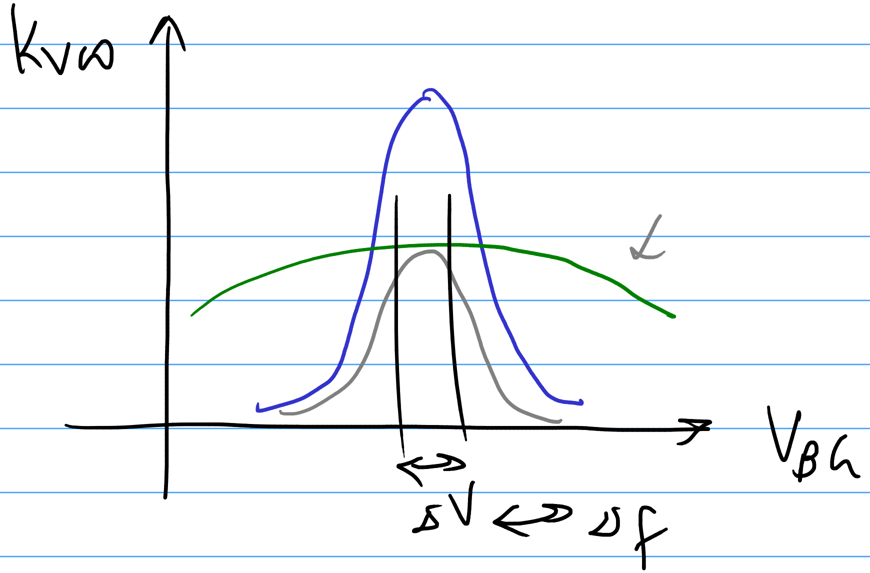


p+
p+
p-well
n-well
p-sub

$$R_{mosvar} \approx \frac{1}{2\mu C_{ox}\left(\frac{W}{L}\right)\left(V_{BL}-V_{Tp}\right)}$$



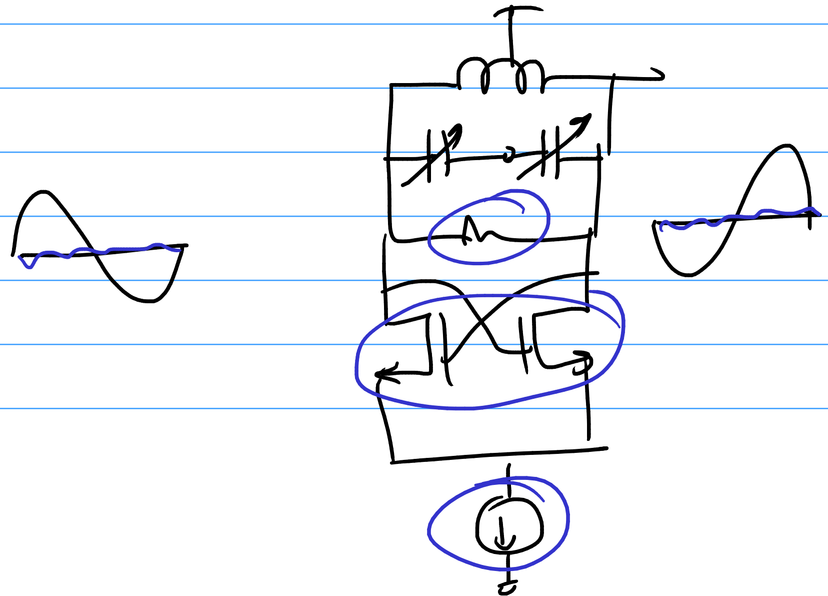


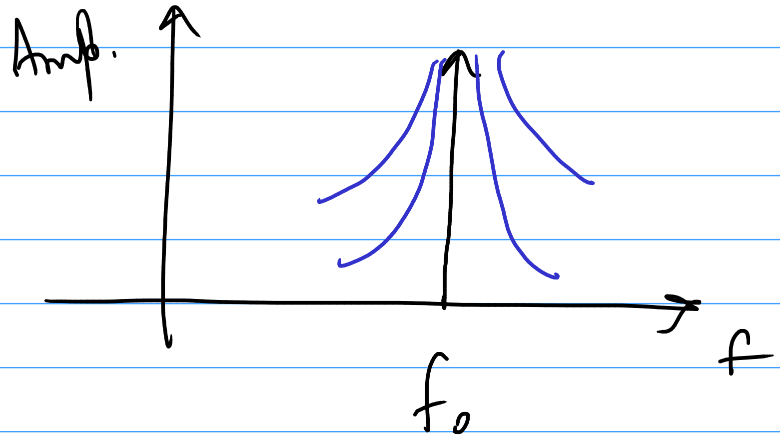


We want Kvco ~ 100MHz/V

large Varactor

AM noise → PM noise

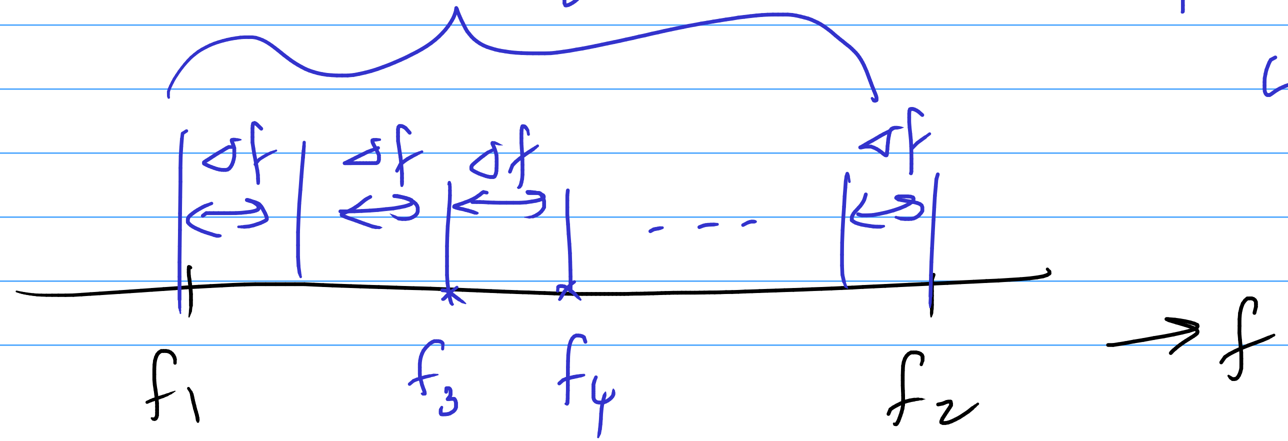


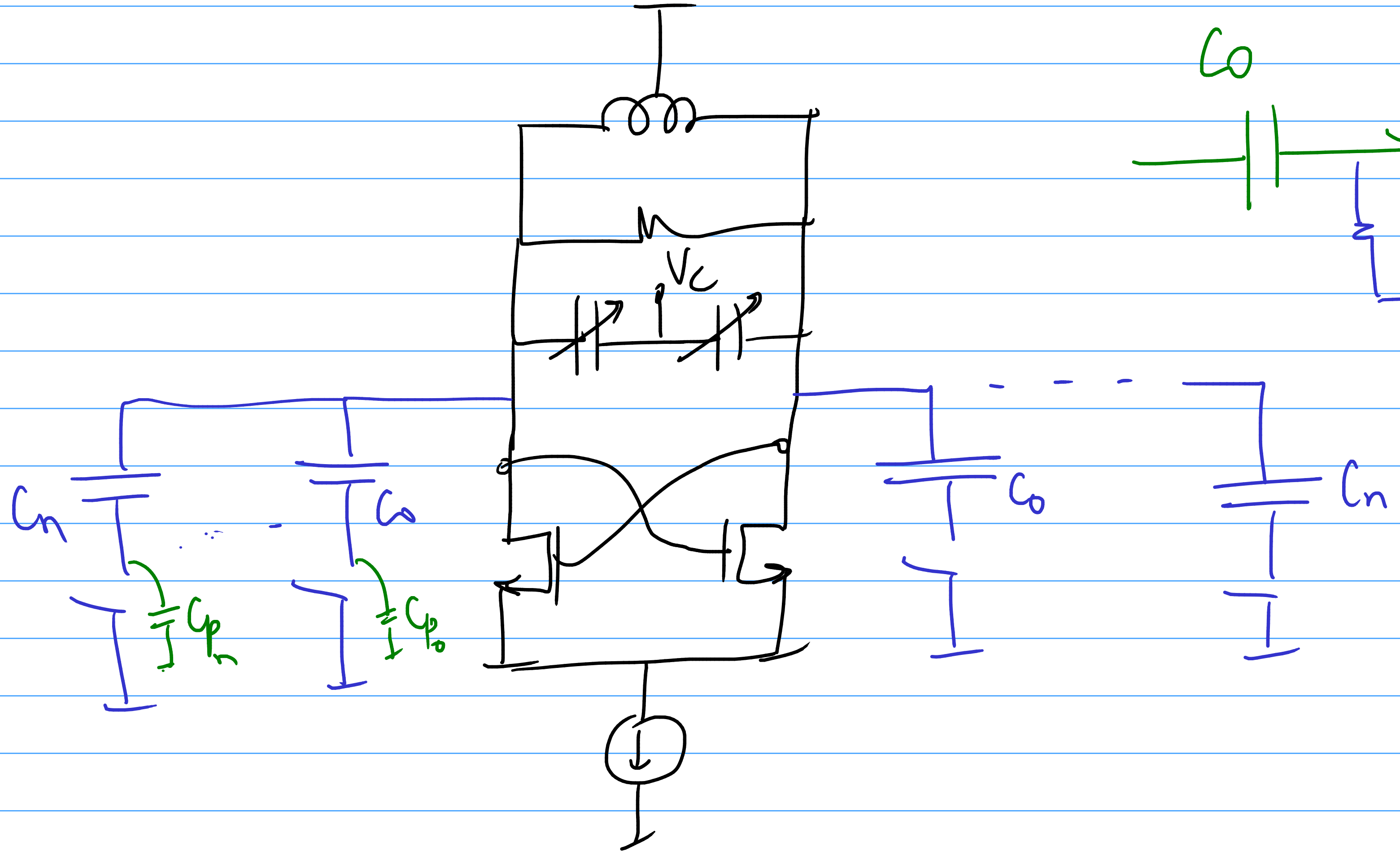


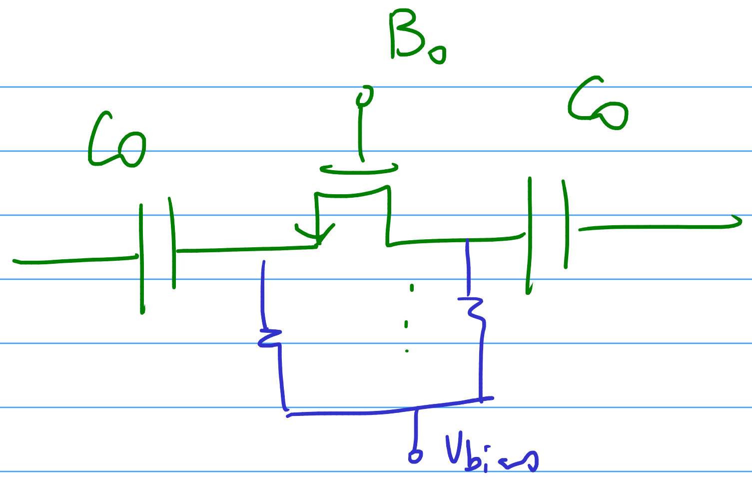
$$\sim \longrightarrow \left[A + a_n(t)\right] \cos(\omega_0 t)$$

$$f_1 \longrightarrow f_2$$

32 pieces $\rightarrow$ 5 bits of digital control



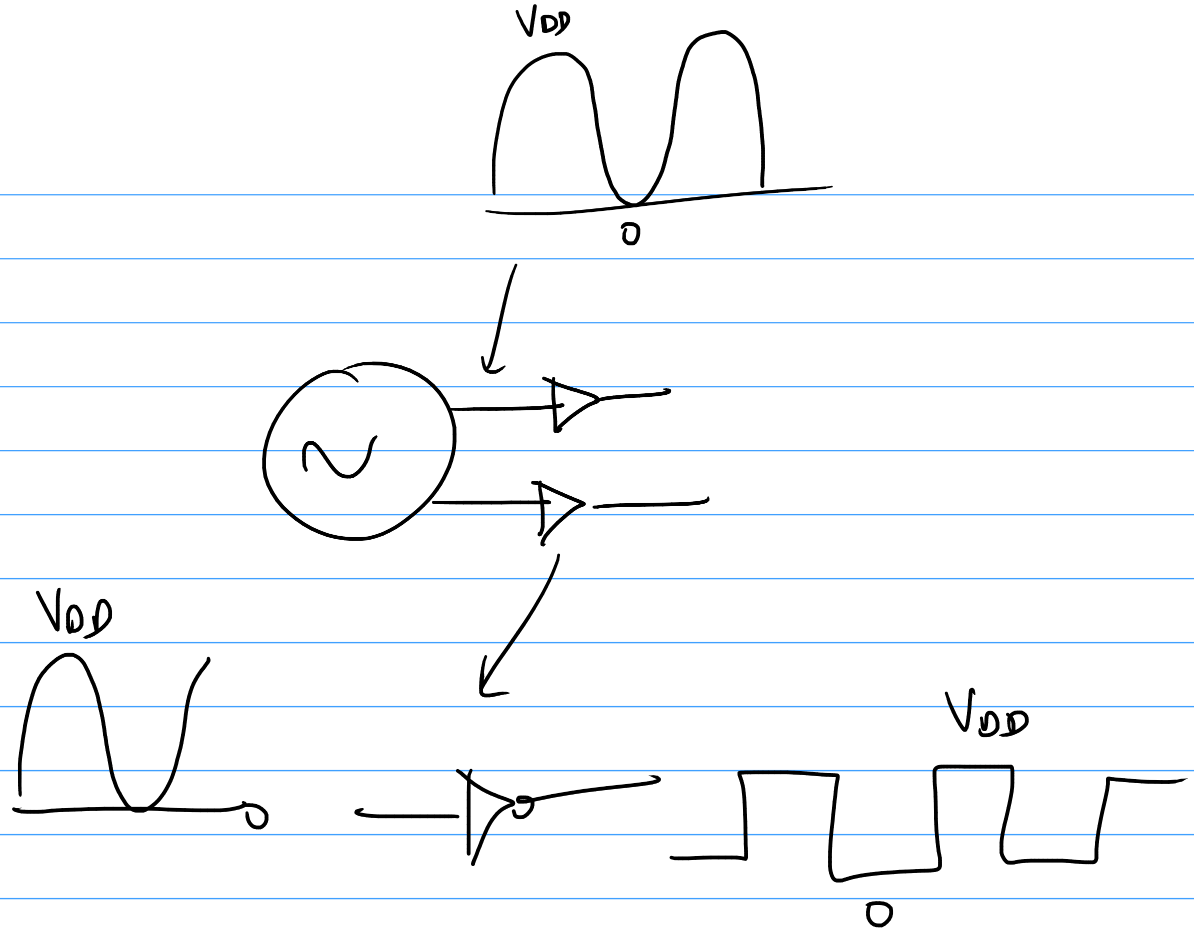


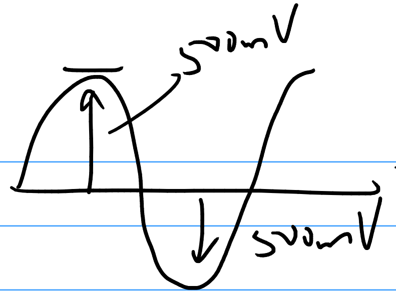


Size of switch

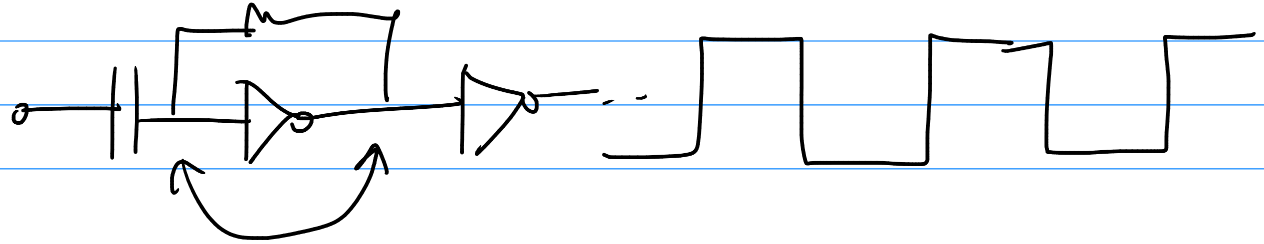
$\frac{C_{max}}{C_{min}}$ | cap bank

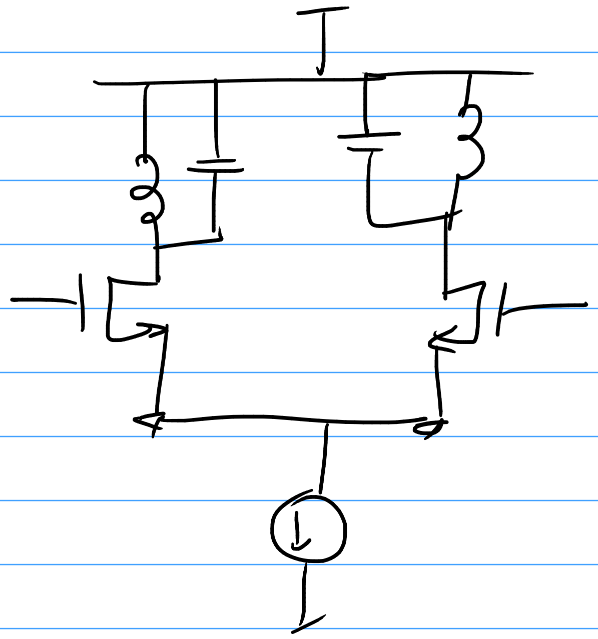
Q cap. bank





$V_{DD} = 1.2V$

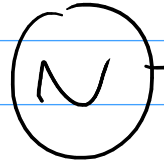




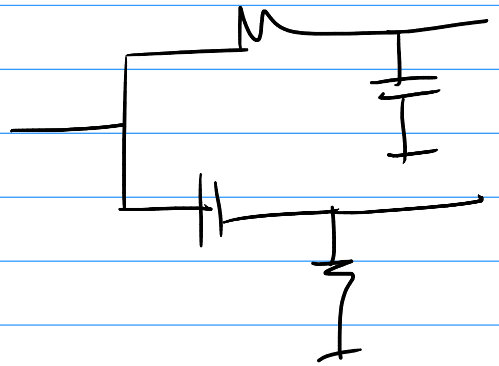
tuned buffer

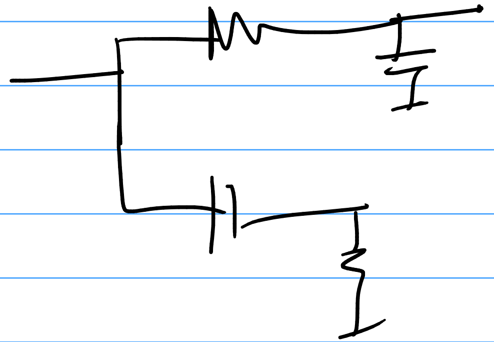
f_{VCO} +

Polyphase filter

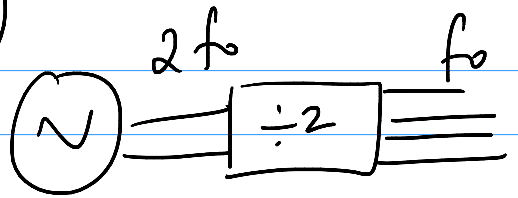
 $\rightarrow$ I & Q $\nearrow$ Polyphase filter

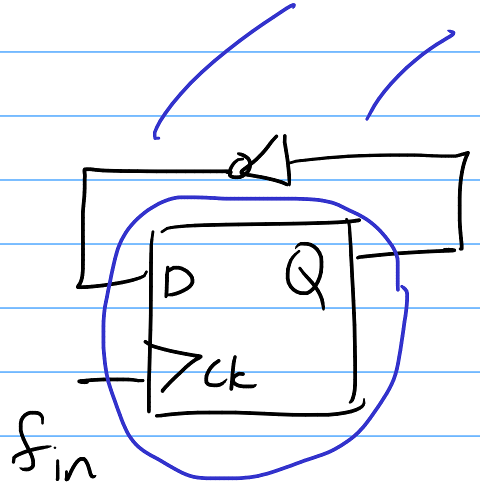
1) ~~Polyphase~~



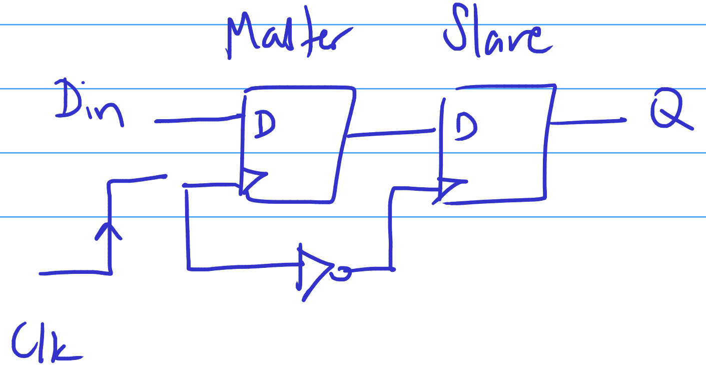


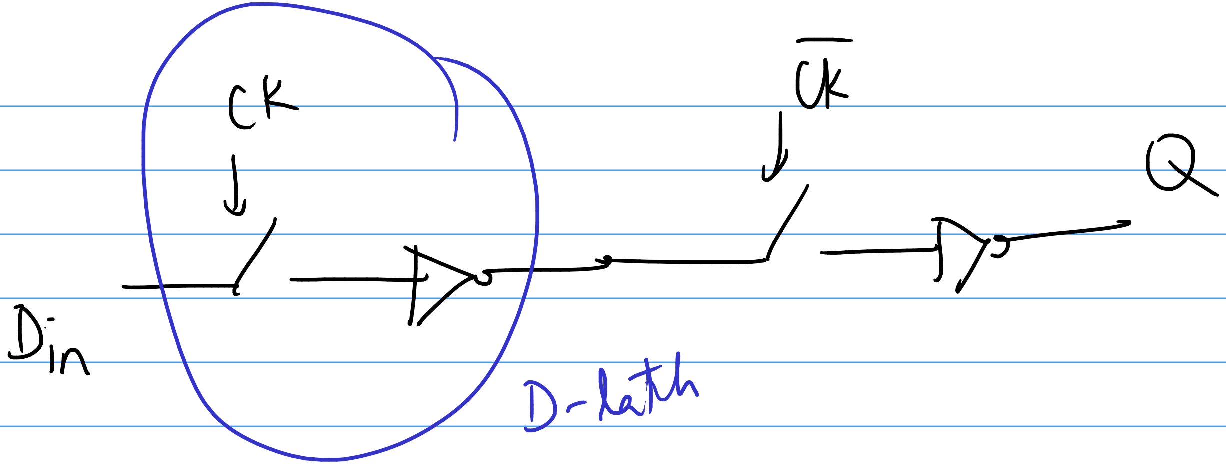
2)

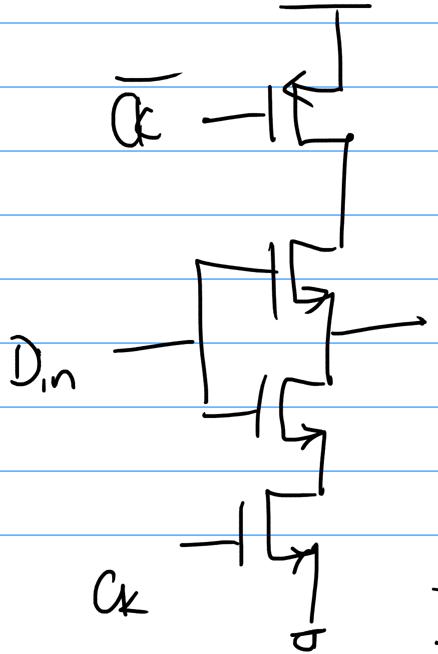




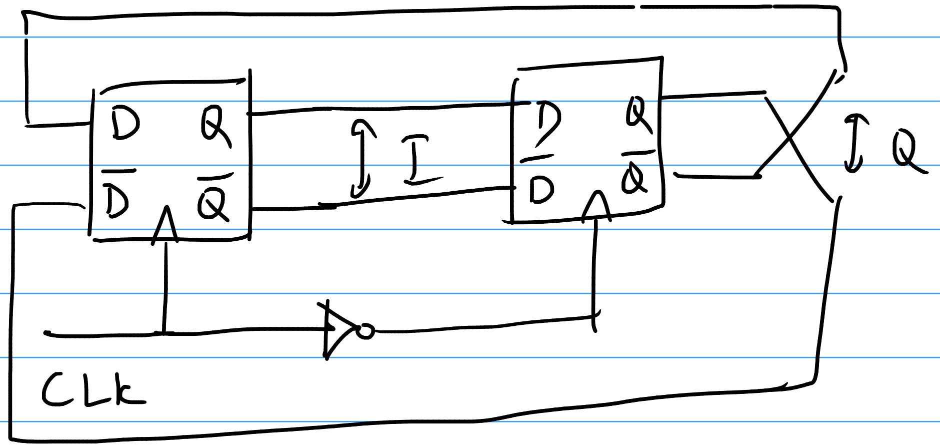
$f_{out} = \frac{f_{in}}{2}$



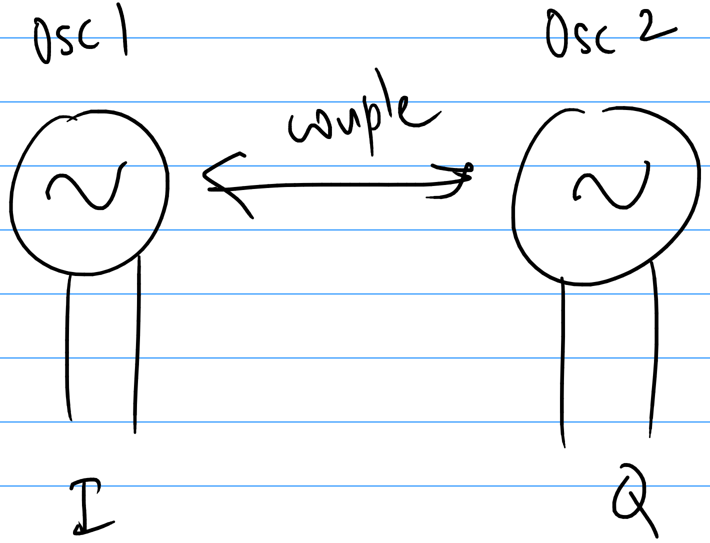

CK
$\overline{CK}$
Q
D_{in}
D-latch


$\overline{CK}$
D_{in}
CK

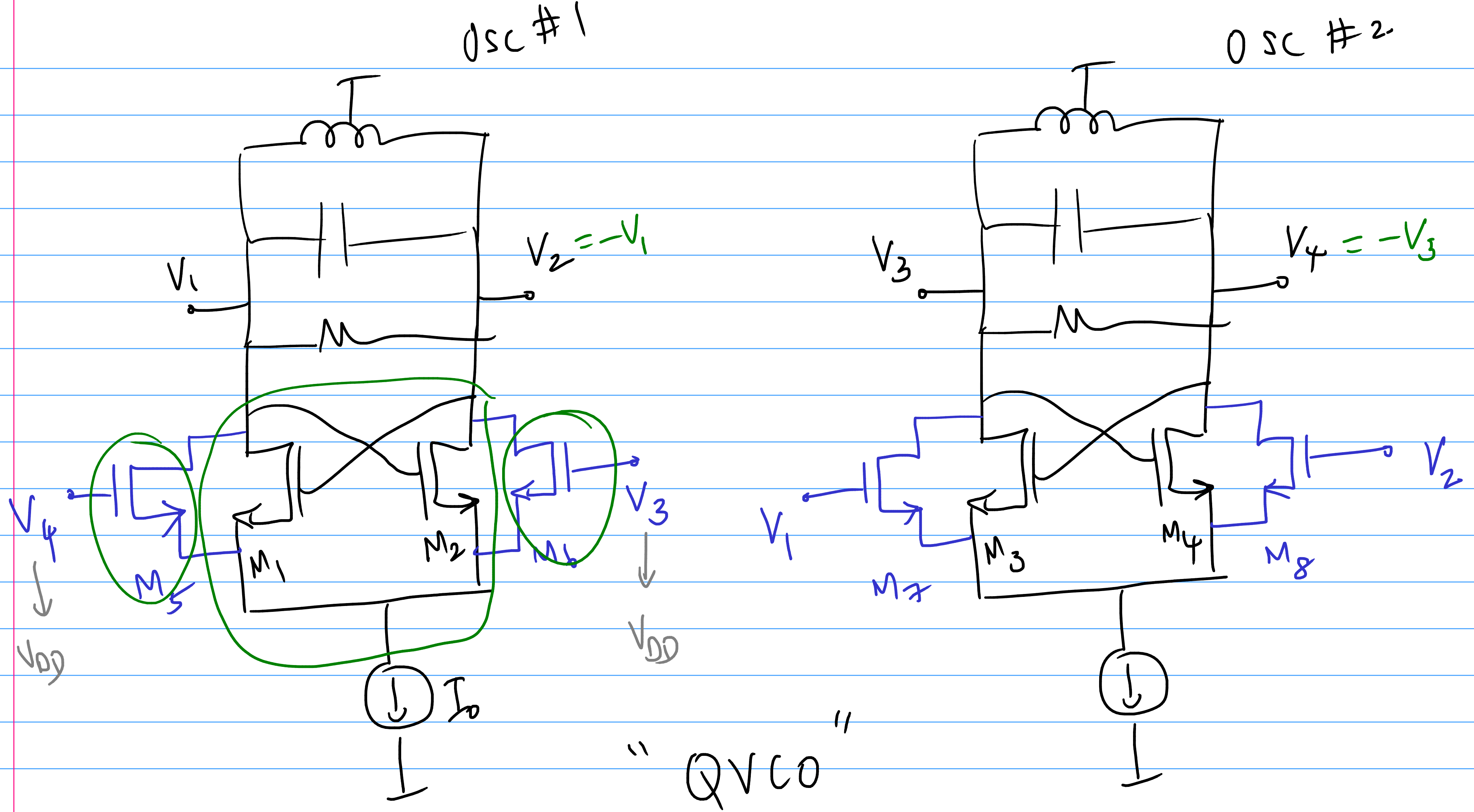
D-latch


D
Q
$\overline{D}$
$\overline{Q}$
D
Q
$\overline{D}$
$\overline{Q}$
CLK
Q

3)



Quadrature VCO



Osc #1
Osc #2
V1
V2 = −V1
V3
V4 = −V3
V4
V3
V1
V2
M1
M2
M3
M4
M5
M6
M7
M8
VDD
VDD
I0
"QVCO"

<u>case (i)</u> : $V_1 - V_2$ & $V_3 - V_4$ are in phase

$$V_1 \equiv V_3 , \quad V_2 \equiv V_4$$

$M_1 - M_8$ are all identical

* Both osc. die down

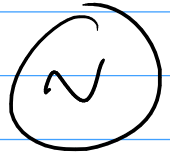
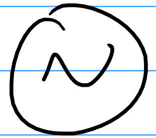
<u>case (ii)</u> $V_1 - V_2$ & $V_3 - V_4$ are out of phase

$$V_1 \equiv V_4 , \quad V_2 \equiv V_3$$

* Both osc. die down

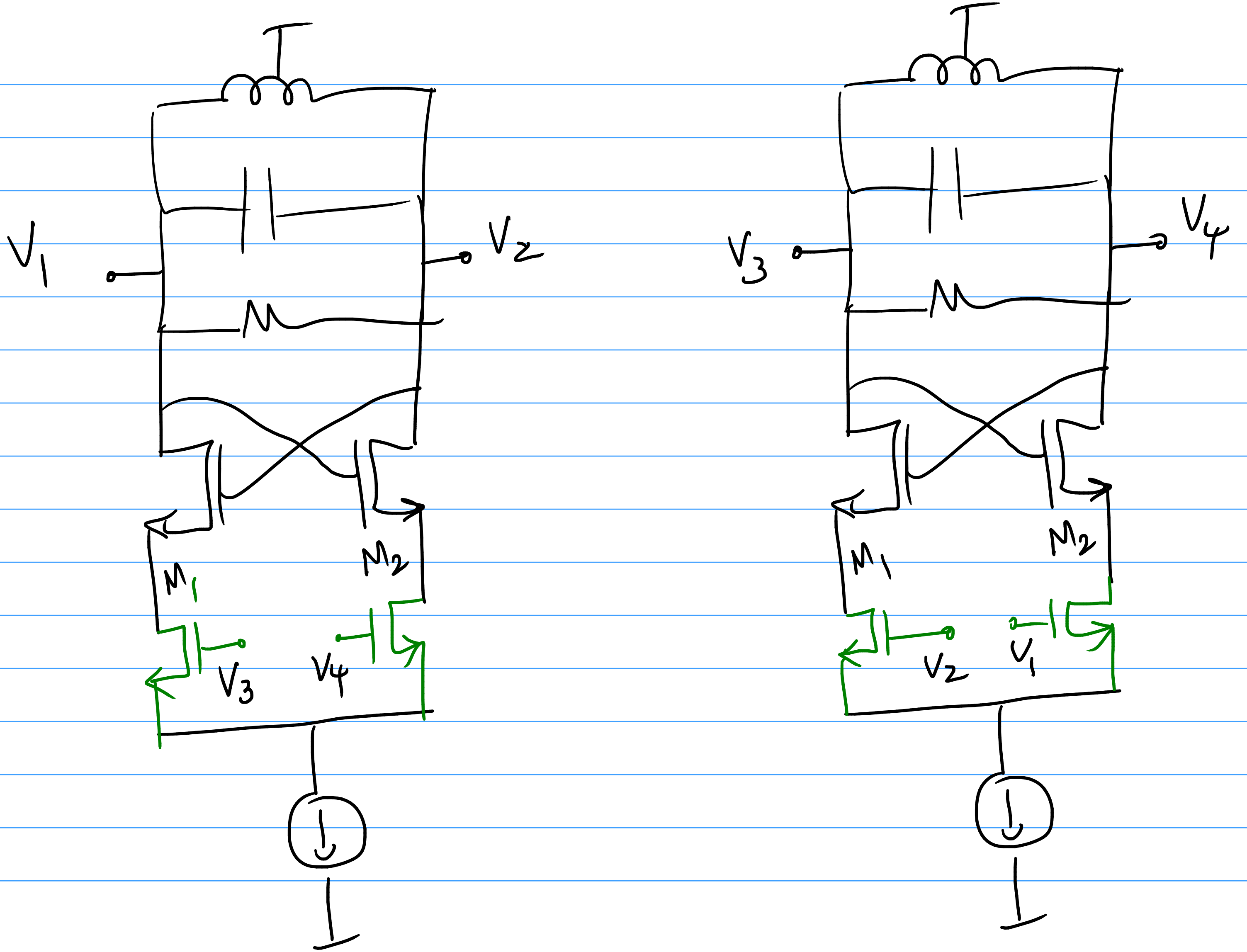
case (iii)

$V\angle 0$ $\qquad$ $V\angle\theta$

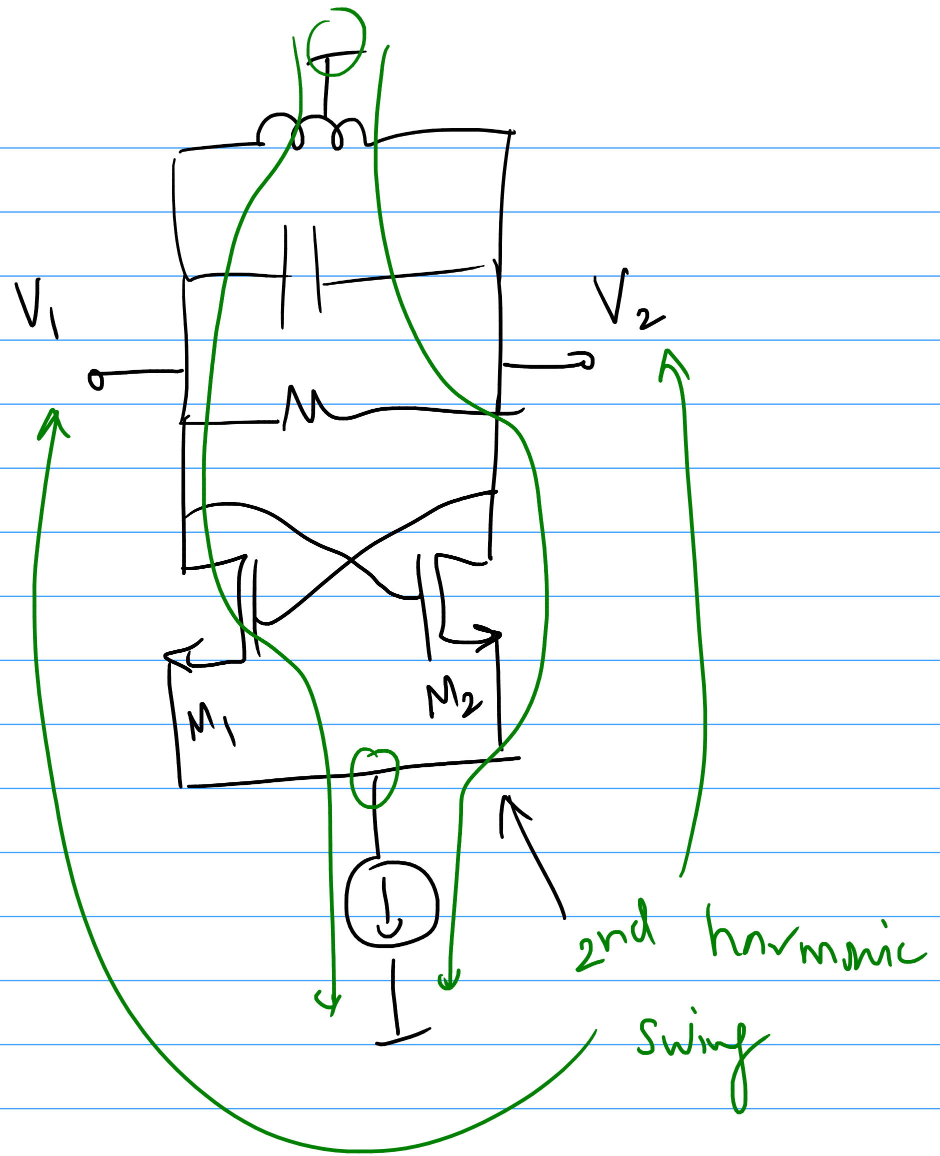
 

* Component of $V\angle\theta$ in- or out-of-phase with $V\angle 0$ will die out

* $\left.\begin{array}{l} V\angle 0 \quad \& \quad V\angle 90^\circ \\ V\angle 0 \quad \& \quad V\angle -90^\circ \end{array}\right\}$ steady state



Series
coupled
QVCO

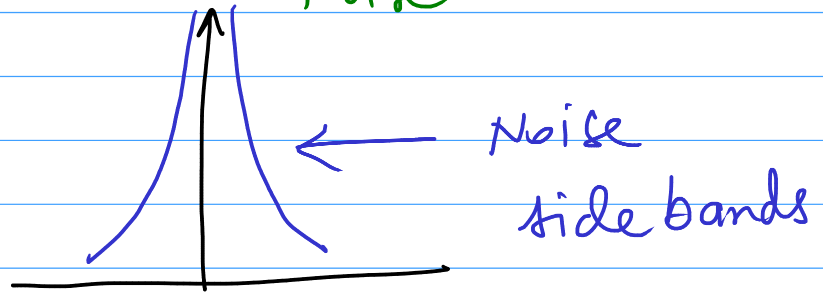

V1
V2
M1
M2
2nd harmonic
Swing

Noise in Oscillators

Ideal (~) → $V_a \cos(\omega_0 t)$

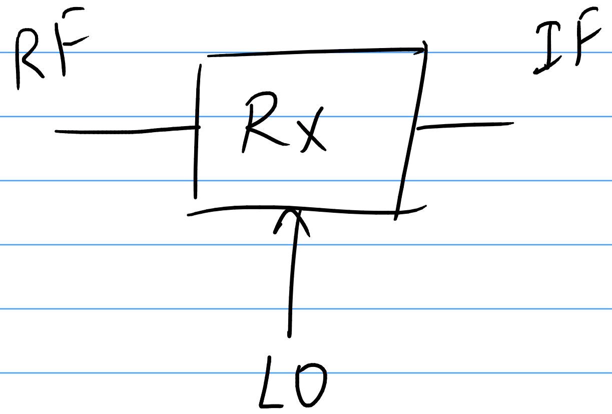
Real (~) →

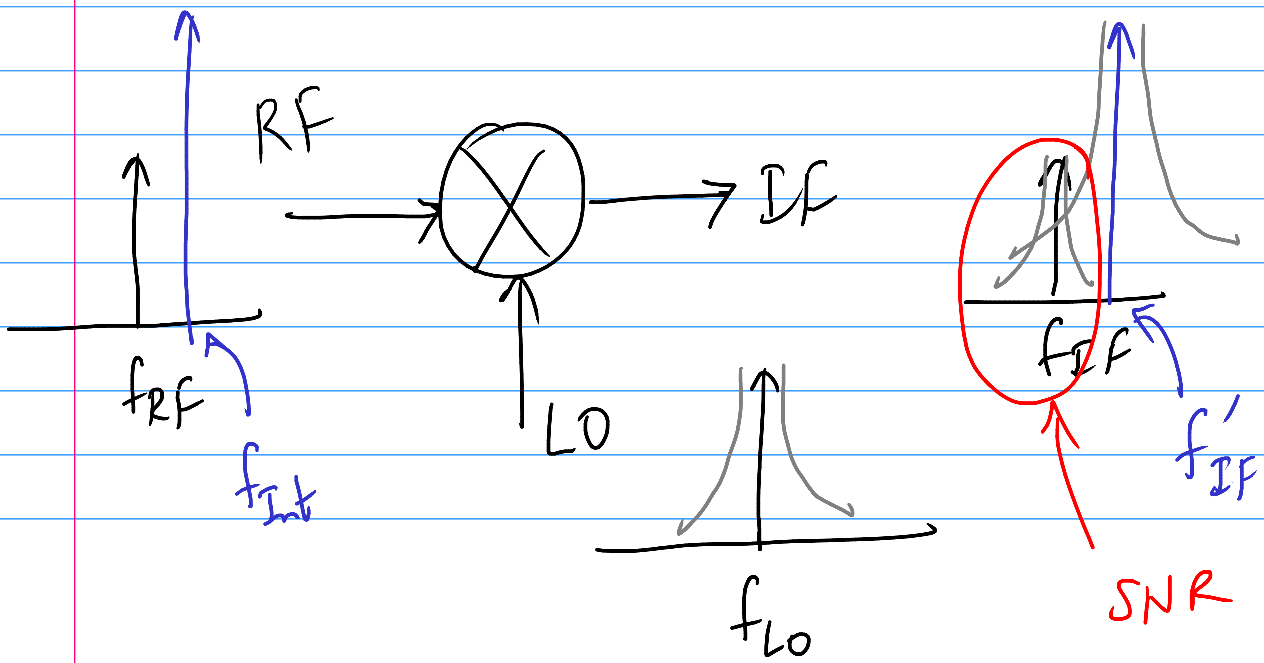
$$\left[V_a + \underbrace{v_n(t)}_{\text{Amplitude Noise}}\right] \cdot \cos\left(\omega_0 t + \underbrace{\varphi_n(t)}_{\text{Phase Noise}}\right)$$



$$\omega(t) = \omega_0 + \frac{d}{dt}\varphi_n(t)$$

1)

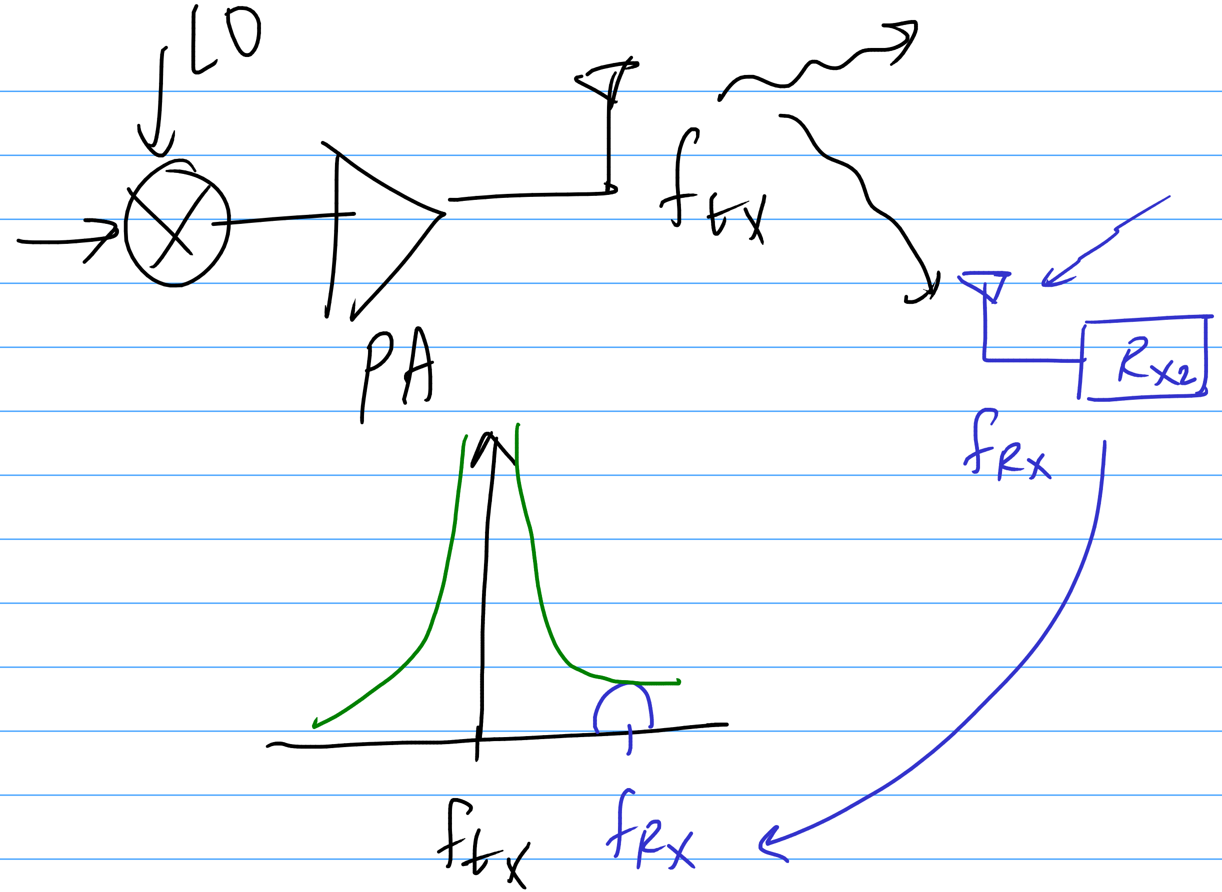


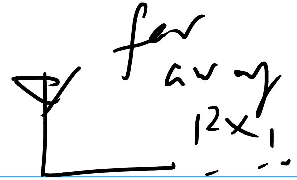


"Reciprocal Mixing"

SNR degradation

2)

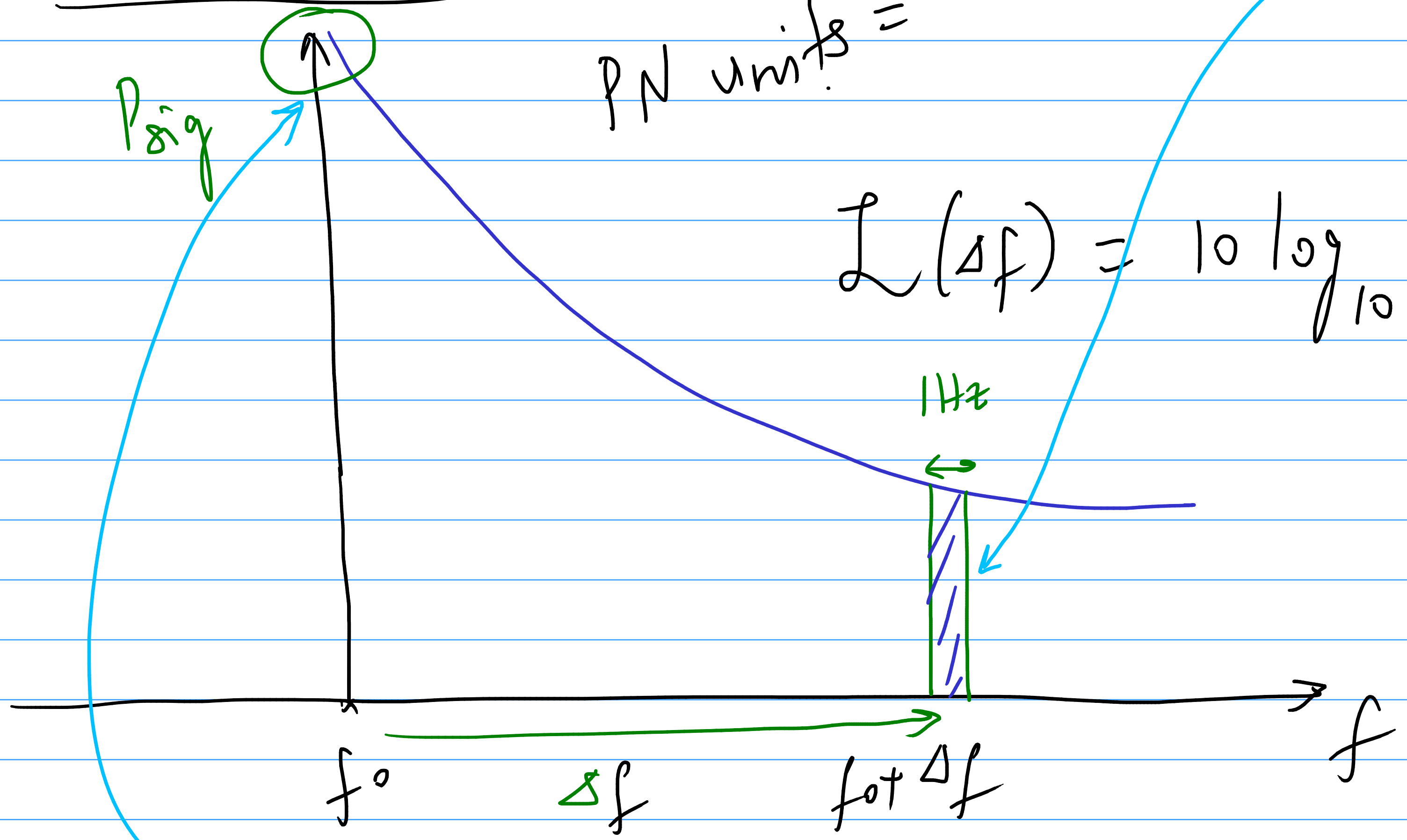


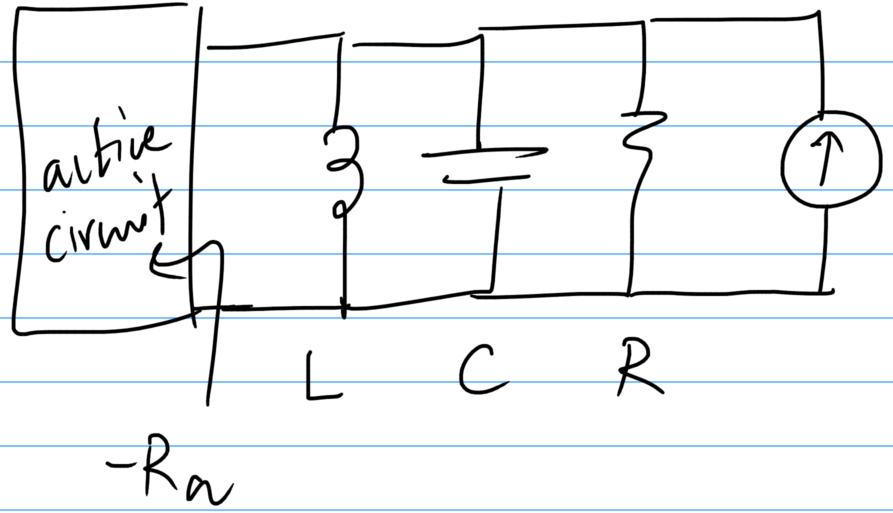


Phase Noise

PN units = dBc/Hz

$$\mathcal{L}(\Delta f) = 10 \log_{10}\left[\frac{P_{1Hz}(f_0+\Delta f)}{P_{sig}}\right]$$ ✓





$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R} = 4kTG$$

@ $(\omega_0 + \Delta\omega)$

$$Y(\omega_0+\Delta\omega) = G + j(\omega_0+\Delta\omega)\cdot C + \frac{1}{j(\omega_0+\Delta\omega)L}$$

$$= G + \frac{j^2(\omega_0+\Delta\omega)^2 LC + 1}{j(\omega_0+\Delta\omega)L}$$

$$= G + \frac{\underbrace{1-\omega_0^2 LC}_{\to 0} - 2\omega_0\Delta\omega LC - \underbrace{\cancel{\Delta\omega^2 LC}}_{\approx 0}}{j(\omega_0+\Delta\omega)L}$$

$$\approx G - \frac{2\omega_0\Delta\omega LC}{j(\omega_0+\Delta\omega)L} \approx G + j\frac{2\omega_0\Delta\omega}{\omega_0+\Delta\omega}C$$

$$Z(\omega_0 + \Delta\omega) = \frac{1}{G + j\,\dfrac{2\omega_0 \Delta\omega\, C}{\omega_0 + \Delta\omega}}$$

$$= \frac{1}{G} \cdot \frac{1}{1 + j\,\dfrac{2\omega_0 \Delta\omega\, RC}{\omega_0 + \Delta\omega}}$$

$$\omega_0 + \Delta\omega \approx \omega_0$$

$$Q_0 = \omega_0 RC$$

$$Z(\omega_0 + \Delta\omega) = R \cdot \frac{1}{1 + j\, 2Q_0 \dfrac{\Delta\omega}{\omega_0}}$$

$$\left| Z(\omega_0 + \Delta\omega) \right| \approx \frac{R\omega_0}{2Q_{eff}\Delta\omega} \quad \text{if } Q \text{ is very high}$$

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot |Z|^2$$

$$= \frac{4kT}{R} \times \frac{R^2 \cdot \omega_0^2}{4Q_{eff}^2 \Delta\omega^2}$$

$$= 4kTR \cdot \left(\frac{\omega_0}{2Q_{eff}\Delta\omega} \right)^2$$

$\rightarrow$ ½ as Ampl. Noise

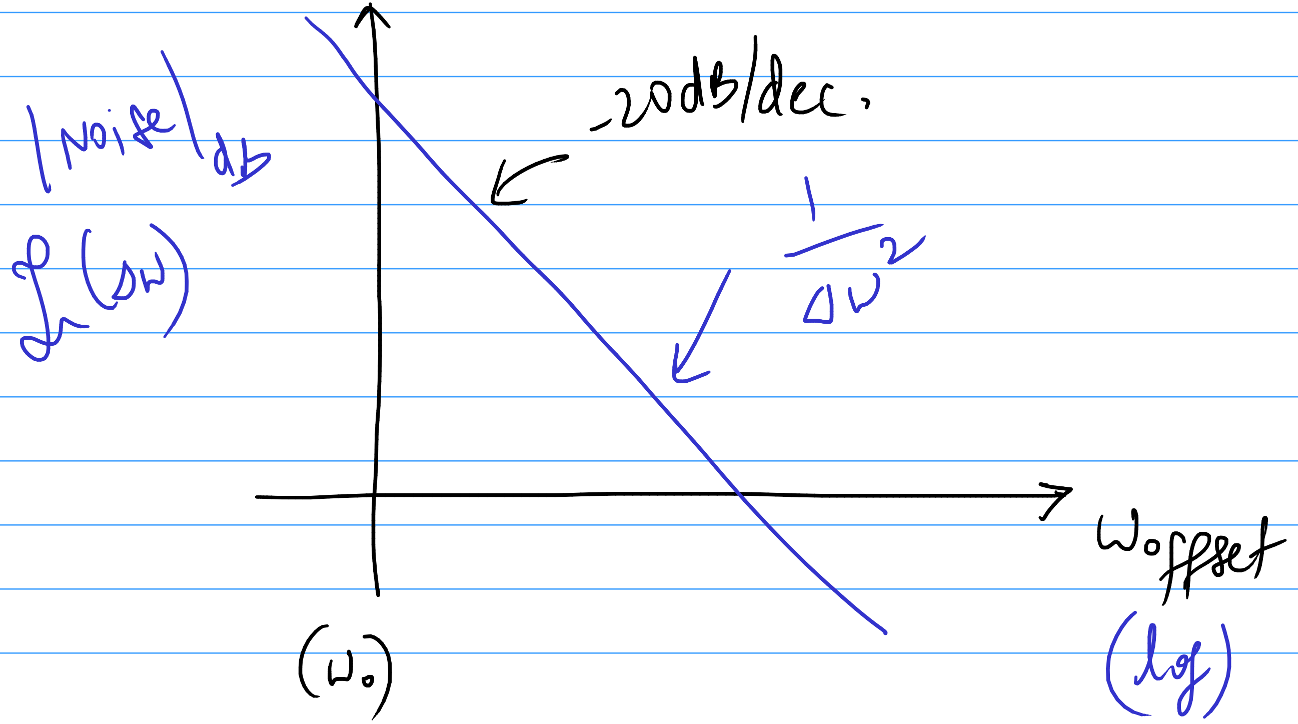
$\rightarrow$ ½ as Phase Noise

$$PN = \frac{1}{2}\frac{\overline{V_n^2}}{\Delta f} = 2kTR\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

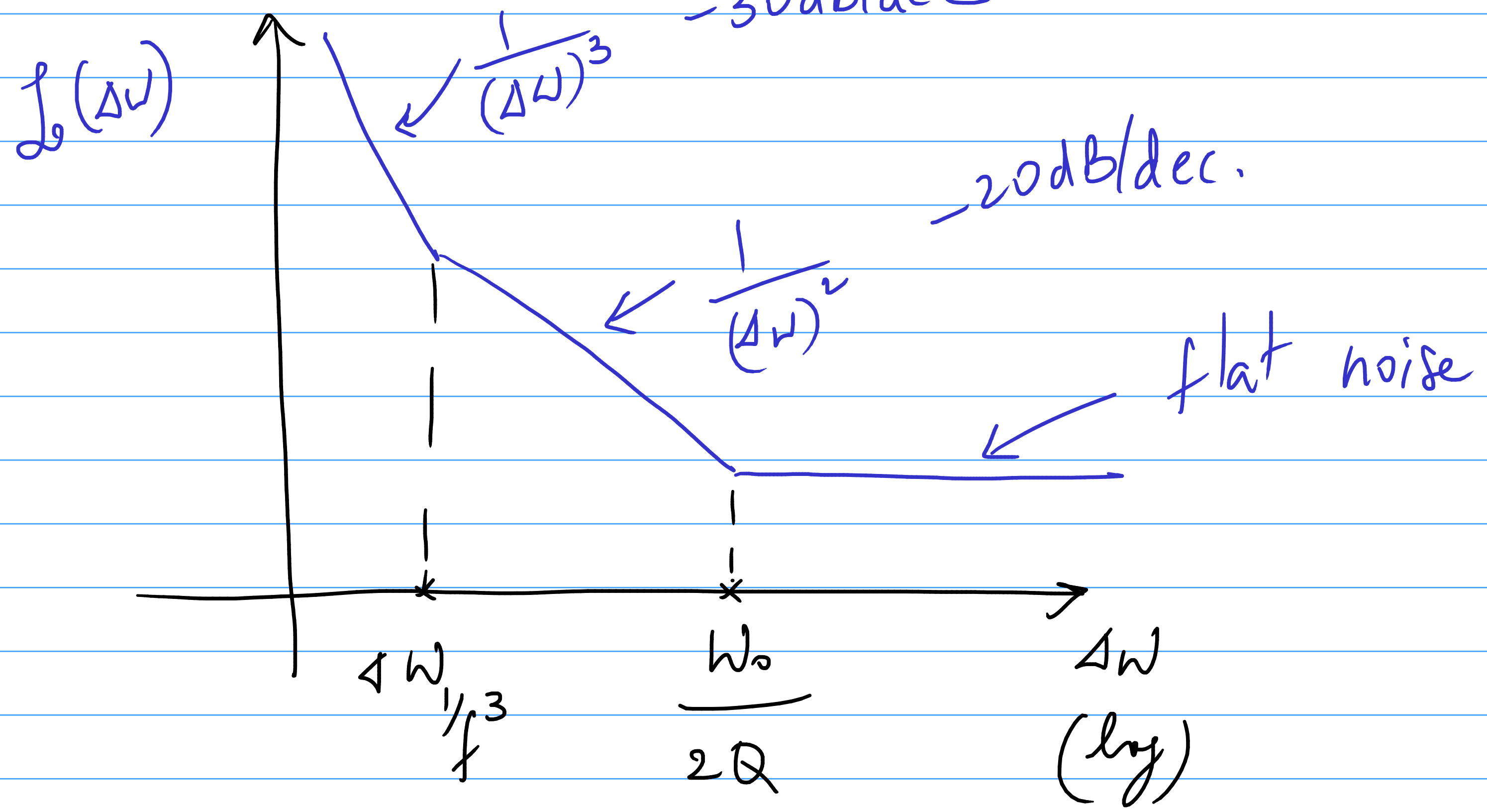
$$\mathcal{L}(\Delta\omega) = 10\log_{10}\left[\frac{\frac{1}{2}\frac{\overline{V_n^2}}{\Delta f}}{V_{sig}^2}\right]$$

$$= 10\log_{10}\left[\frac{2kTR\cdot\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2}{V_{sig}^2}\right]$$

$$\mathcal{L}(\Delta\omega) = 10\log_{10}\left[\frac{2kT}{P_{sig}}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$



Real Osc. Spectrum:



1) D. B. Leeson, "A simple model of feedback oscillator noise spectrum", Proceedings of the IEEE pp 320–330, Feb. 1966

$$\mathcal{L}(\Delta\omega) = 10\log_{10}\left[\frac{2kTF}{P_{sig}}\left\{1+\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right\}\left\{1+\frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right\}\right]$$

Maximise V_{sig}

Maximise Q

F – empirical term
"excess noise number"

2) J. Rael & A. Abidi, "Physical processes of phase noise in differential LC oscillators", IEEE Custom Integrated Circuits Conference, 2000.

Osc. is Nonlinear

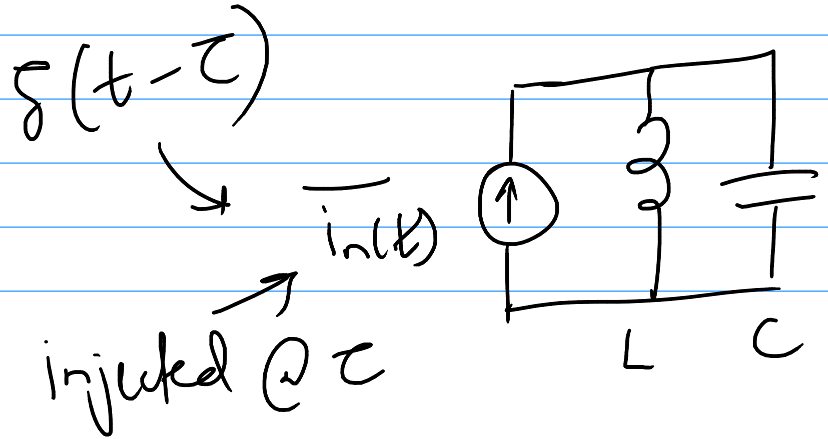
$$\boxed{F = 2 + \frac{8\gamma R I_T}{\pi V_0} + \gamma \cdot \frac{8}{9} g_m R}$$

(CC LC osc.)

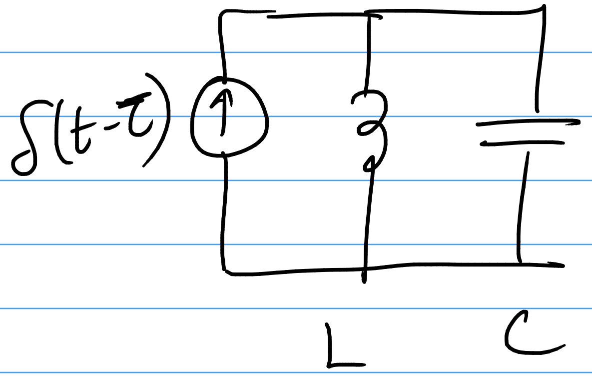
$$V_0 = \frac{2}{\pi} \cdot I_T R$$

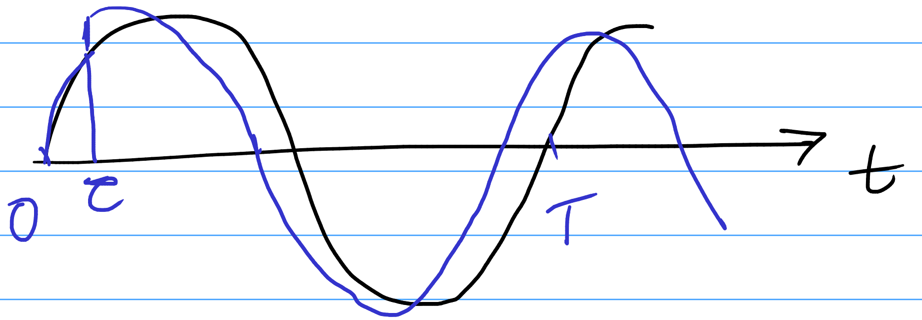
3) Hajimiri-Lee LTV model

Noise is small → "Linear" approx is valid for small deviations around NL v operating point (but periodic)

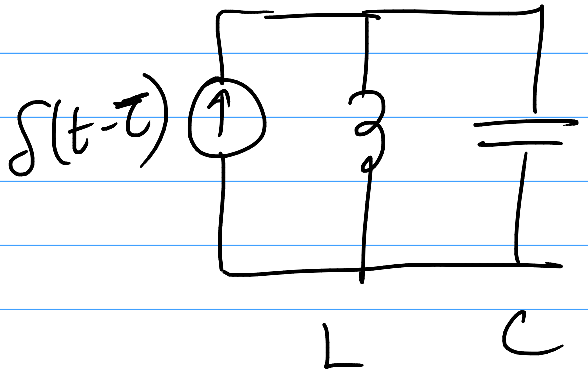


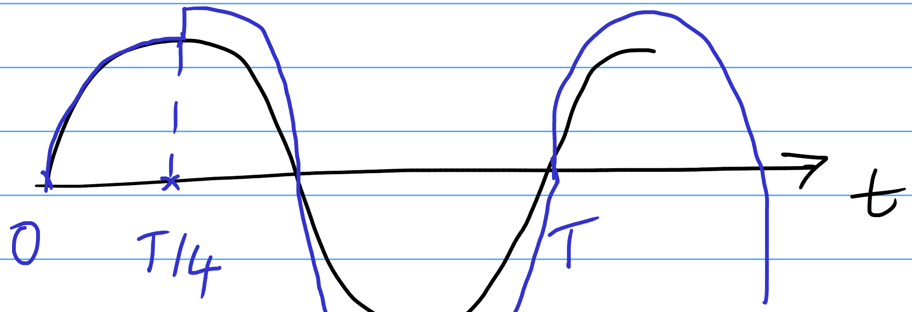
1)





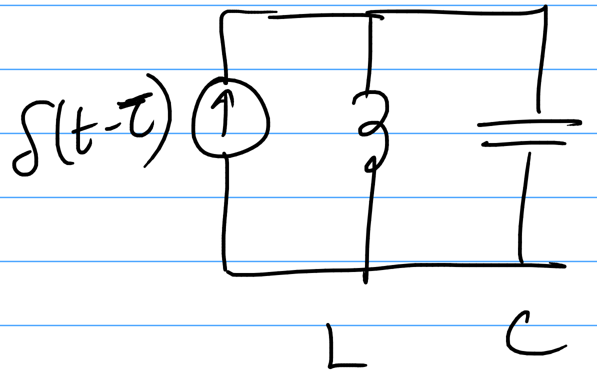
2)

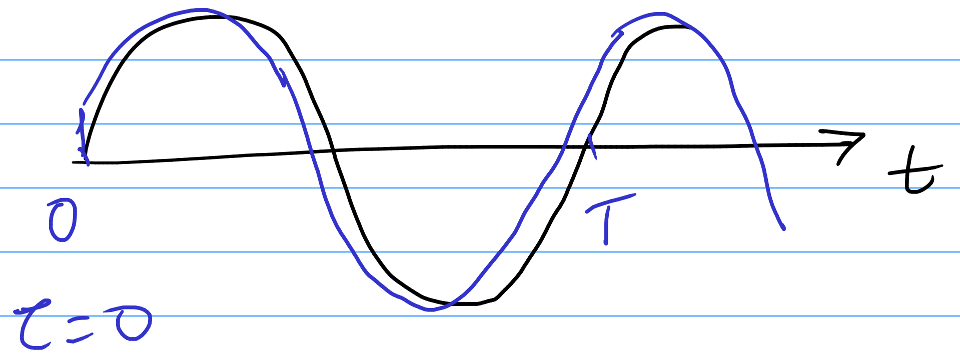




$\tau = T/4$

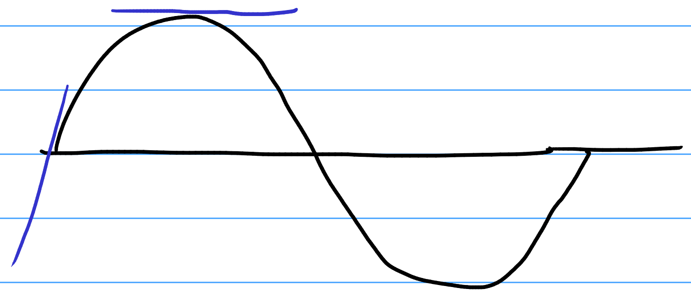
3)





Osc. looks like an LTV system w.r.t. noise

$$v(t) = V_0 \cos(\omega_0 t)$$



$$i_n(t) = \Delta q \, \delta(t - \tau)$$

$$\Delta v = \frac{\Delta q}{C}$$

$$v(t+\Delta t) = v(t) + \frac{dv}{dt} \cdot \Delta t$$

$$\Delta v = \frac{dv}{dt} \cdot \Delta t \longrightarrow \Delta t = \frac{\Delta v}{dv/dt}$$

$$\Delta t(\tau) = \frac{\Delta V}{-\omega_0 V_0 \sin(\omega_0 \tau)}$$

$$\Delta \varphi = \omega_0 \Delta t$$

$$\Delta \varphi(\tau) = \omega_0 \cdot \frac{\Delta q / C}{-\omega_0 V_0 \sin(\omega_0 \tau)} = -\frac{\Delta q}{\underbrace{C V_0}_{q_{max}} \cdot \sin(\omega_0 \tau)}$$

$$\Delta \varphi(\tau) = -\frac{\Delta q}{q_{max} \sin(\omega_0 \tau)}$$

<u>Define</u> $\Gamma(\omega_0\tau)$ = Impulse Sensitivity Function (ISF)

$$\Delta\varphi = \frac{\Delta q}{q_{max}} \cdot \underbrace{\Gamma(\omega_0\tau)}$$

$\rightarrow$ dimensionless, Periodic

$$\Gamma(\omega_0\tau + 2\pi) = \Gamma(\omega_0\tau)$$

$i_n(t) \leftarrow$ noise current flowing into LC tank

$$dq = i_n(t)\,dt$$

$$d\varphi(t) = \frac{\Gamma(\omega_0 t)}{q_{max}} \cdot i_n(t)\,dt$$

$$\frac{d\varphi}{dt} = \frac{\Gamma(\omega_0 t)}{q_{max}} \cdot i_n(t)$$

$$\Gamma(\omega_0 t) = C_0 + \sum_{k=1}^{\infty} C_k \cos(k\omega_0 t + \theta_k)$$

$i_n(t) \leftarrow$ white noise

$$S_i(\omega) = S_i$$

$$\dot{\varphi} = \frac{d\varphi}{dt} = \frac{\Gamma(\omega_0 t)}{q_{max}} \cdot i_n(t)$$

PSD

$$S_{\dot{\varphi}} = \frac{\Gamma_{rms}^2}{q_{max}^2} S_i$$

$$\Gamma_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} \Gamma^2(x)\, dx$$

$$\dot{\varphi} = \frac{d\varphi}{dt}$$

$$\dot{\Phi}(\omega) = j\omega\,\Phi(\omega)$$

$$\Phi(\omega) = \frac{1}{j\omega}\cdot\dot{\Phi}(\omega)$$

Phase PSD

$$S_{\varphi}(\omega) = \frac{1}{\omega^2}\,S_{\dot{\varphi}}(\omega)$$

$$= \frac{1}{\omega^2}\cdot\frac{\Gamma_{rms}^{2}}{q_{max}^{2}}\cdot S_i$$

$$\mathcal{L}(\Delta\omega) \propto \frac{1}{(\Delta\omega)^2}$$

$-20\,dB/dec$ regim

Flicker noise

$$S_i(\omega) \propto \frac{1}{|\omega|} \leftarrow \text{low-freq.}$$

$$S_\varphi(\omega) \propto \frac{1}{|\omega|^3} ,$$

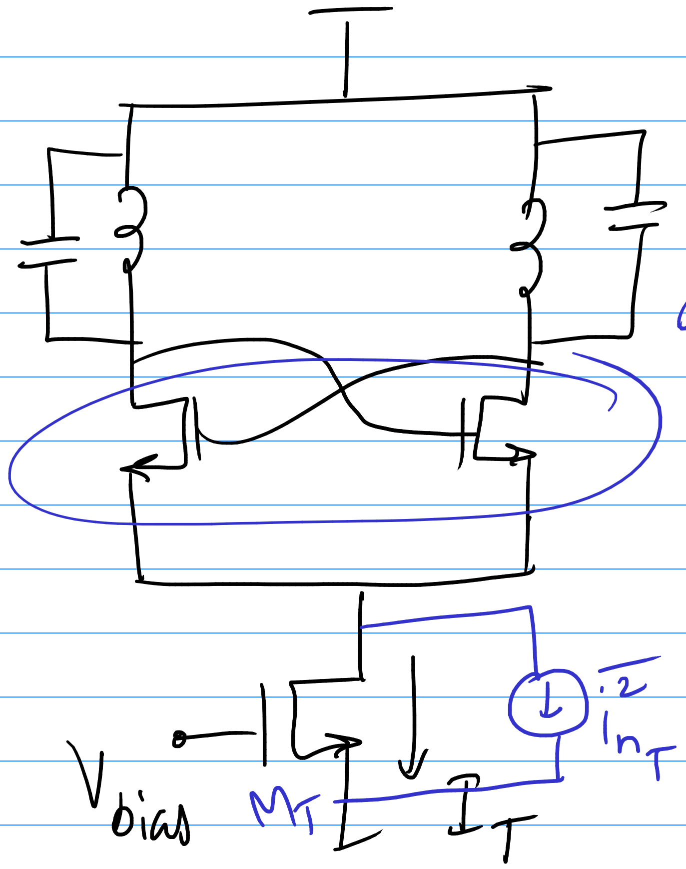
depends on DC value of $\Gamma \rightarrow C_0$

$$S_{\dot{\varphi}}(\omega) \propto \frac{C_0^2}{q_{max}^2} \cdot \frac{1}{|\omega|}$$

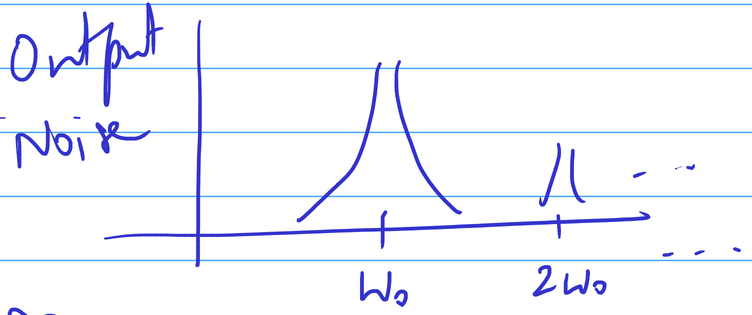
$$\left.S_{\varphi}(\omega) \propto \frac{C_0^2}{q_{max}^2} \cdot \frac{1}{|\omega|^3}\right\} \quad -30 \text{ dB/dec.}$$

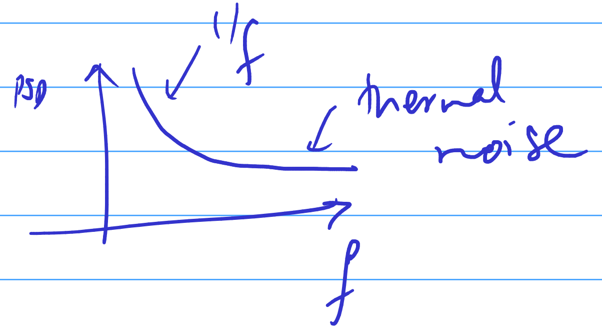
low C_0 $\Longleftrightarrow$ symmetric Γ

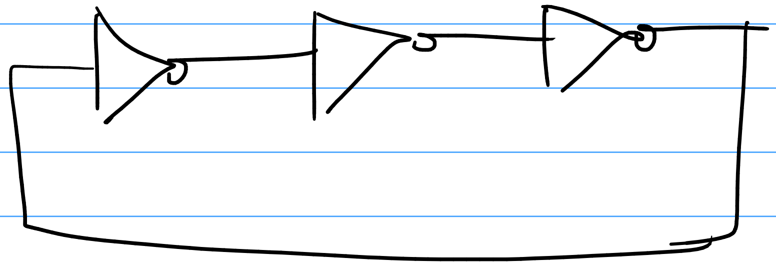
choice of VCO topology matters

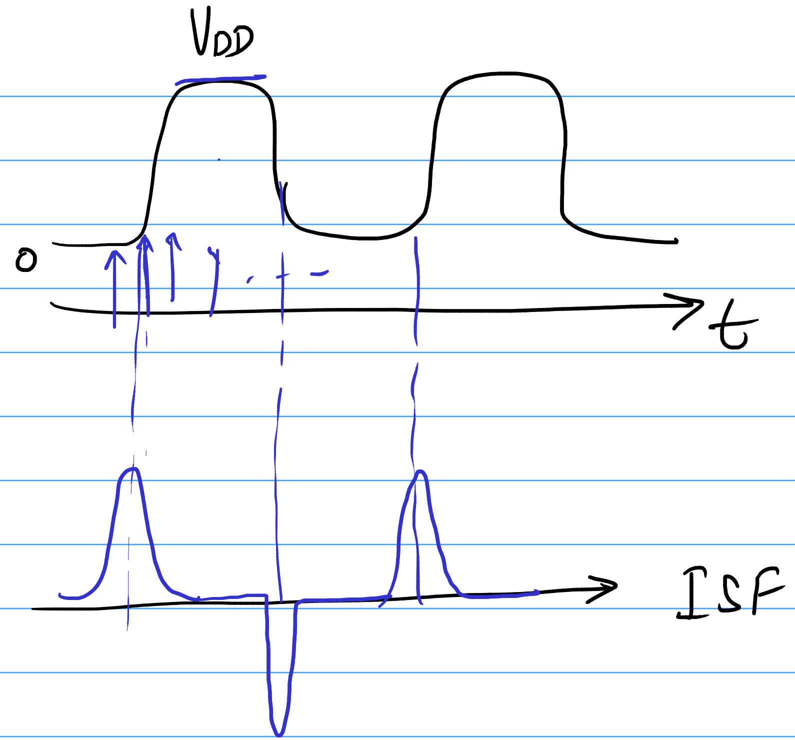


Noise Translation



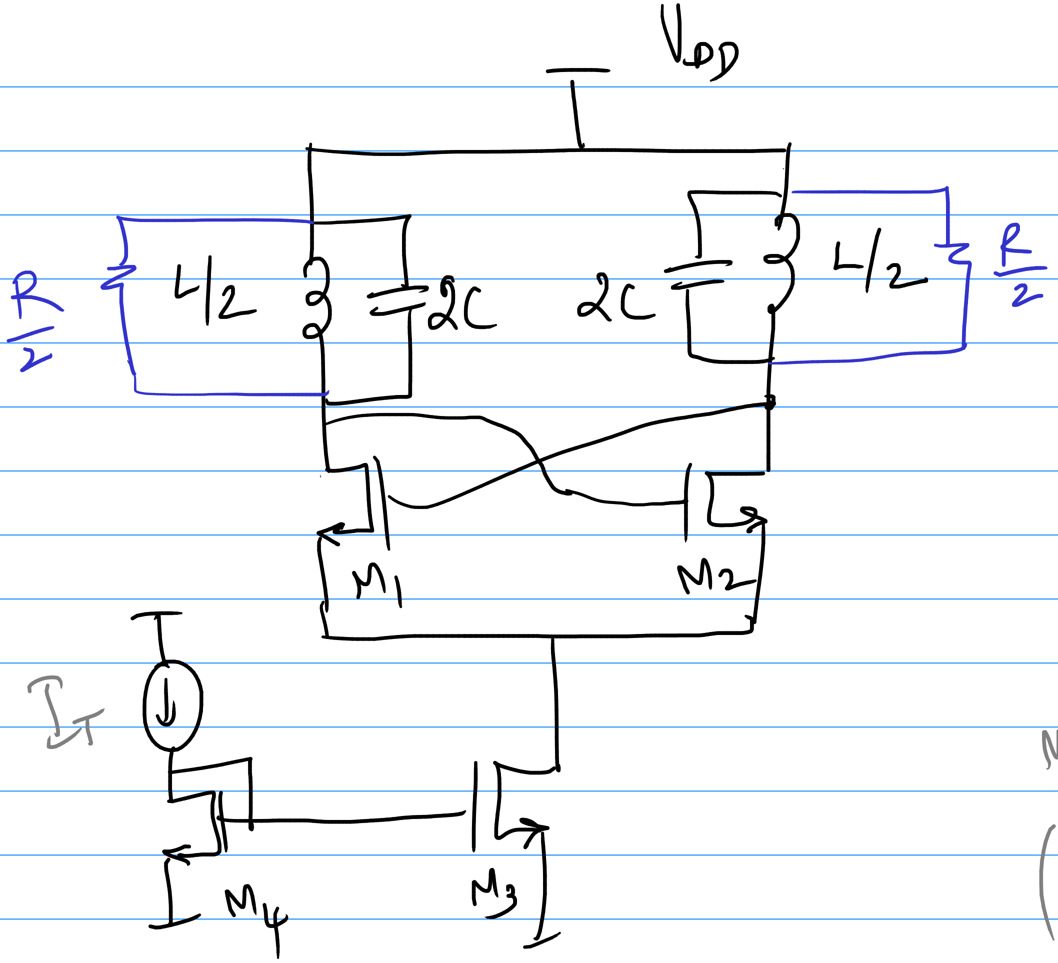






4) Demir - Roy Chowdhury Model

ISF $\Longleftrightarrow$ PPV



$$Q_L = \frac{R/2}{\omega_0 L/2} = \frac{R}{\omega_0 L}$$

$$R = \omega_0 L \cdot Q_L$$

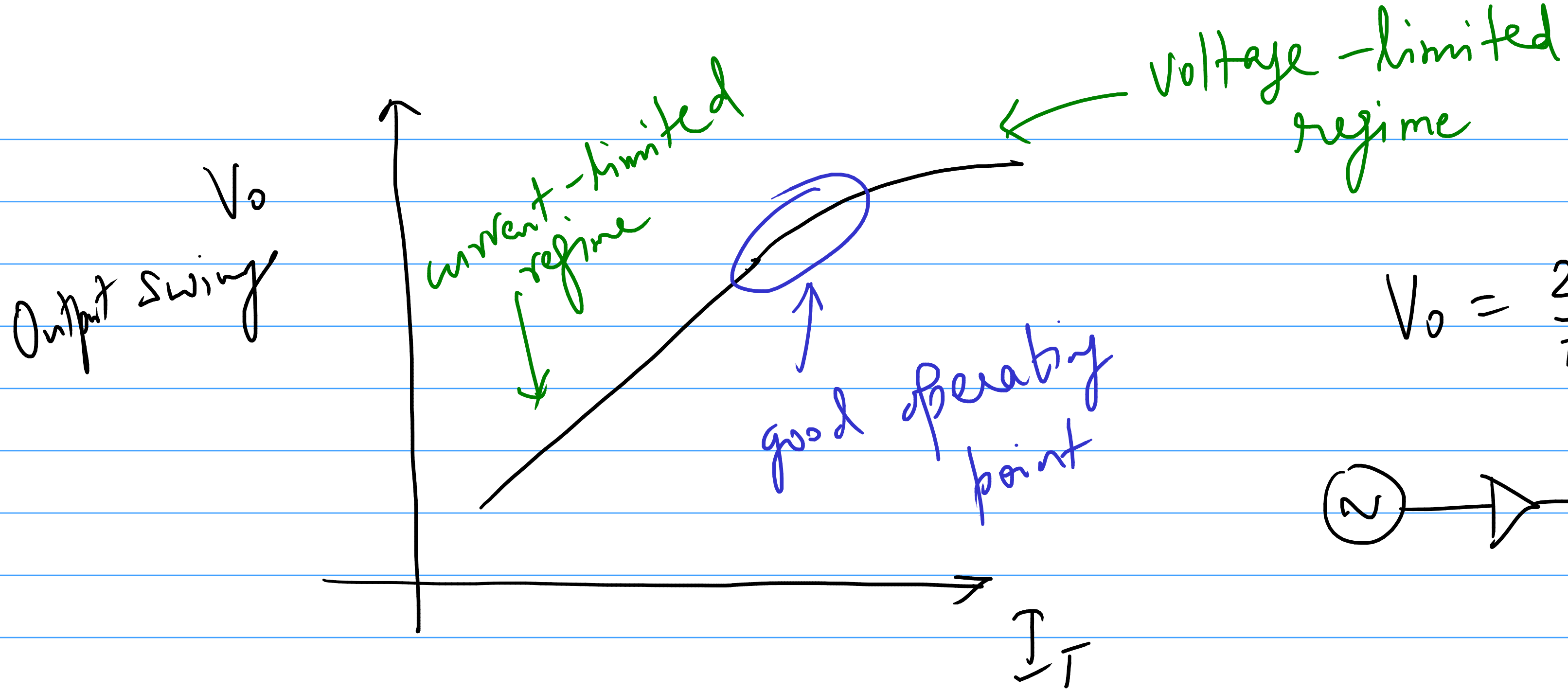
maximise

Max V_{sig} (P_{sig}) for a given I_T

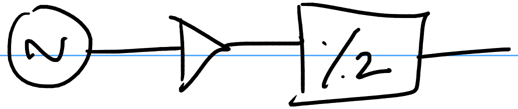
Best PN

maximise L

maximise Q_L



$$V_0 = \frac{2}{\pi} I_T \cdot R$$



Max I_T $\rightarrow$ $\left(\frac{W}{L}\right)_{1,2}$ for startup gain of 2–3
($P_{diss.}$)

choose $C \leftarrow \omega_0 = \frac{1}{\sqrt{LC}} \leftarrow$ optimise for PN.

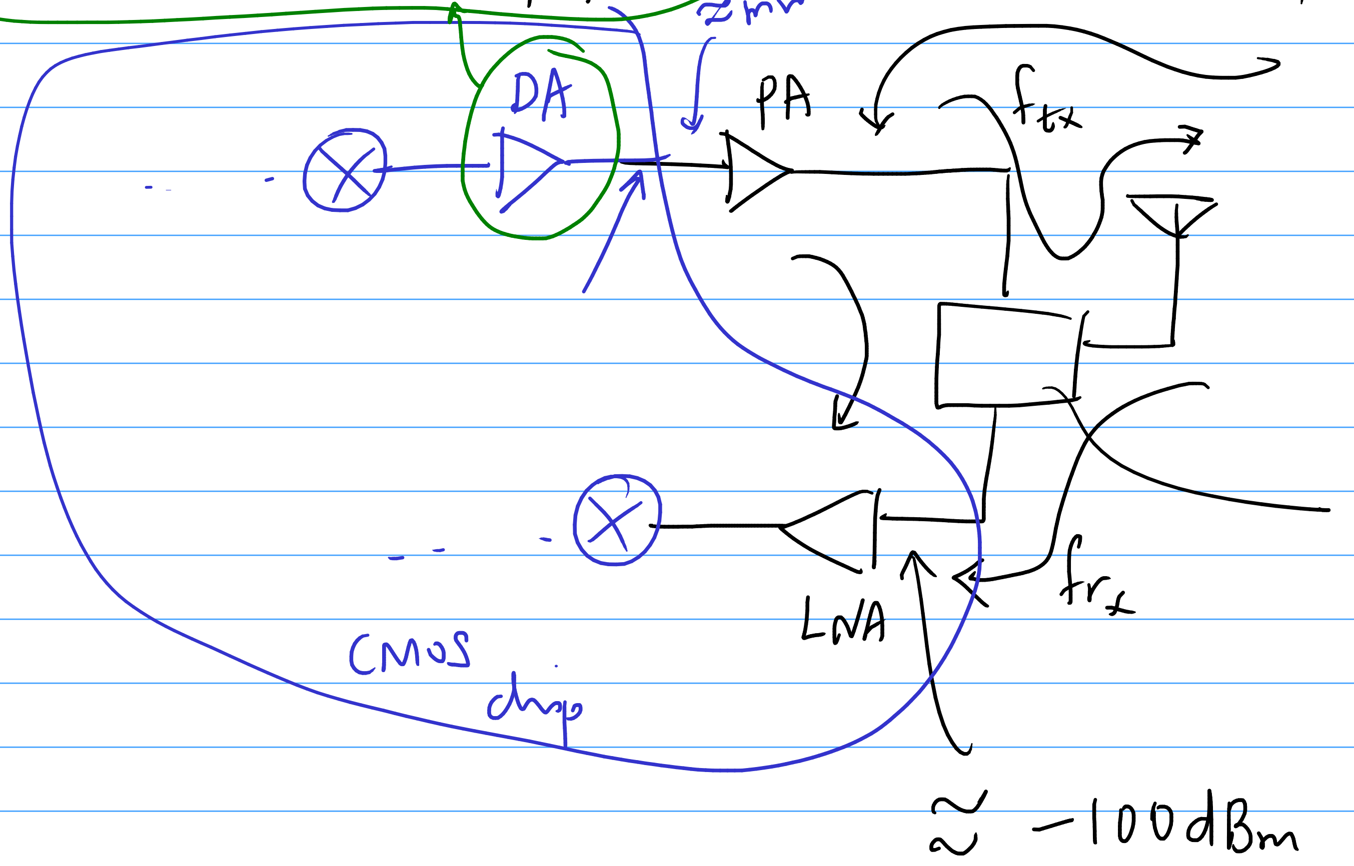
CMOS Power Amplifiers → Driver Amplifiers

GaN, LDMOS etc.

Cellular system > 1W

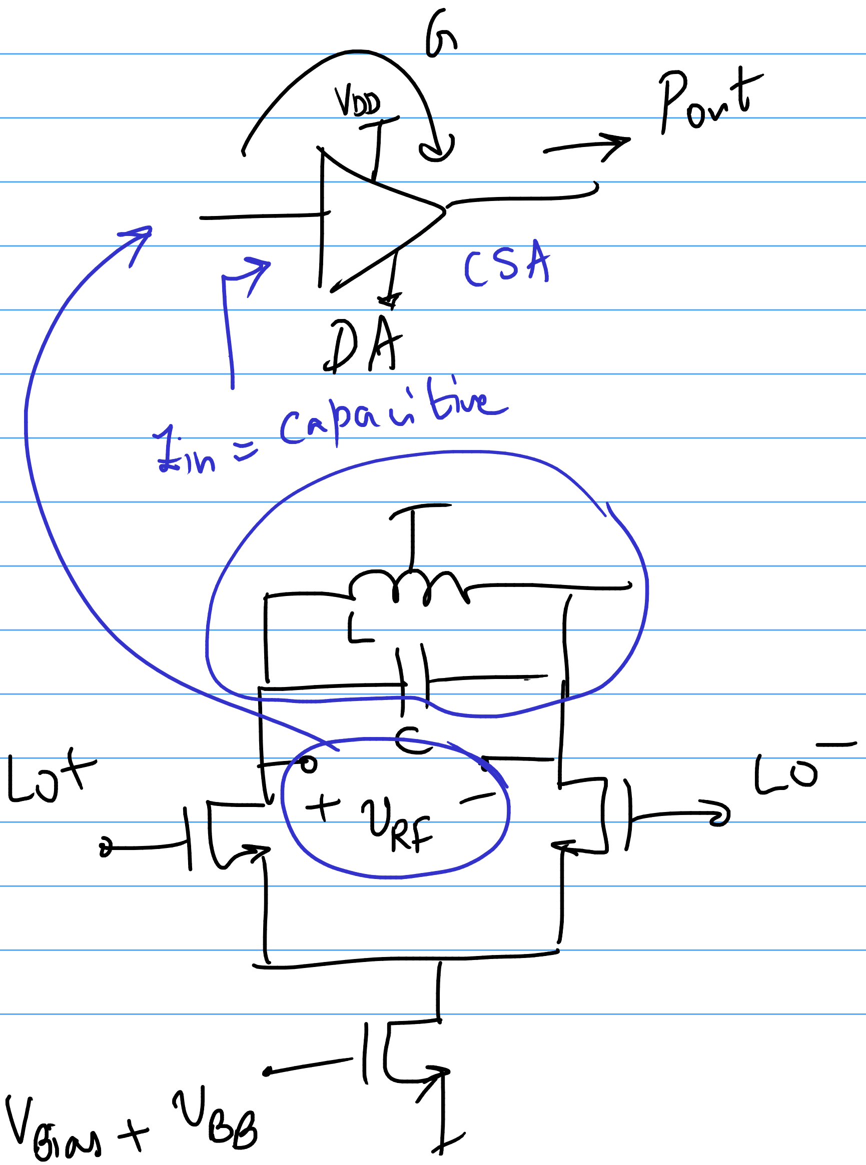
WiFi ≈ 100mW

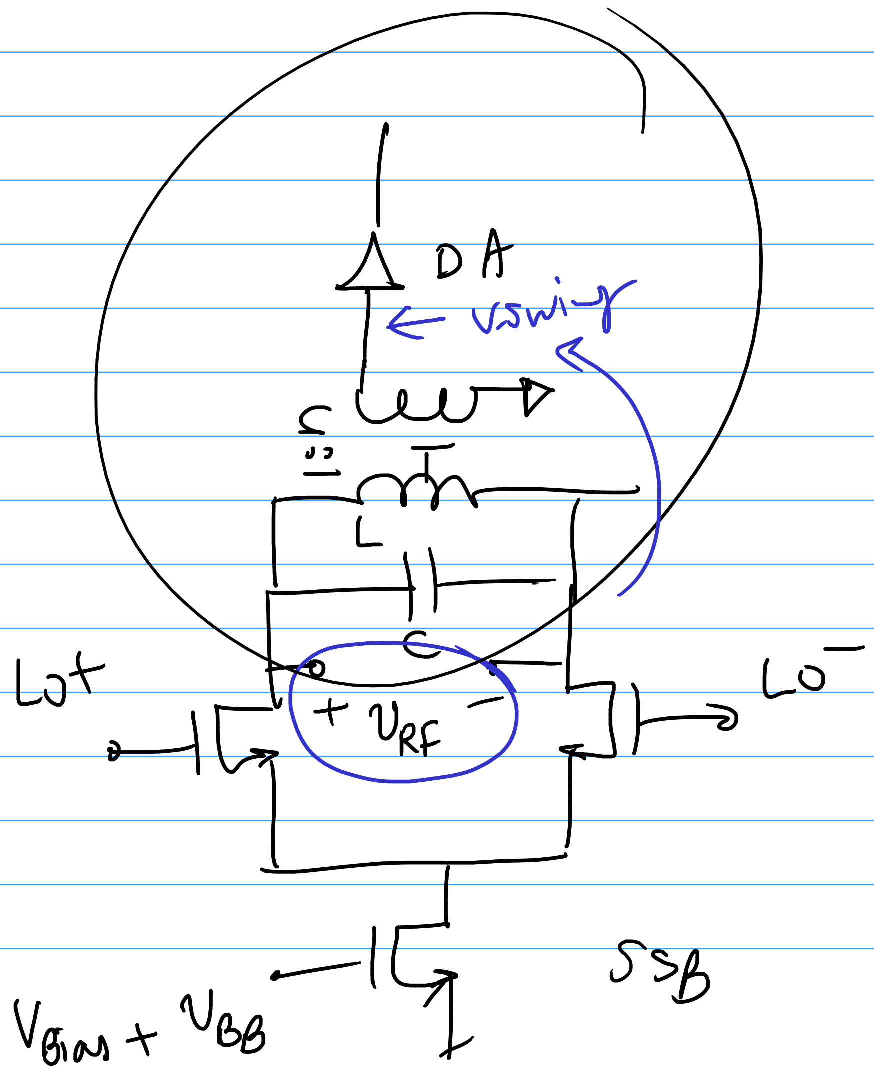
Bluetooth < 10mW



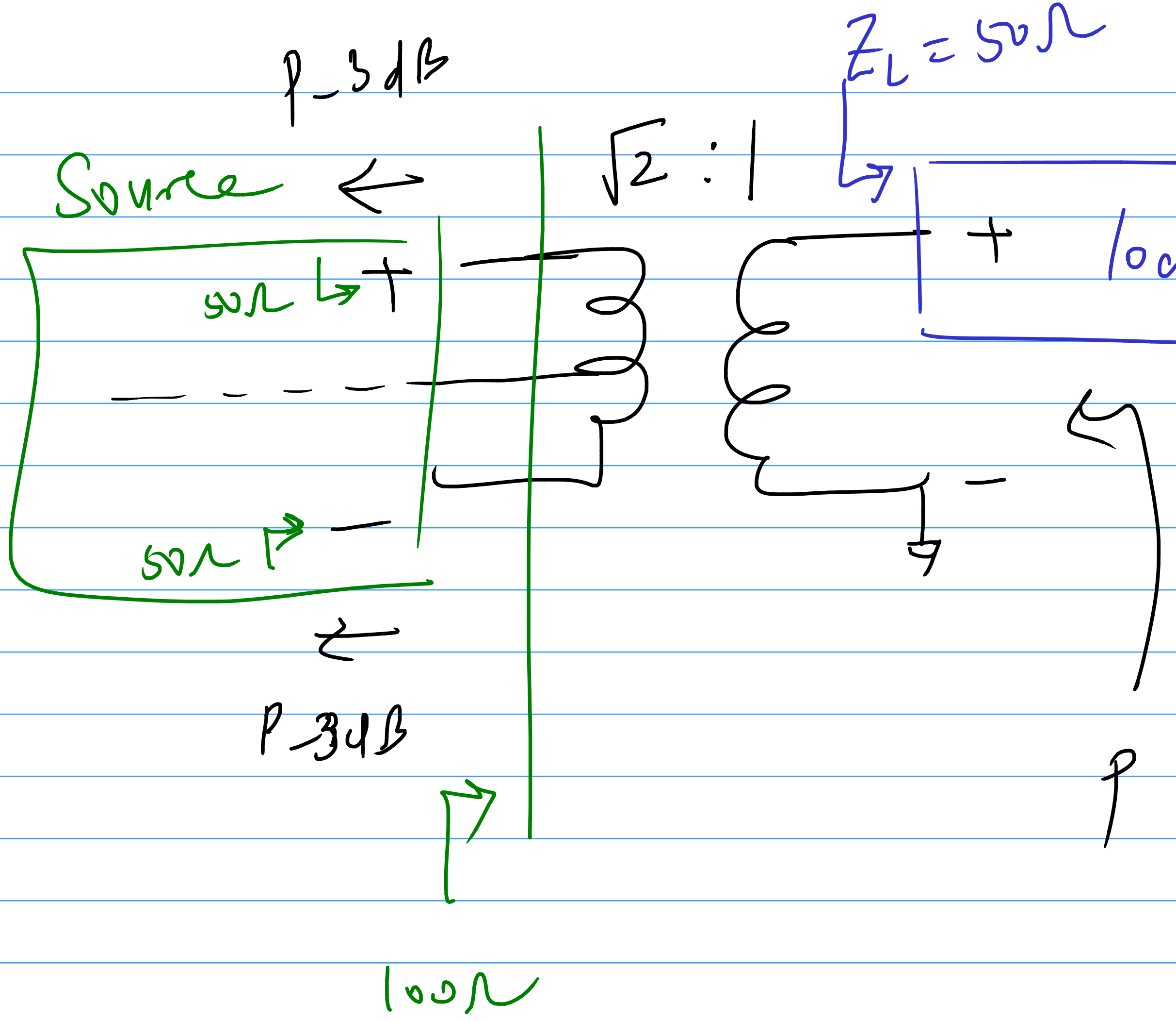
Diplexer / Switch / Duplexer

Tx mixer


G
VDD
Pout
CSA
DA
Zin = capacitive
L
C
Lo+
Lo−
+ vRF −
Vbias + vBB

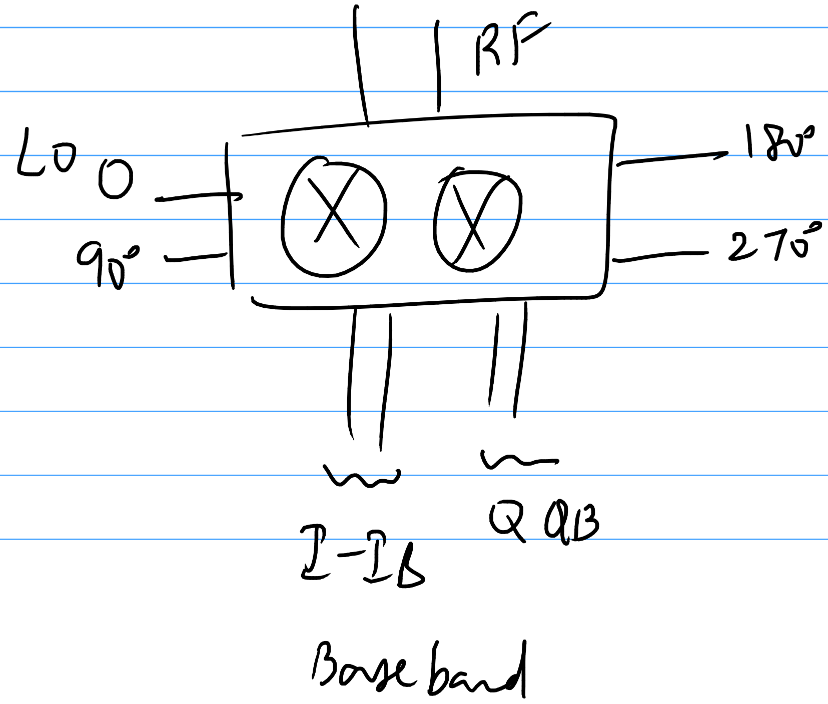

DA
vswing
L
C
Lo+
Lo−
+ vRF −
Vbias + vBB

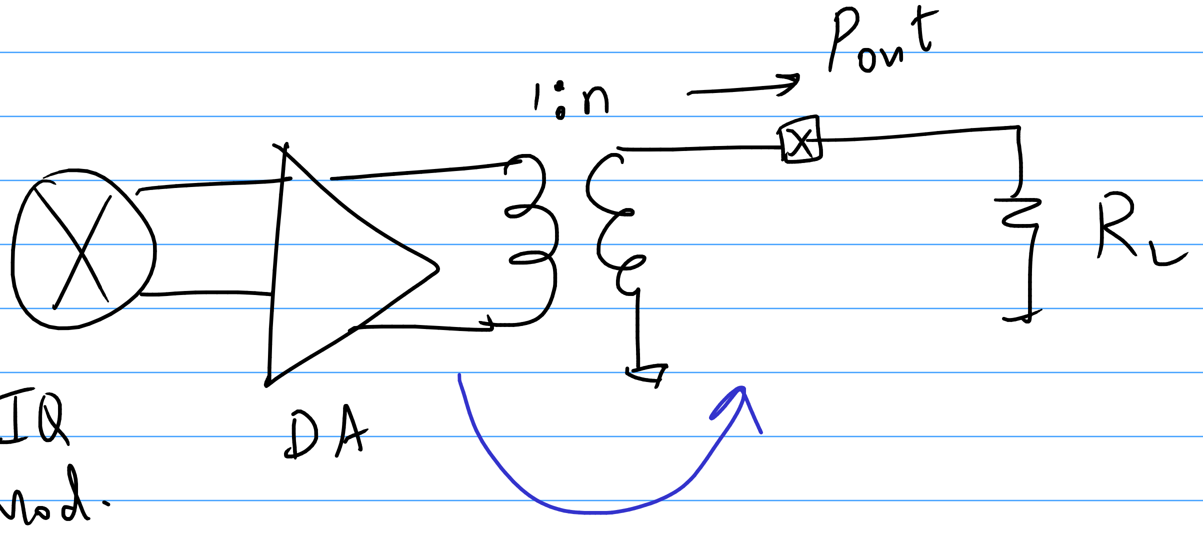
SSB

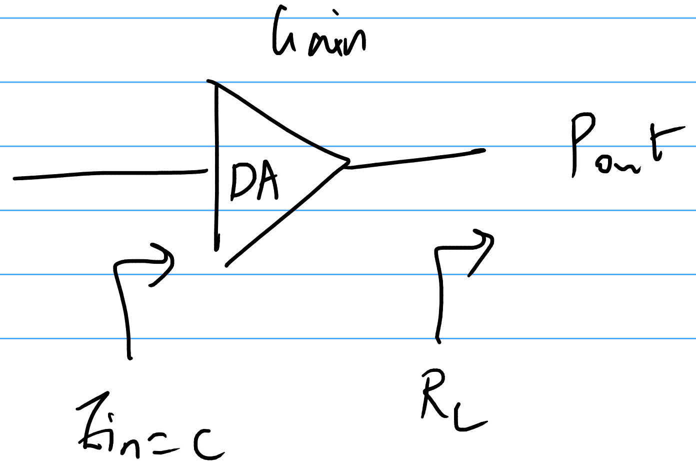


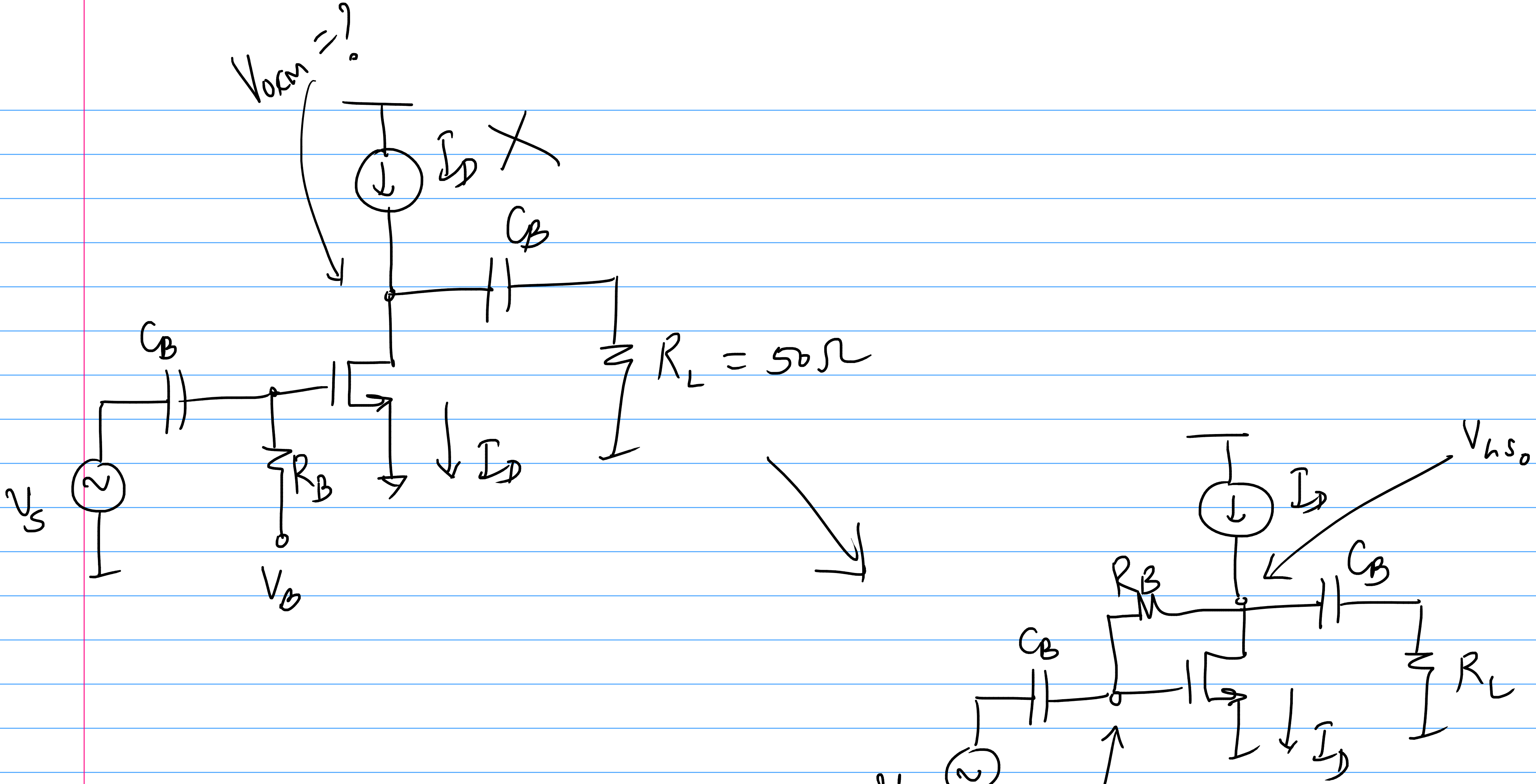
Balun unbalanced

Balanced

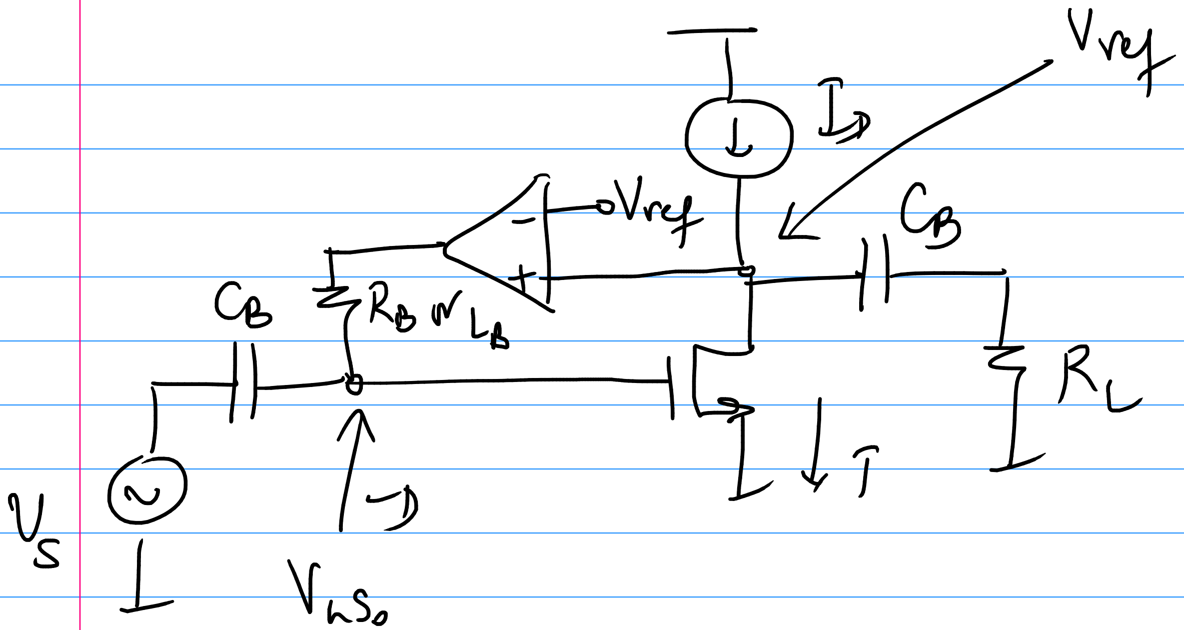


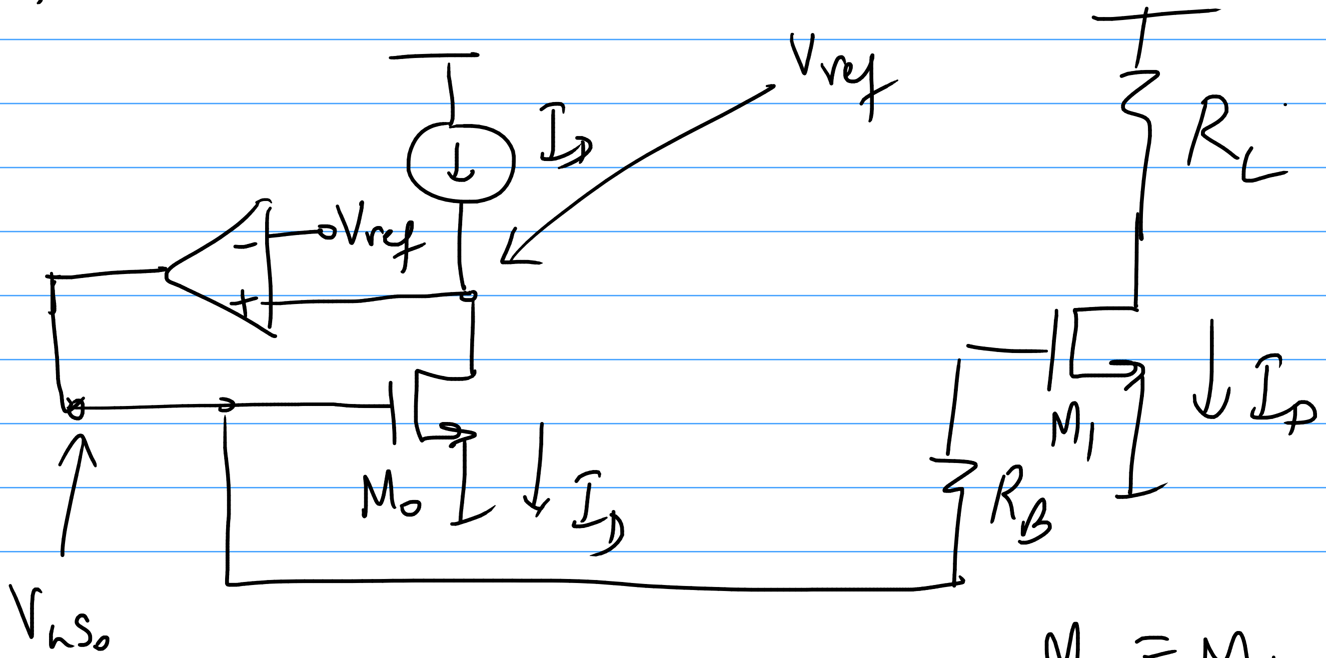

1:n
Pout
RL
IQ Mod.
DA


Gain
DA
Pout
Zin=C
RL

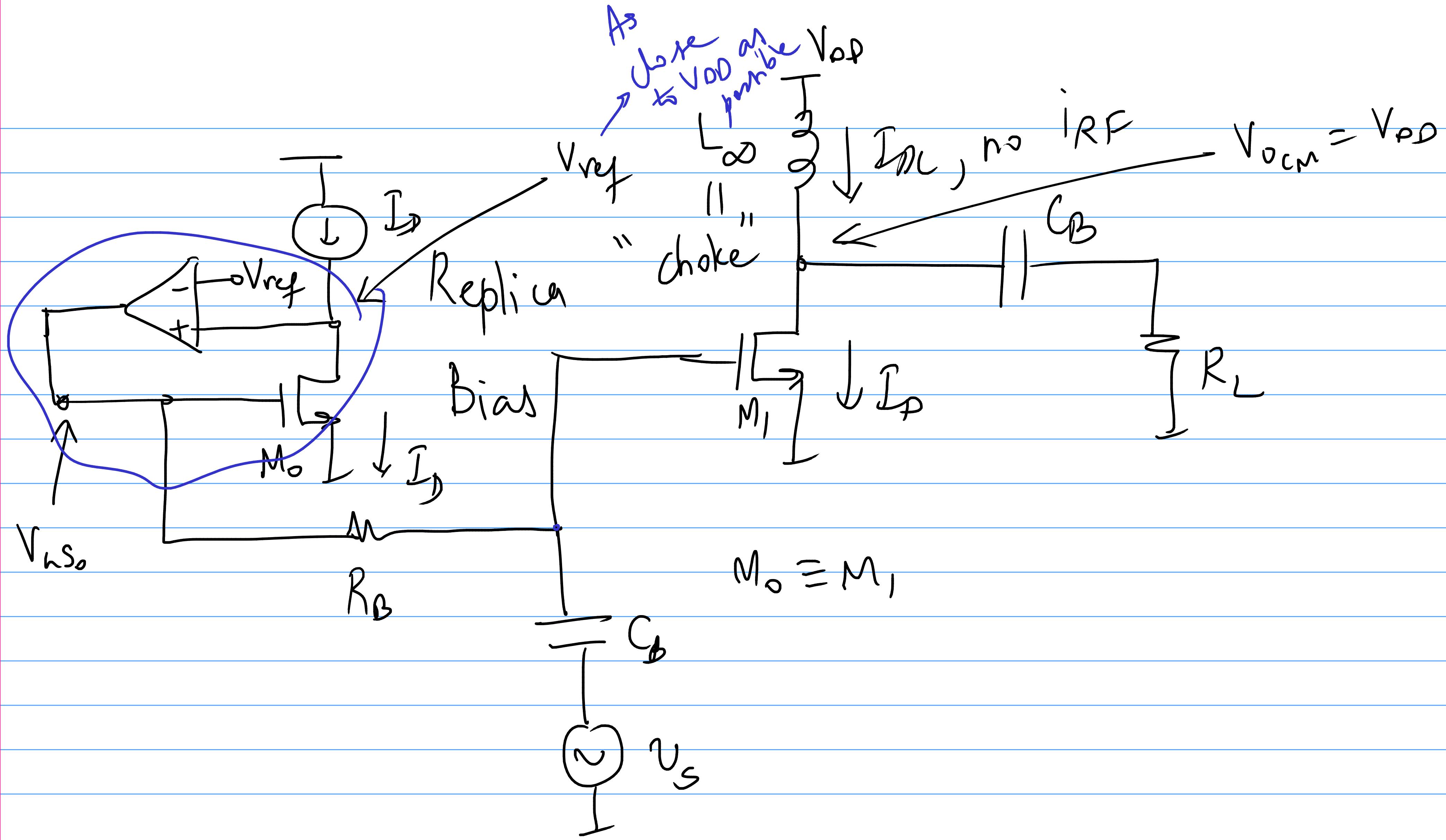


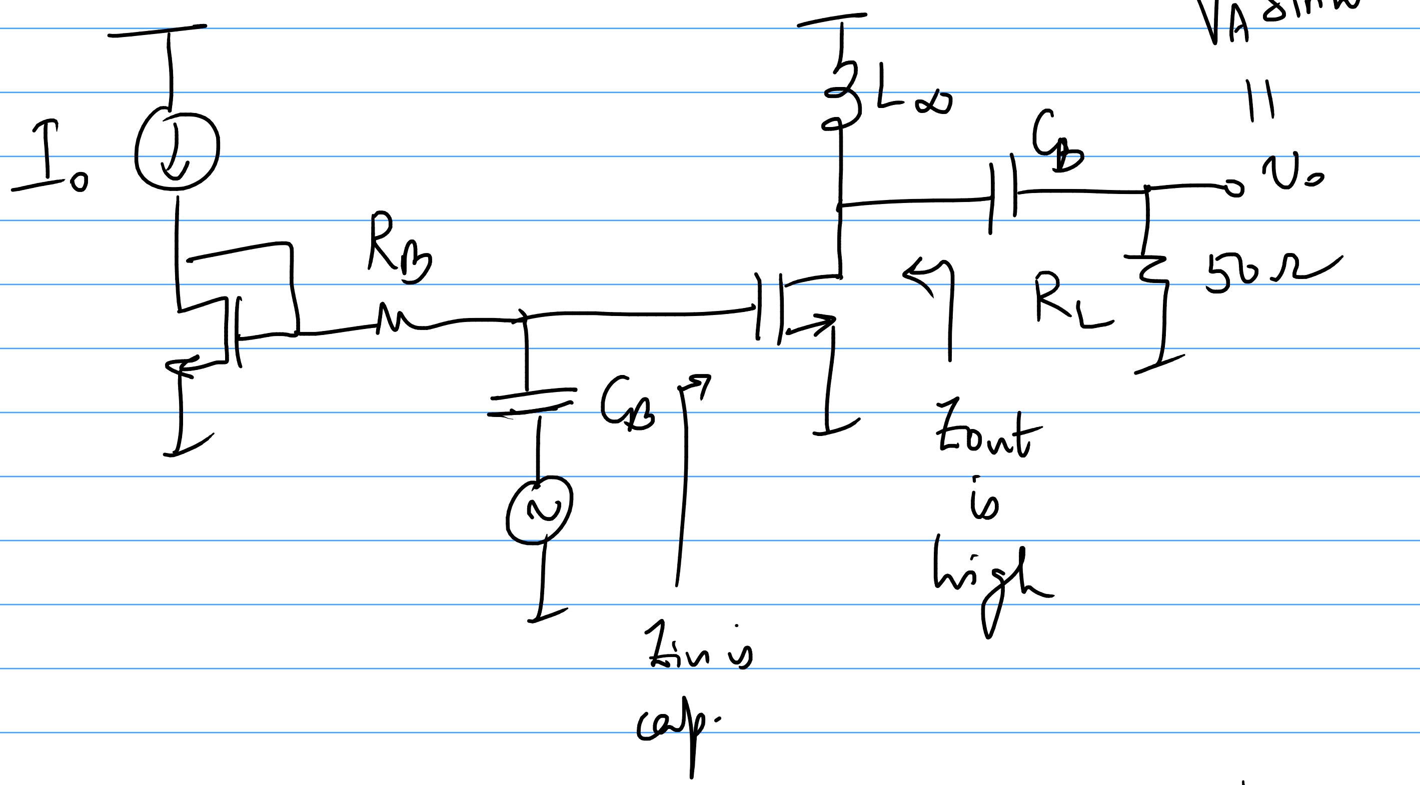
$$V_T + V_{Dsat}\Big|_{I_D} = V_{GS_0}$$


Vref
I_D
V_{ref}
C_B
C_B
R_B
R_L
V_S
V_{nS_0}


Vref
I_D
V_{ref}
R_L
M_1
I_D
R_B
M_0
I_D
V_{nS_0}

$M_0 \equiv M_1$


As close to VDD as possible
V_{DD}
L_∞
I_{DC}, no i_{RF}
$V_{OCM} = V_{DD}$
V_{ref}
I_D
|| "choke"
V_{ref}
Replica
C_B
M_1
I_D
R_L
Bias
M_0
I_D
V_{GS_0}
R_B
$M_0 \equiv M_1$
C_B
v_s

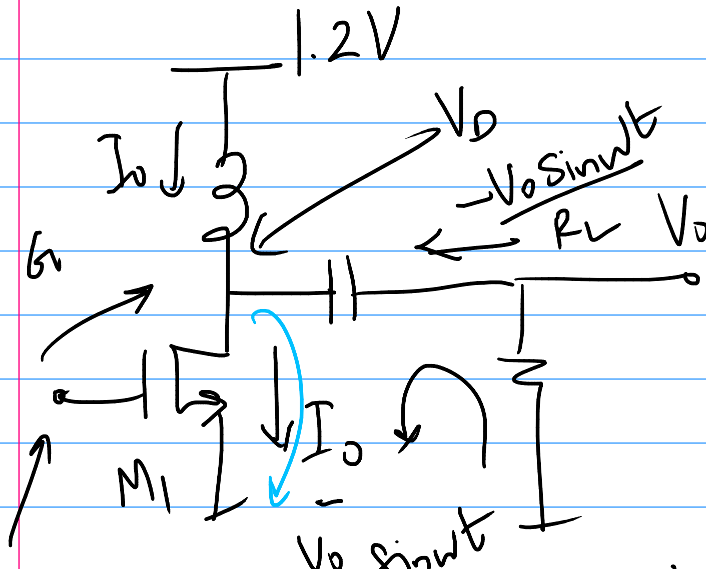


$$\text{Gain} = -g_m \cdot R_L$$

$$P_{out} = \frac{(V_A/\sqrt{2})^2}{R_L} = \frac{V_A^2}{2R_L} = \frac{V_A^2}{100}$$

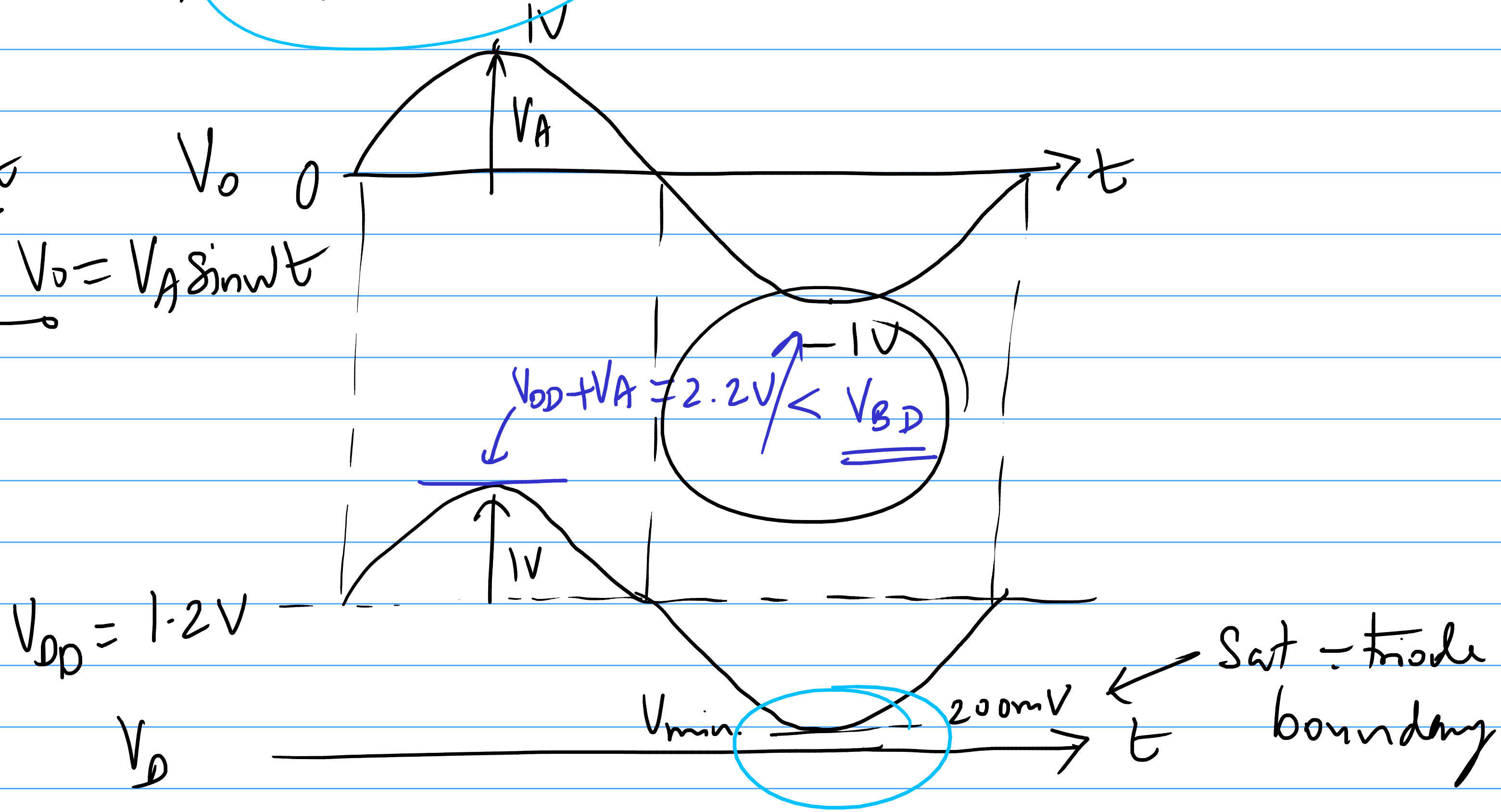
We want P_{out} =	1W	10mV
V_A =	10V	1V

$P_{out} = 10mW \Rightarrow V_A = 1V$



$V_0 = V_A \sin\omega t$

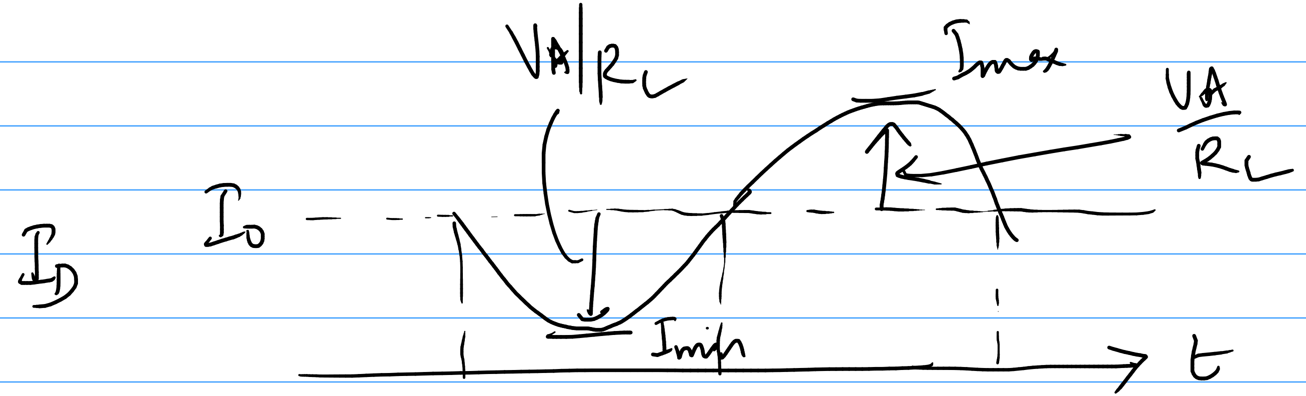
$\frac{V_0 \sin\omega t}{R_L}$



$V_B - V_s \sin\omega t$

$V_G = V_B + V_S = V_B + \frac{V_A}{G}$

$V_D = V_{DD} - V_A$



No Cut-off

$\Downarrow$

$$\frac{V_A}{R_L} \leq I_0$$

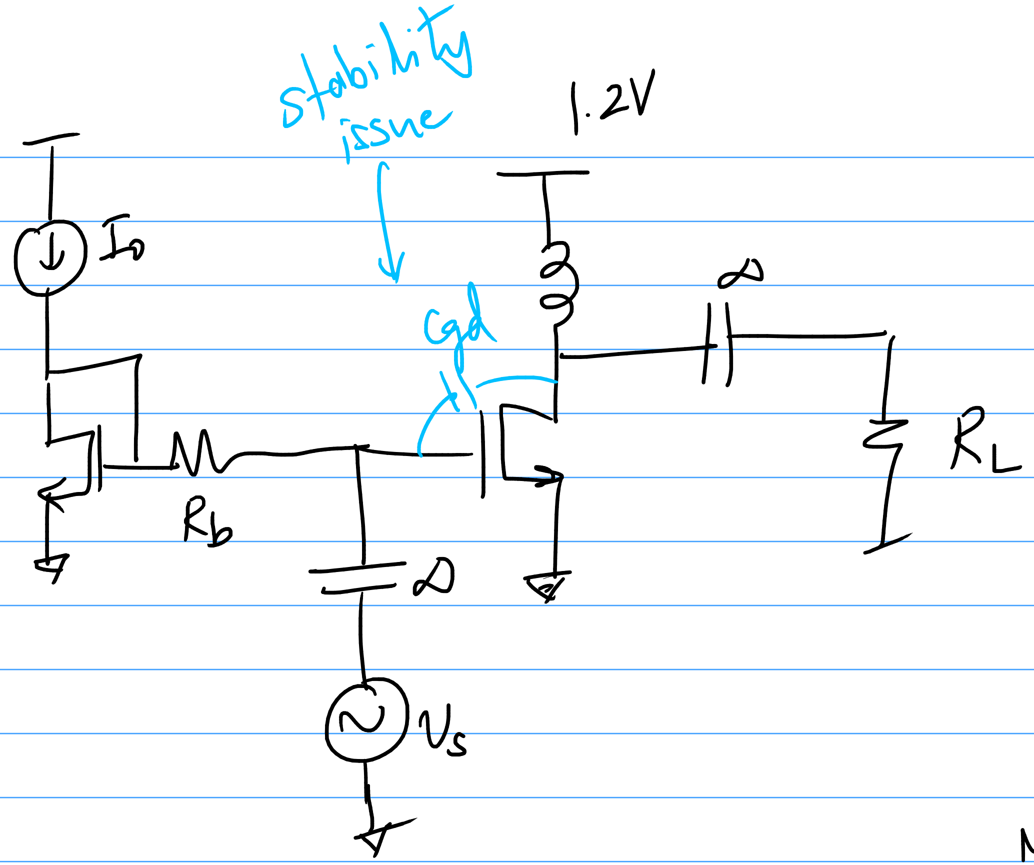
large Gain $\rightarrow$ $V_{min} \approx V_{Dsat}$

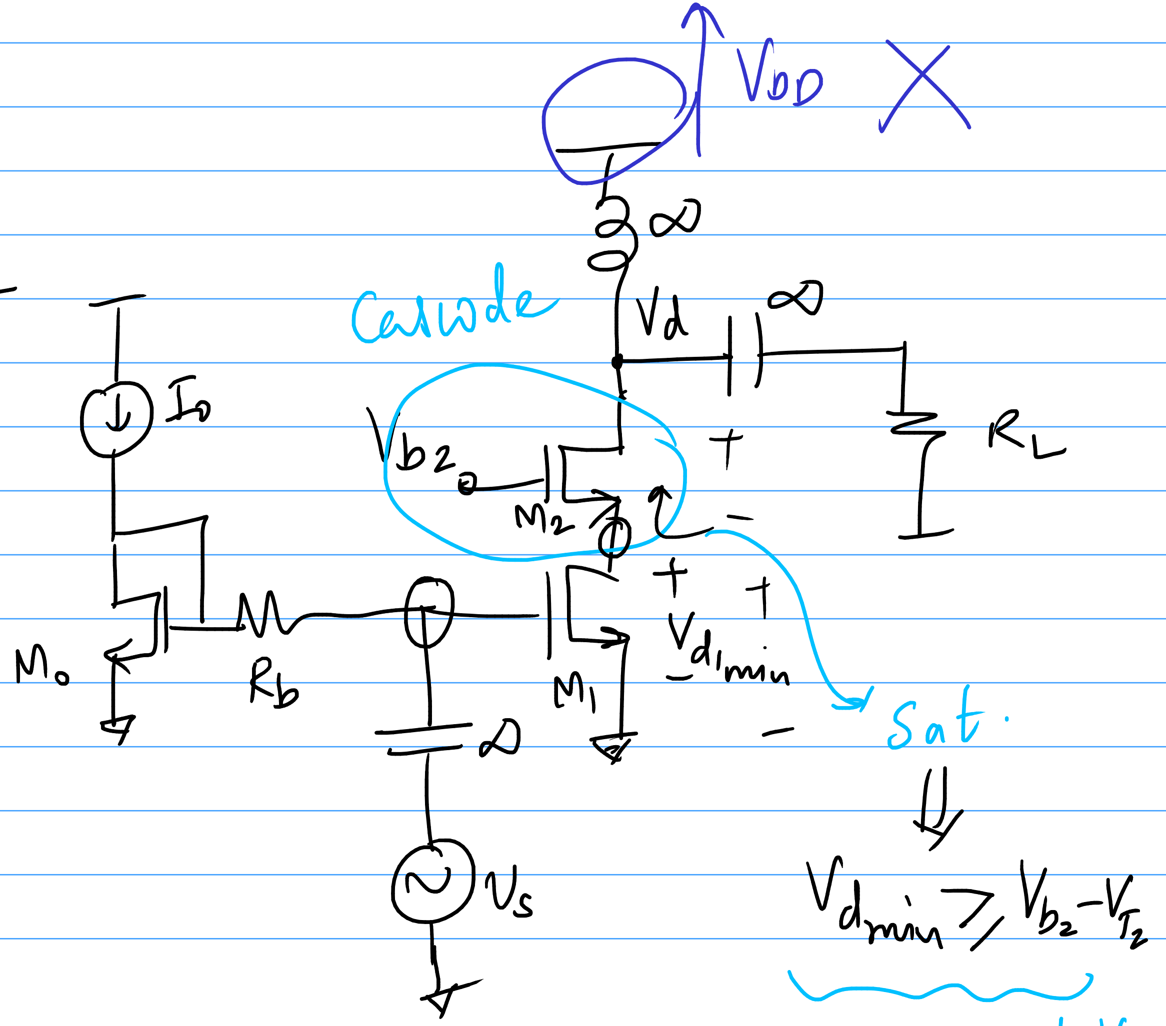
$$V_A = I_0 R_L$$

$$V_{D\,sat} = \sqrt{\frac{2 I_0}{\mu C_{ox} \left(\frac{W}{L}\right)}}$$

e.g. $V_A = 1V \Rightarrow I_0 = 20mA$

$V_{Dsat} = 0.2V \Rightarrow \left(\frac{W}{L}\right) = \dfrac{2 I_0}{\mu C_{ox} \cdot (V_{Dsat})^2}$



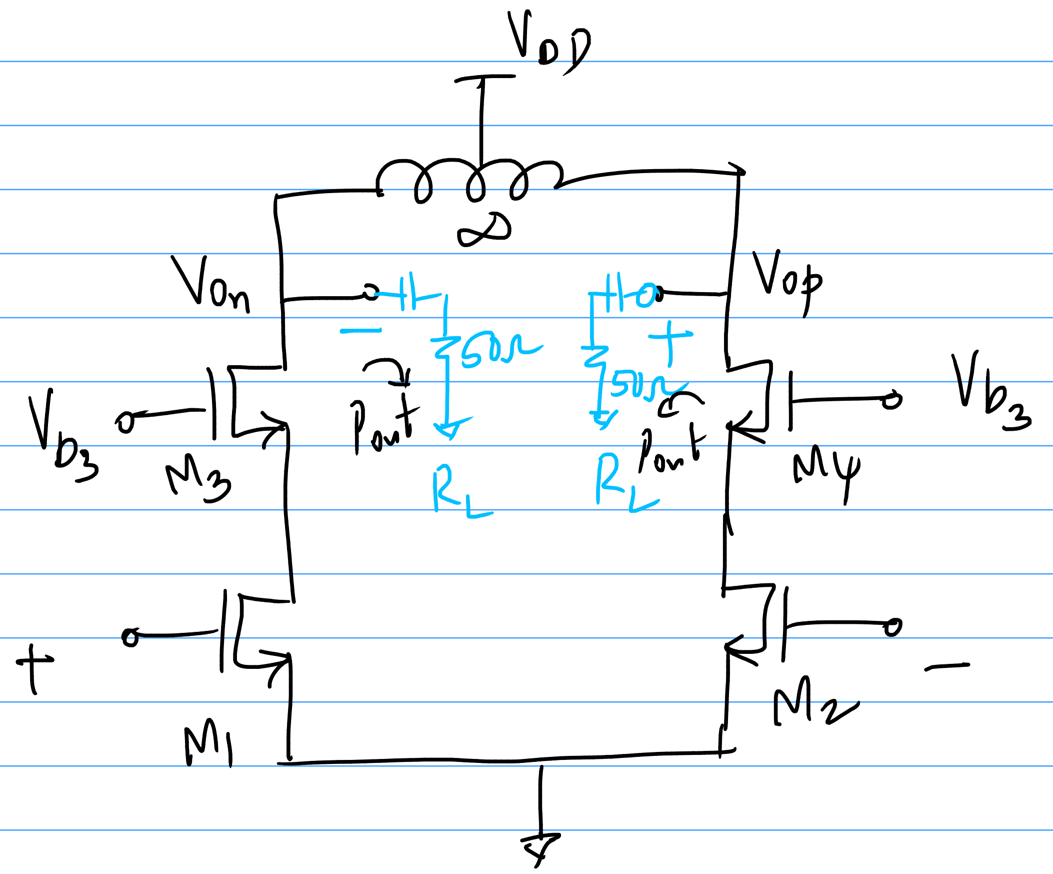


min. $V_{b_2} = \underline{V_{d1,min}} + V_{GS_2}$

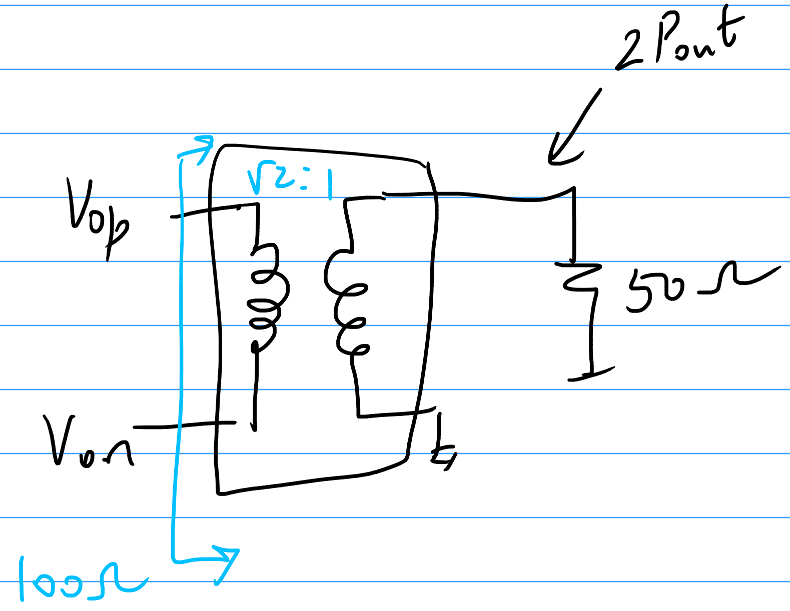
$\Downarrow$

$\underbrace{V_{d_{min}} \geq V_{b_2} - V_{T_2}}$

may not meet V_o req.



V_{DD}
V_{on}
V_{op}
50Ω
50Ω
P_{out}
P_{out}
R_L
R_L
V_{b_3}
V_{b_3}
M_3
M_4
M_1
M_2
+
−
+
−

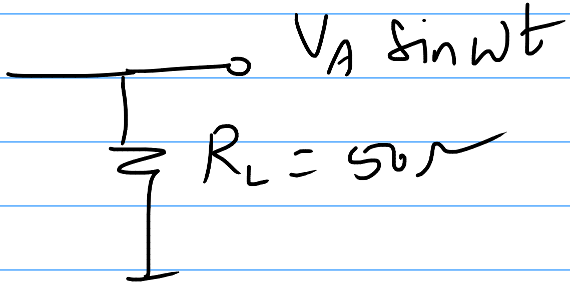


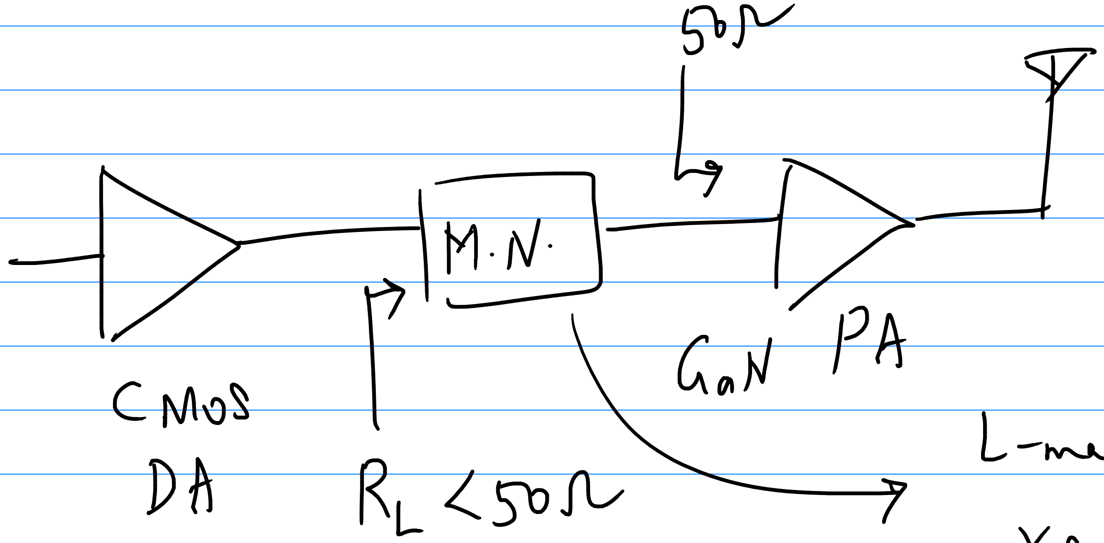
$2P_{out}$
$\sqrt{2}:1$
V_{op}
V_{on}
50Ω
100Ω

$$P_{out} = 10mW$$

$$P_{out} = \frac{\left(V_A/\sqrt{2}\right)^2}{50\Omega} \rightarrow R_L < 50\Omega$$

$$\rightarrow V_A = 1V$$





L-match, π, T, Tapped-C, -L

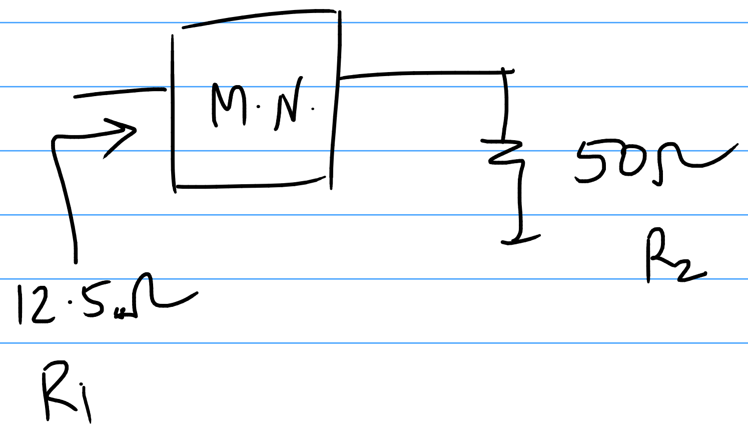
Xfmr . . .

Say $V_{A\,\text{allowed}} = 0.5V$

$V_{BD} = 1.7V$

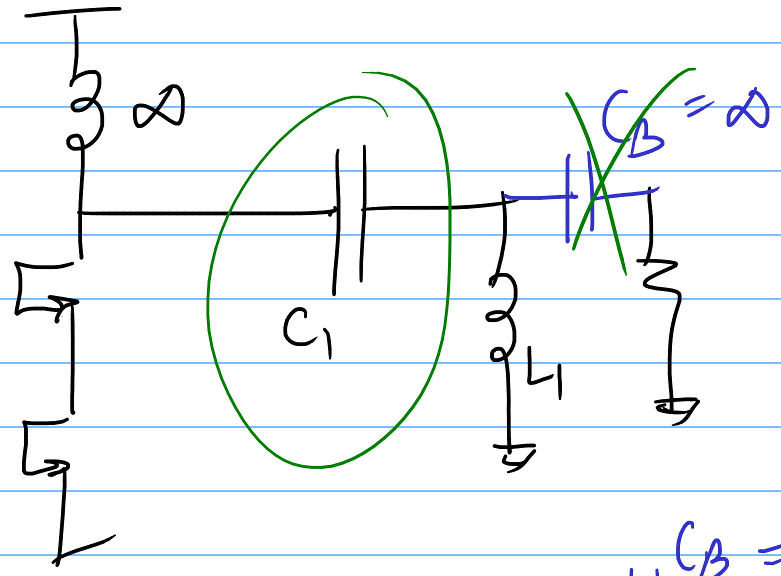
. . . .

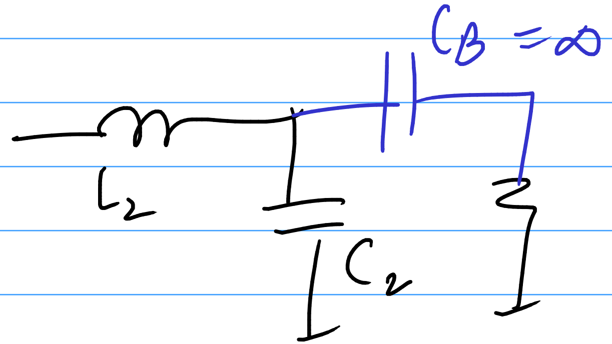
$$\frac{(0.5/\sqrt{2})^2}{R_L} = 10mW \Rightarrow R_L = 12.5\,\Omega$$

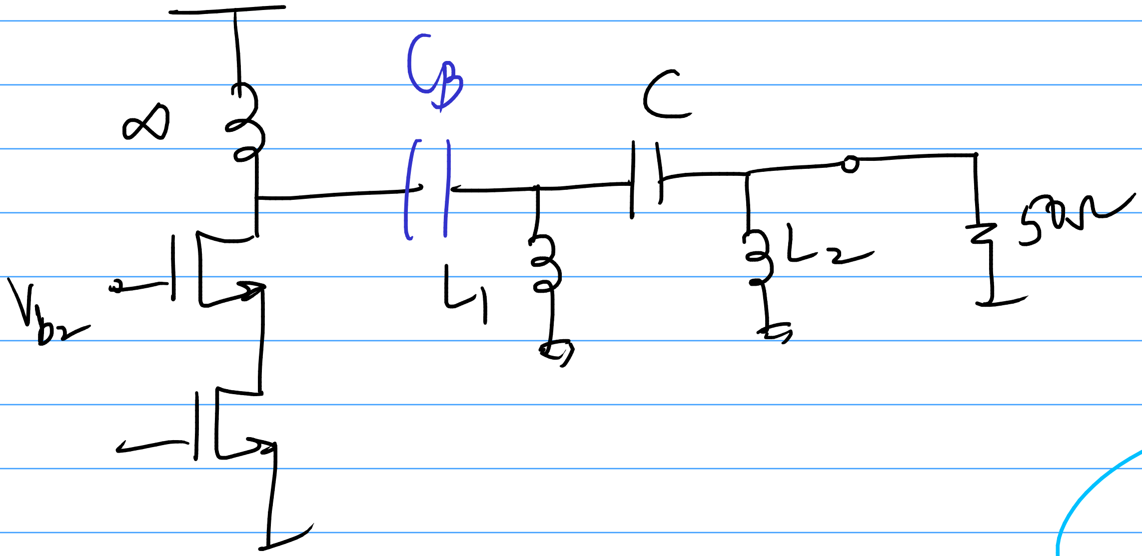


L match → $Q = \sqrt{\frac{R_2}{R_1} - 1}$

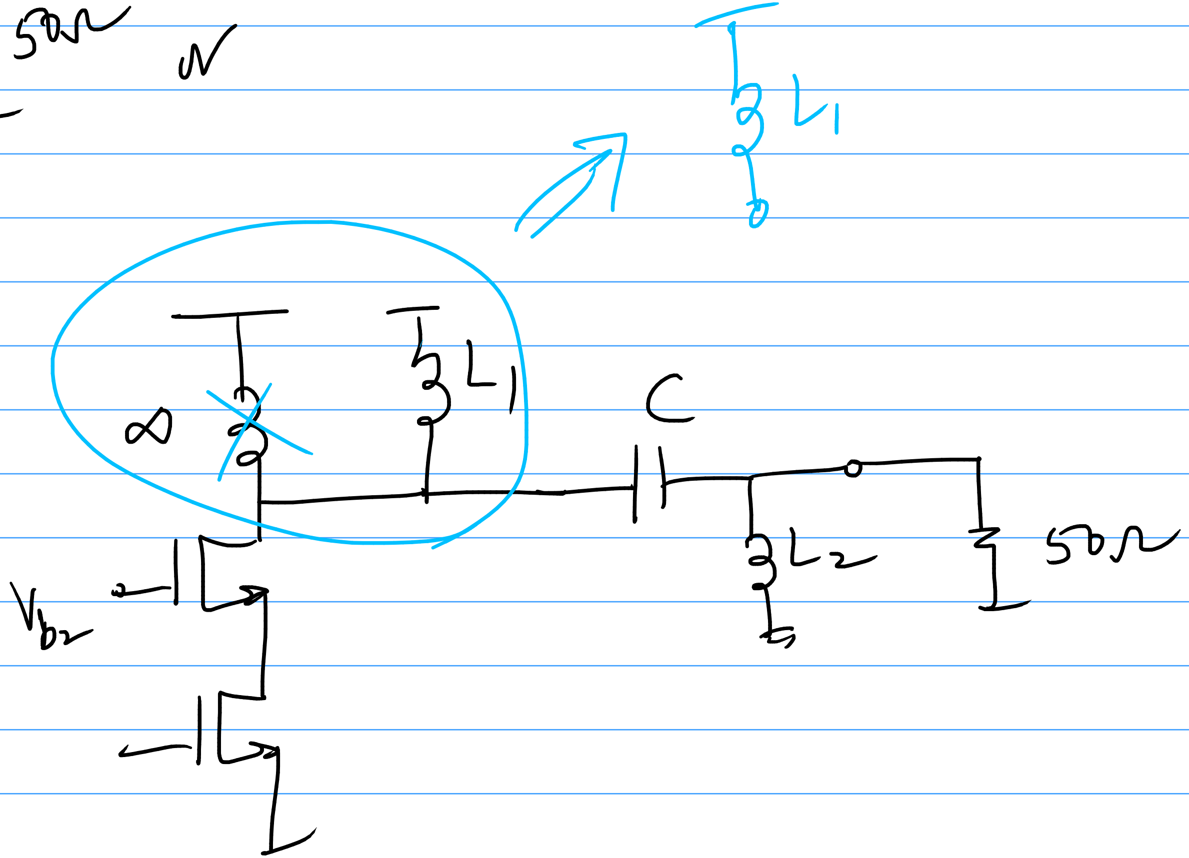
$= \sqrt{3}$


∞
C_1
L_1
$C_B = \infty$

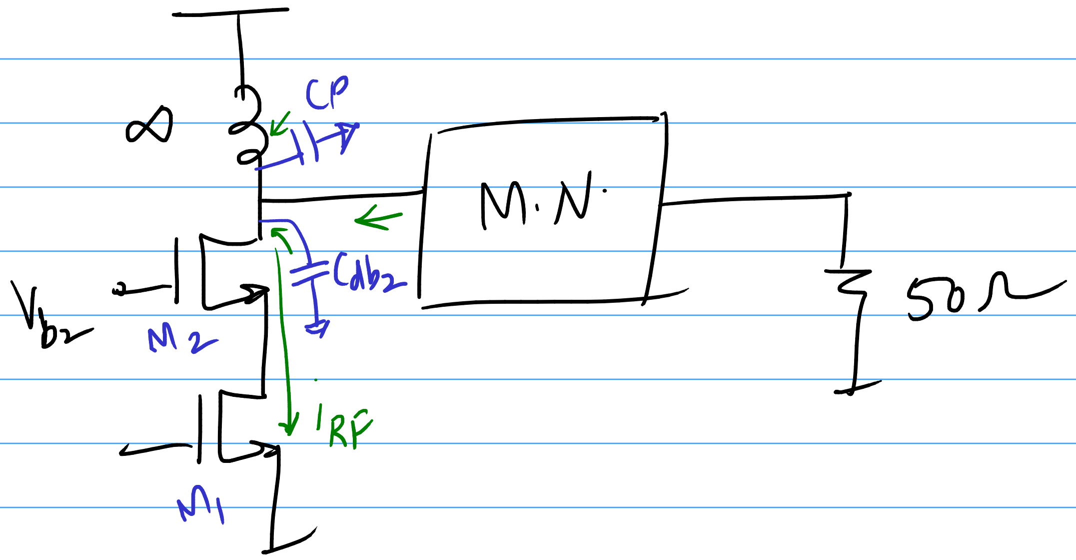

$C_B = \infty$
L_2
C_2



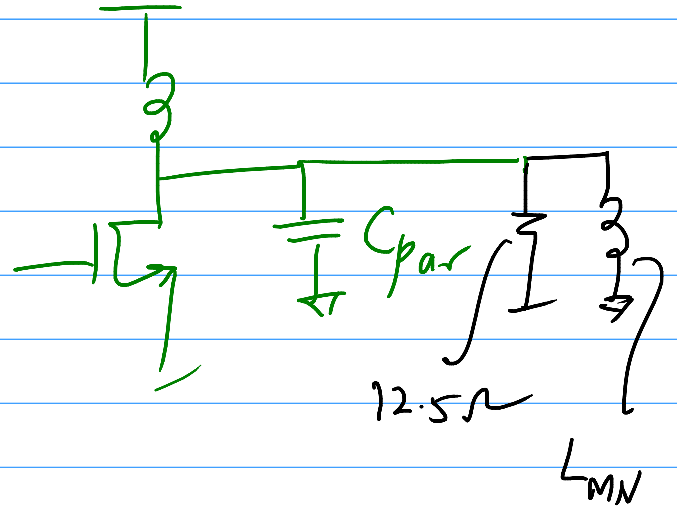
∞
C_B
C
L_1
L_2
50Ω
V_{b2}



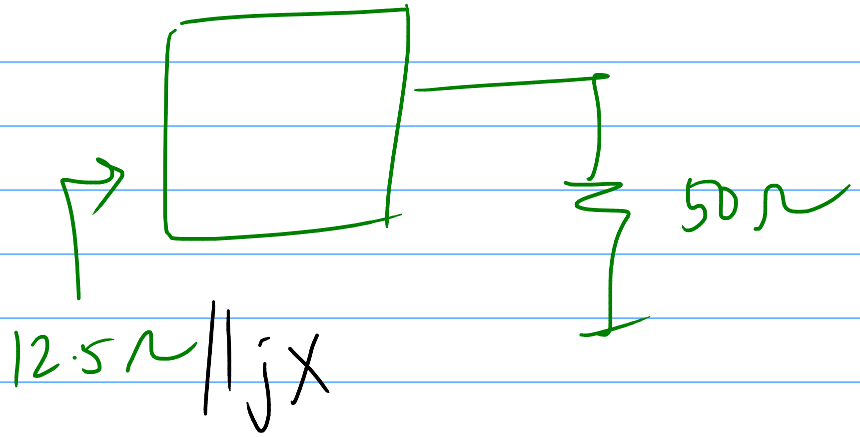
L_1
∞
L_1
C
L_2
50Ω
V_{b2}



∞
CP
M.N.
Cdb2
Vb2
M2
50Ω
iRF
M1

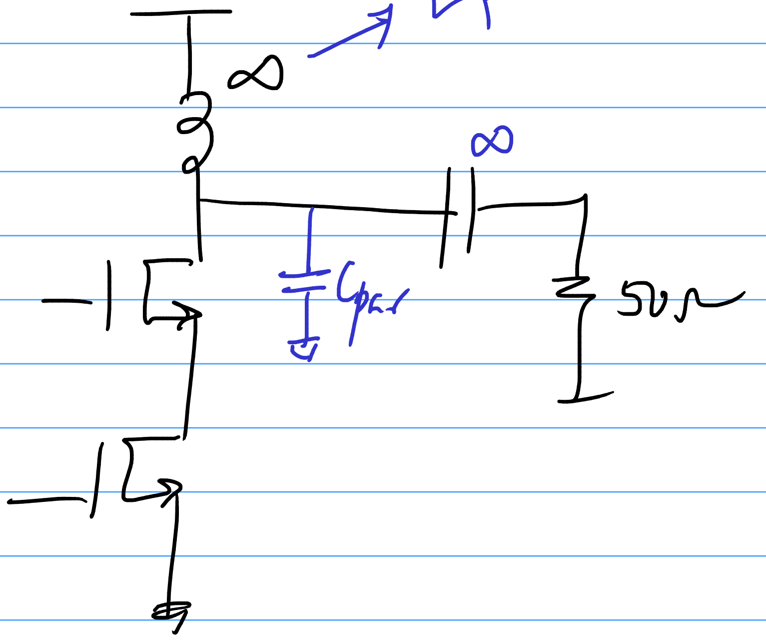


Cpar
12.5Ω
LMN



50Ω
12.5Ω||jX

$$\infty \to L_p \to \omega_0 = \frac{1}{\sqrt{L_p \cdot C_{par}}}$$

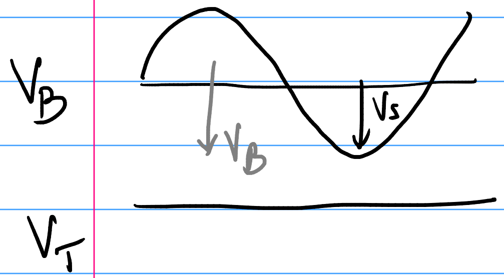


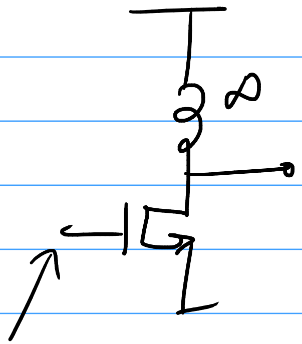
Linear (class-A)

PA

$\Downarrow$

device is always in sat.

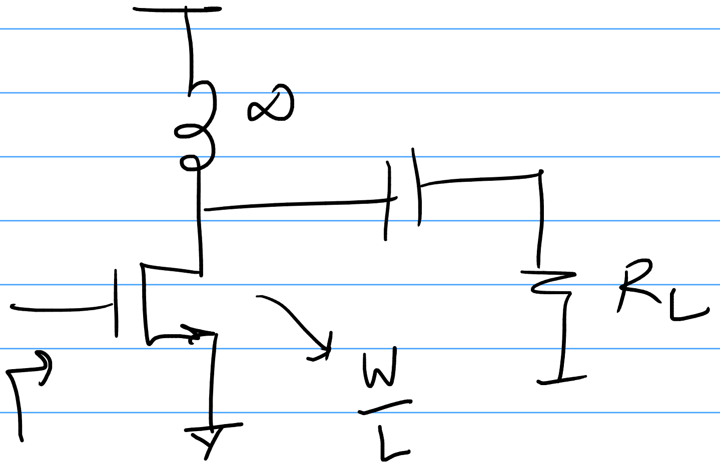




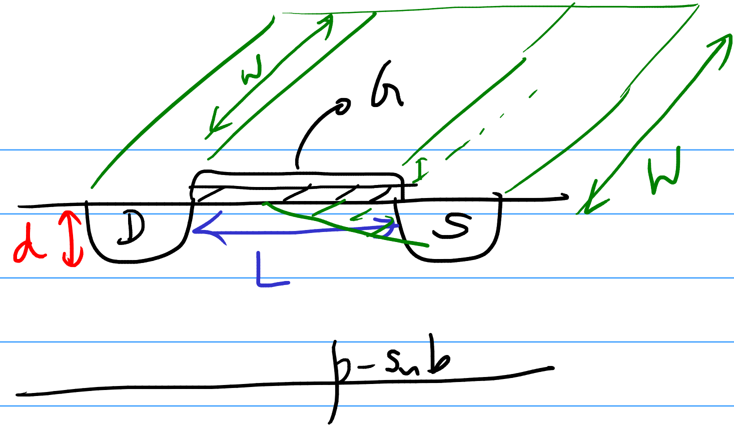
$$\left.\begin{array}{l} V_B + V_s \sin\omega t \\ \\ V_B + V_s \sin\omega t \geq V_T \end{array}\right\}$$

class-A

Conduction angle $= 360^\circ$

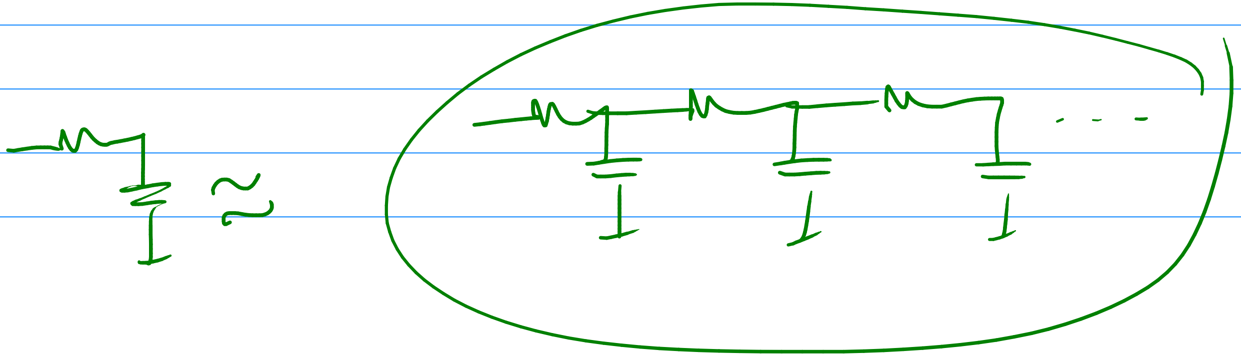


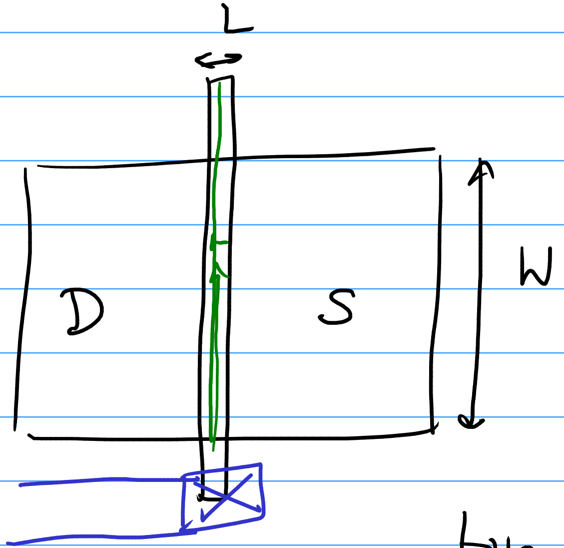
$$Z_{in} = \frac{1}{j\omega C_{gs}}$$



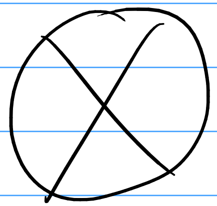
side view

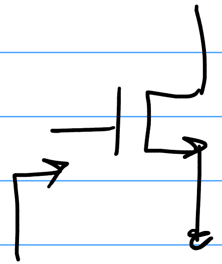
$\frac{W}{L} \times R_{sheet,poly}$

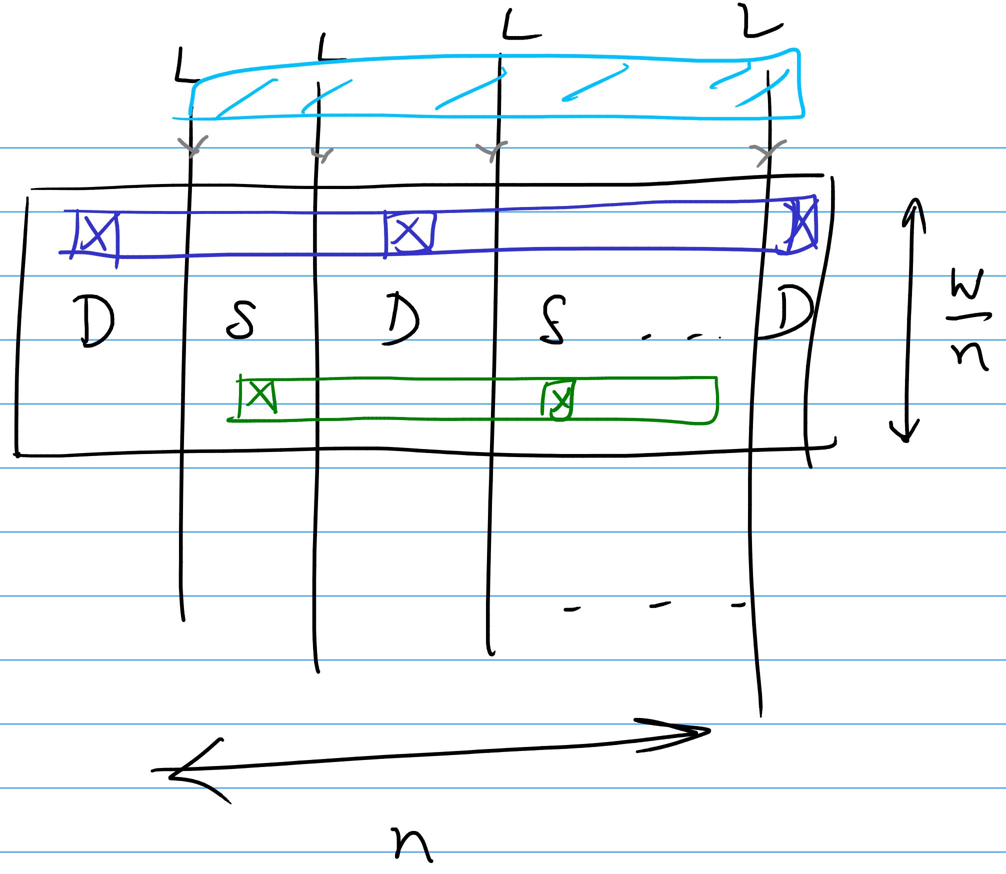




top view





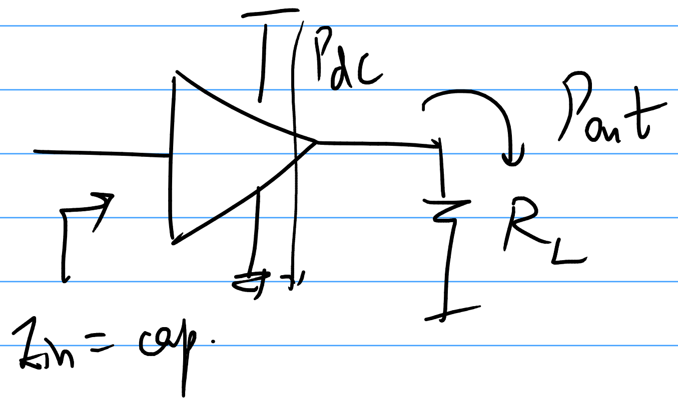


$$Z_{in} = R_g + \frac{1}{j\omega C_{gs}}$$

$$Y_{in} = G_g + j\omega C_{gs}$$

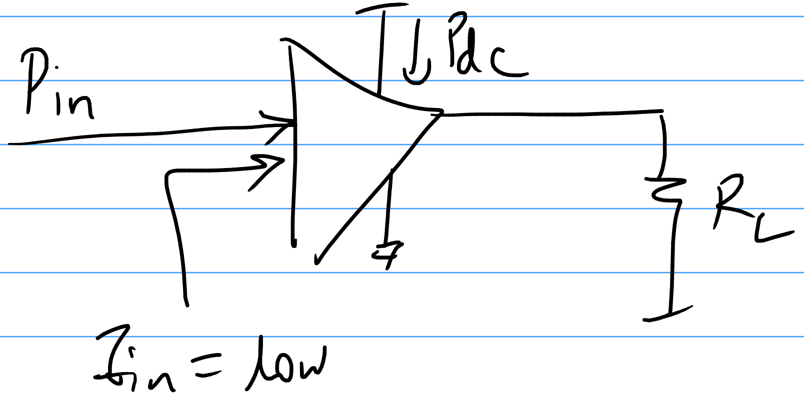
Voltage gain $\longleftrightarrow$ Power Gain

GaAs, GaN



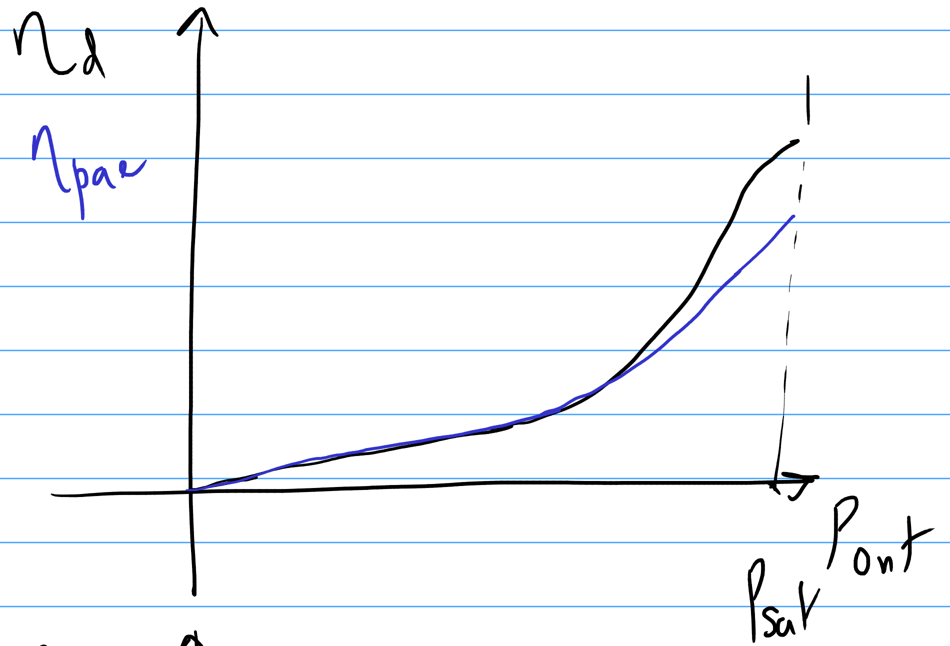
"Drain efficiency"

$$\eta_d = \frac{P_{out}}{P_{dc}}$$

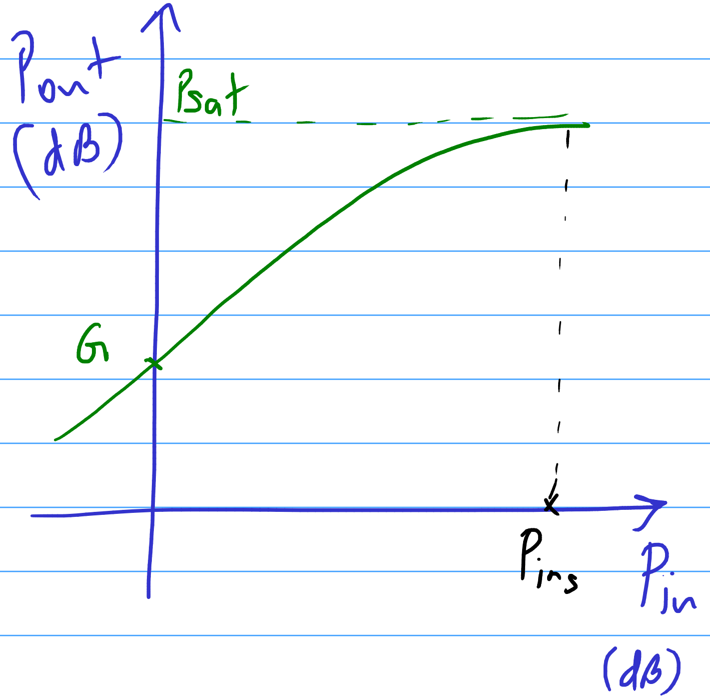


"power added efficiency"

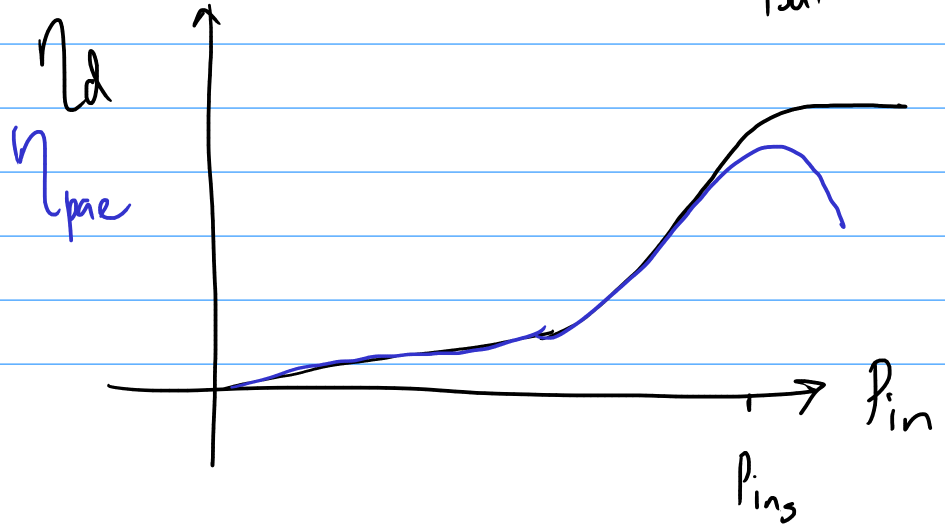
$$\eta_{pae} = \frac{P_{out}}{P_{in} + P_{dc}}$$



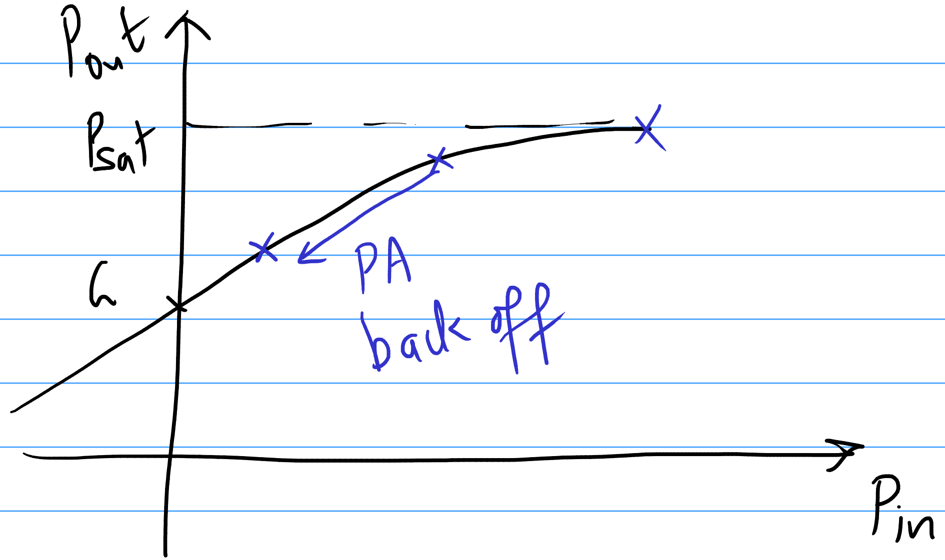
η_d
η_{pae}
P_{out}
P_{sat}

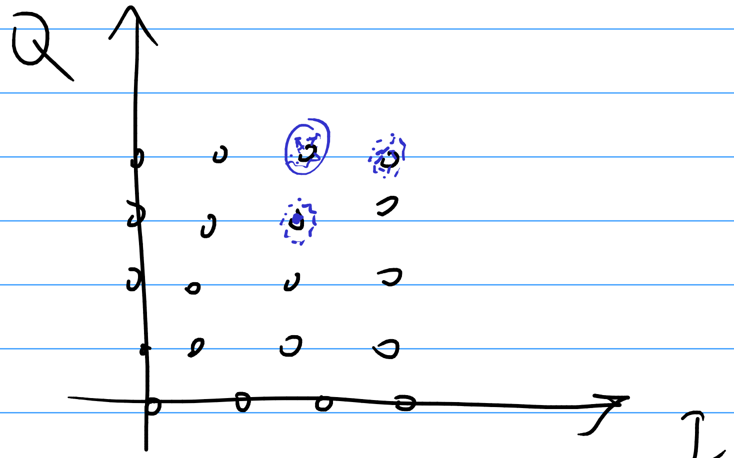


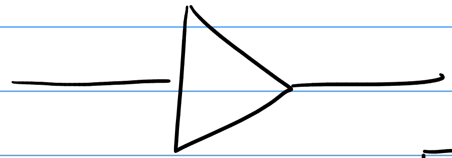
P_{out} (dB)
P_{sat}
G
P_{ins}
P_{in} (dB)



η_d
η_{pae}
P_{in}
P_{ins}







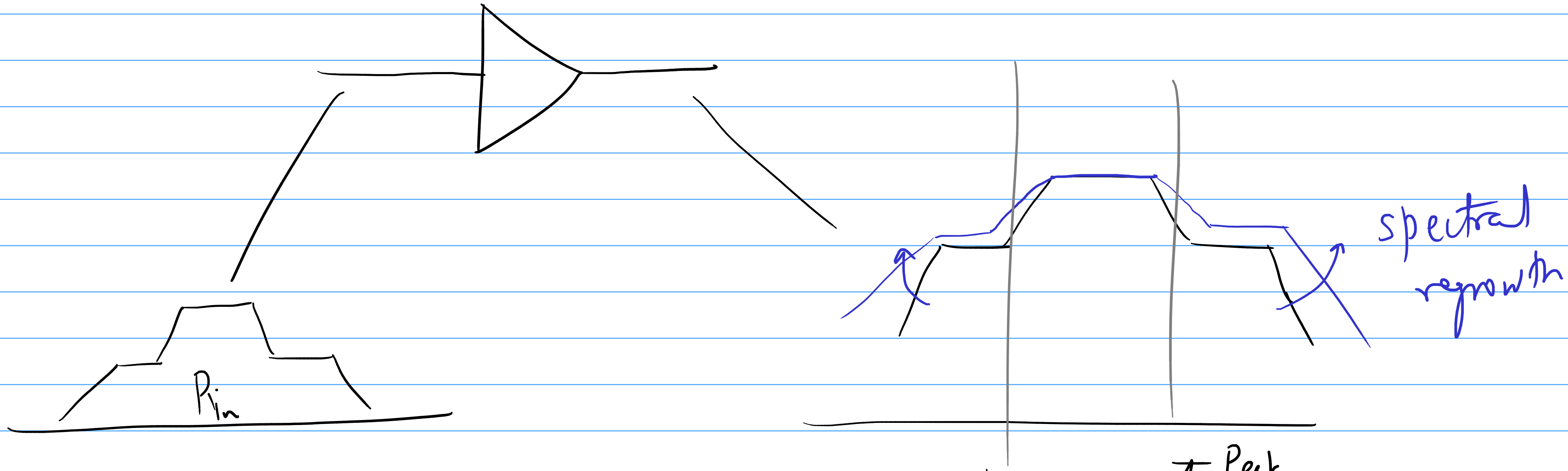
Linearity

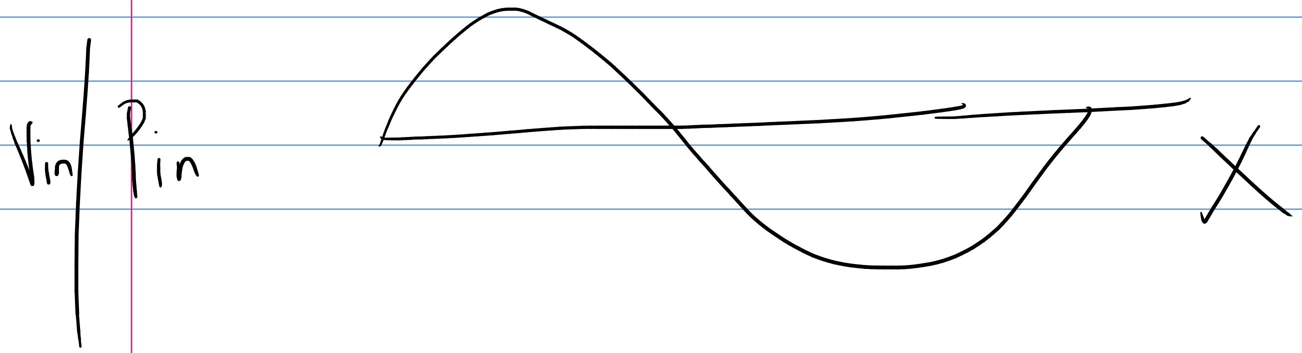
EVM
Error Vector
Magnitude

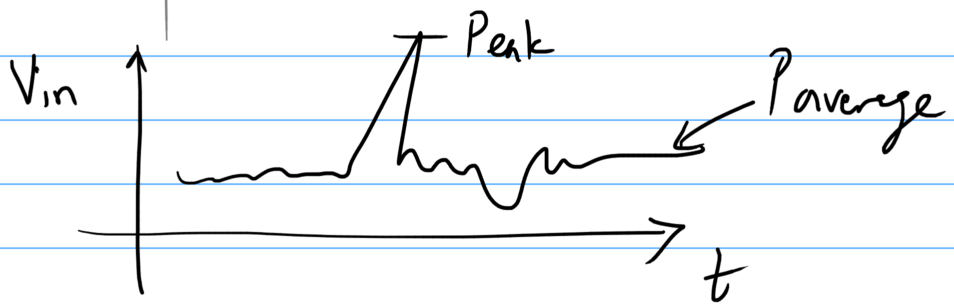
Harmonics

ACPR/
ACLR

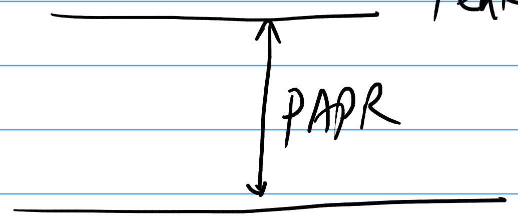
Adjacent channel
Power/leakage
Ratio







PAPR ≡ peak-to-average power ratio



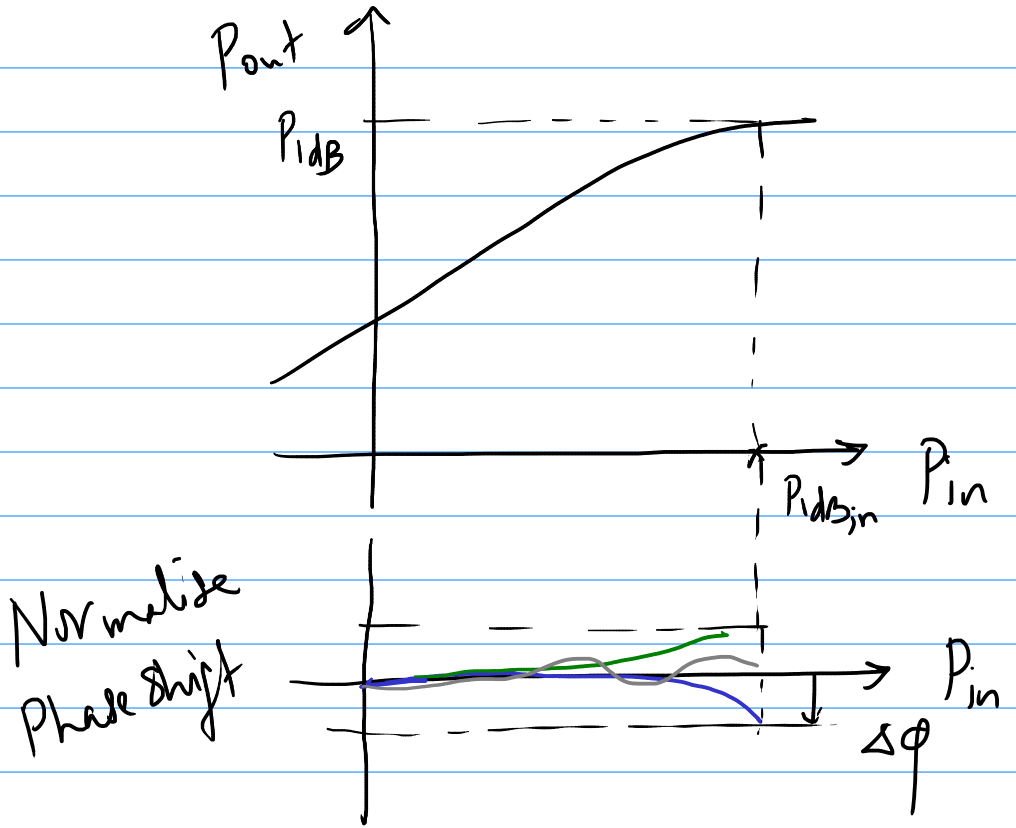
Peak Power $= P_{1dB}$

Average power $\leftarrow P_{1dB} - \text{PAPR}$

Desired RF $P_{out} = 10mW = 10dBm$

PAPR $= 10dB \leftarrow$ WiFi, 5G etc

PA $P_{1dB_{out}} = 20dBm = 100mW$!

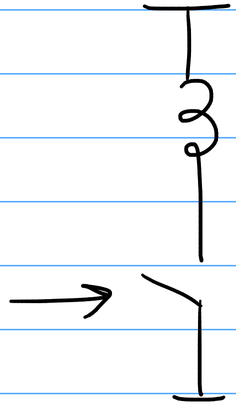


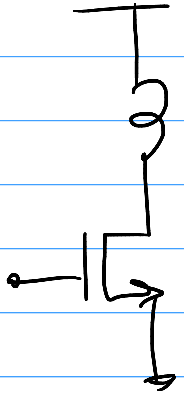
AM - AM

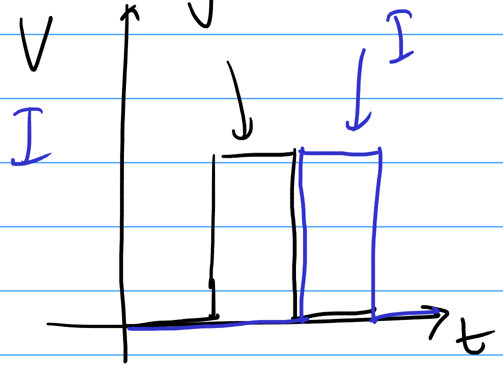
AM - PM

conduction angle

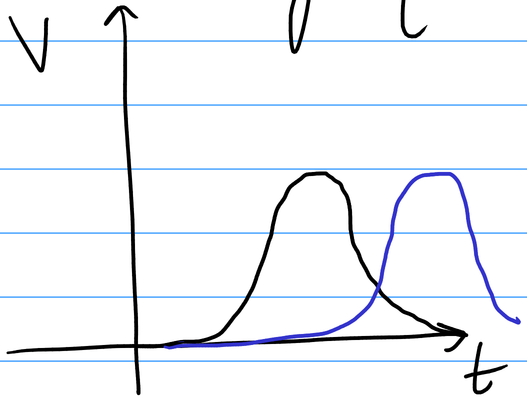
θ_c	class of PA
360°	Class – A
$180^\circ - 360^\circ$	class – AB
180°	Class – B
$<180^\circ$	class – C
	class → D
	– E
	F
	⋮





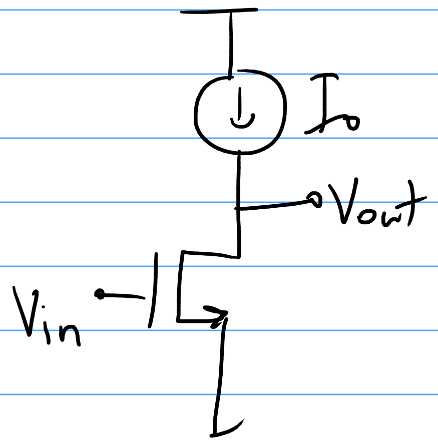


High η

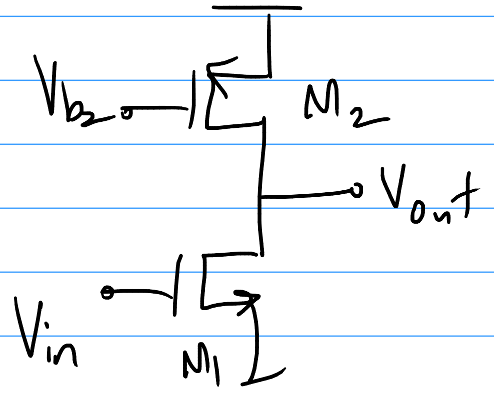


Class-E PA

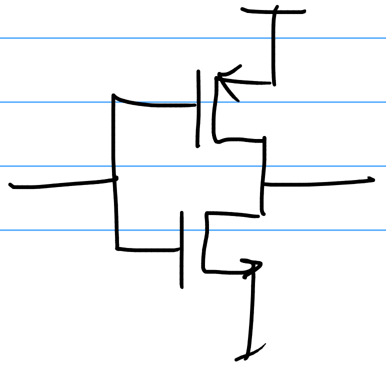
Phase Modulated Signals



$\equiv$



$\downarrow$



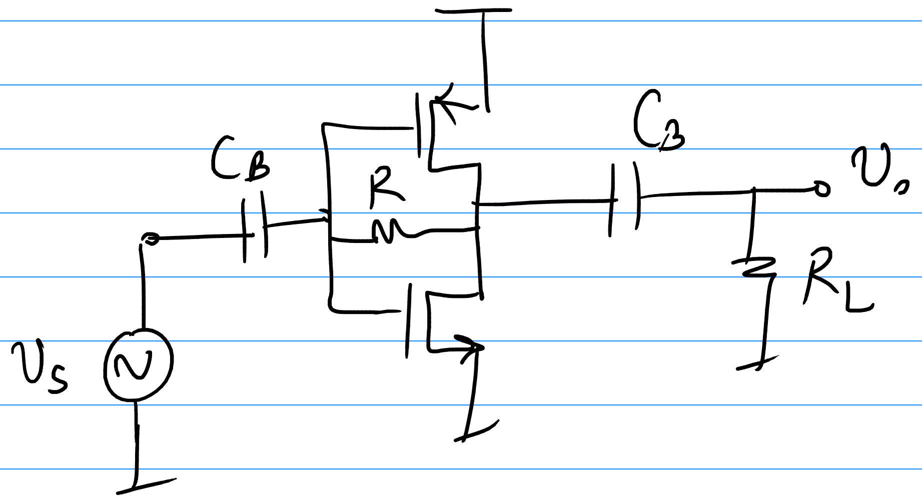
$$\text{gain} = \frac{g_{m_n} + g_{m_p}}{g_{ds_n} + g_{ds_p}}$$

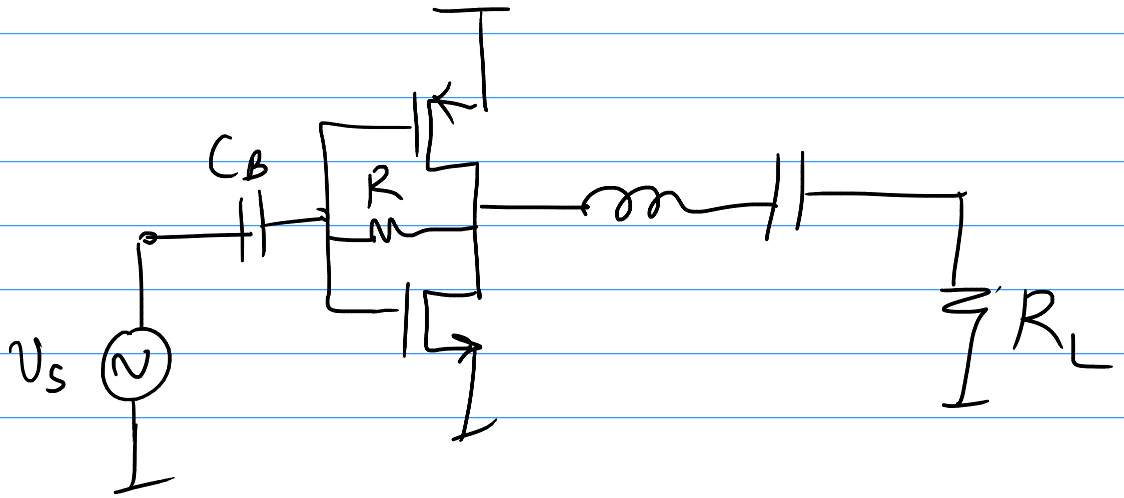
CMOS

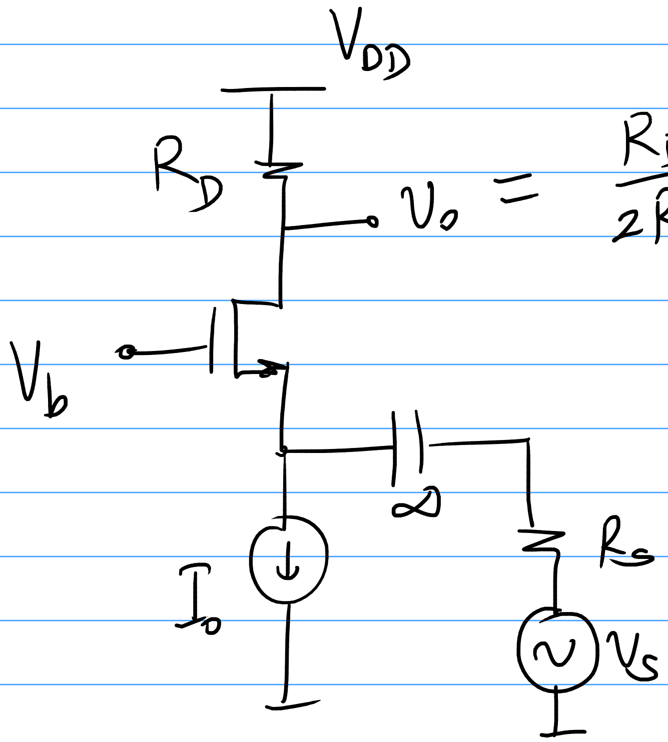
Inverter

based

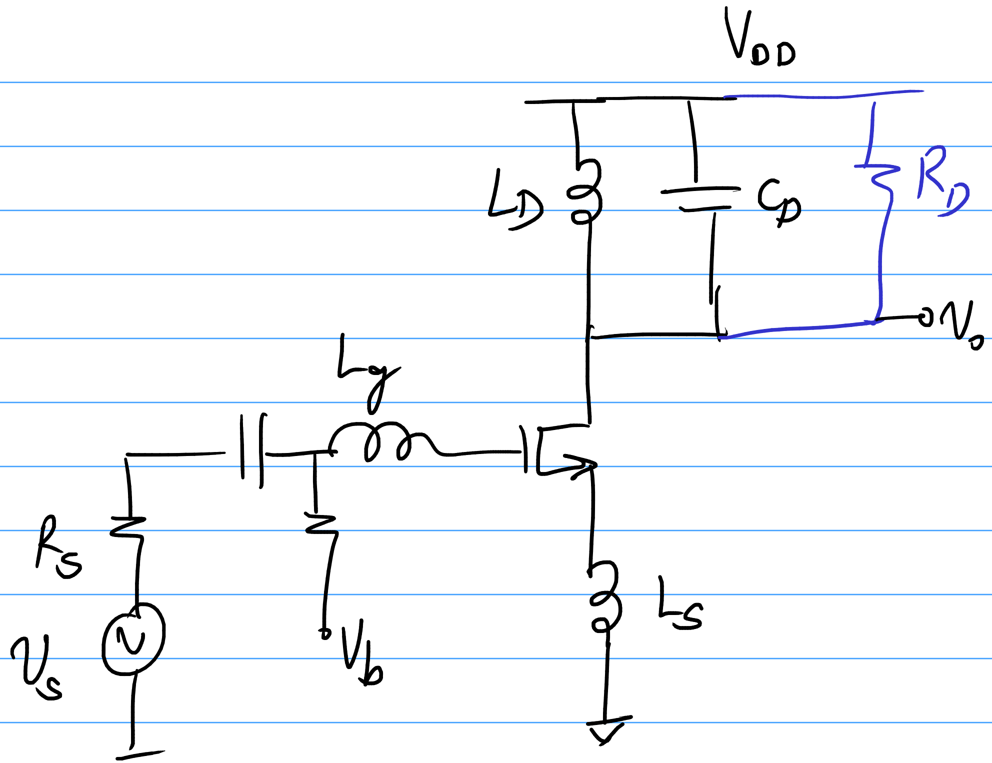
DA







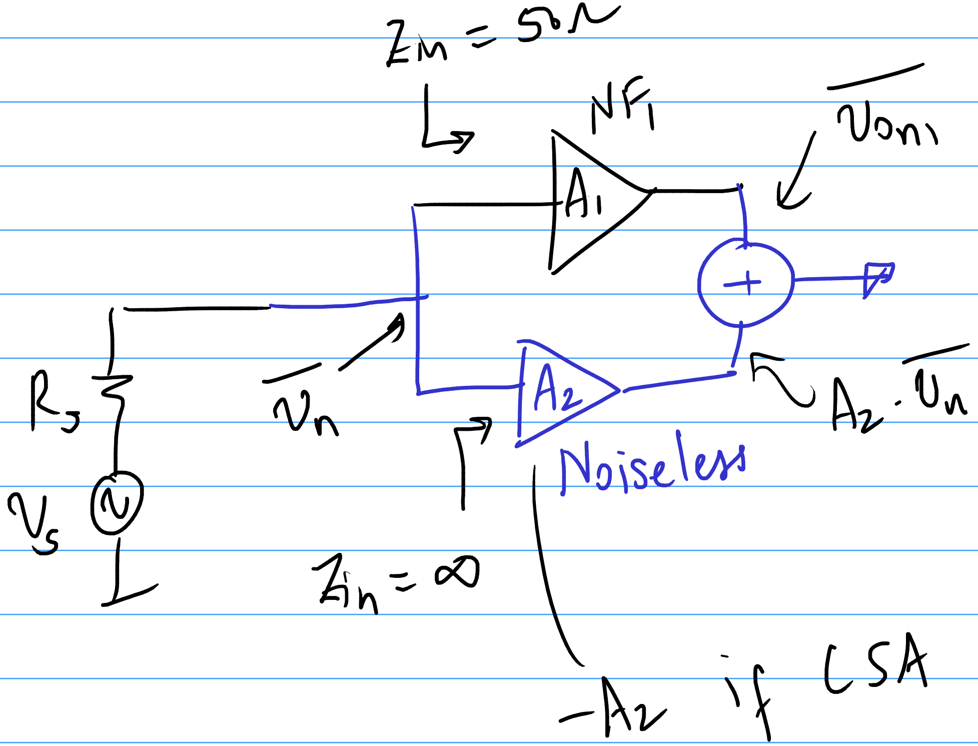
$$v_o = \frac{R_D}{2R_s} \cdot v_s$$



$$\frac{1}{g_m} = 50\Omega = R_s$$

CG LNA
- high NF
- Broadband
- low gain

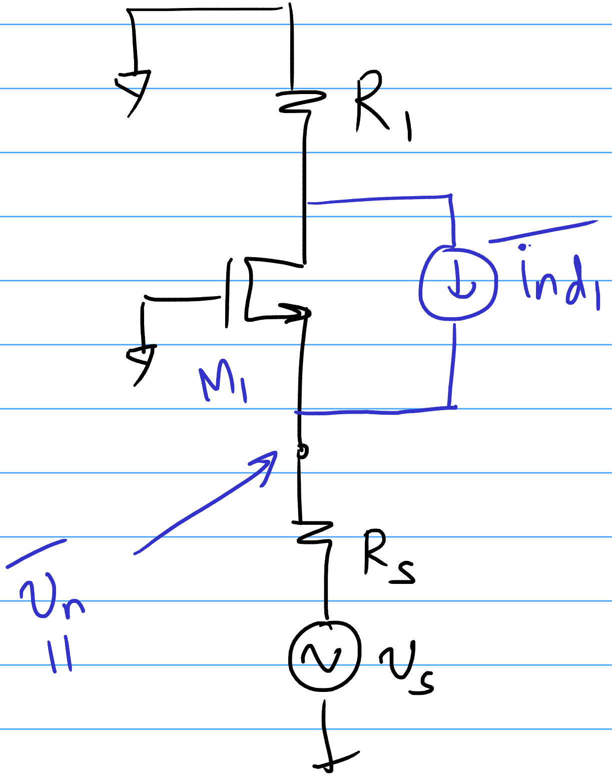
CS LNA
- high gain
- low NF
- narrowband



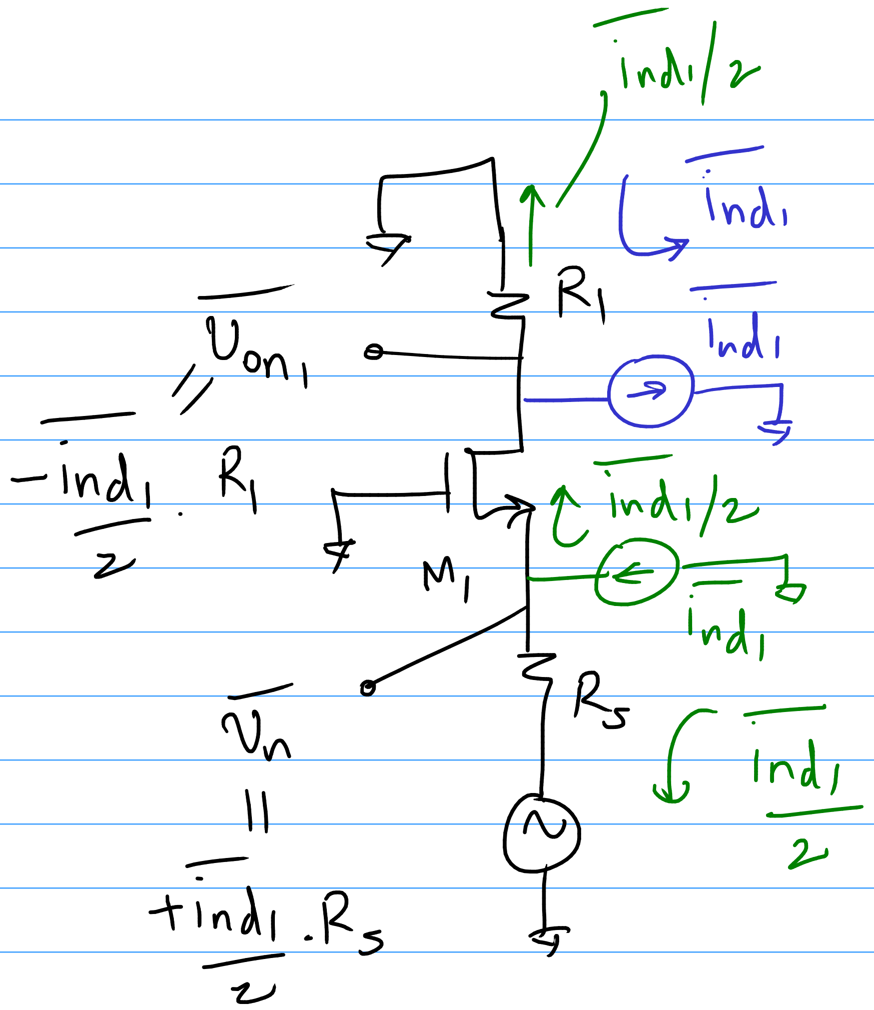
$A_2 \overline{v_n} + \overline{v_{on1}} \rightarrow$ lower NF

$NF < NF_1$

$A_2 \cdot v_n + \overline{v_{on1}} < \overline{v_{on1}}$

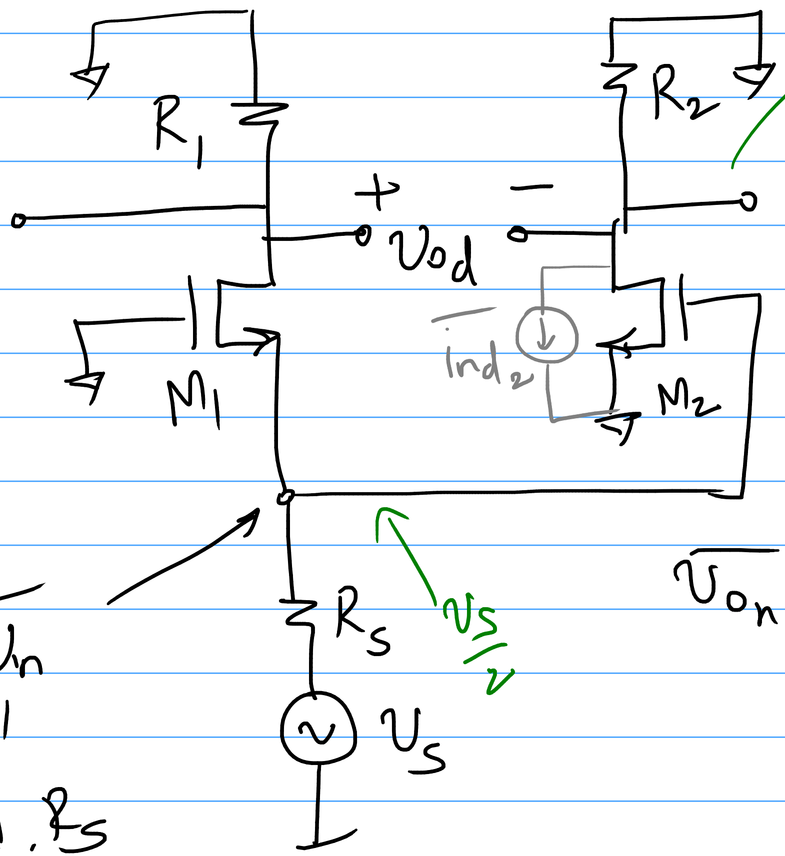


R1
ind1
M1
vn
Rs
vs



ind1/2
ind1
R1
von1
-ind1/2 . R1
M1
vn
ind1/2
Rs
+ind1/2 .Rs

$$v_{o_2} = -g_{m_2} R_2 \cdot \frac{v_s}{2}$$



$$\overline{V_{on_1}} = -\frac{\overline{i_{nd_1}}}{2} \cdot R_1$$

$$\overline{V_n} = +\frac{\overline{i_{nd_1}}}{2} \cdot R_s$$

$$\overline{V_{on_2}} = -g_{m_2} R_2 \cdot \overline{V_n}$$

$$= -g_{m_2} R_2 \cdot \frac{\overline{i_{nd_1}}}{2} \cdot R_s$$

$$\overline{V_{ond_{M_1}}} = \overline{V_{on_1}} - \overline{V_{on_2}}$$

$$= \left(g_{m_2} R_2 R_s - R_1\right) \cdot \frac{\overline{i_{nd_1}}}{2}$$

Set $\overline{V_{ond_{M_1}}} = 0$

"Noise Cancelling" LNA

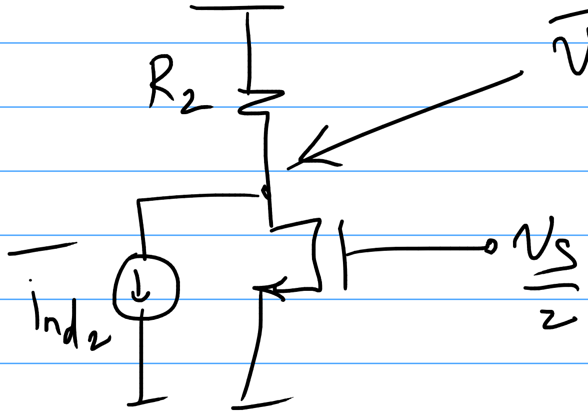
$$\boxed{g_{m_2} R_2 = \frac{R_1}{R_s}}$$ for Noise cancellation

$$\frac{v_{o1}}{v_s} = \frac{R_1}{2R_s}$$

$$\frac{v_{o2}}{v_s} = -\frac{g_{m_2} R_2}{2}$$

$$\frac{v_{od}}{v_s} = \frac{R_1}{2R_s} + \frac{g_{m_2} R_2}{2}$$

$$\boxed{\frac{v_{od}}{v_s} = \frac{R_1}{R_s}}$$



$$\overline{v_{on_2}} = \overline{i_{nd_2}^2} \cdot R_2^2$$

$$\left.\begin{array}{l}\text{gain} \\ \text{from} \\ G_2 \\ \text{to } D_2\end{array}\right\} = -g_{m_2} R_2$$

input referred noise due to M_2 $= \dfrac{\overline{i_{nd_2}^2}\, R_2^2}{g_{m_2}^2 R_2^2} = \dfrac{4kT\gamma}{g_{m_2}} \Delta f$

large $g_{m_2} \longrightarrow$ low NF contribution from M_2

@ output

* Noise from M_1 @ output $= 0$ (CG device)

* Noise from M_2 $= 4kT\gamma g_{m_2} R_2^2$ (CS device)

* Noise from R_1 $= 4kTR_1$

* " " $R_2 = 4kTR_2$

* Noise from R_S $= \left(\frac{R_1}{R_S}\right)^2 \times 4kTR_S = \frac{4kTR_1^2}{R_S}$

$(gain)^2 \times \overline{v_{nR_S}^2}$

$$F = 1 + \frac{4kT\gamma g_{m_2} R_2^2 + 4kTR_1 + 4kTR_2}{4kTR_1^2 / R_S}$$

apply $g_{m_2} R_2 = \frac{R_1}{R_S}$

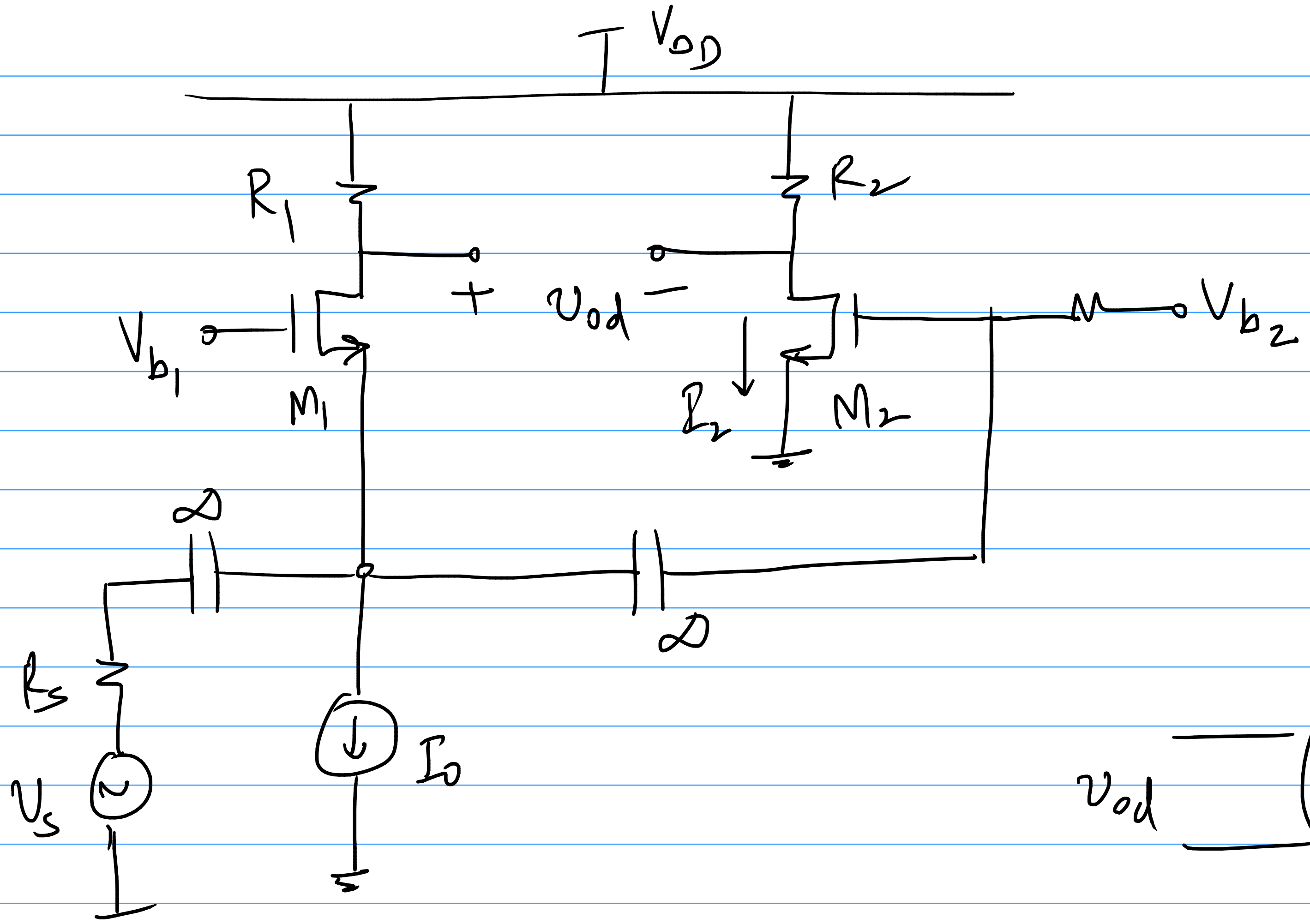
$$= 1 + \gamma \cdot g_{m_2} \cdot R_S \cdot \frac{R_2^2}{R_1^2} + \frac{R_S}{R_1} + \frac{R_S R_2}{R_1^2}$$

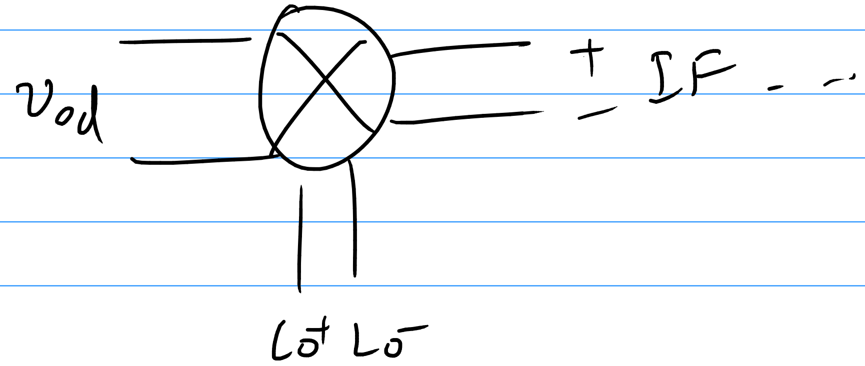
$$= 1 + \gamma \cdot \frac{R_2}{R_1} + \frac{R_S}{R_1} + \frac{R_S R_2}{R_1^2}$$

min F $\rightarrow$ min. R_2 $\longleftrightarrow$ max. g_{m_2}

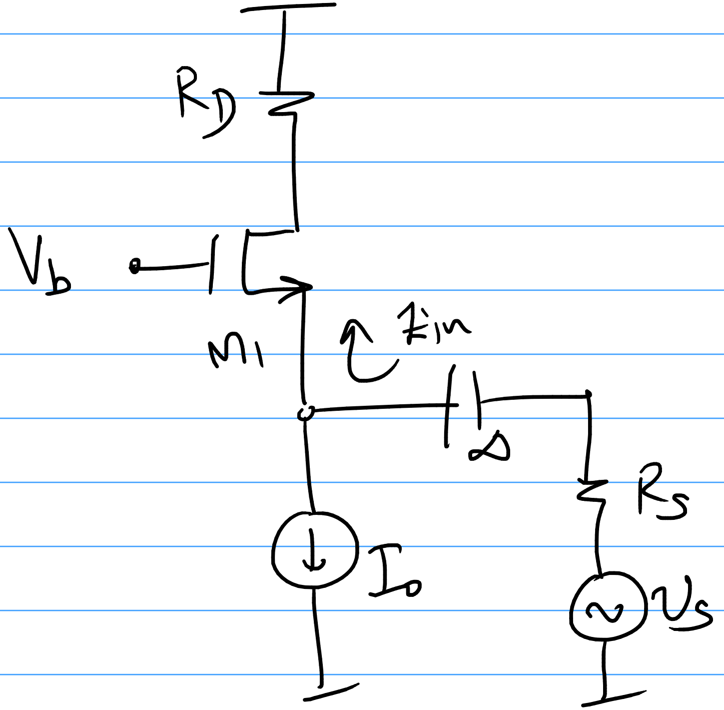
$\rightarrow$ max R_1

Noise Cancelling LNA

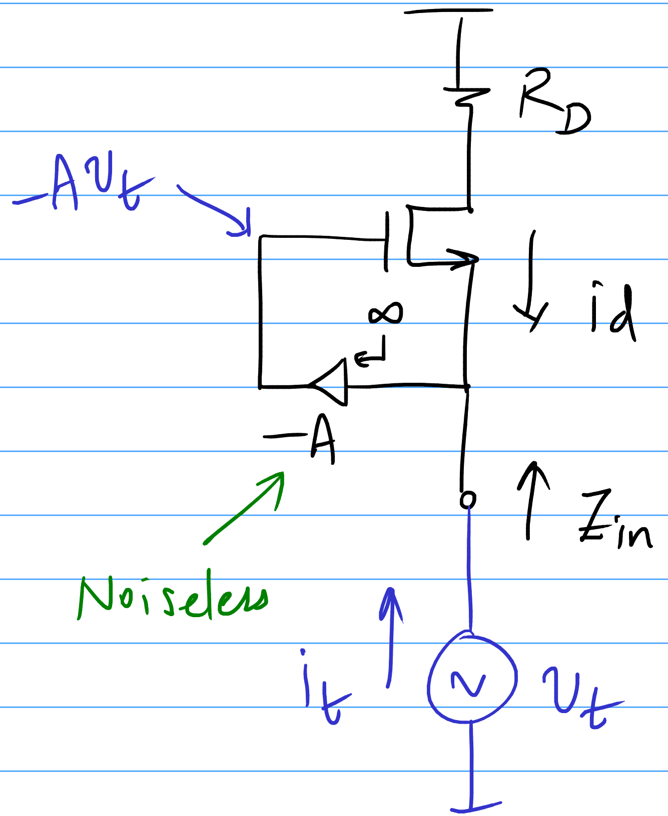




"Gain Boosted" CG LNA



$$Z_{in} = \frac{1}{g_{m_1}} = R_S$$



$$i_d = g_m \cdot v_{gs}$$
$$= -i_t$$

$$-i_t = g_{m_1}(-Av_t - v_t)$$

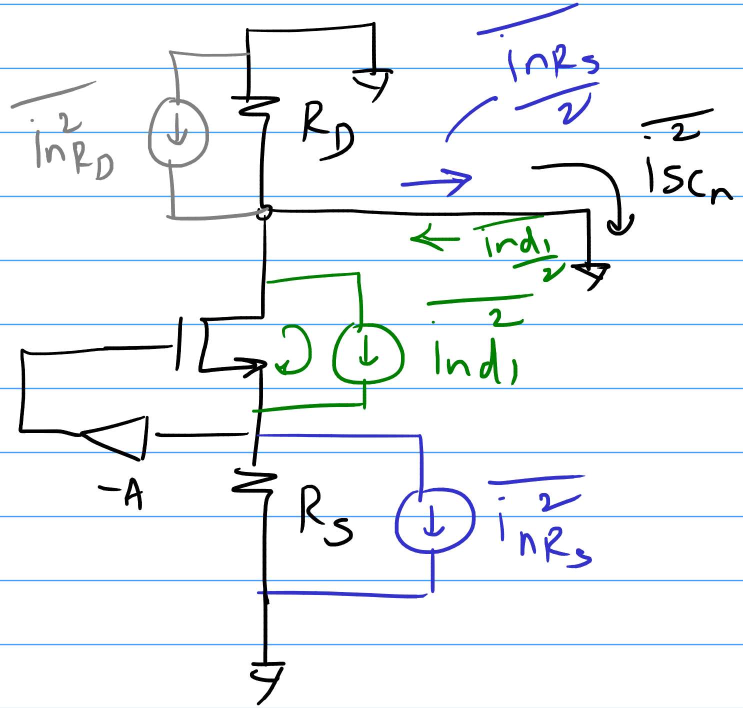
$$Z_{in} = \frac{v_t}{i_t} = \frac{1}{(1+A)\,g_m} = R_s$$

e.g. CGA $\rightarrow$ $g_m = \frac{1}{R_s} = 20\,mS$

Gain Boosted CGA $\rightarrow$ $g_m = \frac{1/R_s}{(1+A)} = \frac{20\,mS}{1+A}$

gain $\frac{v_o}{v_s}$ is same

$$R_{in} = R_s = \frac{1}{g_m(1+A)}$$



$\frac{kT}{R_s}$

$\frac{4kT}{R_1}$

$kT\gamma g_{m_1}$

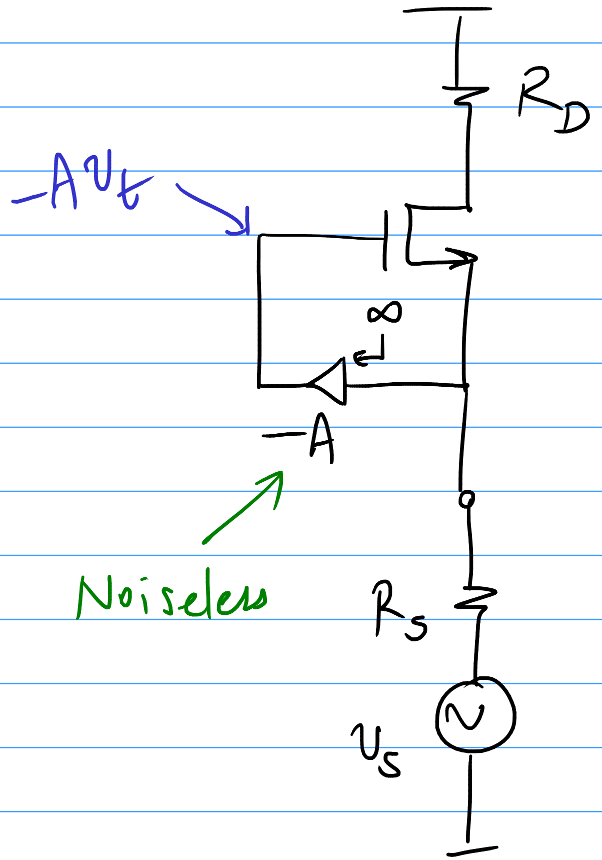
$$F = 1 + \frac{kT\gamma g_{m_1} + 4kT/R_1}{kT/R_s} = 1 + \gamma g_{m_1} R_s + \frac{4R_s}{R_1}$$

$$g_{m1} R_S = \frac{1}{1+A}$$

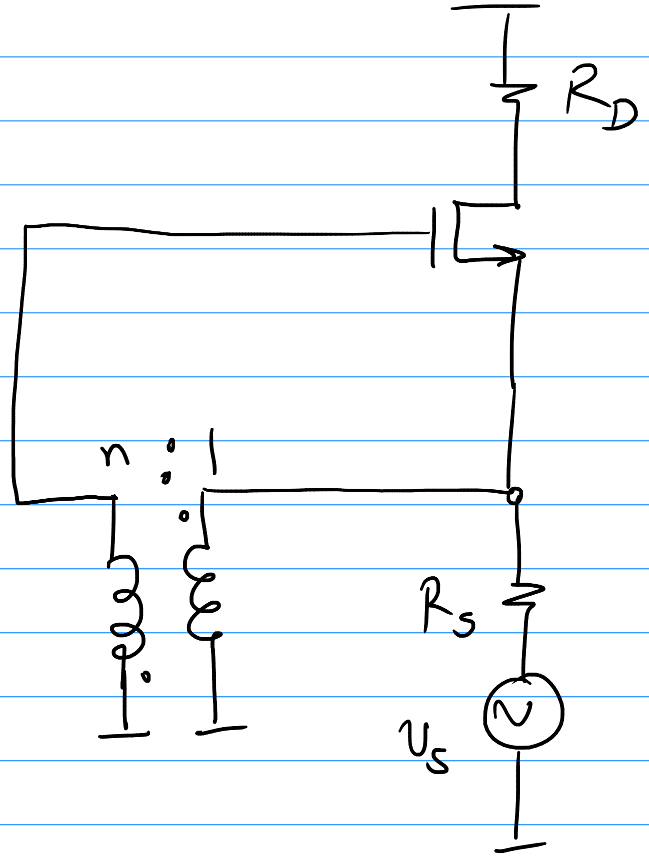
$$F = 1 + \frac{\gamma}{1+A} + \frac{4R_S}{R_1}$$

A = Noiseless amp.

L M C } give passive Noiseless voltage gain

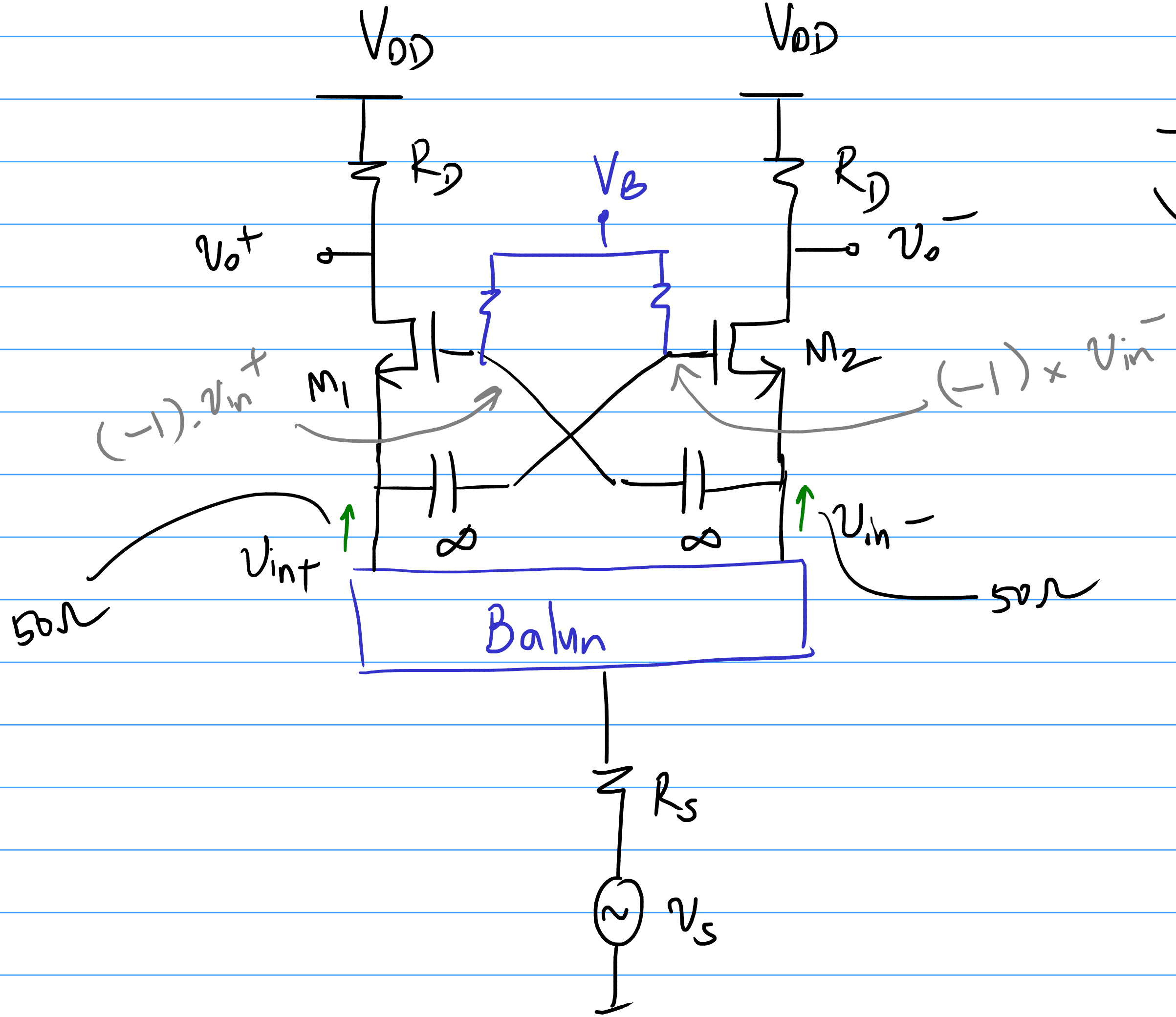


1)



$A = n$

$n = 1$ is common

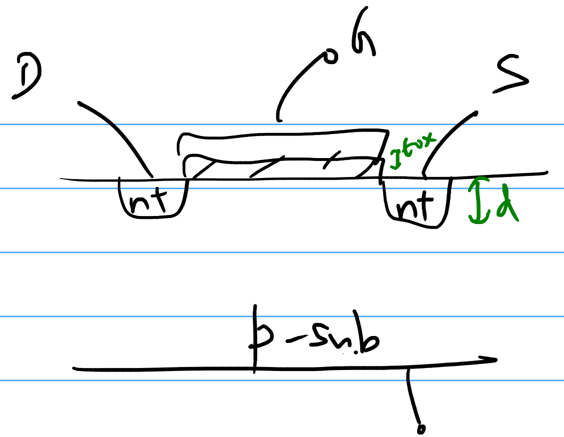


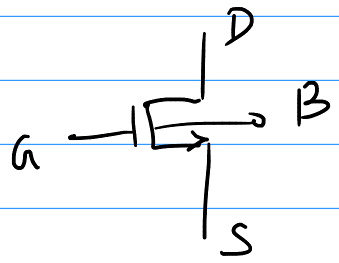
$$\underbrace{-A = -1}$$

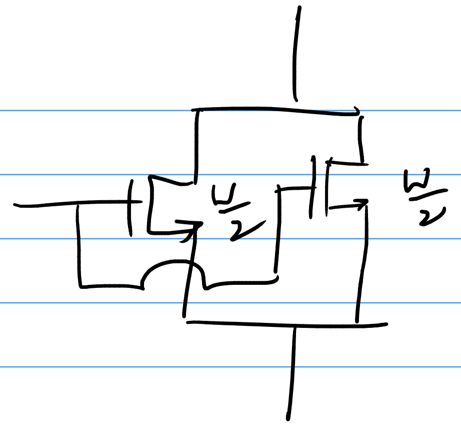
$$g_{m_1} = g_{m_2} = 10\,mS$$

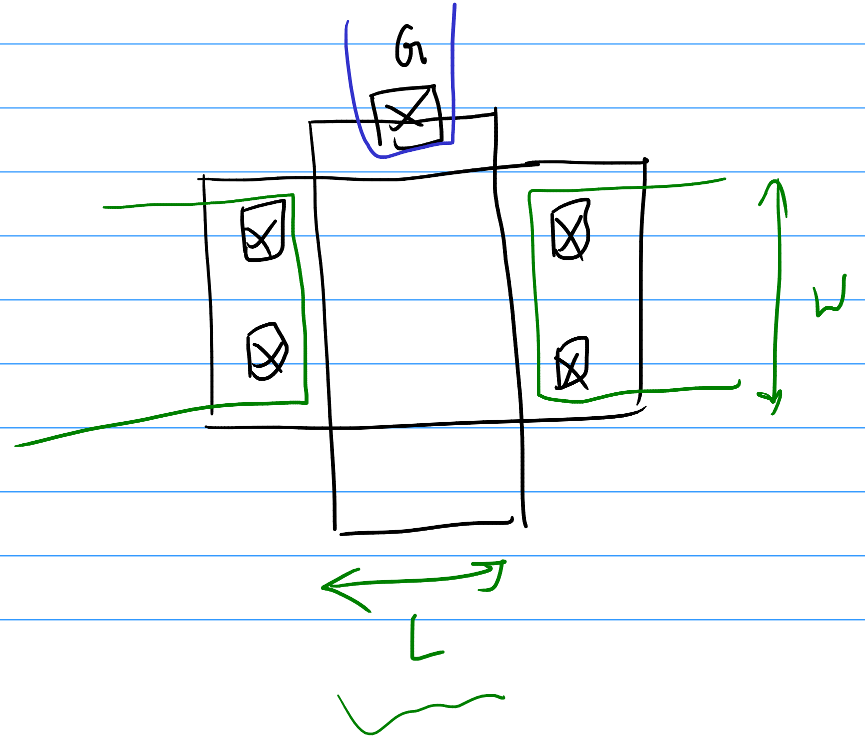
Compact (No L,M)

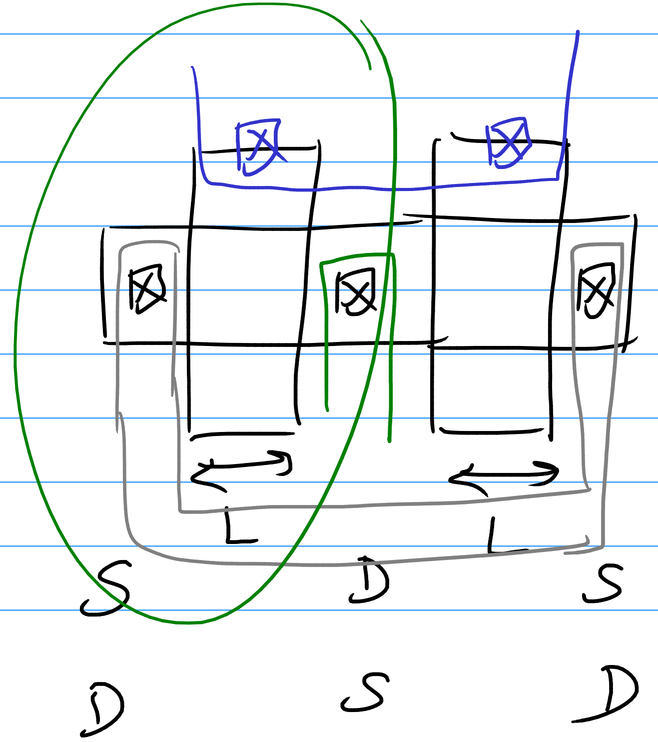
Broadband

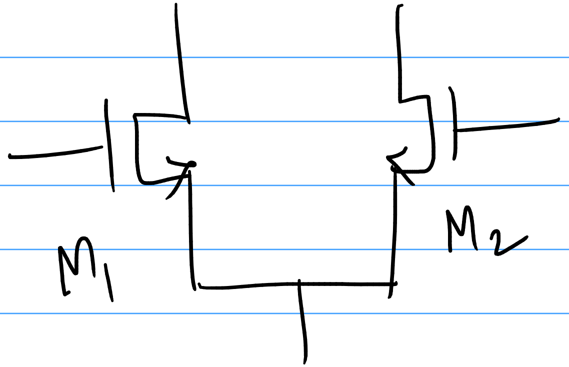

D
G
S
n+
n+
tox
d
p-sub

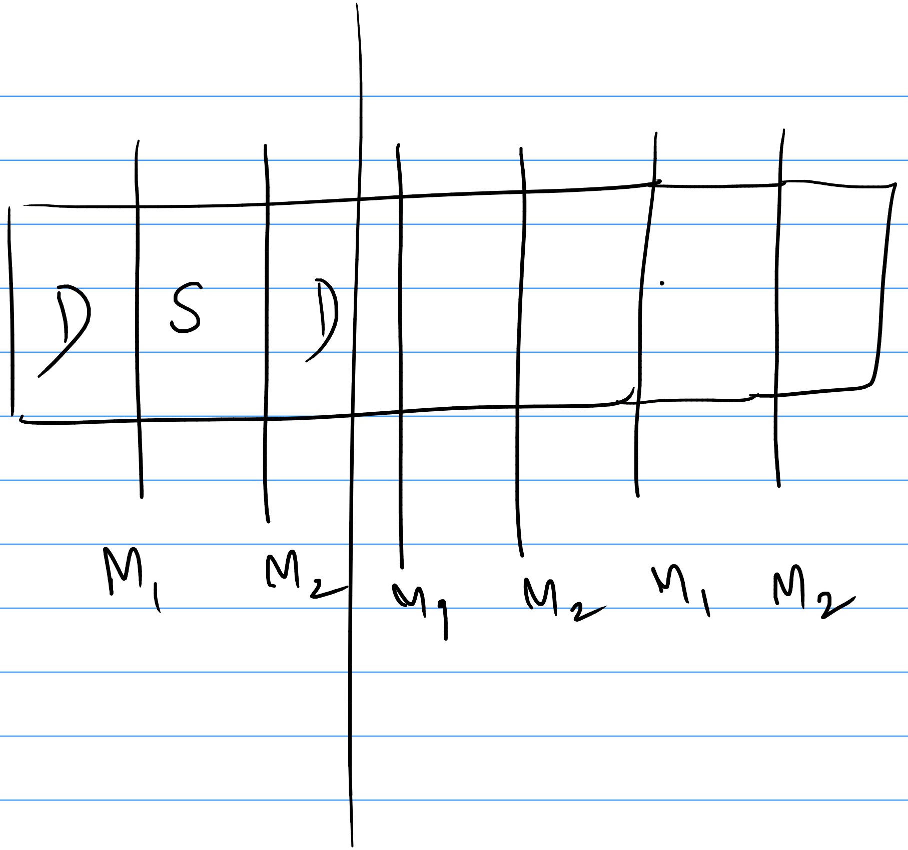

D
G
B
S

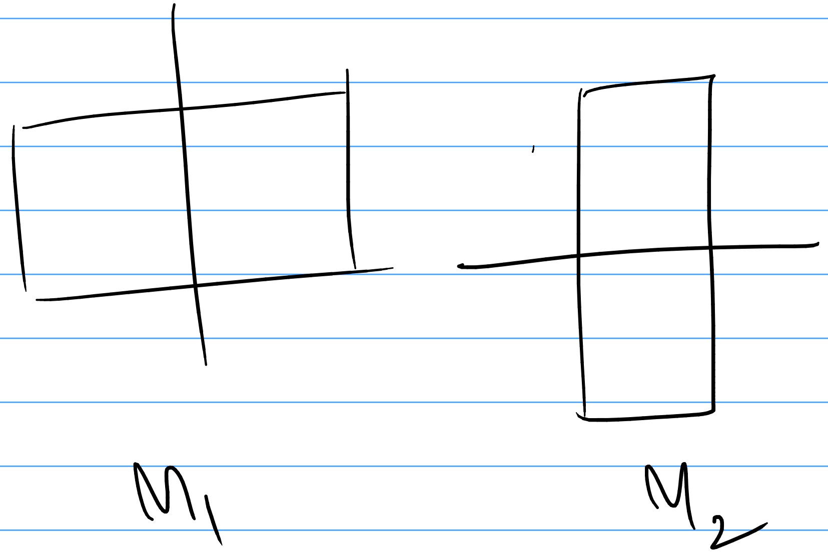

W/2
W/2

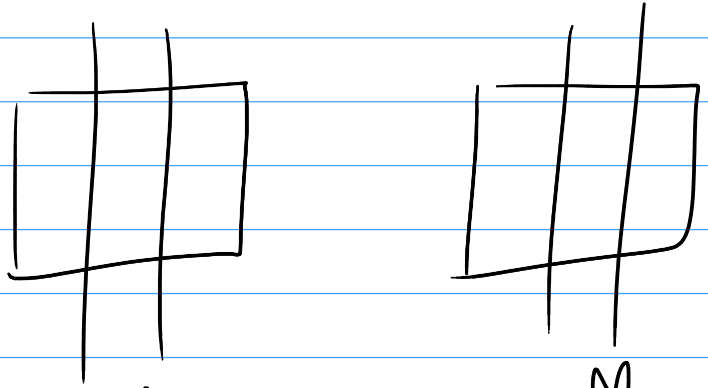

G
W
L


L
L
S
D
S
D
S
D


M1
M2

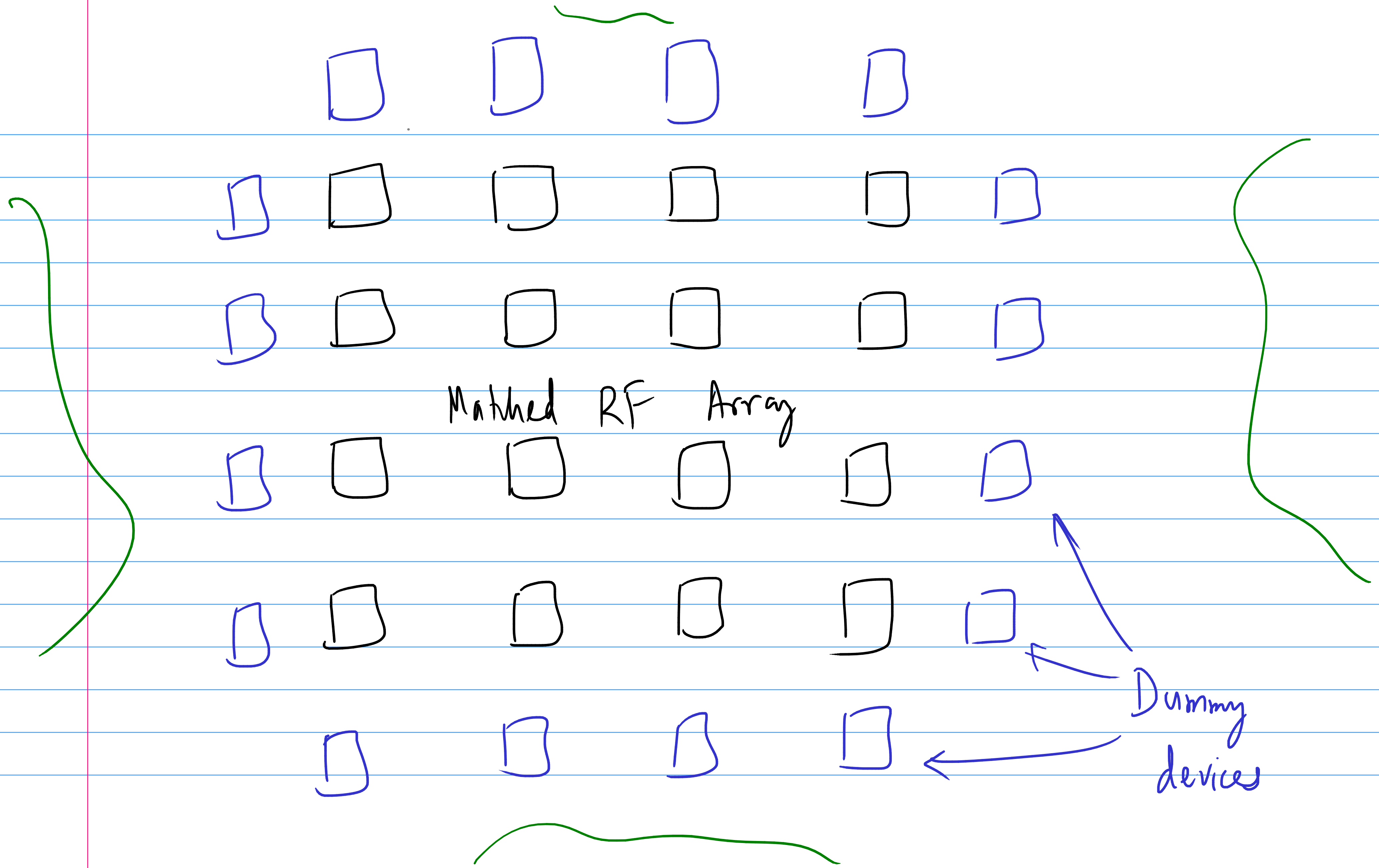

D
S
D
M1
M2
M1
M2
M1
M2

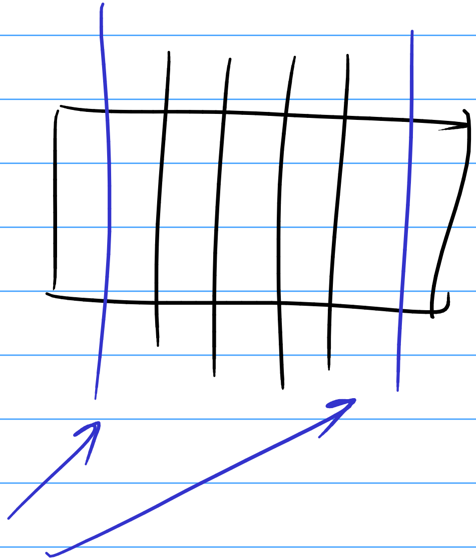

M1
M2



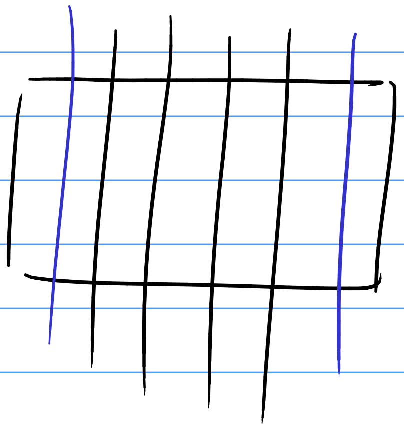
M_1 $\quad$ M_2

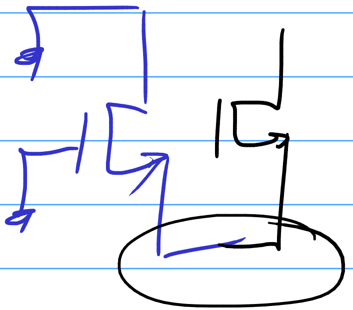
$M_1 \quad M_2 \cdots$

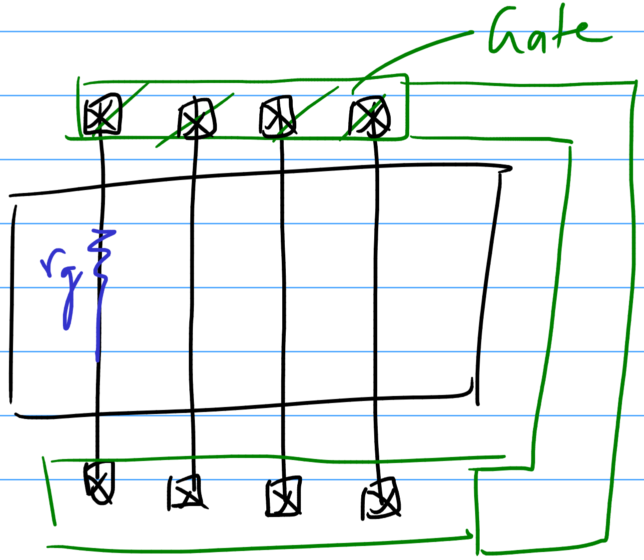

Matched RF Array
Dummy devices



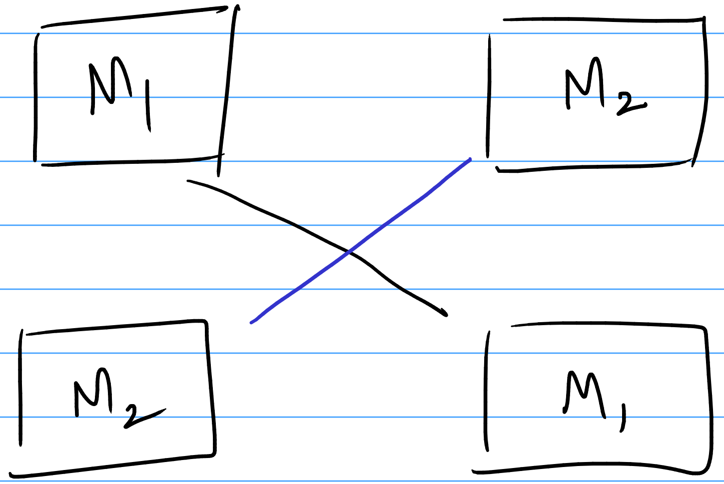
Dummy Finger



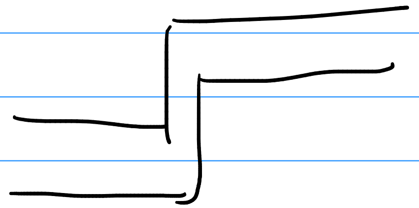


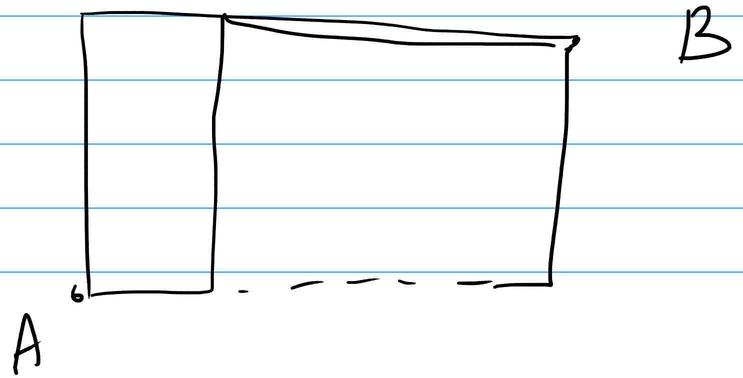


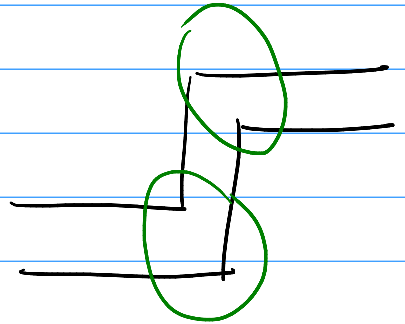
Dual-sided gate contacts



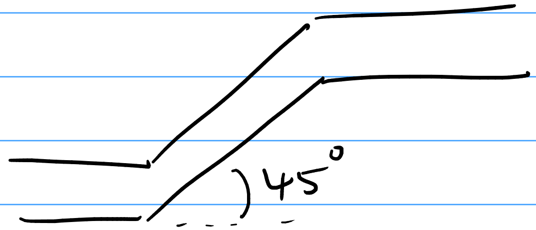
common-centroid

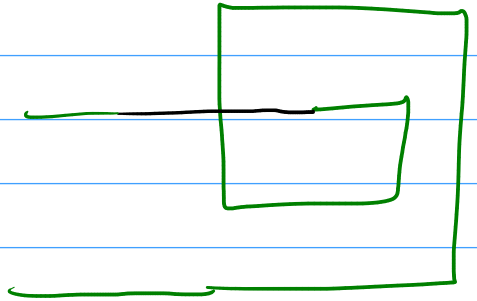


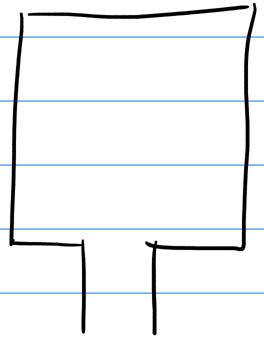


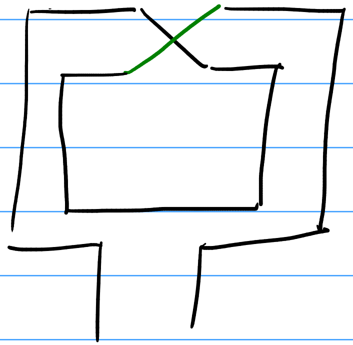


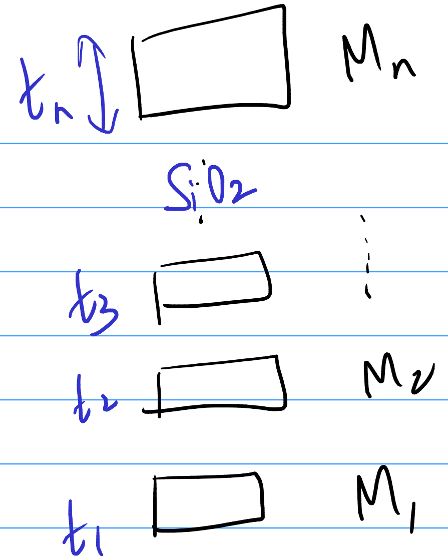
X

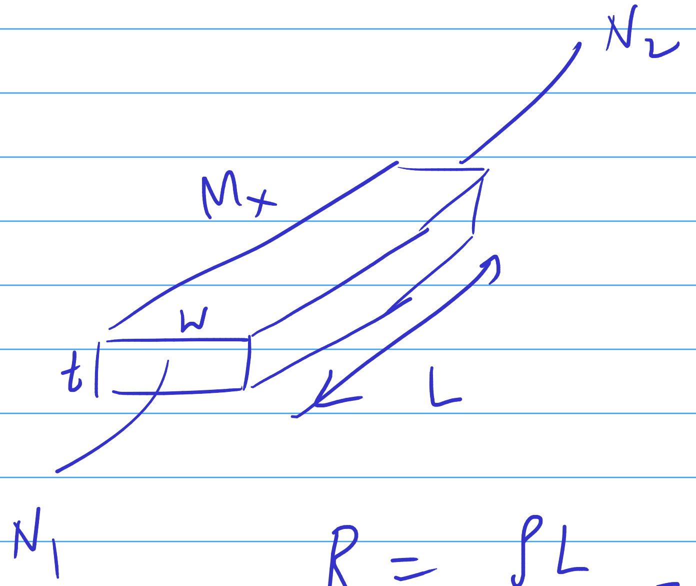






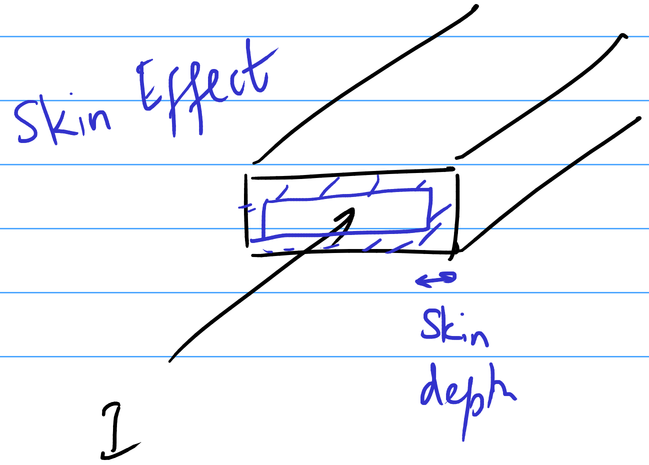


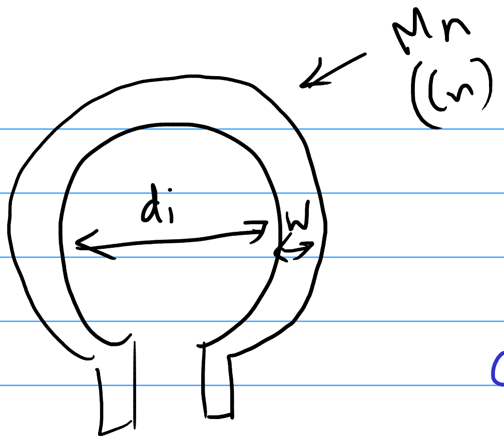




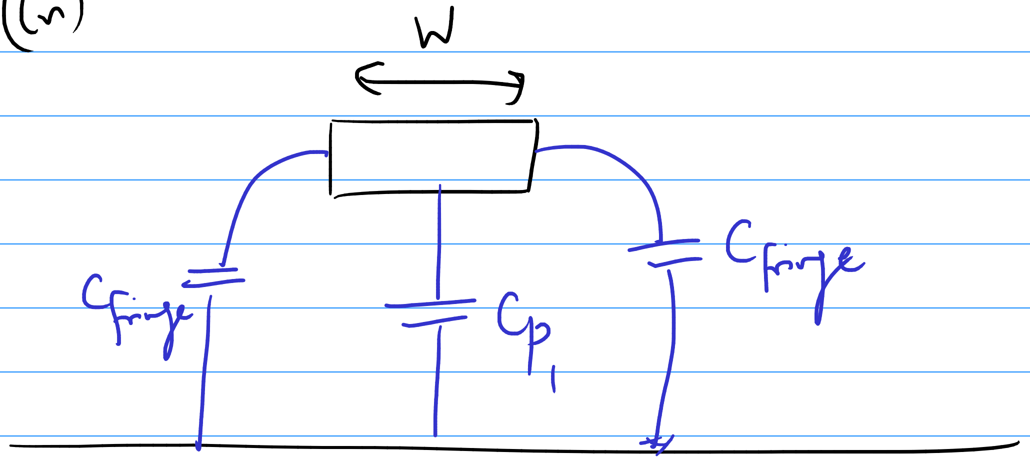
$$R = \frac{\rho L}{A} = \frac{\rho L}{W \cdot t}$$

$$= \underbrace{R_{sheet}(M_x)}_{\rho / t} \cdot \underbrace{\#\text{ of squares}}_{\frac{L}{W}}$$

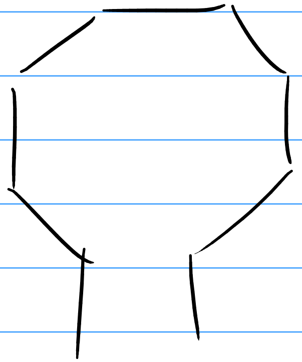




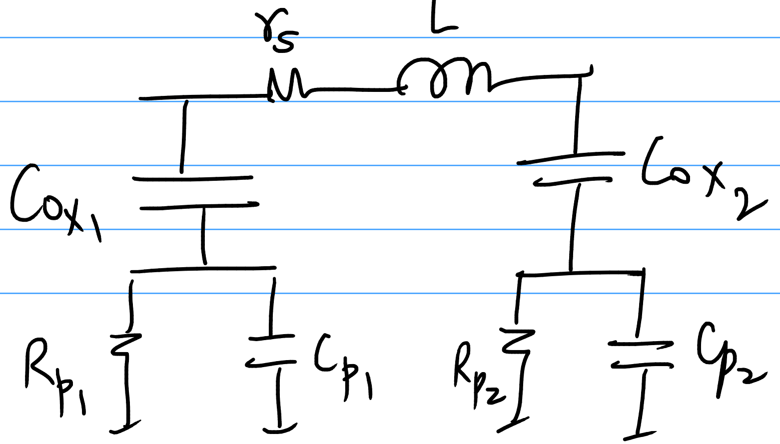
Circular



p-sub

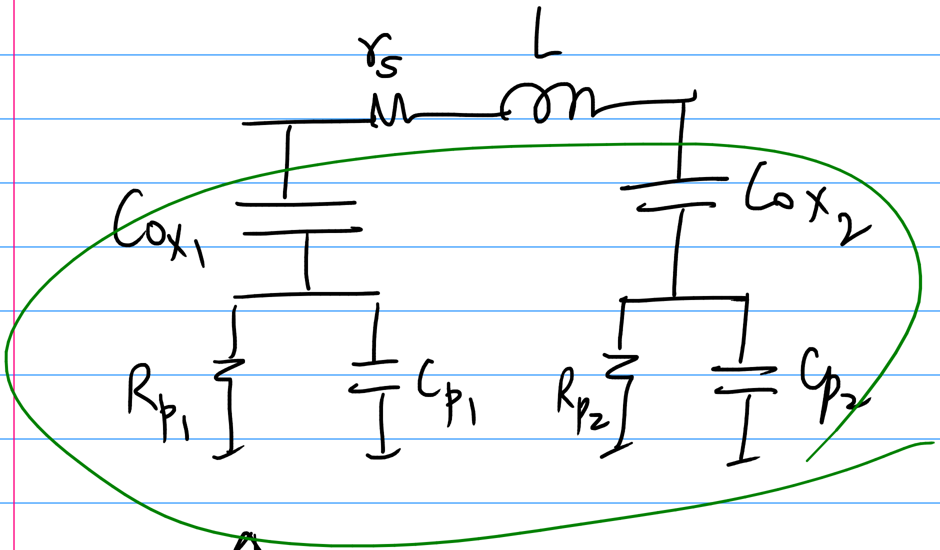


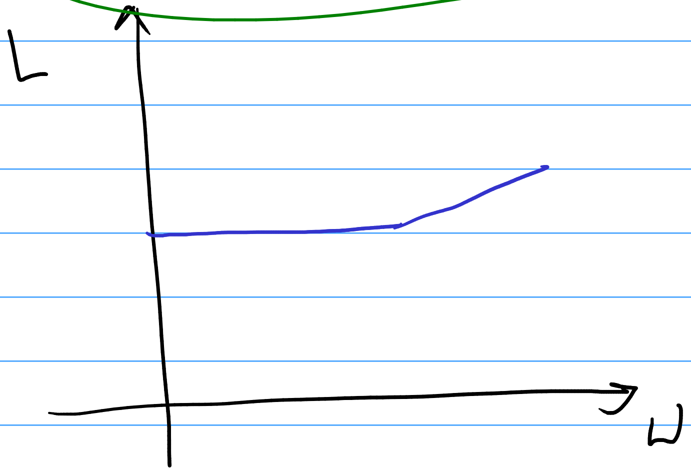
octagon

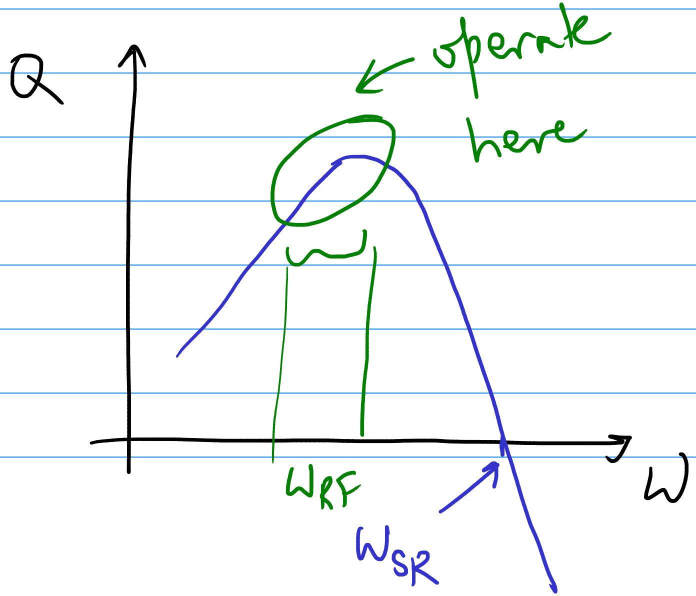


Lumped Element Model of a spiral inductor (Narrowband)

Lumped Element Model of a spiral inductor (Narrowband)

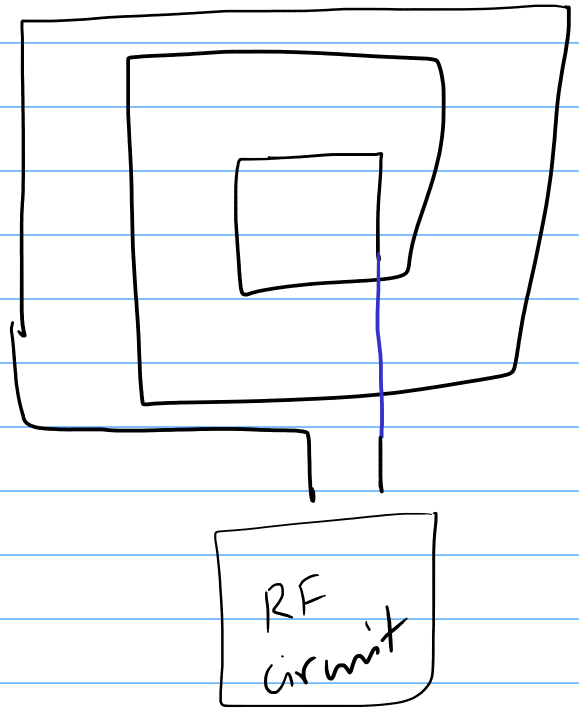


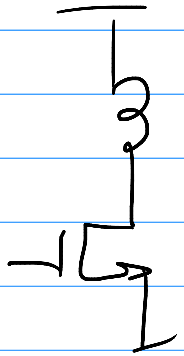


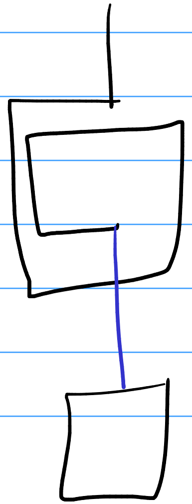


$$Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)}$$

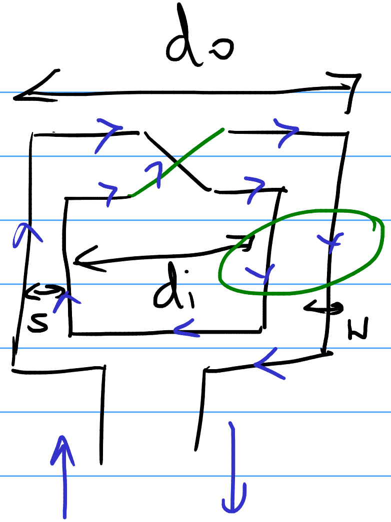
ω_{SR} = self resonant frequency







1.5 turn



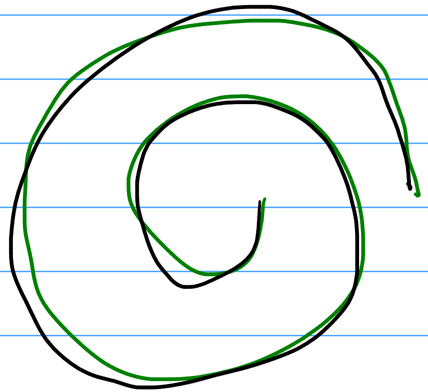
\# turns n

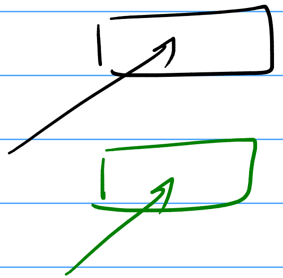
trace width w

turn-to-turn spacing s

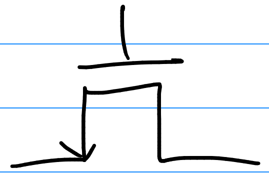
inner diameter d_i

outer diameter d_o



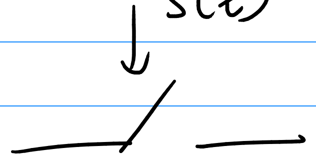


Passive Mixers



triode

$S(t)$ → periodic signal switching @ ω_{LO}



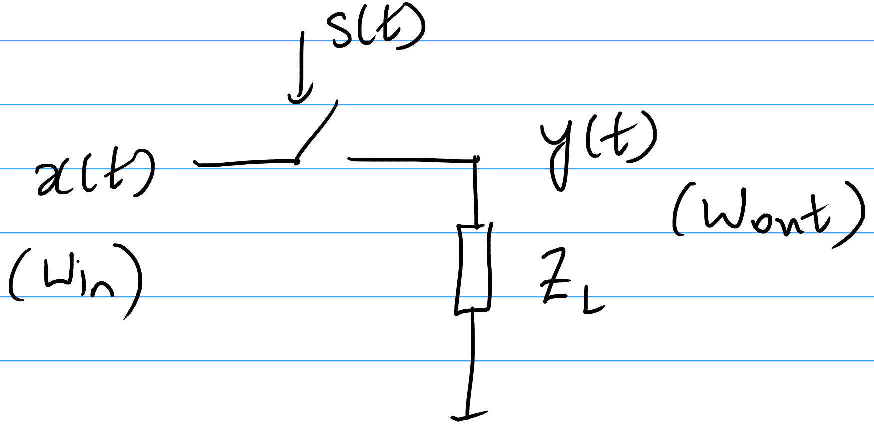
In $x(t)$

Out $y(t)$

LPTV analysis

* $S(t + T_{LO}) = S(t)$
* Linearity is valid
* LO is ideal

* Z_S & Z_L connected to LPTV mixer



$$s(t) = \sum_{k=-\infty}^{\infty} s_k e^{jk\omega_{Lo}t}$$

$$y(t) = x(t) \cdot s(t)$$

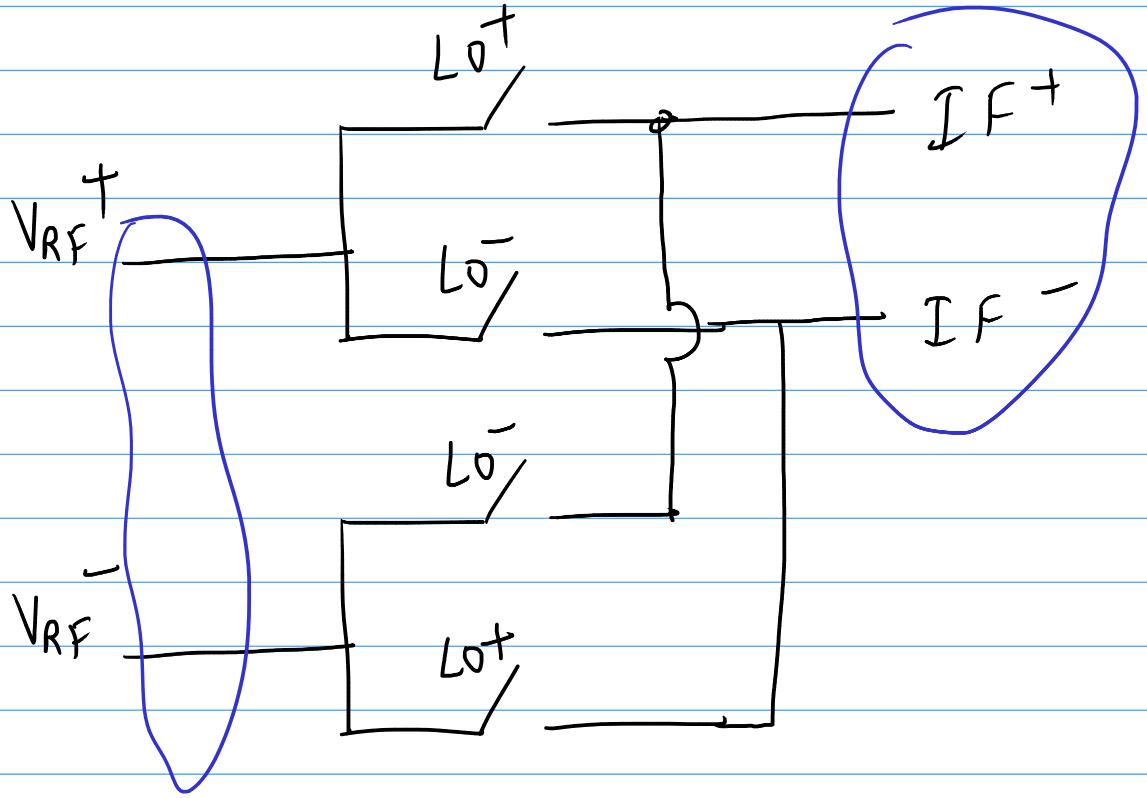
$$Y(\omega_{out}) = \sum_{k} s_k X(\omega_{in} - k\omega_{Lo})$$

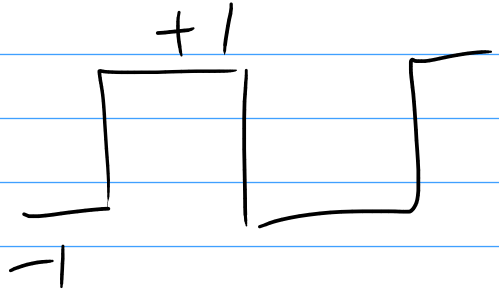
$$\omega_{out} = \omega_{in} + k\omega_{Lo}$$

R_X : $k = -1$

T_X : $k = +1$

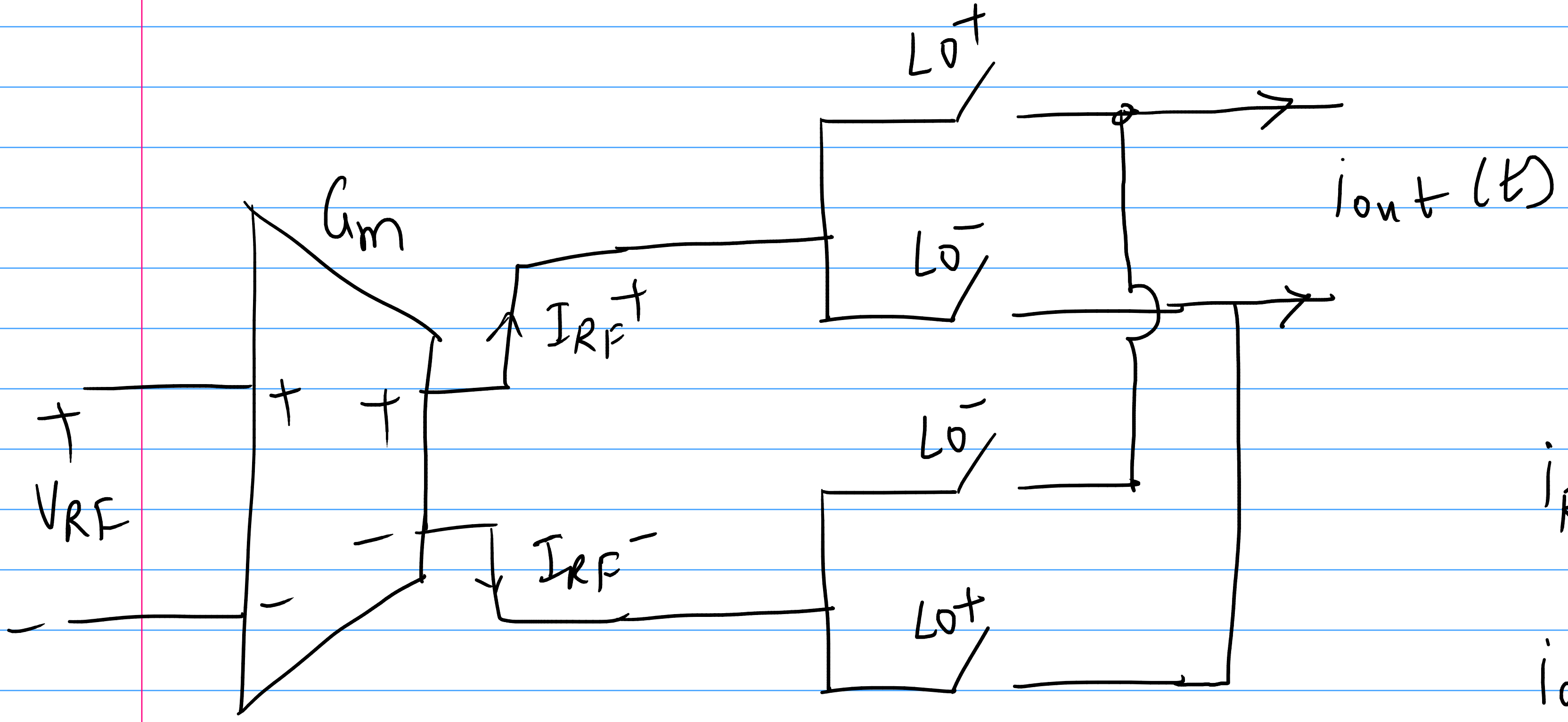
Conversion gain $= S_k$



$S(t) =$ 

$$S_k = \begin{cases} \dfrac{2}{\pi k} & \text{odd } k \\ 0 & \text{even } k \end{cases}$$

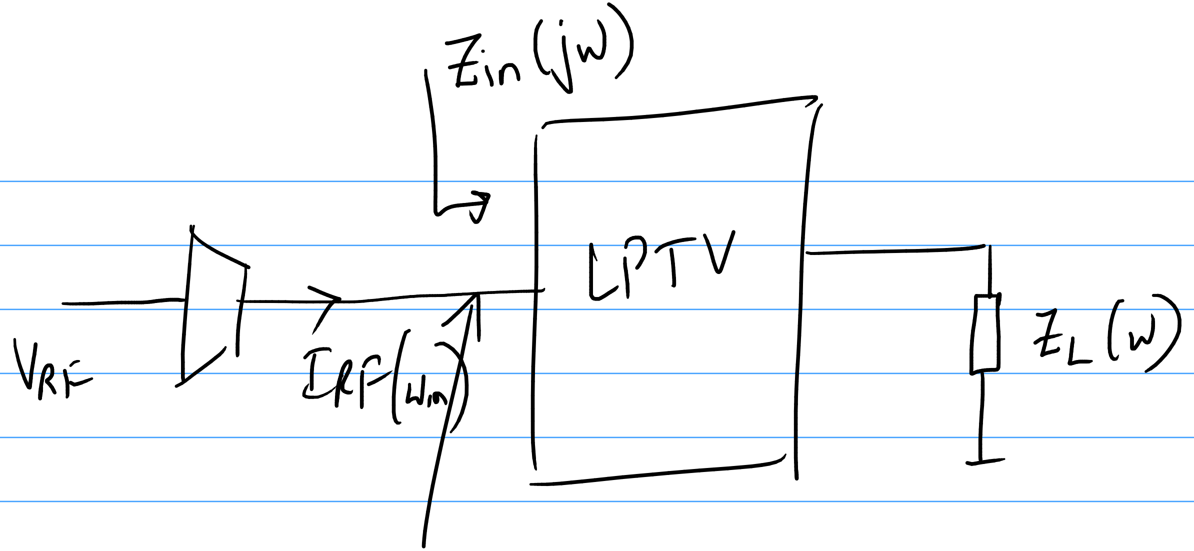
Voltage mode Passive Mixer



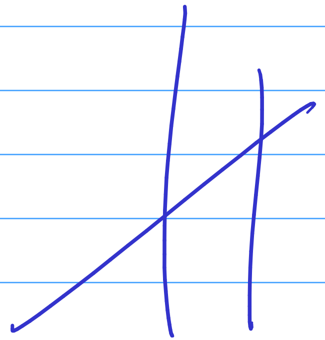
$$i_{RF}(t) = G_m \, v_{RF}(t)$$

$$i_{out}(t) = s(t) \, i_{RF}(t)$$

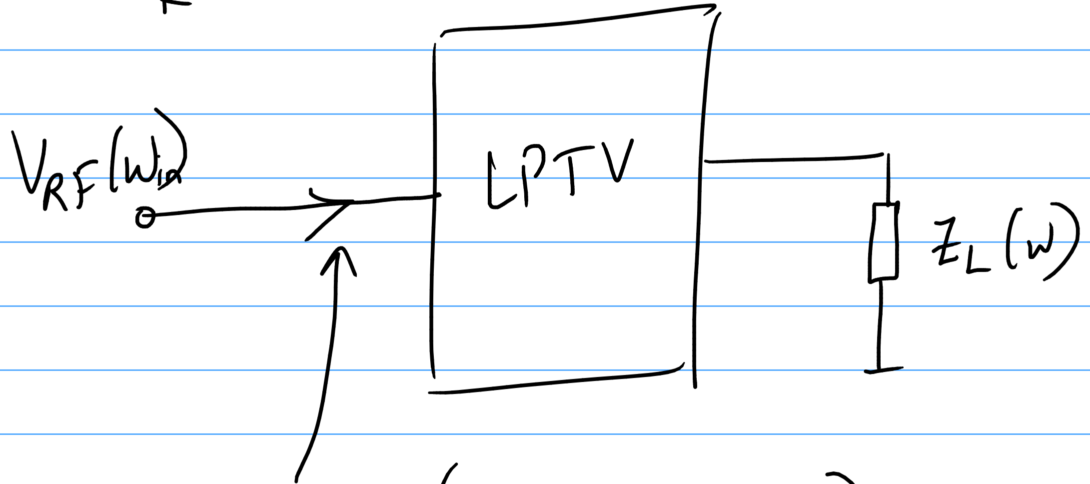
Current Mode Passive Mixer



$$Y_{in}(\omega_{in}) = \frac{I_{RF}(\omega_{in})}{V_{RF}(\omega_{in})}$$



$$\sum_k V_{RF}(\omega_{in} - k\,\omega_{LO})$$



$$Z_{in}(\omega_{in}) = \frac{V_{RF}(\omega_{in})}{I_{RF}(\omega_{in})}$$

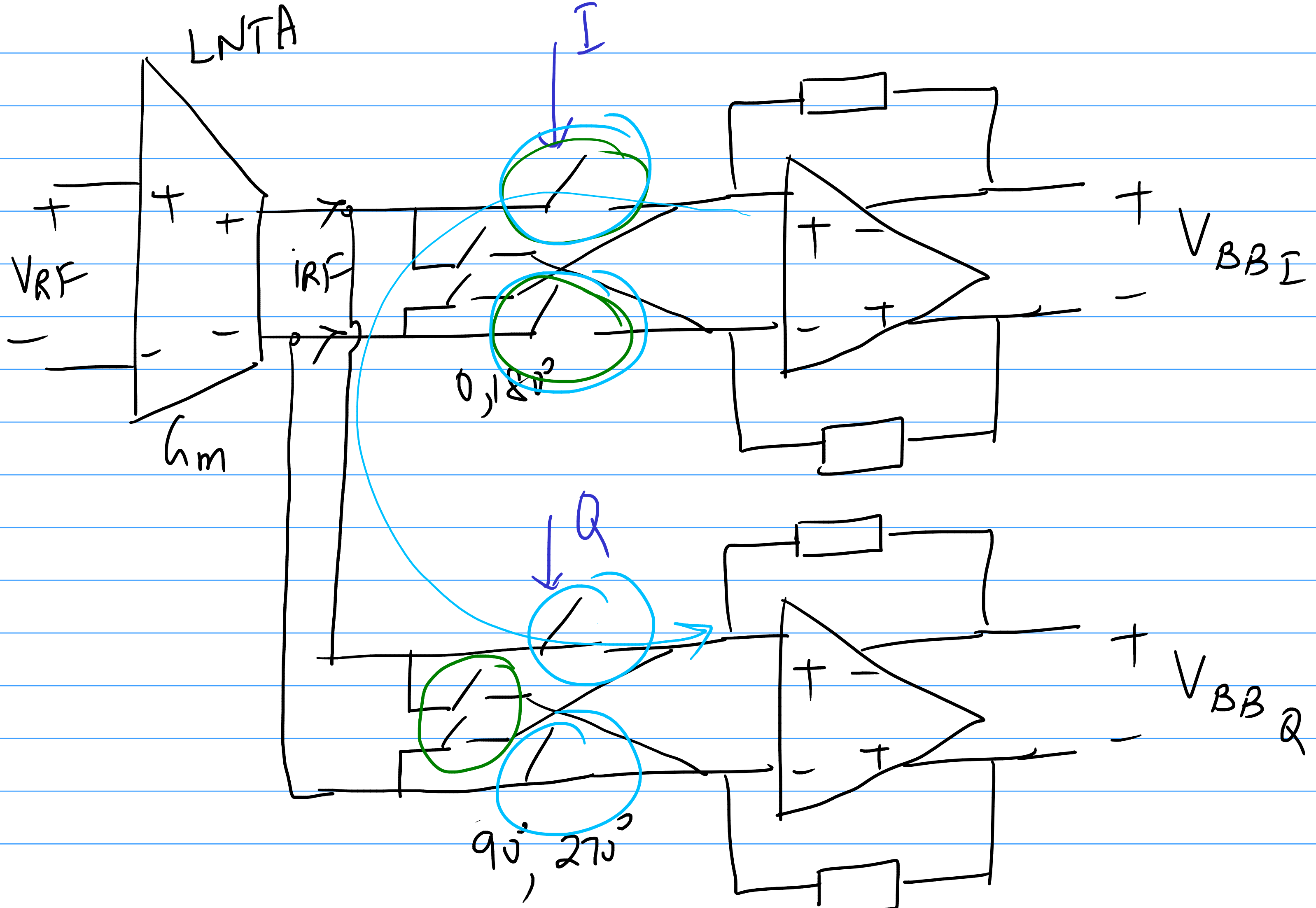
$$\sum_k I_{RF}(\omega_{in} - k\,\omega_{LO})$$

Noise

$$S_y(\omega_{out}) = \sum_k |S_k|^2 \cdot S_x(\omega_{in} - k\omega_{LO})$$

Noise Folding

$$F = \frac{\sum_k |S_k|^2}{|S_1|^2} = \frac{\pi^2}{4}$$


LNTA
+
+
+
V_{RF}
−
−
−
G_m
i_{RF}
I
$0, 180°$
+
−
−
+
+
$V_{BB\,I}$
−
Q
$90°, 270°$
+
−
−
+
+
$V_{BB\,Q}$
−

RF Simulation

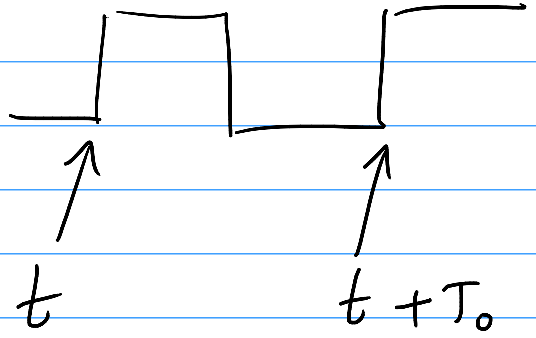
DC op pt

AC small-signal

Noise "

transient large signal

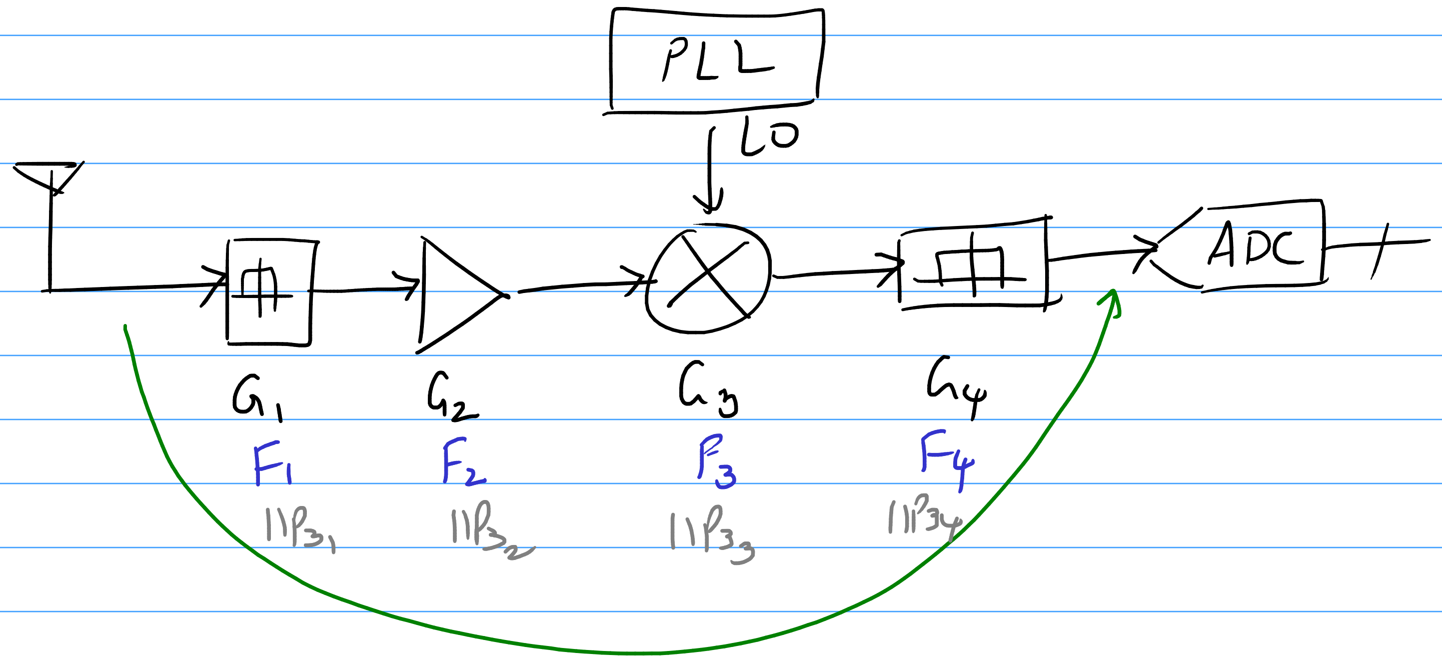
Analog sims



S-parameter small signal

PSS ← periodic steady state

HB ← Harmonic Balance



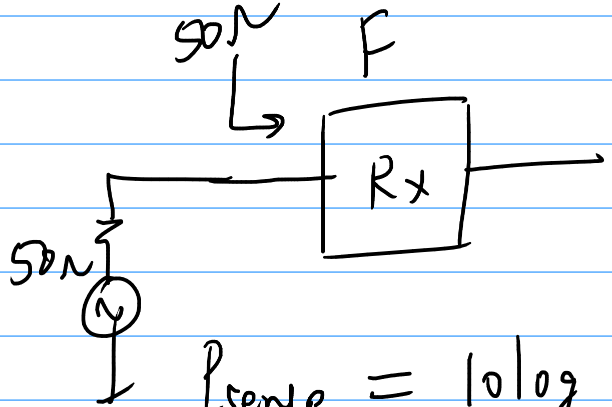
Overall gain G $= G_1 + G_2 + G_3 + G_4$ (all in dB)

$= ?$

Overall NF $= ?$

Overall IIP_3 or $P_{1dB} = ?$

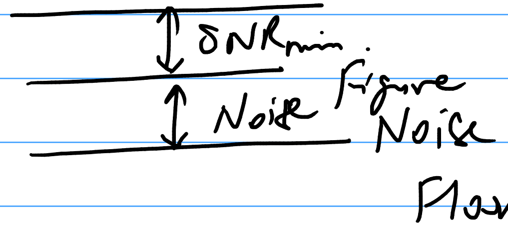
P_{sense} = Sensitivity of Rx $\equiv$ lowest P_{in} @ Rx input that can be demodulated with a specific desired BER (i.e. SNR_{min})



available Noise power = kTB

$$P_{sense} = 10\log_{10}(kT) + 10\log_{10}(B) + NF_{dB} + SNR_{min}\,(dB)$$

in dBm



$$P_{sens.} = -174\ dBm/Hz + 10\log(B) + NF + SNR_{min}.$$

WiFi 802.11ag system design example

Ref: Chap 13 in RF Microelectronics by B. Razavi 2nd ed.

gain, NF, IIP3/P1dB

↓

gain range (AGC)

1) NF

802.11ag std. specifies: Packet Error Rate = 10%.

↓

$BER = 10^{-5}$

↓

(64 QAM) $SNR = 18.3\,dB$

Tx BB pulse shaping $\longrightarrow$ $BW = 16.6\,MHz$

$P_{sens} = -174\,dBm/Hz + NF + 10\log(16.6\,MHz) + 18.3\,dB$

@ 52 Mbps $\Rightarrow$ $P_{sens.} = -65\,dBm$ from standard

$NF = 18.4\,dB$

$\downarrow$ Non-idealities / Margins / Competitiveness

$NF = 10\,dB$

@ 6 Mbps $\Rightarrow$ $P_{sense} = -82\,dBm$ from standard

$\hookrightarrow NF = 10\,dB$

2) <u>Non linearity</u>

(i) # 52 sub channels $\rightarrow$ PAPR = 9 dB

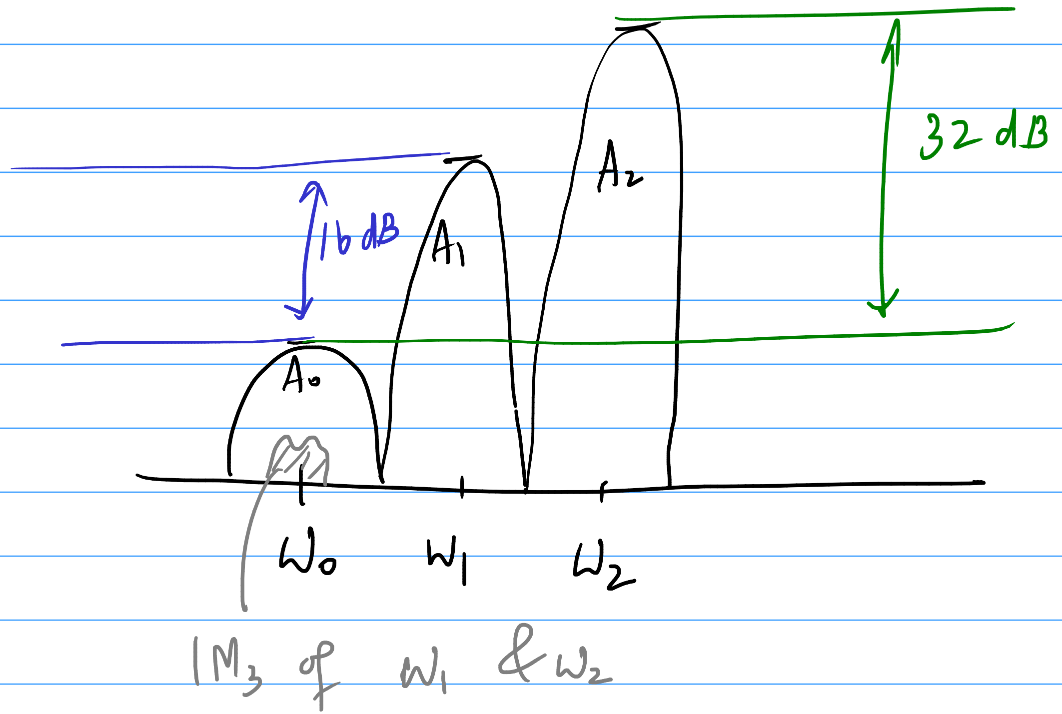
<u>Standard specifies Pin (max)</u> = −30 dBm

Peak input power = −21 dBm

Envelope variation due to pulse shaping $\sim$ 2 dB

$P_{1dB} = -19\ \text{dBm} \Rightarrow IIP_3 \approx -9\ \text{dBm}$

(ii) IIP_3 due to intermodulation



Specified by standard

$P_{in_{min}} = P_{sense} + 3\,dB$

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$IM_3 = \frac{3\alpha_3 A_1^2 A_2}{4}$$; Desired signal $= \alpha_1 A_0$

$P_{sense} = -82$ dBm (BPSK)

$SNR = 4\text{-}5$ dB

IM_3 should lie 15dB below desired signal

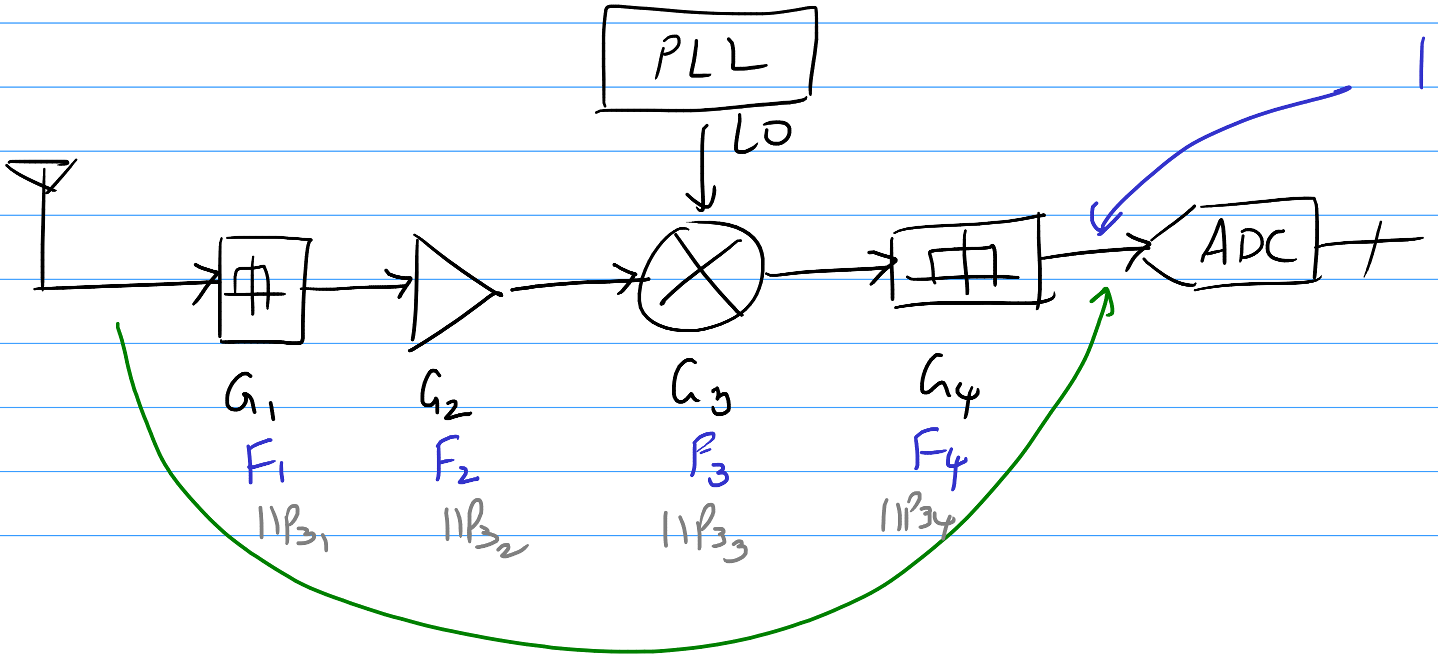
$$20\log\left(\frac{3\alpha_3 A_1^2 A_2}{4\alpha_1 A_0}\right) = -15\text{dB}$$

$$20\log\left(\frac{3\alpha_3}{4\alpha_1}\right) = -15\text{dB} - 40\log A_1 - 20\log A_2 + 20\log A_0$$

$A_0 \Leftrightarrow -79$ dBm, $A_1 \Leftrightarrow -63$ dBm, $A_2 \Leftrightarrow -47$ dBm

$$20\log\left(\frac{3\alpha_3}{4\alpha_1}\right) = +79\ dBm$$

$$IIP_3\Big|_{dBm} = 20\log\sqrt{\frac{4\alpha_1}{3\alpha_3}} = -39.5\ dBm$$



1 Vpp Diff.

||

0.25V S.E.

ampl.

$P_{in} = -82\,dBm$ 6 Mbps

$P_{in} = -65\,dBm$ 54 Mbps

$NF = 10\,dB$

$V_{out} = \underbrace{1\,V_{p-p}}_{+4\,dBm\ @50\Omega}$

		$P_{sens.}$	PAPR	Env. Var.
64QAM	54 Mbps	-65	9	2
BPSK	6 Mbps	-82	2	0

$\underline{P_{sens.} = -82\,dBm}$ Rx gain $= +84\,dB$

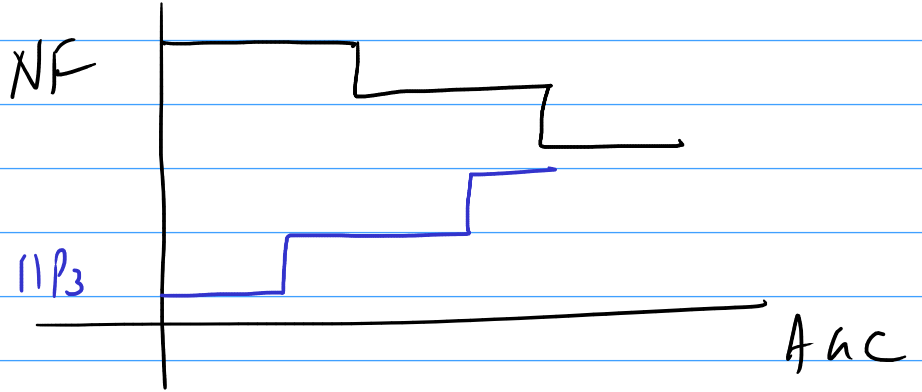
$P_{sens.} = -65\,dBm$ Rx gain $= +4 - (-65+9+2) = 58\,dB$

Max Gain of Rx

When $P_{in_{max}} = -30\,dBm$

Rx gain $= 32\,dB$ for BPSK

Rx gain $= 23\,dB$ for 64 QAM



Tx