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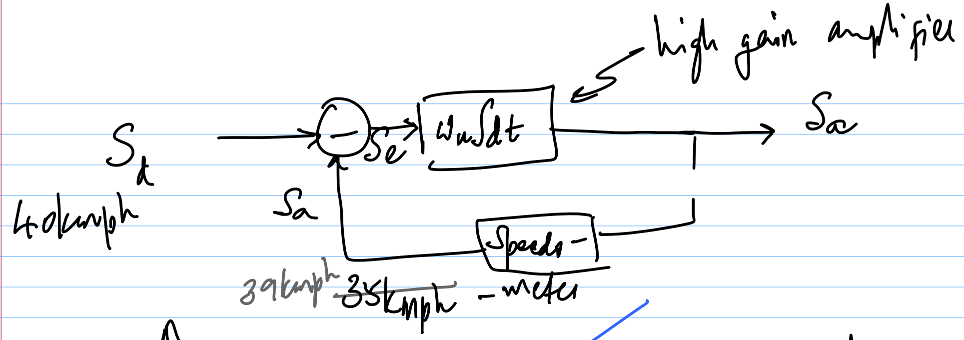
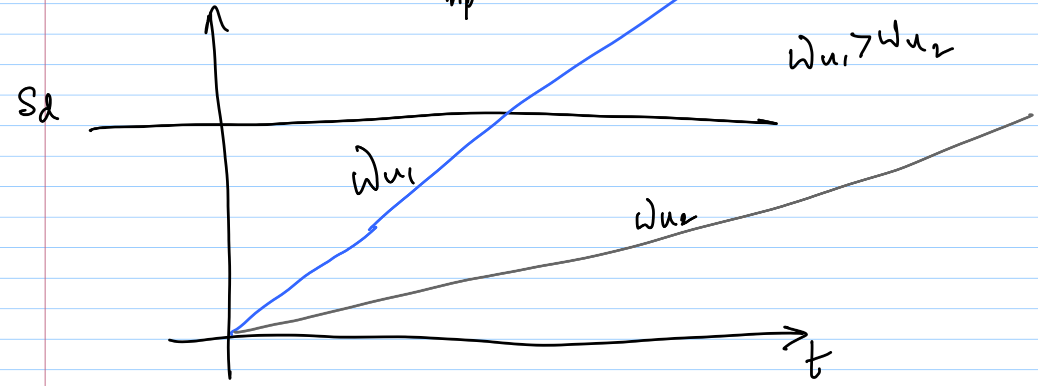
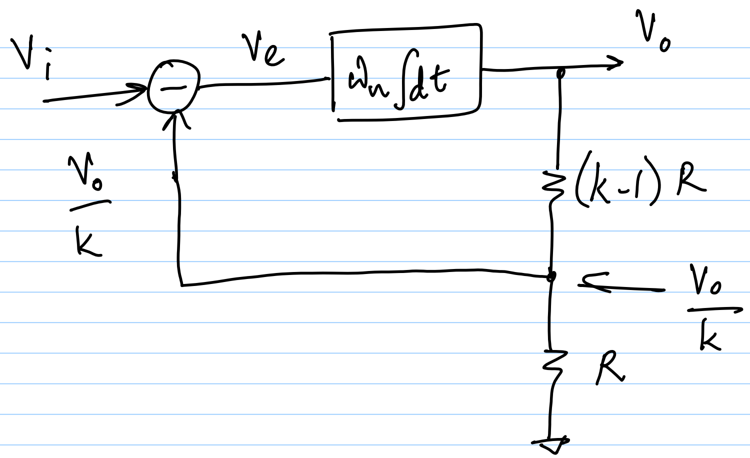
Lec 15

Feedback

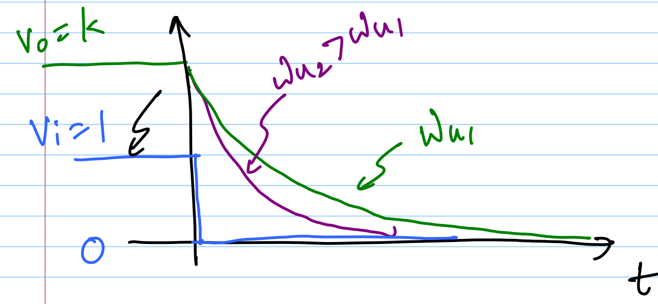
* Desired value
* Compare actual value w/ desired value
* drive actual towards desired







$$V_e = V_i - \frac{V_o}{k} \longrightarrow 0$$



1) $V_e = -\frac{V_o}{k}$ @ $t > 0$

$$V_o = \omega_u \int V_e \, dt \Rightarrow \frac{dV_o}{dt} = \frac{-\omega_u}{k} \cdot V_o$$

$$\frac{dV_o}{V_o} = \frac{-\omega_u}{k}\cdot dt$$

$$V_o = k e^{-\omega_u/k\, t}$$

2) $V_{i_{final}} = V_i$

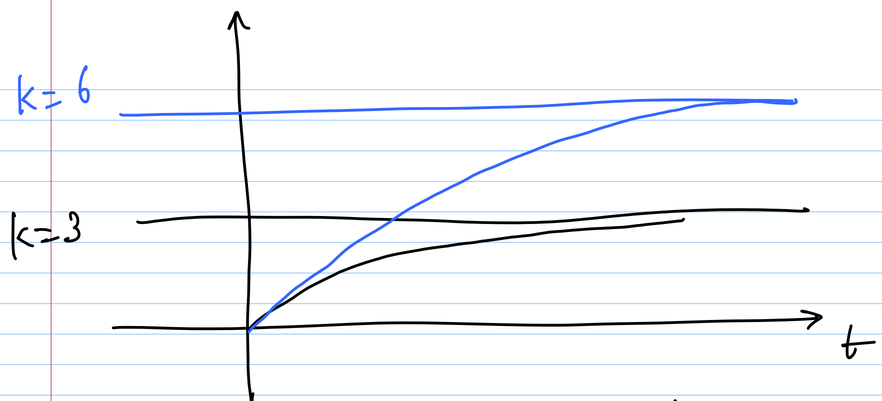
$$V_e = V_i - \frac{V_o}{k}$$

$$\frac{dV_o}{V_i - \frac{V_o}{k}} = \omega_u\, dt$$

$$V_o = kV_i + e^{-\omega_u/k\, t}\, k\left[\frac{V_o(0)}{k} - V_i\right]$$

$$= V_o(0)\, e^{-\frac{\omega_u}{k}\cdot t} + kV_i\left[1 - e^{-\frac{\omega_u}{k}\cdot t}\right]$$

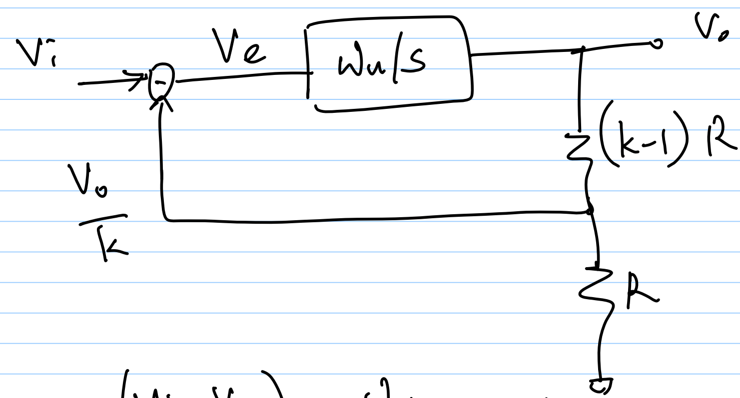
forced response



initial slope $= \omega_u \cdot V_i$

higher k ⇒ slower settling

3) Laplace solution



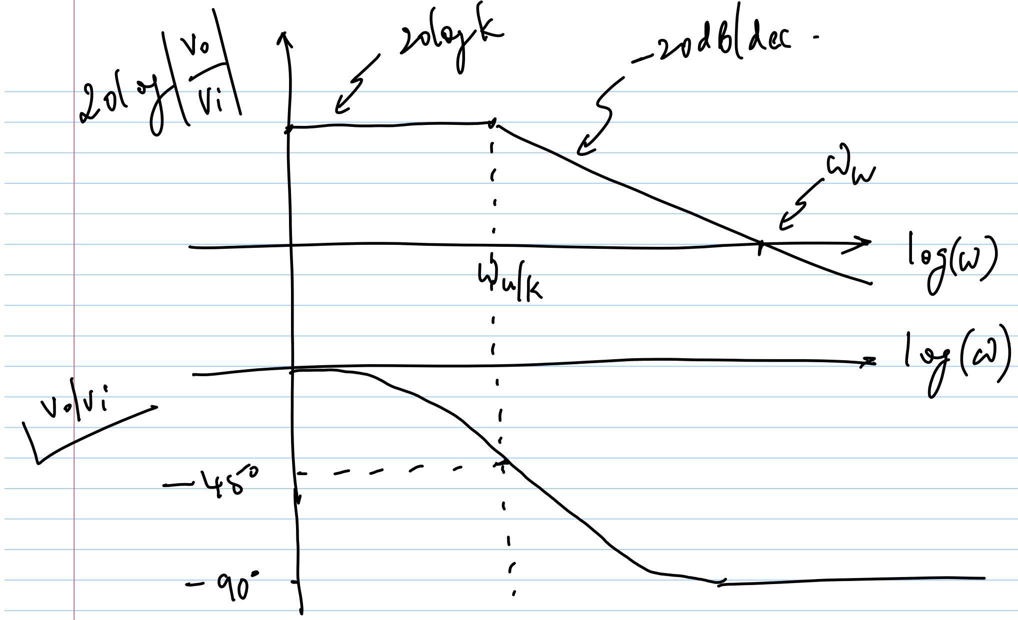
$$\left(V_i - \frac{V_o}{k}\right)\cdot\frac{\omega_u}{s} = V_o$$

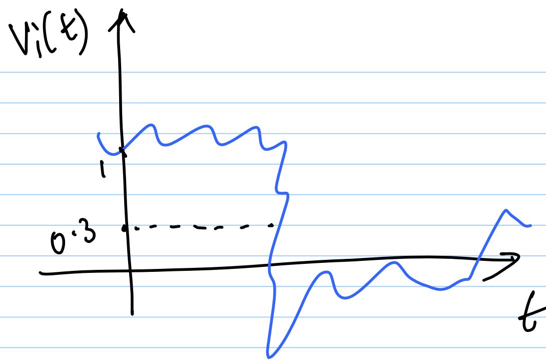
$$\frac{V_o}{V_i}(s) = \frac{\omega_u/s}{1+\frac{\omega_u}{ks}}$$

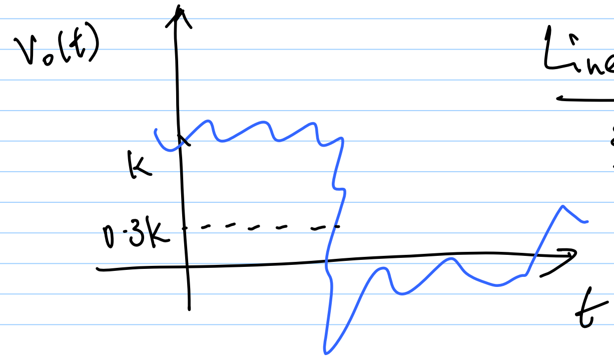
$$\frac{V_o}{V_i}(s) = k \cdot \frac{1}{1+\frac{s}{(\omega_u/k)}}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{k}{\sqrt{1+\frac{\omega^2 k^2}{\omega_u^2}}}$$

$$\angle V_o/V_i = -\tan^{-1}\left(\frac{k\omega}{\omega_u}\right)$$







Linear phase system

Linear phase $\Rightarrow$ $\omega << \omega_u/k$

$\frac{V_o}{V_i} = k \implies \omega << \omega_u/k$

$\frac{\omega_u}{k}$ = BW of amplifier

$\omega >> \frac{\omega_u}{k} \Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{\omega_u}{\omega}$

$\angle V_o/V_i = -90^\circ$

$$V_i \longrightarrow \boxed{\omega_u / s} \longrightarrow V_o$$

@ $\omega >> \frac{\omega_u}{k} \Rightarrow$ no feedback

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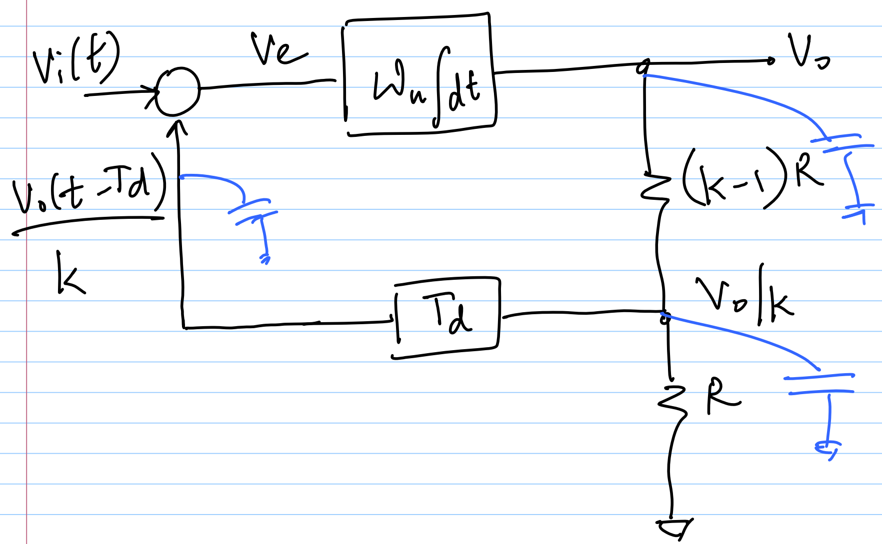
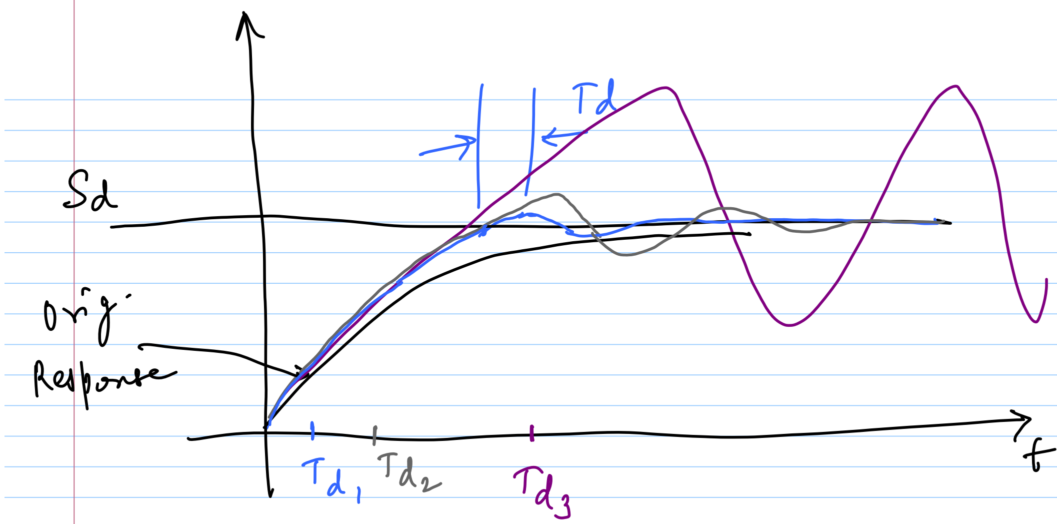
$$\boxed{\begin{aligned} \text{pole freq.} &= -\frac{\omega_u}{k} \\ BW &= \frac{\omega_u}{k} \\ \tau &= \frac{k}{\omega_u} \end{aligned}}$$



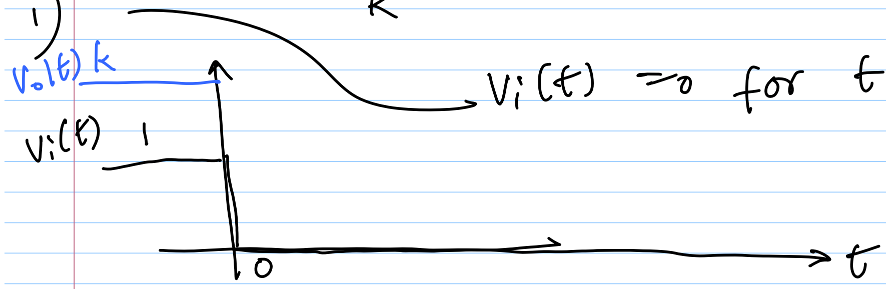


$$V_e(t) = V_i(t) - \frac{V_o(t-T_d)}{k}$$

$$V_o(t) = \omega_n \int V_e(t)\,dt$$

$$\frac{dV_o}{V_i(t) - \frac{V_o(t-T_d)}{k}} = \omega_n\,dt$$

1)



$$\frac{dV_o}{V_o(t-T_d)} = -\frac{\omega_n}{k}\,dt$$

$$\frac{dV_o}{dt} = -\frac{\omega_n}{k} V_o(t-T_d)$$

$$e^{t+\tau} = e^{\tau}\cdot e^{t}$$

1) exponential solution possible

2) sinusoidal solution possible

assume solution is of the form $e^{\sigma t}$

Diff. equation is :

$$\sigma e^{\sigma t} = -\frac{\omega_n}{k} e^{\sigma(t-T_d)}$$

$$\boxed{\sigma + \frac{\omega_n}{k} e^{-\sigma T_d} = 0}$$

$f_1(\sigma)$ $f_2(\sigma)$

$$\sigma' = \frac{k\sigma}{\omega_n}$$

$$\sigma' + e^{-T_d\cdot\left(\frac{\omega_n \sigma'}{k}\right)}$$

$$\tau = \frac{T_d\,\omega_n}{k}$$

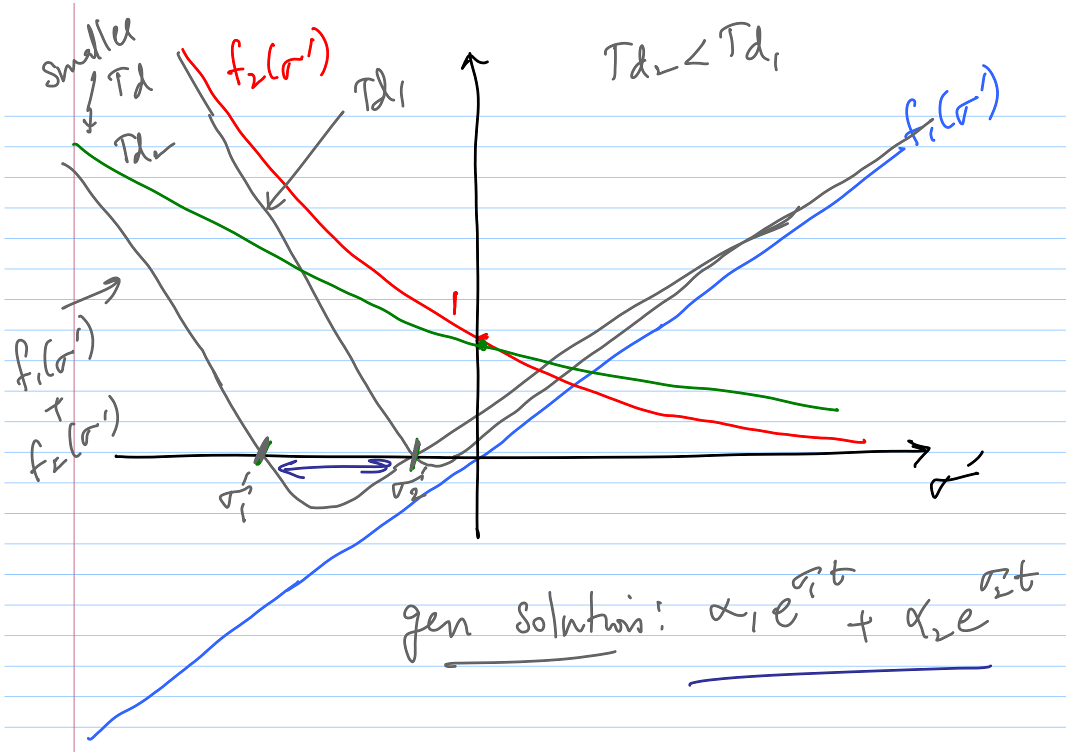
$$\boxed{\sigma' + e^{-\sigma'\tau} = 0}$$

$f_1(\sigma')$ $f_2(\sigma')$

σ_1' & σ_2'

$\Downarrow$

σ_1 & σ_2



gen solution: $\alpha_1 e^{\sigma_1 t} + \alpha_2 e^{\sigma_2 t}$

1) for large enough Td, ← Td_{c_1}

$f_1(\sigma') + f_2(\sigma')$ may not touch x-axis

* no real solutions
* $\sigma' = \sigma + j\omega$

2) Td even larger ← Td_{c_2}

$\sigma = 0$, $j\omega$ solutions

3) Critical $Td > 0$ for which settling is fastest

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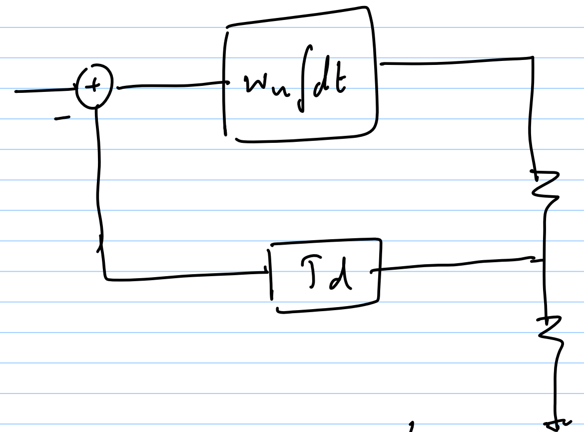
Lec 17



1) Root locus plot / Nyquist plot
2) Gain & phase margin

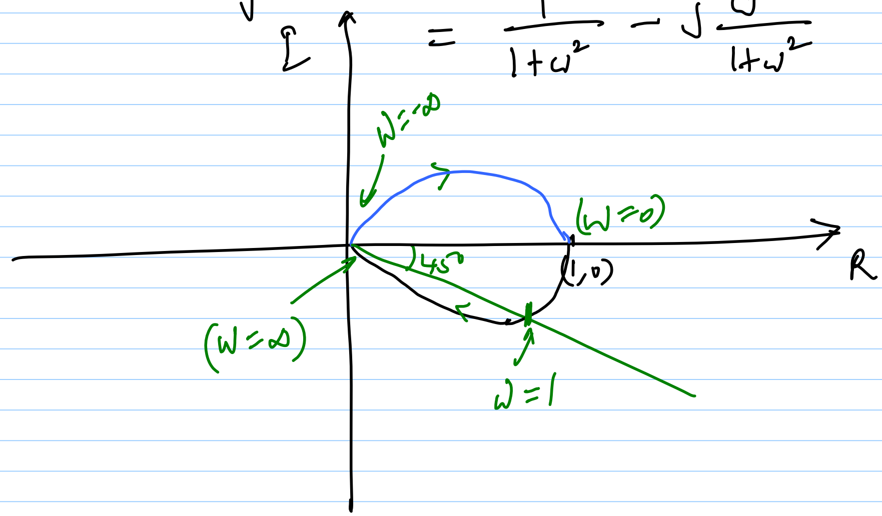
$\frac{1}{\text{gain}}$ @ phase $= -180°$ is Gain Margin

$(180° + \text{phase})$ @ gain $= 1$ is phase margin

→ good for rational polynomial systems

$$H(j\omega) = \frac{1}{1+j\omega} = R(\omega) + j\,I(\omega)$$

$$= \frac{1}{1+\omega^2} - j\frac{\omega}{1+\omega^2}$$

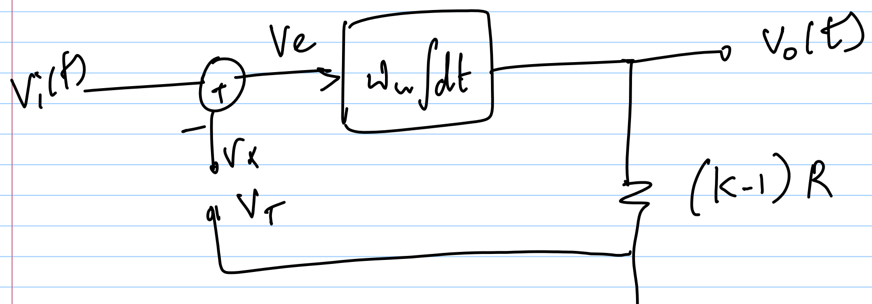


* # of encirclements of $(-1,0)$ ←

= # of RHP

$$CLTF = \frac{1}{f} \cdot \frac{L(s)}{1+L(s)}$$

opamp



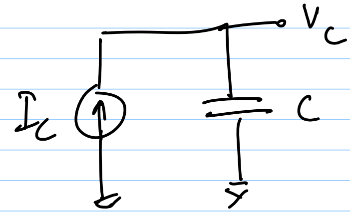
$$\left|\frac{V_T}{V_x}\right| = \frac{1}{k} \cdot \frac{\omega_u}{\omega}$$

ideal opamp ; $\omega_u = \infty$

* Integration:

use capacitors

$$I = C\frac{dV}{dt}$$

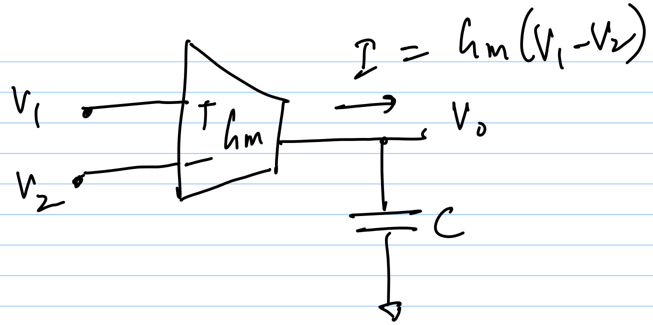


1) $$V_c = \frac{1}{C}\int I_c\,dt$$

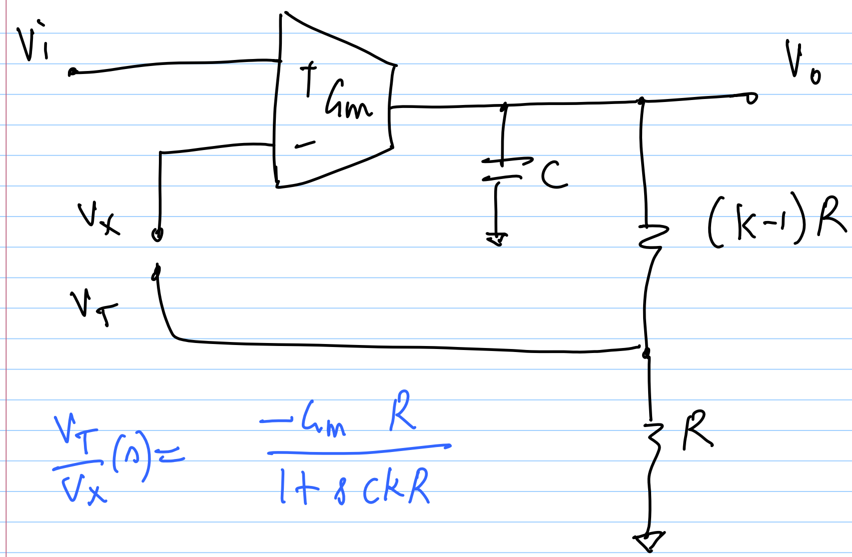
2) $$I_c \propto (V_1 - V_2) \leftarrow \text{VCCS}$$

or

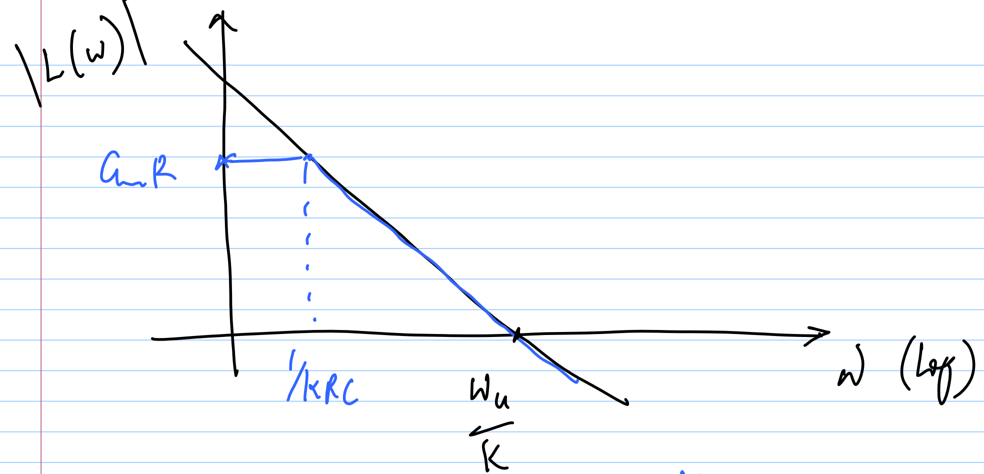
$$I_c = G_m(V_1 - V_2) \quad \text{transconductor}$$



$$V_o = \underbrace{\frac{G_m}{C}}_{=\,\omega_u}\int (V_1 - V_2)\,dt$$



$$\frac{V_T}{V_X}(s) = \frac{-G_m R}{1 + sCkR}$$



* DC gain = $G_m R$
* pole @ $\frac{1}{kRC}$
* $\frac{V_T}{V_X}(\omega) = 1$ when $\omega \approx \frac{G_m}{kC}$ if $G_m R \gg 1$