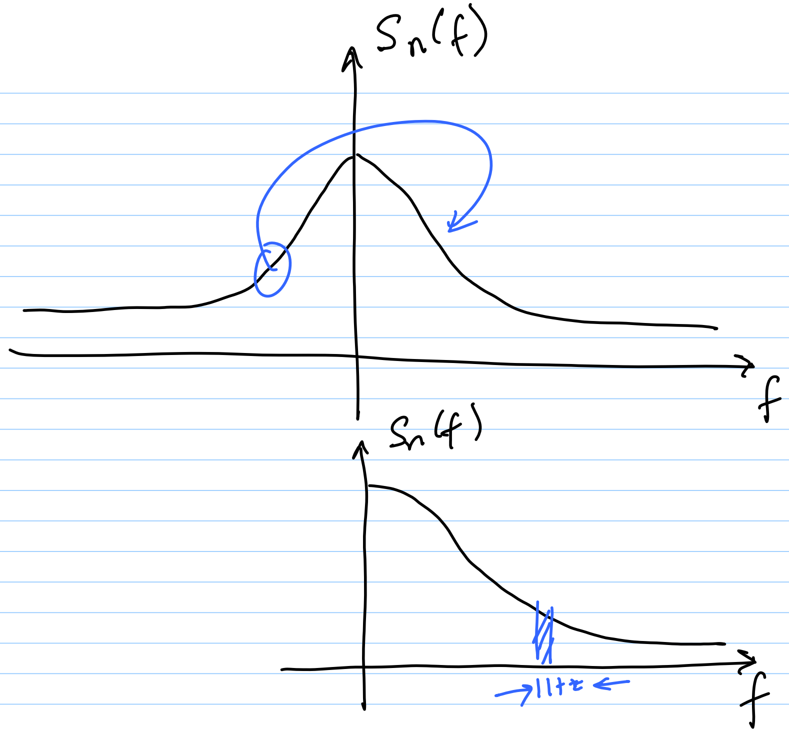
27/1/14

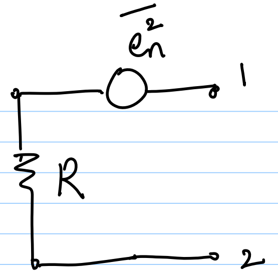
Lec 6

Noise

* Disturbances on top of desired signal
* Unwanted signal
* Random in nature ← Statistical representation
* Can have specific freq. characteristics

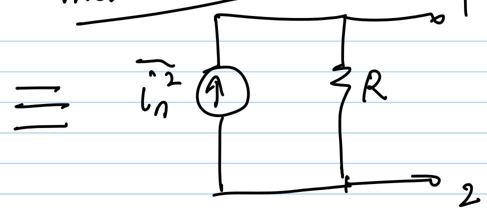


1)



Thermal Noise

≡



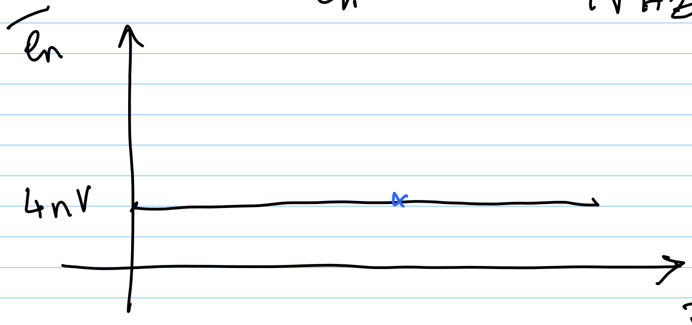
$$\overline{e_n^2} = R^2\,\overline{i_n^2}$$

$$\frac{\overline{e_n^2}}{\Delta f} = 4kTR \qquad \frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$$

1.38×10^{-23} J/K

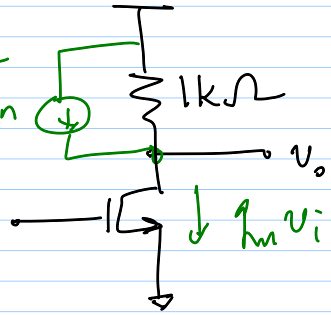
$R = 1k\Omega \Rightarrow \overline{e_n^2} = 1.656\times10^{-17}\ V^2/Hz$

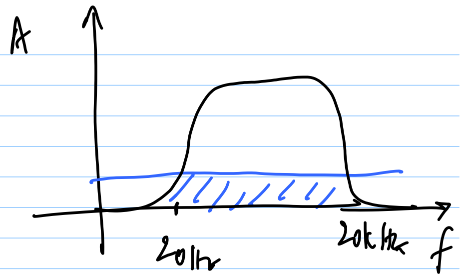
$\overline{e_n} \approx 4\ nV/\sqrt{Hz}$



$\overline{i_n} = 4pA/\sqrt{Hz}$

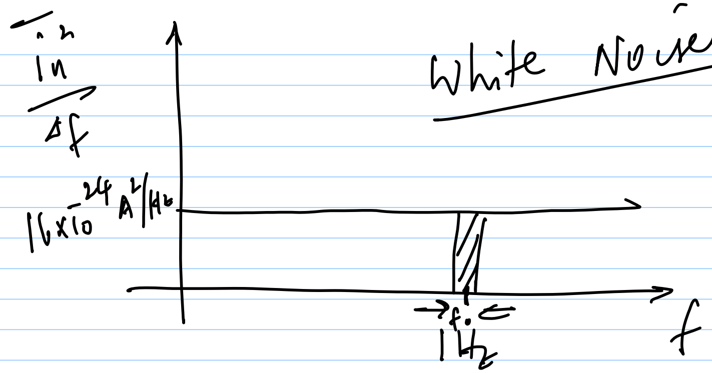
$\overline{i_n} = 4pA/\sqrt{Hz}$





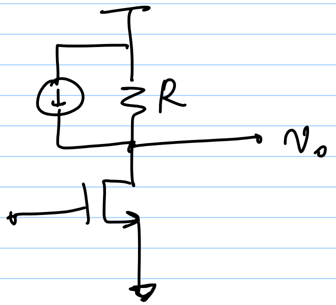
$$SNR_o = \frac{\cancel{|v_o|^2}}{\cancel{\overline{v_{no}^2}}} = \frac{\int_{f_1}^{f_2} S_s(f)\, df}{\int_{f_1}^{f_2} S_n(f)\, df}$$

White Noise

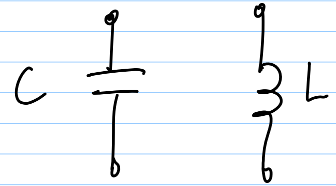


$$\overline{i_n}\Big|_{f=f_0} = 4pA$$

$$\overline{i_n^2} = \frac{4kT}{R}$$



2)



Noiseless

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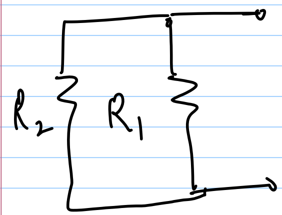
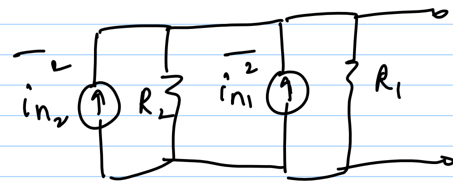
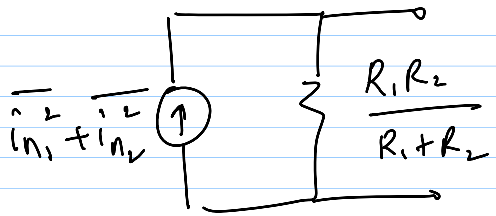
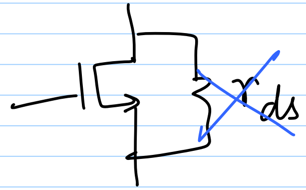
Lec 7

* Noise sources ← use superposition
* Independent noise sources add as powers
* Correlated noise sources add as v/i and then squared to get power





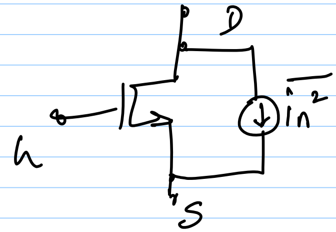




no noise

3) MOSFETs

a) OFF: no noise

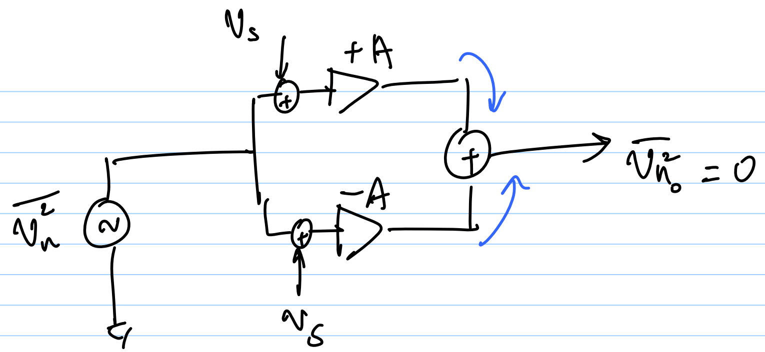


b) Triode

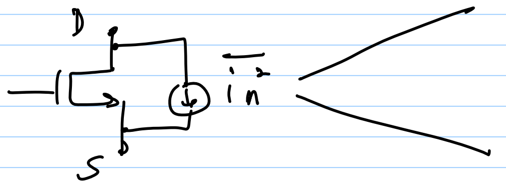
$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{r_{ds}} = 4kT\, g_{do}$$

$$g_{ds} = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)$$

$$= g_m$$



c) Saturation



drain thermal noise $\overline{i_{n_d}^2}$

flicker noise $\overline{i_{n_f}^2}$

Drain Thermal Noise

$$\frac{\overline{i_{n_d}^2}}{\Delta f} = 4kT \cdot \left(\frac{2}{3}\right) \cdot g_m = \frac{8kT}{3} g_m$$

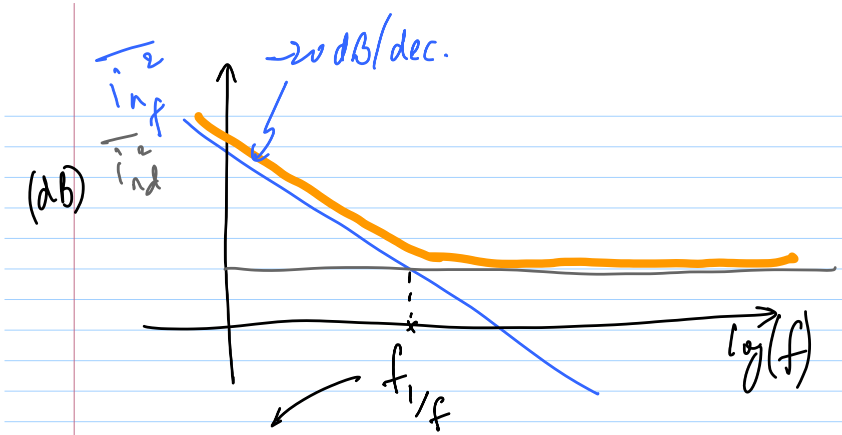
?

Flicker Noise

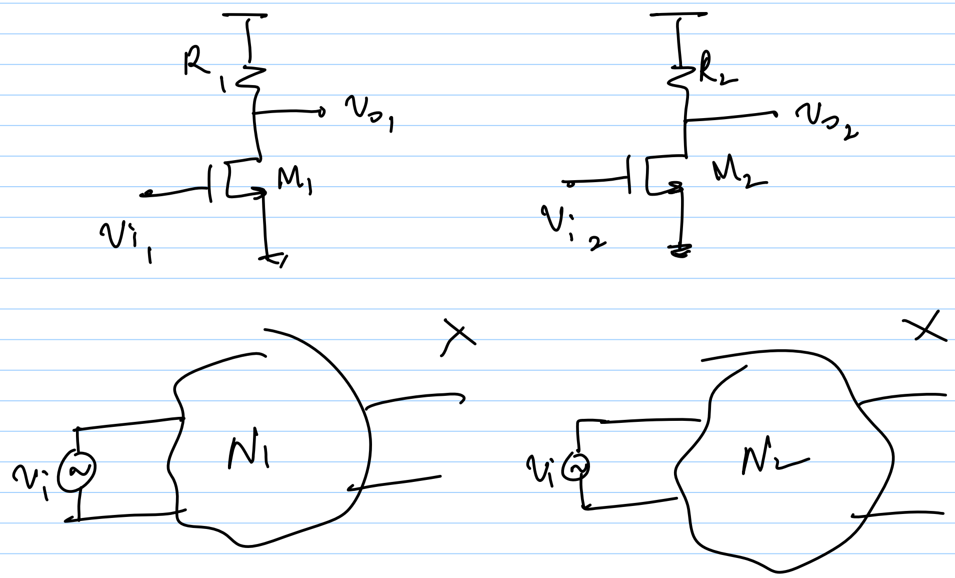
$$\frac{\overline{i_{n_f}^2}}{\Delta f} = \frac{K}{WL\,C_{ox}} \cdot \frac{g_m^2}{f}$$

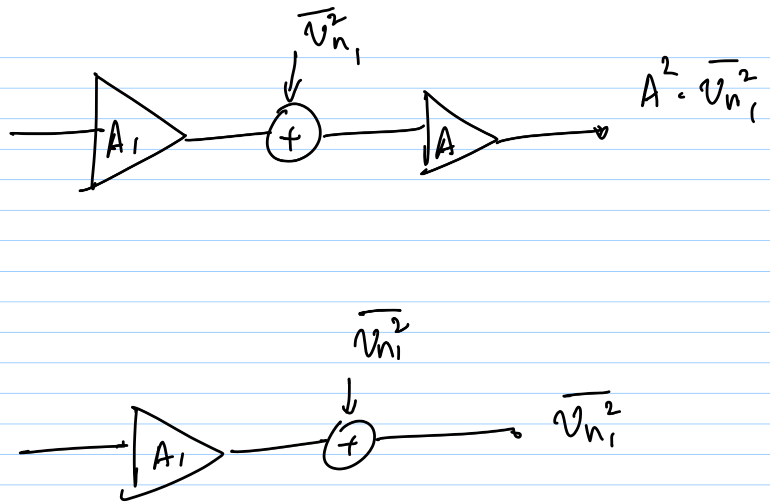
device parameter

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{K}{WL\,C_{ox}} \cdot \frac{1}{f}$$

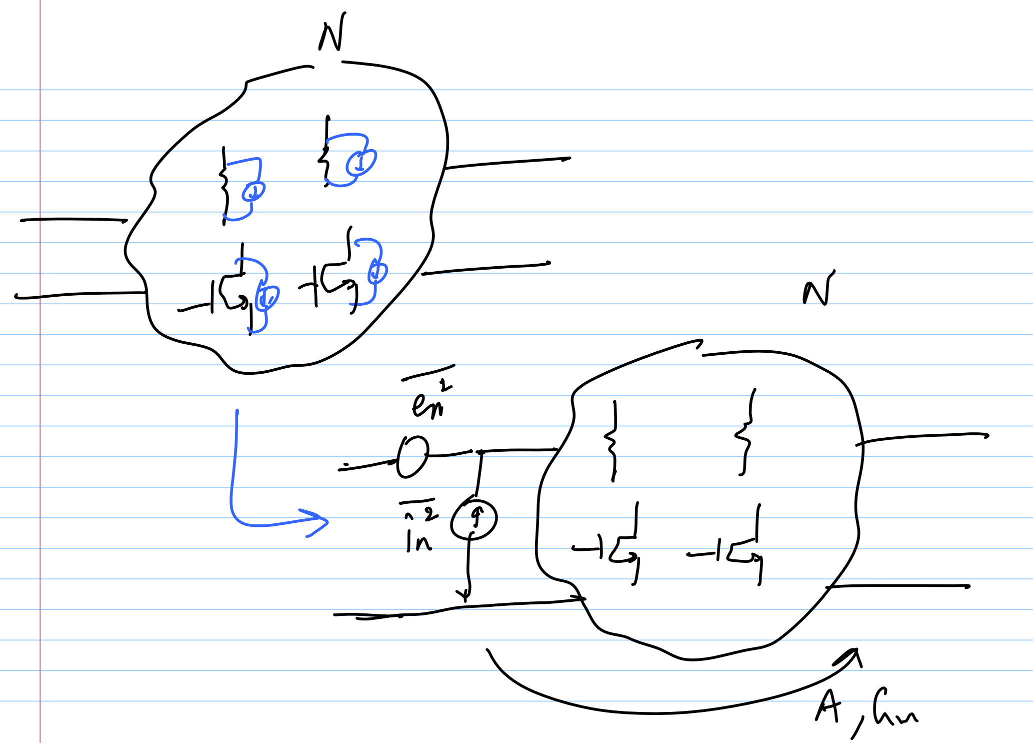


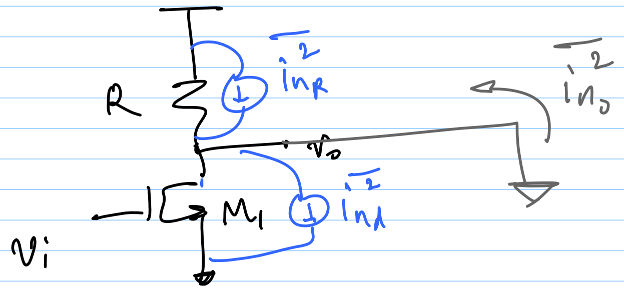
$1/f$ noise corner of device





* Input-referred noise





G_m

$$\frac{\overline{i_{no}^2}}{G_m^2} = \overline{v_{ni}^2}$$

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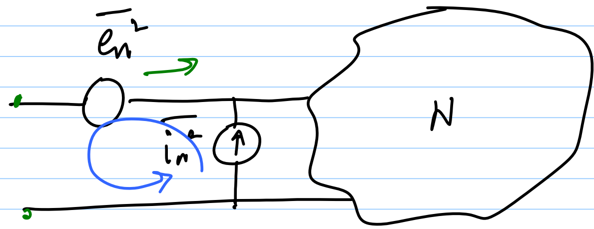
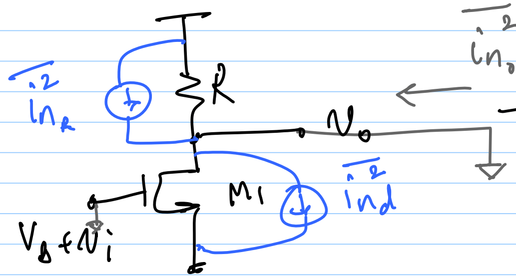
Lec 8

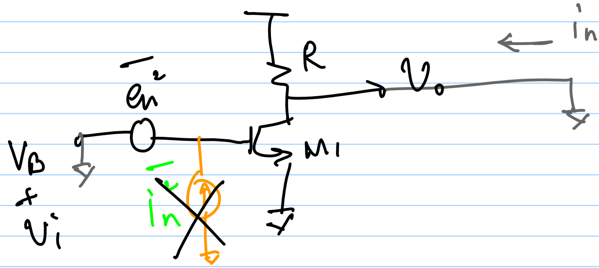


* In general, $\overline{e_n}$ & $\overline{i_n}$ are correlated

1)



$$\overline{i_{no}^2} = \overline{i_{nR}^2} + \overline{i_{nd}^2}$$

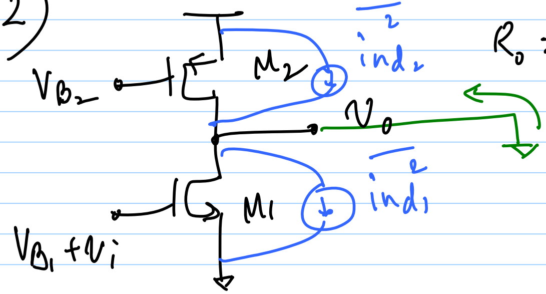


$$\overline{i_{no}^2} = g_m^2 \cdot \overline{e_n^2}$$

$$g_m^2\,\overline{e_n^2} = \frac{4kT}{R} + \frac{8kT\,g_m}{3}$$

$$\overline{e_n^2} = \frac{4kT}{g_m^2 R} + \frac{8kT}{3g_m}$$

2)



$R_o = r_{ds_1} \| r_{ds_2}$

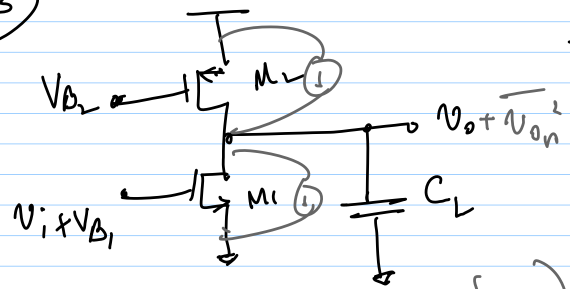
$$\overline{i_{no}^2} = \overline{i_{nd_1}^2} + \overline{i_{nd_2}^2}$$

$$= \frac{8kT}{3}(g_{m1} + g_{m2})$$

$$g_{m1}^2 \overline{e_n^2} = \frac{8kT}{3}\left(g_{m1} + g_{m2}\right)$$

$$\overline{e_n^2} = \frac{8kT}{3}\left[\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right]$$

3)



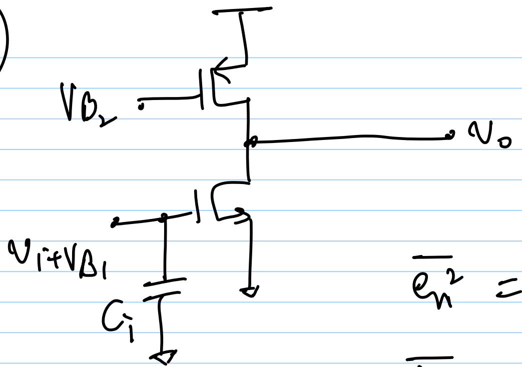
$\overline{e_n^2} =$

$$A\ (\text{gain}) = g_{m1} \times \left(R_o \,\|\, \frac{1}{j\omega C_L}\right)$$

$$\overline{v_{on}^2} = \left(\overline{i_{nd1}^2} + \overline{i_{nd2}^2}\right) \times \left(R_o \,\|\, \frac{1}{j\omega C_L}\right)^2$$

$$e_n^2 = \frac{\overline{v_{on}^2}}{A^2}$$

4)

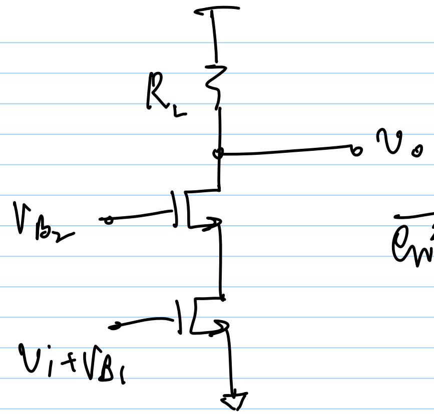


$\overline{e_n^2}$ = same as before

$$\overline{i_n^2} = \omega^2 C_i^2 \cdot \overline{e_n^2}$$

choose directions carefully

5)



$\overline{e_n^2} = ?$

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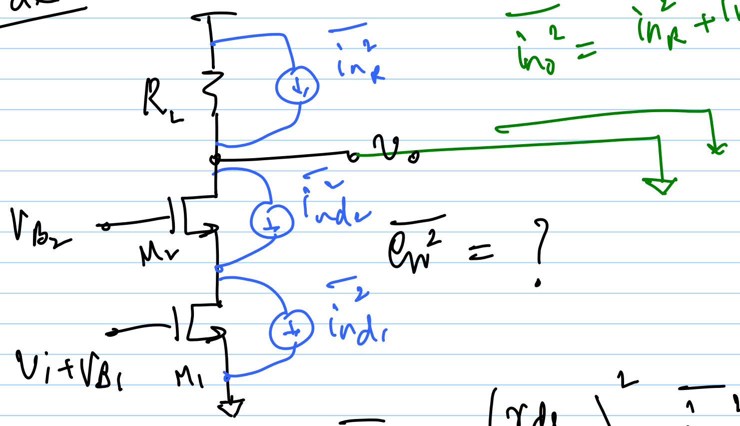
Lec 9

Cascode



$\overline{i_{no}^2} = \overline{i_{nR}^2} + \overline{i_{nd_2}^2}$

$\overline{e_n^2} = ?$

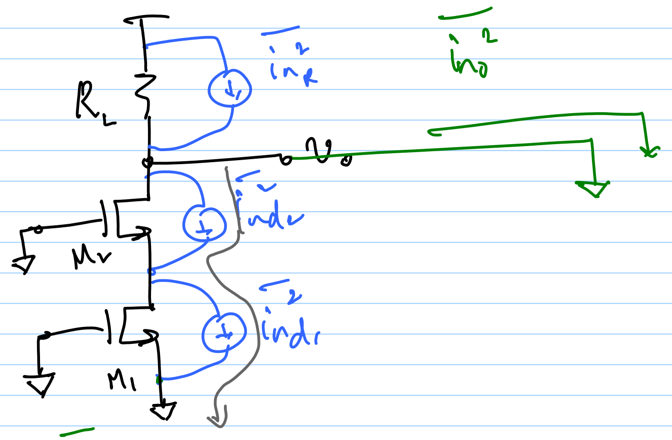
i) $\overline{i_{no}^2} = \overline{i_{nR}^2} + \dfrac{\overline{i_{nd_2}^2}\cdot\left(\dfrac{r_{ds_2}}{r_{ds_1}}\right)^2 + \overline{i_{nd_1}^2}}{(1 + r_{ds_2}/r_{ds_1})^2}$

ii) $\overline{e_n^2} = \dfrac{4kT}{g_{m_1}^2 R_D} + \dfrac{8kT}{3}\cdot\dfrac{(g_{m_1}+g_{m_2})}{g_{m_1}^2}$

$\overline{i_{no}^2} = \overline{i_{nR}^2} + \overline{i_{nd_1}^2} +$

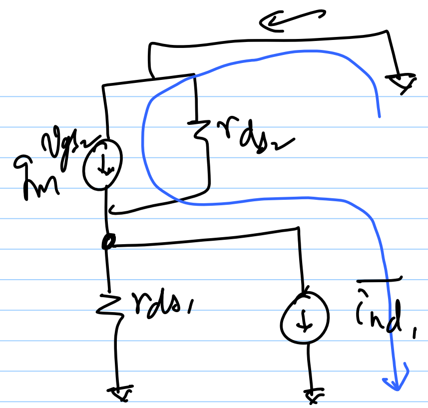
iii) $\overline{i_{no}^2} = \overline{i_{nR}^2} + \overline{i_{nd_1}^2}$

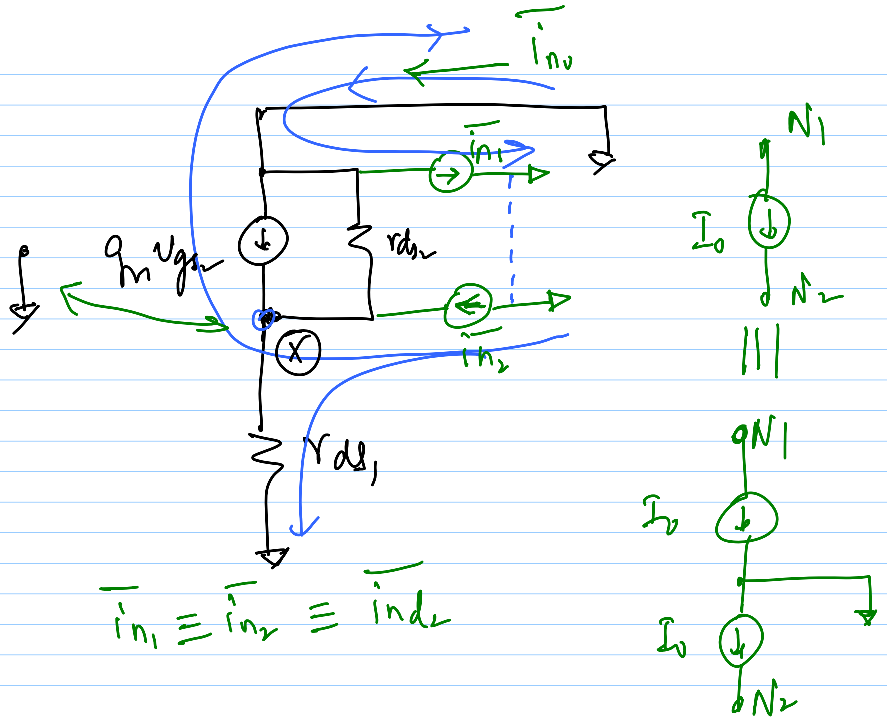
iv) $\overline{i_{no}^2} = \overline{i_{nR}^2} + \overline{i_{nd_1}^2}\cdot\left[\dfrac{r_{ds_1}}{r_{ds_1} + (1/g_{m_2} \,\|\, r_{ds_2})}\right]^2 + \overline{i_{nd_2}^2}\left[\dfrac{\cdot\left(\dfrac{1}{g_{m_2}} \,\|\, r_{ds_2}\right)}{r_{ds_1} + \ldots}\right]^2$



a) $\overline{i_{nR}^2}$

b) $\overline{i_{nd_1}^2}$





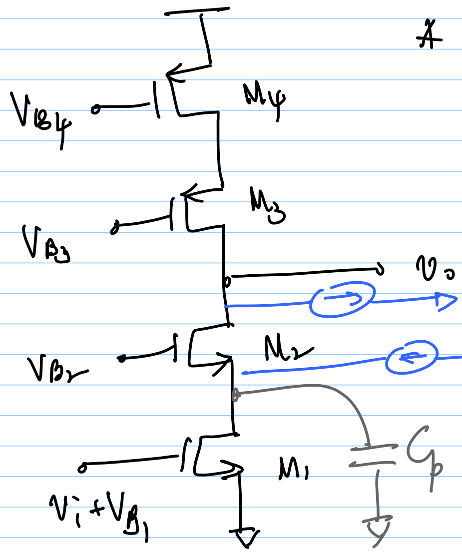
$$\overline{i_{n_1}} \equiv \overline{i_{n_2}} \equiv \overline{i_{nd_2}}$$

$$\overline{i_{n_{o,1}}} = \overline{i_{n_1}} = \overline{i_{nd_1}}$$

$$\overline{i_{n_{o_2}}} = -\overline{i_{nd_1}}$$

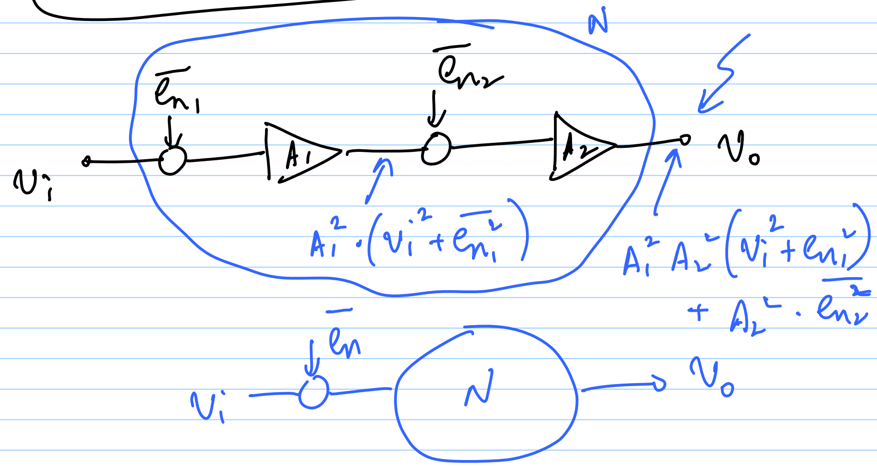
$$\overline{i_{n_{o_{M_2}}}} = 0 \Rightarrow \overline{i^2_{n_{o_{M_2}}}} = 0$$

$$\boxed{\overline{i_{n_o}^2} = \overline{i_{n_R}^2} + \overline{i_{nd_1}^2}}$$

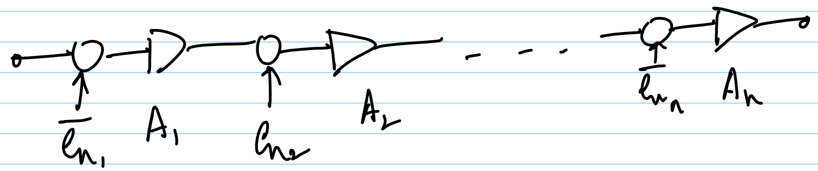


$*\ r_{ds_1} \gg \frac{1}{g_{m_1}}$

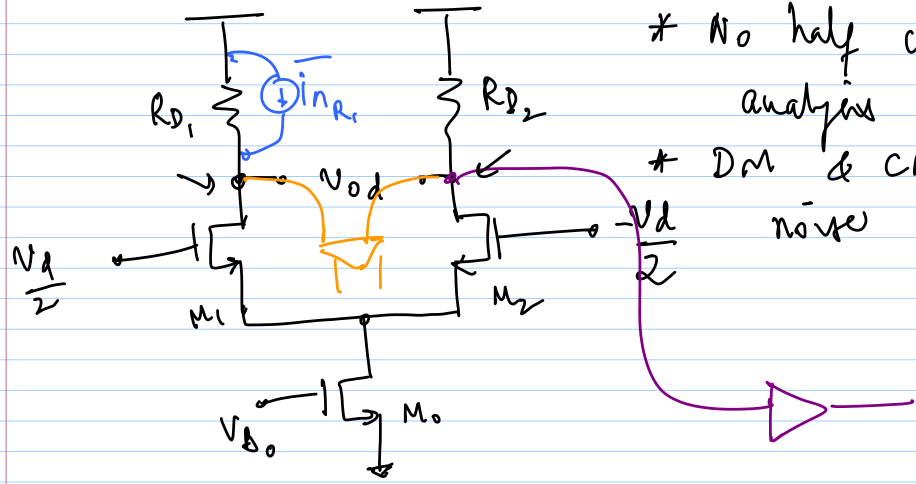
$$\boxed{\overline{e_n^2} = \frac{\overline{i_{n_o}^2}}{g_{m_1}^2}}$$



$$\overline{e_n^2} = \overline{e_{n1}^2} + \frac{\overline{e_{n2}^2}}{A_1^2}$$



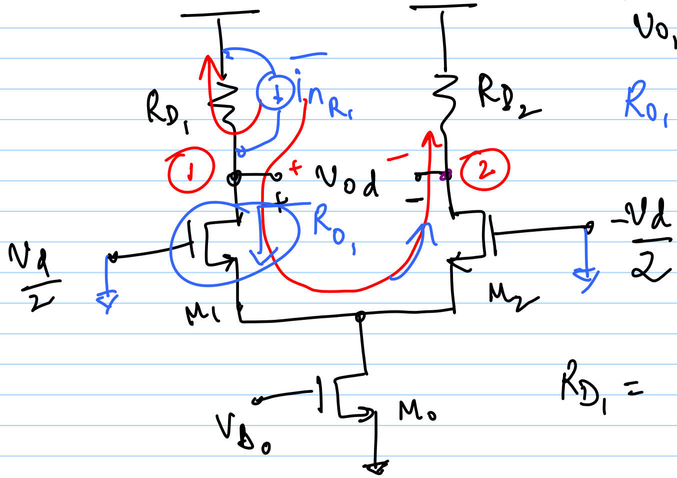
$$\overline{e_n^2} = \overline{e_{n1}^2} + \frac{\overline{e_{n2}^2}}{A_1^2} + \frac{\overline{e_{n3}^2}}{A_1^2 A_2^2} + \dots \frac{\overline{e_{nn}^2}}{(A_1 \cdots A_{n-1})^2}$$



* No half ckt analysis
* DM & CM noise

3/2/14

Lec 10



$\overline{v_{on}^2}$

$R_{o1} = 2r_{ds}$

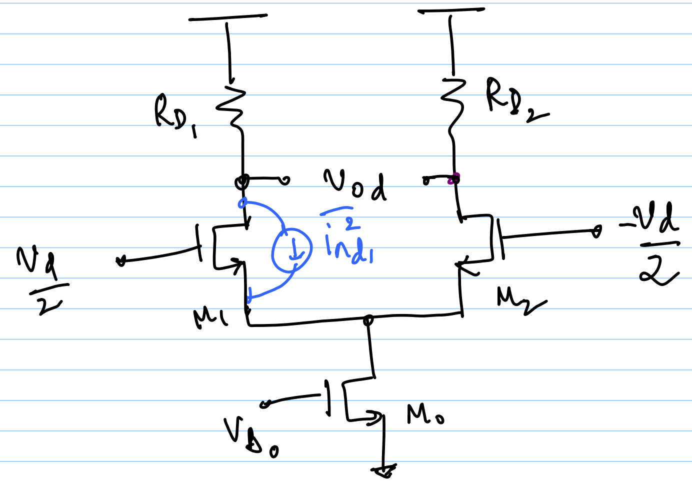
$R_{D1} = R_{D2} = R_D$

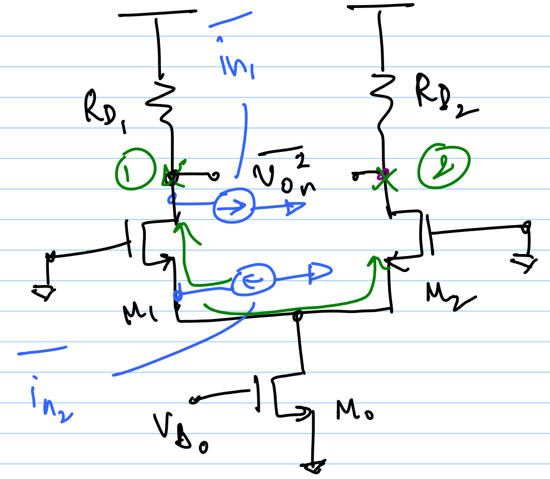
1) $\dfrac{\overline{v_{on}^2}_{R_{D1}}}{\Delta f} = 4kT R_{D1}$

2) $\dfrac{\overline{v_{on}^2}_{R_{D2}}}{\Delta f} = 4kT R_{D2}$

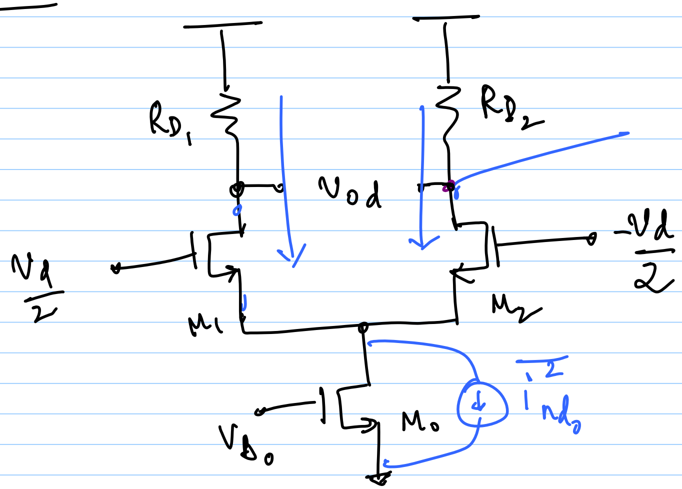
3) $\overline{v_{on}^2}_{M_{1,2}} = \overline{i_{n1}^2} \cdot R_{D1}^2 + \cancel{\left[\overline{i_{n2}}\left(R_{D1} - R_{D2}\right)\right]^2}$

$$= \frac{8kT}{3} g_{m1,2} \, R_{D1}^2$$





4) M_0 :



$$\overline{e_n^2} = \frac{\overline{v_{on}^2}}{(g_m R_D)^2}$$

$$= \frac{8kT R_D}{(g_m R_D)^2} + \frac{\frac{16kT}{3} \cdot g_m R_D^2}{(g_m R_D)^2}$$

$$= \frac{8kT}{g_m^2 R_D} + \frac{16kT}{3 g_m}$$

Mismatch

* IC vs discrete

identical devices available

"Matched" devices

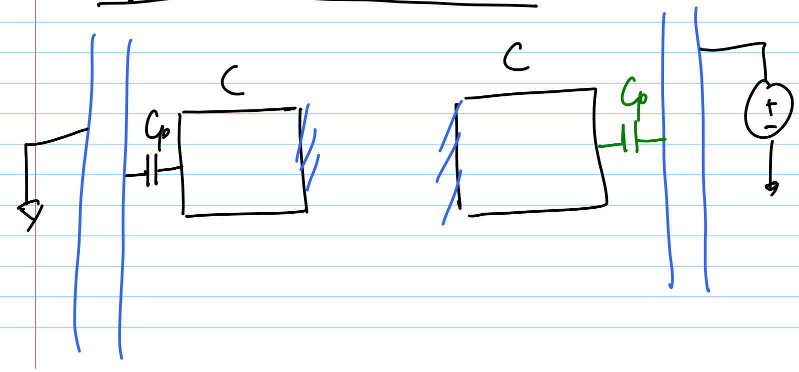
4/2/14

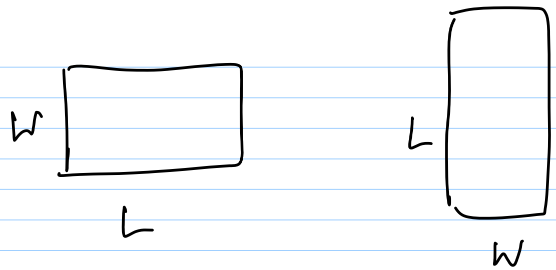
Lec 11

Mismatch

- → systematic mismatch
- → random mismatch

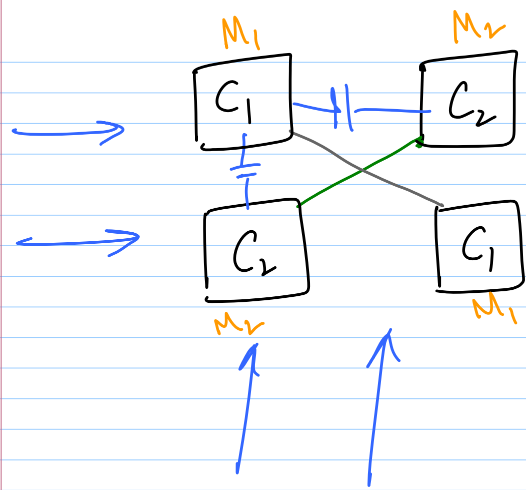
Systematic Mismatch



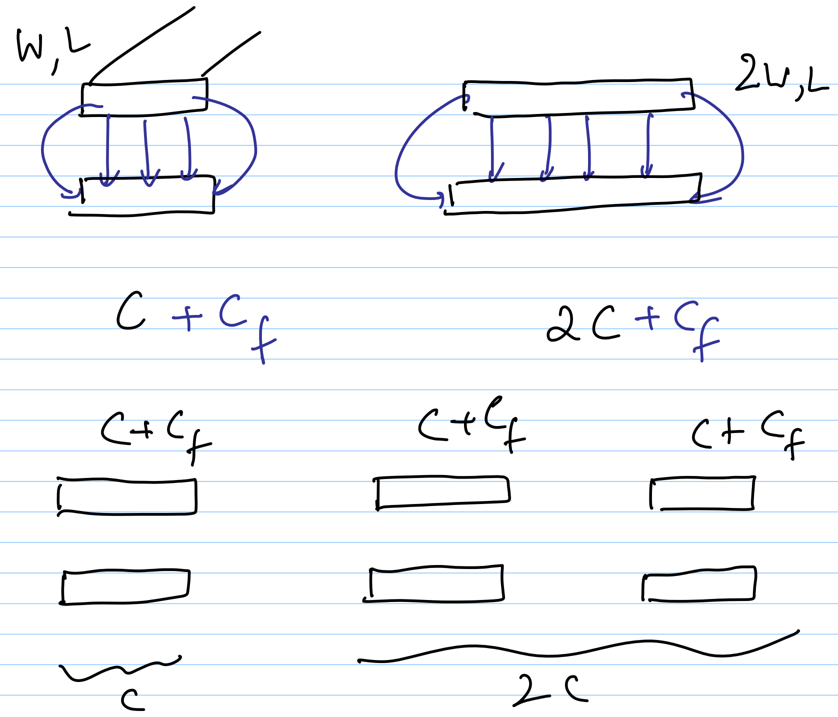


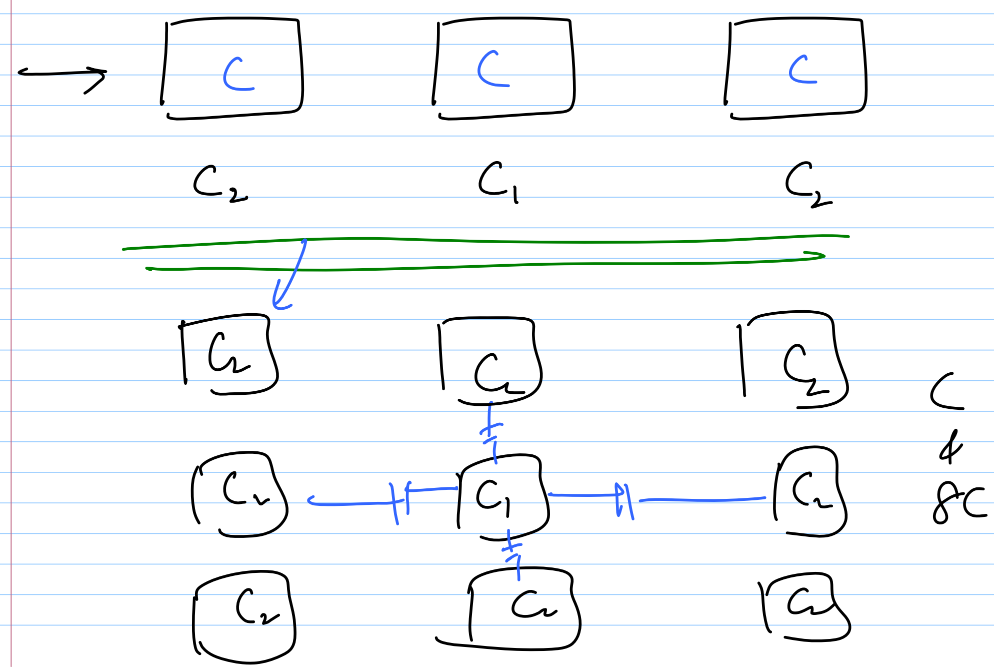
1) Metal line
2) Process directionality < temp / diffusion

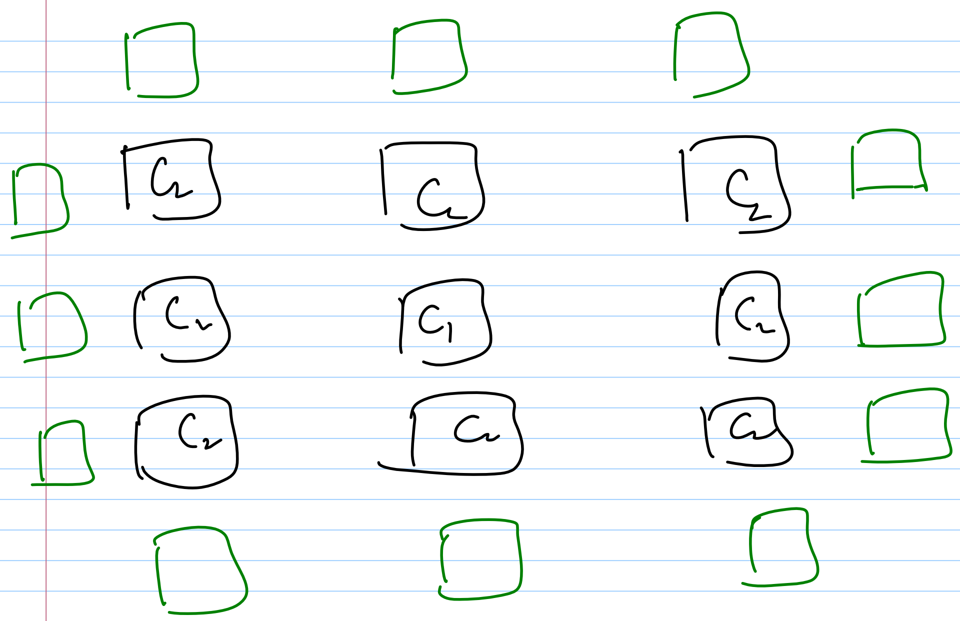
* Same Orientation

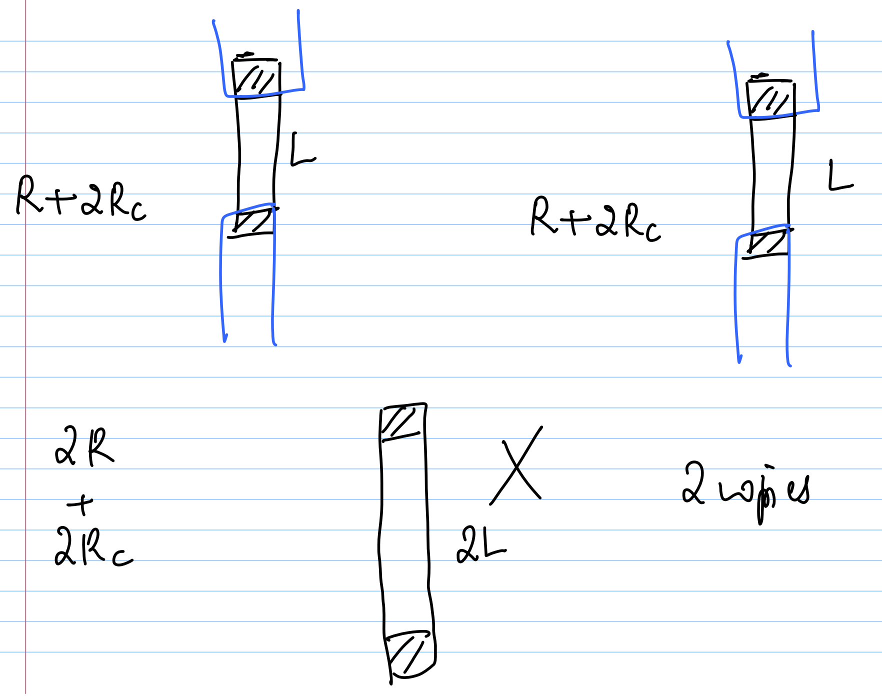


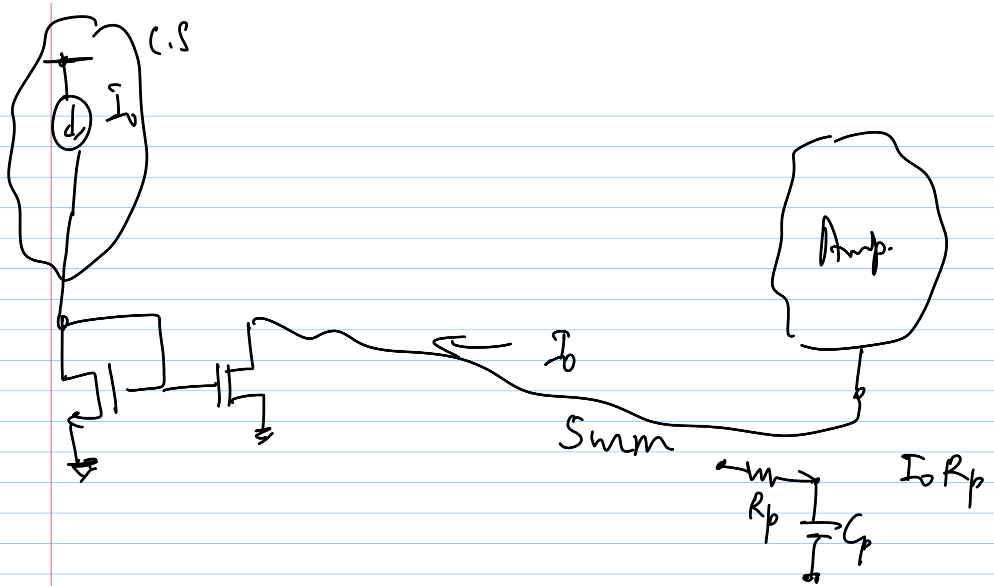
Common Centroid

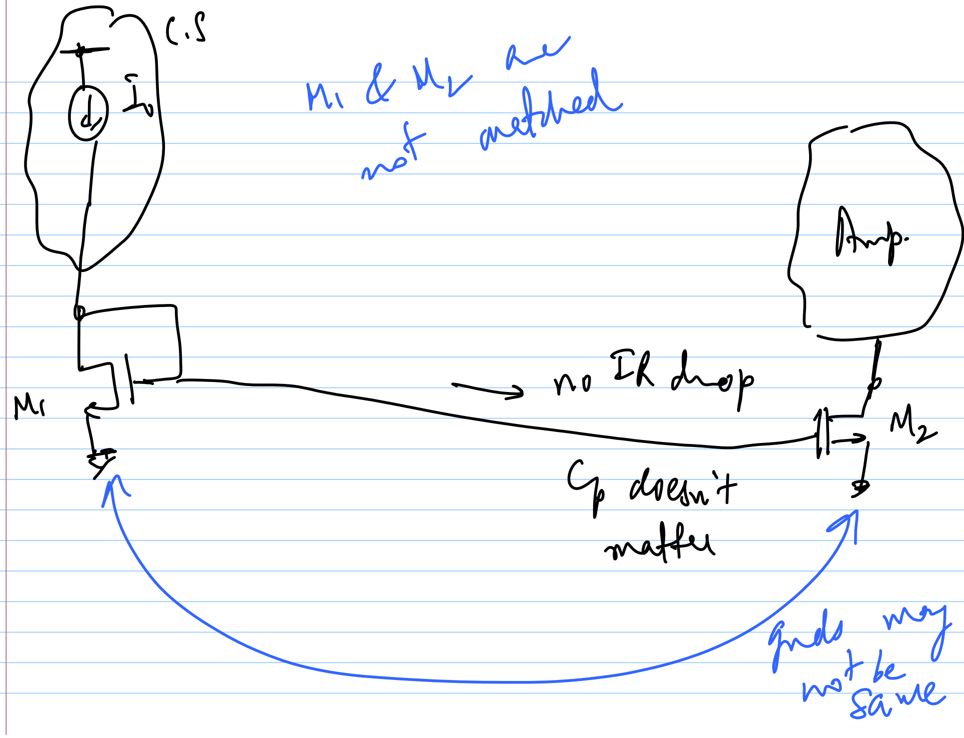

W,L
2W,L
C + C_f
2C + C_f
C+C_f
C+C_f
C+C_f
C
2C

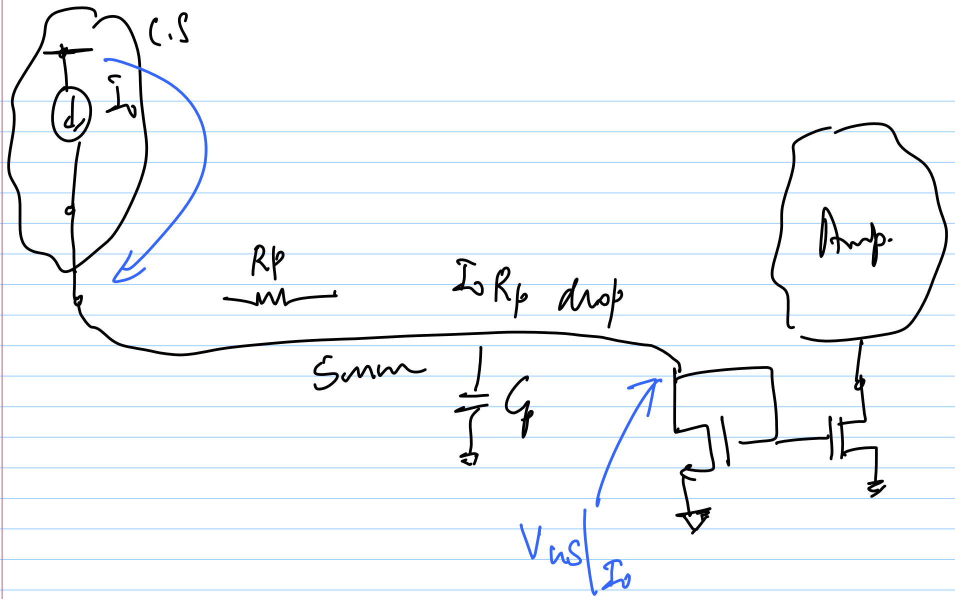

C
C
C
C_2
C_1
C_2
C_2
C_2
C_2
C
4
8C
C_2
C_1
C_2
C_2
C_2
C_2


C_2
C_2
C_2
C_2
C_1
C_2
C_2
C_2
C_2


R+2R_c
L
R+2R_c
L
2R
+
2R_c
2L
X
2 copies

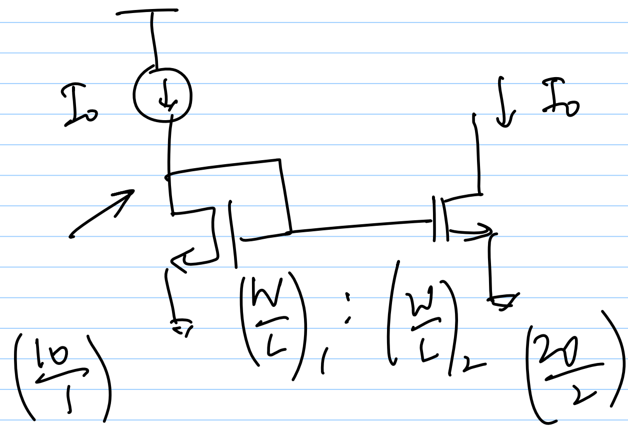

C.S
d_1
I_0
I_0
Amp.
Smm
R_p
C_p
$I_0 R_p$

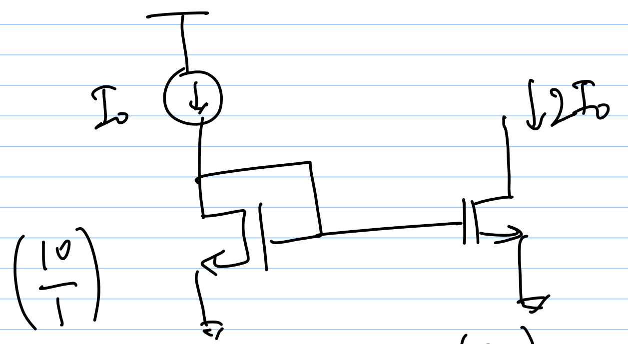

C.S
d_1
I_0
M1 & M2 are not matched
Amp.
no IR drop
M_1
M_2
C_p doesn't matter
gnds may not be same


C.S
d_1
I_0
Amp.
Rp
$I_0 R_p$ drop
Smm
C_p
$V_{GS}|_{I_0}$

5/2/14

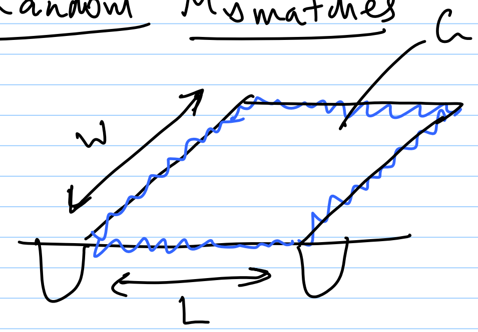
Lec 12


I_0
I_0
$\left(\frac{10}{1}\right)$
$\left(\frac{W}{L}\right)_1 : \left(\frac{W}{L}\right)_2$
$\left(\frac{20}{2}\right)$



$\left(\frac{20}{1}\right)$, $2\times\left(\frac{10}{1}\right)$

Random Mismatches



"averaging effect"

$f(W, L)$

$\frac{\Delta W}{W}$, $\frac{\Delta L}{L}$,

"large area"

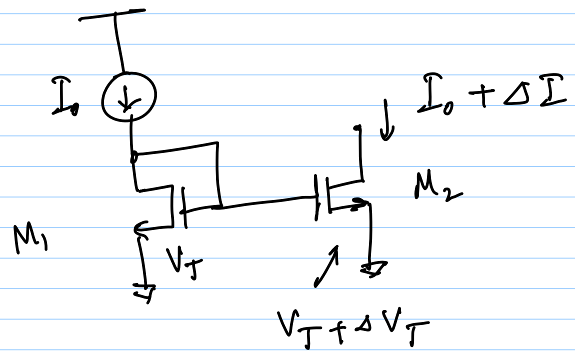
$$\sigma_{V_T} = \frac{A_{V_T}}{\sqrt{W \cdot L}} \leftarrow \text{mV}\cdot\mu\text{m}$$

$\beta = \mu C_{ox}$

$$\sigma_{\beta} = \frac{A_{\beta}}{\sqrt{W \cdot L}}$$

$$\sigma_{\frac{\Delta R}{R}} = \frac{A_R}{\sqrt{WL}}$$

$$\sigma_{\frac{\Delta C}{C}} = \frac{A_C}{\sqrt{WL}}$$



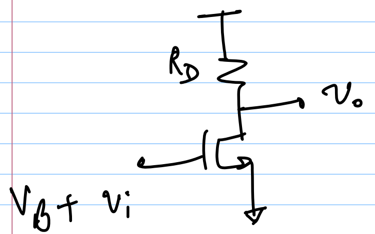
$$\Delta I = -g_m \Delta V_T$$

$$\sigma_{\Delta I}^2 = g_m^2 \, \sigma_{V_T}^2$$

$$\frac{\sigma^2_{\Delta I}}{I} = \frac{g_m^2}{I_0^2} \cdot \frac{A_{V_T}^2}{WL}$$

$$= \frac{4}{V_{DSAT}^2} \cdot \frac{A_{V_T}^2}{WL}$$

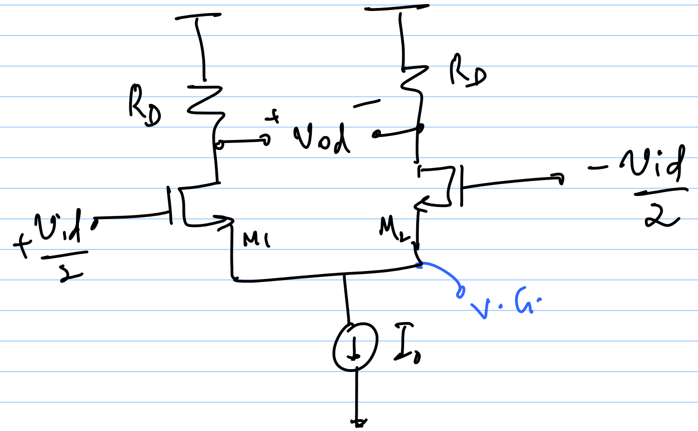
Effects of mismatch



gain = $g_m R_D$

$\hookrightarrow \propto \sqrt{I_D}$

many cases: Open loop gain of amp need not be precise



$$A_1 = g_{m_1} R_D$$

$$A_2 = g_{m_2} R_D$$

$g_{m_2} > g_{m_1}$

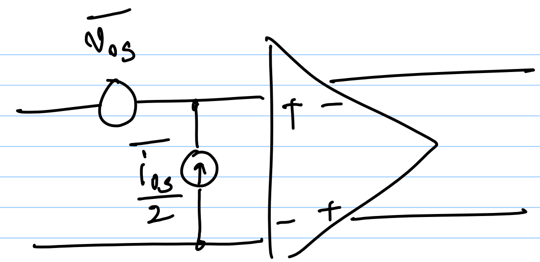
$I_2 > I_1$

Output referred DC offset

$$v_{os,out_1} = \frac{I_1}{2} \Delta R_D$$

$$v_{os,out_2} = \frac{I_2}{2} \cdot \Delta R_D$$

2 > 1



for MOS ckts driven @ gate : $\overline{i_{os}} = 0$

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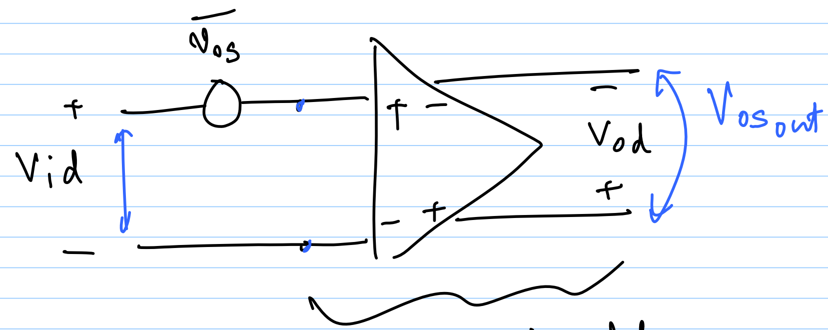
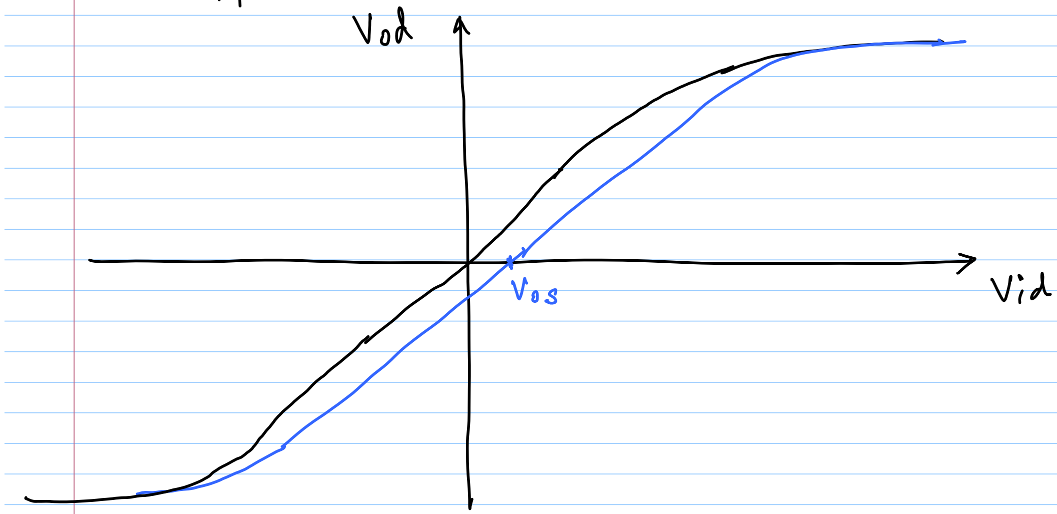
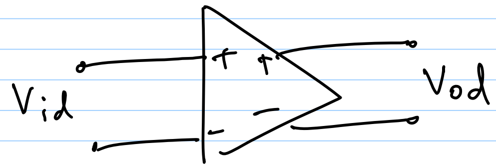
Lec 13



no mismatches

* Affects I/O characteristic





$$V_{od} = A_0 + A_1 V_{id} + A_2 V_{id}^2 + A_3 V_{id}^3 + \dots$$

Ideal diff amp : $A_0, A_2, A_4 \dots = 0$

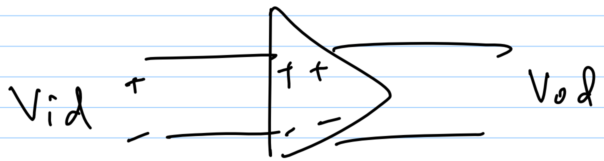
DC offset

* DC offset : DC voltage to be applied @ input so that $V_{odc} = 0$

$$\boxed{V_o = A v_i}$$

$V_o = A_0 + A_1 v_i$ X

$V_o = A_1 v_i + A_2 v_i^2$ X

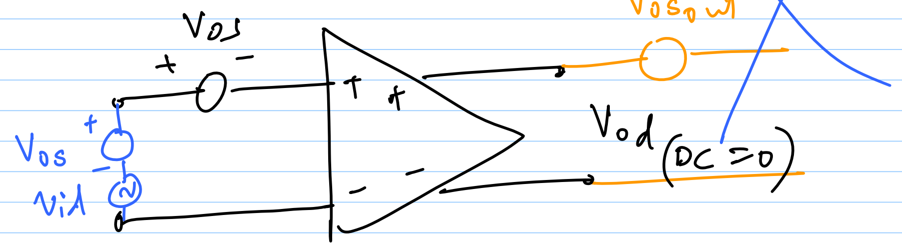


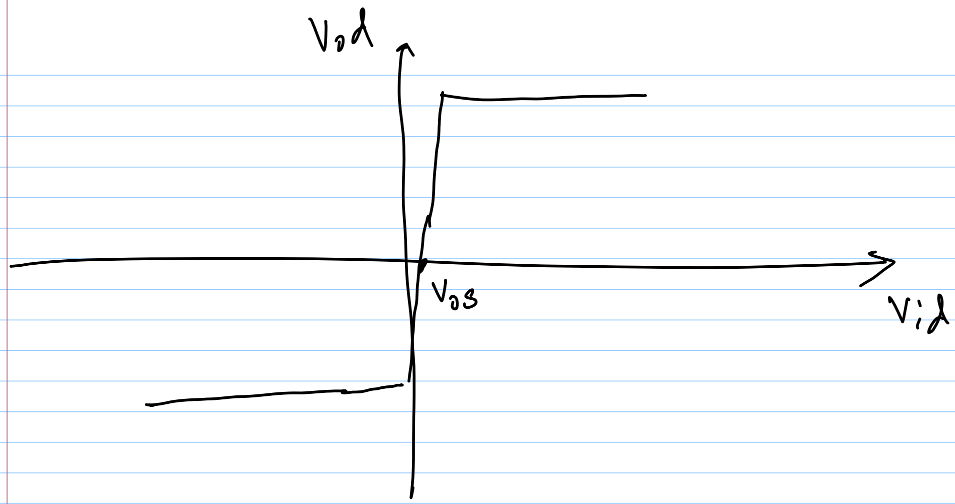
$$V_{od} = A_1 V_{id} + A_2 V_{id}^2$$

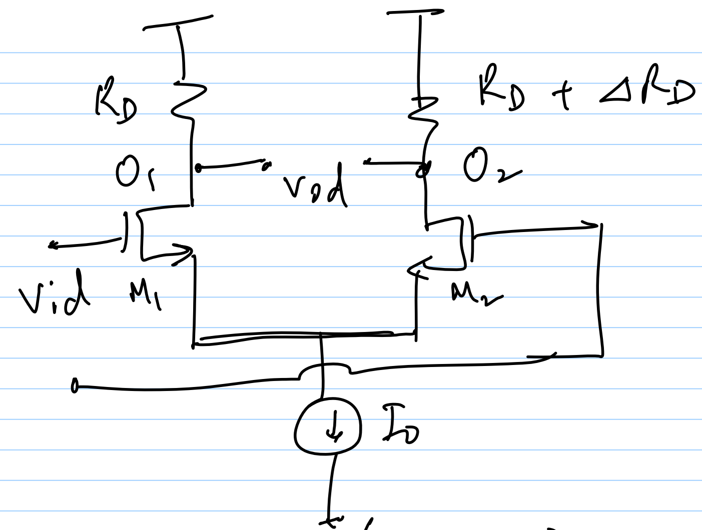
DC 2W

V_{os}

* Even order distortion
* Desensitization





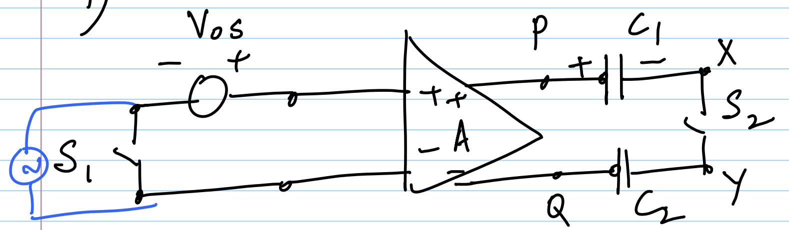


$$I_1 R_D = I_2 (R_D + \Delta R_D)$$

10-2-14

Lec 14

1)

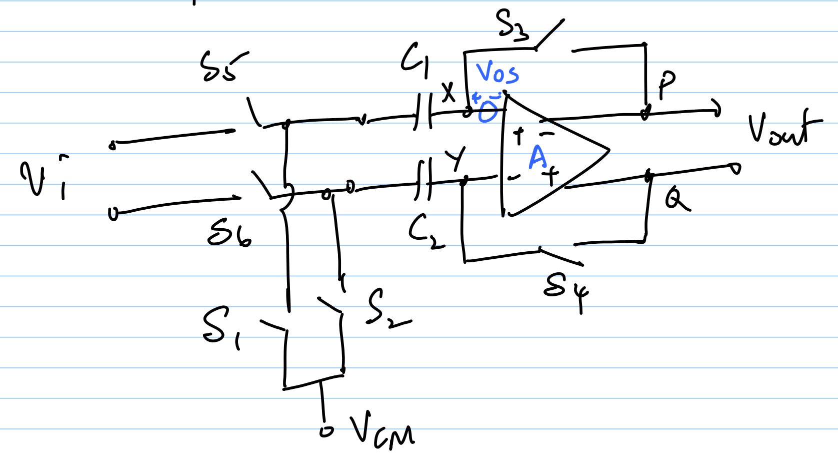


S_1/S_2 closed $V_{PQ} = A V_{os}$

S_1/S_2 open $V_{PQ} = A v_i + A V_{os}$

$V_{xy} = A v_i$

2) Input-referred Offset storage - periodic



(i) $S_1 - S_4$ closed - unity gain config.

$V_{PQ} = V_{XY}$

$$V_{out} = -A\,(V_{out} - V_{os})$$

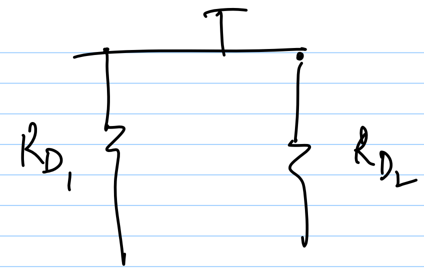
$$V_{out} = \frac{A}{A+1} \cdot V_{os} \approx V_{os}$$

V_{xy}

$$\boxed{V_{xy} = V_{os}}$$

(ii) Open $S_1 - S_4$ & close $S_5 - S_6$

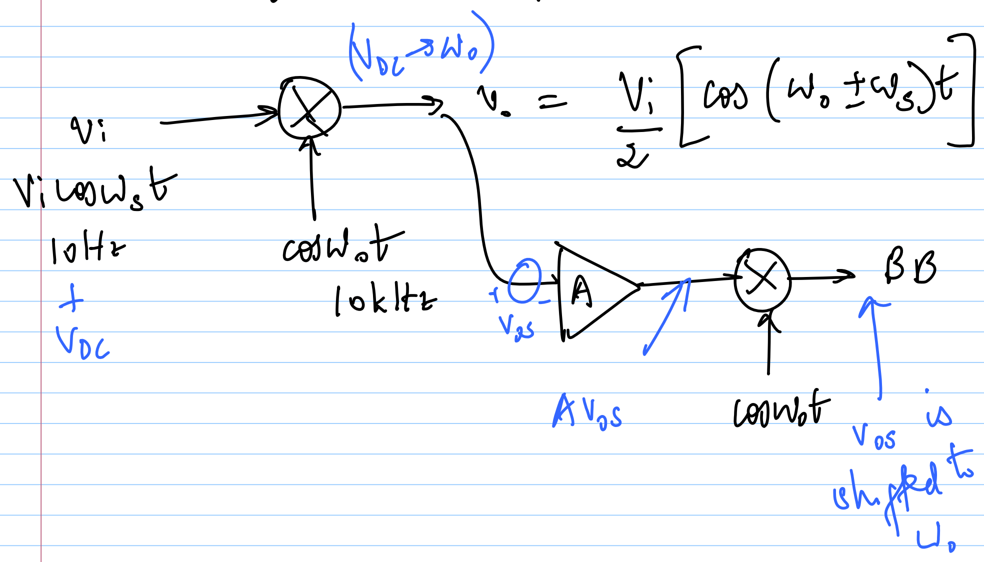
Input to ideal amp $= v_i + V_{os} - V_{os}$
$= v_i$

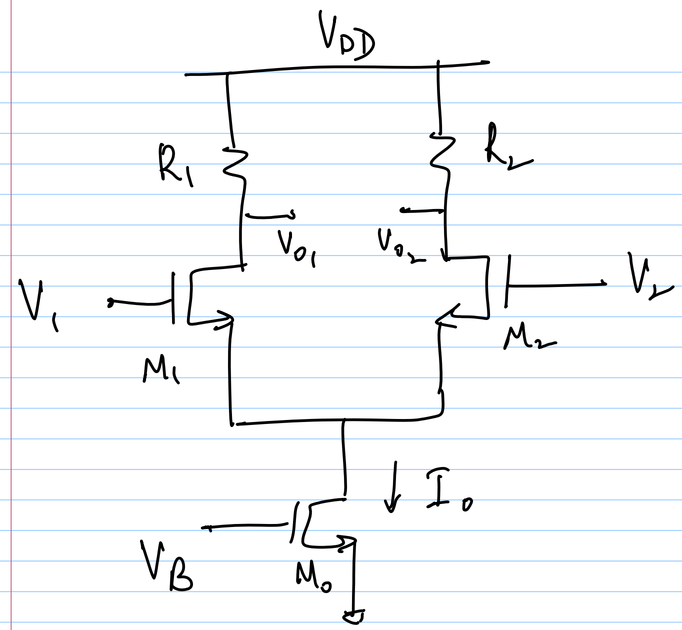


Dynamic Offset

* offset - time varying
* Problem in case of e.g. bio signals

* Chopping — DC offsets & 1/f noise





$R_1 \neq R_2$

$V_{T_1} \neq V_{T_2}$

$\left(\frac{W}{L}\right)_1 \neq \left(\frac{W}{L}\right)_2$

$$R = \frac{R_1+R_2}{2} \; ; \; \Delta R = \frac{R_1-R_2}{2}$$

$$V_{ID} = V_1 - V_2$$

$$= V_{T_1} + \sqrt{\frac{2I_1}{\beta\left(\frac{W}{L}\right)_1}} - \left[V_{T_2} + \sqrt{\frac{2I_2}{\beta\left(\frac{W}{L}\right)_2}}\right]$$

$$= \Delta V_T + \left[\sqrt{\frac{2I_1}{\beta\left(\frac{W}{L}\right)_1}} - \sqrt{\frac{2I_2}{\beta\left(\frac{W}{L}\right)_2}}\right]$$

$$V_{OD} = 0 \Rightarrow I_1 R_1 = I_2 R_2$$

$$V_{os} = \Delta V_T + \frac{(V_{GS}-V_T)}{2}\left[-\frac{\Delta R}{R} - \frac{\Delta(W/L)}{(W/L)}\right]$$