ACTIVE FILTER DESIGN : PROBLEM SET 1

Due 15th January, 11.59pm

1 Problem 1

In class, we derived the transfer function for an n^{th} order Butterworth filter by making $K(\omega^2)$ have $\omega = 0$ as the point of maximum flatness. We could have chosen an arbitrary frequency ω_o in the passband as the point of maximum flatness. This problem develops that line of thought. The desired $K(\omega^2)$ is shown in Figure 1. As in the case of the Butterworth approximation, we constrain $K(1)$ to be unity.

a. Are there any restrictions on the order of $K(\omega^2)$, and thereby on the order of the filter ?

b. What is $K(\omega^2)$ for a filter or order n ?

c. Design a fourth order filter of this type, with $\omega_o = 0.65$. After you find $K(\omega^2)$ proceed just as we did in class - that is, find the LHP roots of D(s)D(-s), and the resulting D(s). You might find it useful to use a computer to find the roots of the appropriate polynomial. Once you have found the denominator polynomial, normalize it so that the maximum transmission of the filter is unity. Plot the resulting magnitude response. On the same graph, plot the response of a fourth order Butterworth filter. Which of the filters has a better stop band response ? Can you intuitively explain why ?

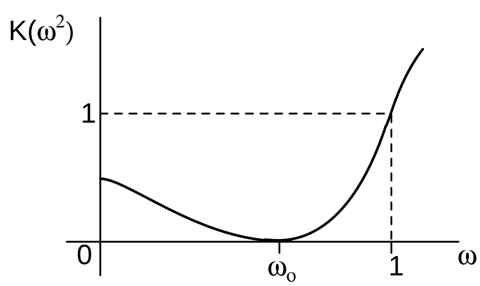

Figure 1: $K(\omega^2)$ is maximally flat about $\omega = \omega_o$.

2 Problem 2

Deriving the Butterworth from the Chebyshev : A Butterworth filter may be seen as a special case of a Chebyshev filter. This problem pursues this line of thought. Derive the squared magnitude of an Nth order Butterworth filter from the squared magnitude of an Nth order Chebyshev filter. One way of doing this is to proceed as follows.

- Determine the 3 dB bandwidth of an Nth order Chebyshev filter in terms of ϵ, and N.
- Evaluate the bandwidth in limit as ϵ tends to zero.
- Now scale the Chebyshev response so that its 3 dB bandwidth is unity, and evaluate this in the limit that ϵ tends to zero.
- Why does this strategy make sense ?

Verify the reasoning above by plotting the magnitude response (linear x and y axes) of a fifth order Chebyshev filter with $\epsilon = 10^{-4}$ and a fifth Butterworth filter. Both filters must have a 3 dB bandwidth of 1 rad/s. For the x-axis, use the range $0 < \omega < 3$ rad/s.

3 Problem 3

A Chebyshev lowpass filter satisfying the following frequency mask is to be designed.

a. In the frequency range $0 < \omega < 100\,\mathrm{rad\,s^{-1}}$, the magnitude of the the transfer function should have a maximum ripple of $0.5\,\mathrm{dB}$. The maximum gain of the filter should be unity.

b. At $\omega = 150\,\mathrm{rad\,s^{-1}}$, the transfer function magnitude must be smaller than 0.02.

Find the minimum order of the filter that will satisfy the above mask, the transfer function and the frequencies of maximum and minimum transmission in the passband. Find the order and bandwidth of a Butterworth filter required to satisfy the same frequency mask. For this filter, find the transfer function. Plot the two responses on the same graph. Plot the group delay $-\frac{d\phi(\omega)}{d\omega}$ for both the filters. Which of the filters has smaller group delay distortion, and why ?

4 Problem 4

A transfer function is given by

$$H(s) = \frac{s^2 + sb_1 + a_1 b_1}{s^3 + s^2(1 + b_1) + sa_1 + a_1 b_1} \qquad (1)$$

Determine the values of a_1 and b_1 if $|H(j\omega)|$ has to have a maximally flat magnitude response.