ADVANCED ELECTRICAL NETWORKS : PROBLEM SET 2

Due 7th February, 11.59pm

Problem 1

In class, we saw that the response of an LPTV system to a complex excitation $e^{j\omega_1 t}$ is of the form

$$\sum_{k=-\infty}^{\infty} H_k(j\omega_1) e^{j(\omega_1 + k\omega_s)t},$$

where ω_s denotes the frequency at which the system is varying. Derive expressions for $H_k(j\omega)$ for the following system. $p(t) = p(t+T_s)$, $q(t) = q(t+T_s)$ and $\omega_s = 2\pi/T_s$. Further, for simplicity, you may assume that $p(t)$ has 3 harmonics, so that

$$p(t) = \sum_{m=-3}^{3} P_m e^{jm\omega_s t}$$

and $q(t)$ has 3 harmonics so that

$$q(t) = \sum_{l=-3}^{3} Q_l e^{jl\omega_s t}$$

The filter is linear and time invariant, with transfer function $F(j\omega)$. N is an integer.

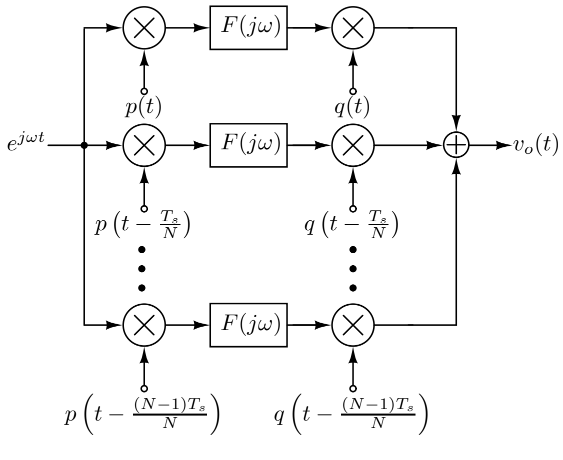

Figure 1: Figure for problem 1.

Problem 2

Design 4th order Gm-C and active-RC Butterworth filters with 3 dB bandwidth 1 MHz. The DC gain of the filters must be 1, and must be realized as a cascade of biquads. The sum total of all capacitances in the filter should not exceed 100 pF. Further, the sum of capacitances in each biquad must be the same. The peak gains (across frequency) of all nodes in the filters must be identical, as discussed in class. Assume that ideal opamps and transconductors are available.

a. Draw the schematics of the Gm-C and active-RC filters with all component values marked. Plot the frequency response at all nodes over a 0-3 MHz range. Both x and y axes must use a linear scale.

b. Now for both filters, add *random* capacitors from every node to ground. The capacitor values must be uniformly distributed in the range 0-0.1 pF. Run 100 instances of each filter (this is what is called a Monte Carlo simulation). Plot all the 100 frequency response curves from the filter input to output of the Gm-C filter on the same figure. Do the same in the active-RC case. Both x and y axes must use a linear scale. The x-axis must range from 0-3 MHz, while the y-axis must range from 0-2. What do you observe ? Explain what you see.

The easiest way of doing this in MATLAB is to write the MNA equations, and perturb the C matrix by adding appropriate random entries corresponding to the random capacitances to ground. Alternatively, you can use SPICE.

Problem 3

Determining means square noise at the output of a network involves computation of integrals that take on the form

$$I = \int_0^\infty |H(j\omega)|^2 \, d\omega \tag{1}$$

Recognizing that $|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega}$, and assuming that $|H(j\omega)| \to 0$ as $\omega \to \infty$, it is easier to determine I using Cauchy's integral formula, which states that

$$\oint F(z)\, dz = j2\pi \sum_k residue_k \tag{2}$$

where $residue_k$ correspond to the residues of the poles enclosed by the contour. Use this to evaluate

$$I_1 = \int_0^\infty \frac{df}{1 + (f/f_0)^2} \tag{3}$$

$$I_2 = \int_0^\infty \frac{df}{\left[1 - (f/f_0)^2\right]^2 + \left(\frac{f}{f_0 Q}\right)^2} \tag{4}$$

and

$$I_3 = \int_0^\infty \frac{\left(\frac{f}{f_0 Q}\right)^2}{\left[1 - (f/f_0)^2\right]^2 + \left(\frac{f}{f_0 Q}\right)^2} df \tag{5}$$

Problem 4

The circuit diagram below shows a second order bandpass filter. The opamps are ideal. Use the results of the previous problem to determine the mean square noise at the bandpass output. The amplifier of gain -1 is noiseless. The only noisy elements are the resistors. Assume $Q \gg 1$.

The opamps have a peak signal swing of $\pm 1\,\text{V}$ at their outputs. The input $|V_{in}| < 1\,\text{V}$. It is desired to maximise the dynamic range of the filter. The dynamic range is the maximum possible SNR at the filter output. The peak signal amplitude at the filter output is restricted to $1\,\text{V}$, but can be lower if one of the amplifiers saturates before the other. The noise power at the filter output is its mean square noise. The dynamic range, therefore, is given by

$$\text{Dynamic Range} = \left(\frac{\text{Max. Signal Power at Output}}{\text{Mean Square Output Noise}}\right) \tag{6}$$

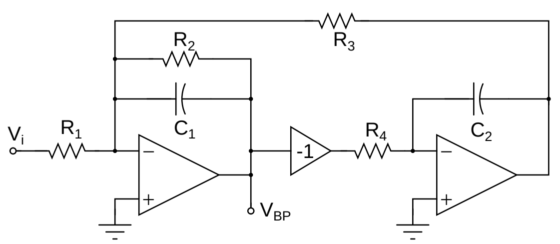

Figure 2: Figure for problem 4.

The aim of this problem is to illustrate the fundamental difficulty of realizing a high-Q active bandpass filter with a high dynamic range. Choose component values so that all the following constraints are satisfied. Assume that the Q is high, so that the peaks of the lowpass and bandpass outputs occur at the same frequency.

- Center frequency f_0 is $1\,\text{MHz}$.
- Q = 20
- Gain at f_0 of 1.
- $C_1 + C_2 = 100\,\text{pF}$.

Since there are six component values but only four constraints, many component choices can realize the desired transfer function.

a. Choose component values so that the output dynamic range is maximized. Do not skip steps - show your reasoning clearly. What is the maximum dynamic range in dB? Verify your noise calculations using SPICE.

b. Repeat part (a) if $C_1 + C_2 = 200\,\text{pF}$.

c. Repeat part (a) if $Q = 40$, but all other constraints remained as in part (a).

d. Repeat part (a) if $f_0 = 2\,\text{MHz}$, but all other constraints remained as in part (a).