ADVANCED ELECTRICAL NETWORKS : PROBLEM SET 1

Due 31st January, 11.59pm

1   Problem 1

In class, we saw that the response of an LPTV system to a complex excitation $e^{j\omega_1 t}$ is of the form

$$\sum_{k=-\infty}^{\infty} H_k(j\omega_1)e^{j(\omega_1+k\omega_s)t},$$

where ω_s denotes the frequency at which the system is varying. Derive expressions for $H_k(j\omega)$ for the following systems. In all the systems shown below, $p(t) = p(t+T_s)$, $q(t) = q(t + T_s)$ and $\omega_s = 2\pi/T_s$. Further, for simplicity, you may assume that $p(t)$ has 3 harmonics, so that

$$p(t) = \sum_{m=-3}^{3} P_m e^{jm\omega_s t}$$

and $q(t)$ has 3 harmonics so that

$$q(t) = \sum_{l=-3}^{3} Q_l e^{jl\omega_s t}$$

The filter is linear and time invariant, with transfer function $F(j\omega)$. N is an integer.
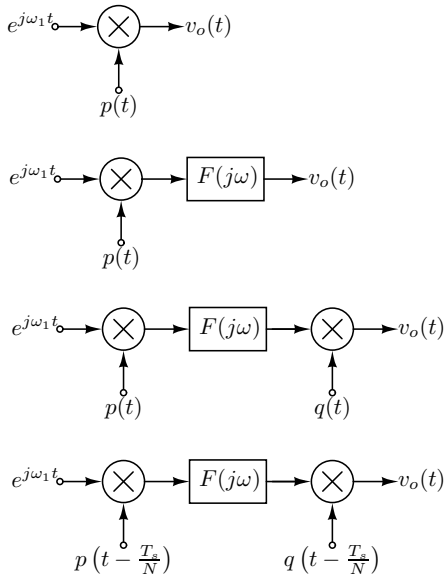

Figure 1: Figure for problem 1.

2   Problem 2

A weakly nonlinear voltage amplifier without memory can be modeled by the following relationship.

$$v_{out} = a_1 v_{in} - a_3 v_{in}^3, \tag{1}$$

where, v_{in} is restricted in amplitude so that $|a_3 v_{in}^3| \ll |a_1 v_{in}|$. Suppose $v_{in}(t) = A_1 \sin(\omega_1 t) + A_2 \sin((\omega_1 + \Delta\omega)t)$, where $A_2 \ll A_1$. Determine the amplitude of the component of v_{out} at $\omega_1 - \Delta\omega$.

   In this problem, we see how one can derive the same result in another way by proceeding as follows. The system in the dotted box is excited by $v_i(t)$, which is treated a small signal. What is the "small signal gain" from $v_i(t)$ to v_{out}? Is this system linear? Time invariant? Use the LPTV nature of the system to determine the component of v_{out} at $\omega_1 - \Delta\omega$.
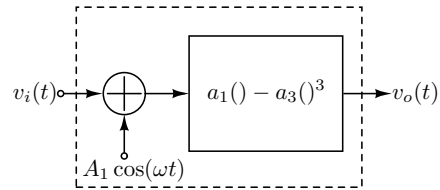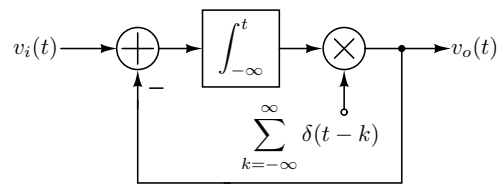

Figure 2: Figure for Problem 2.

3   Problem 3

Figure 3: Figure for Problem 3.

   For the LPTV system of Fig. 3, determine $H_{-l}(j2\pi l)$ for

- $l = 0$.

- $l \neq 0$.

Recall that $H_k(j\omega)$ is the response of an LPTV system at the frequency $\omega + k\omega_s$ to an input of the form $e^{j\omega t}$.

4 Problem 4

In class, we derived the transfer function for an nth order Butterworth filter by assuming that its magnitude squared response is of the form

$$|H(j\omega)|^2 = \frac{1}{1 + F(\omega^2)}$$

$F(\omega^2)$ had $\omega = 0$ as the point of maximum flatness. We could have chosen an arbitrary frequency ω_o in the passband as the point of maximum flatness. This problem develops that line of thought. The desired $F(\omega^2)$ is shown in Figure 4. As in the case of the Butterworth approximation, we constrain $F(1)$ to be unity.

a. Are there any restrictions on the order of $F(\omega^2)$, and thereby on the order of the filter ?

b. What is $F(\omega^2)$ for a filter or order n ?

c. Design a fourth order filter of this type, with $\omega_o = 0.65$. After you find $F(\omega^2)$ proceed just as we did in class - that is, find the LHP roots of D(s)D(-s), and the resulting D(s). You might find it useful to use a computer to find the roots of the appropriate polynomial. Once you have found the denominator polynomial, normalize it so that the maximum transmission of the filter is unity. Plot the resulting magnitude response. On the same graph, plot the response of a fourth order Butterworth filter. Which of the filters has a better stop band response ? Can you intuitively explain why ?
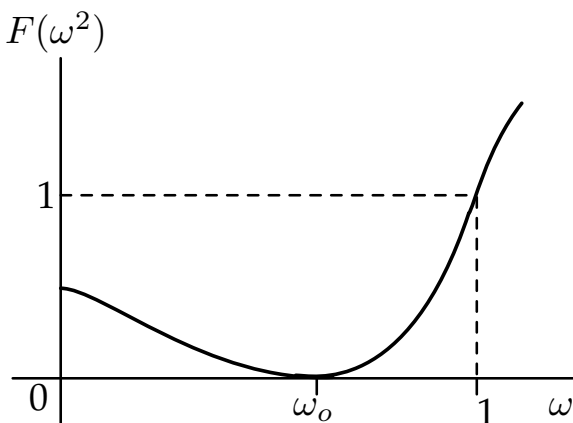

Figure 4: $F(\omega^2)$ is maximally flat about $\omega = \omega_o$.

5 Problem 5

A signal with bandwidth 1 MHz needs to be sampled by an analog-to-digital converter. For a sampling rate of 5 MHz, determine the minimum order and 3 dB bandwidth of a Butterworth filter needed so that any signal that can potentially alias into the desired signal band is attenuated by atleast 40 dB. Repeat the exercise if the sampling rate is increased to 25 MHz. For each of the sampling rates

1. Plot the magnitude responses of the filters in both cases, on the same graph. In the figure, the y-axis must be in dB, and must have a range -60 dB to 10 dB. The x-axis must be in a linear scale, with range 0-30 MHz.

2. Determine the spread in cutoff frequency allowable, if the order was chosen to be 1 higher than the minimum order you determined above.

6 Problem 6

Write a MATLAB program to determine the denominator polynomial of an nth order Butterworth filter, with a specifiable cutoff frequency ω_c. Using your routine to generate the transfer function, plot the impulse response of a 10th order Butterworth filter with a 3 dB bandwidth of 1 Hz. What do you observe ? You might want to use the "impulse" function in MATLAB to generate the impulse response.

7 Problem 7

A linear time invariant filter has an impulse response $h(t)$. The input to the filter, $x(t)$ is constrained so as to never exceed 1 in absolute value - i.e $|x(t)| \leq 1$. Determine the maximum value that $y(t)$, the filter output, can ever attain, for arbitrary values of $x(t)$ satisfying the constraint above. What $x(t)$ will drive the filter output to this maximum ?