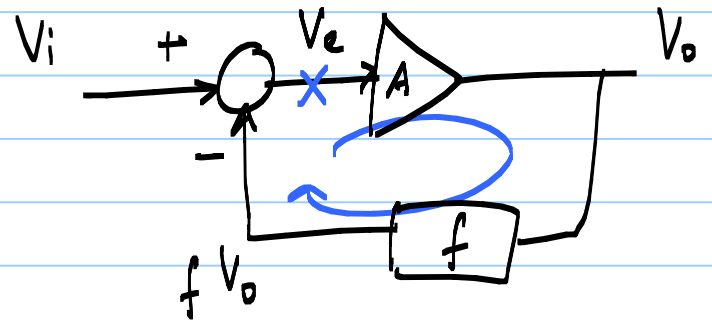
16|10|2020

Lecture 40

Feedback Systems



$$CLG = \frac{V_o}{V_i} = \frac{1}{f}\,\frac{Af}{1+Af}$$

$$\approx \frac{1}{f} \text{ if } \underbrace{Af}_{\text{loop gain (LG)}} \text{ is large}$$

If $A = A(s)$, f is freq.-indep.

$$CLG = \underbrace{\frac{1}{f}}\,\frac{A(s)\cdot f}{1+A(s)f} = CLG(s)$$

1) 1st order : $A(s) = \dfrac{A_o}{1+s/\omega_p}$ ← "DC" gain

(single pole amp.)

@ low freq. $A(s) \simeq A_o \Rightarrow LG \simeq A_o f \Rightarrow CLG \simeq \frac{1}{f}$

Assume $CLG \approx \frac{1}{f}$ is valid till $|LG| \sim 1$

$$CLG(s) = \frac{1}{f} \frac{A(s) \cdot f}{1 + A(s) \cdot f}$$

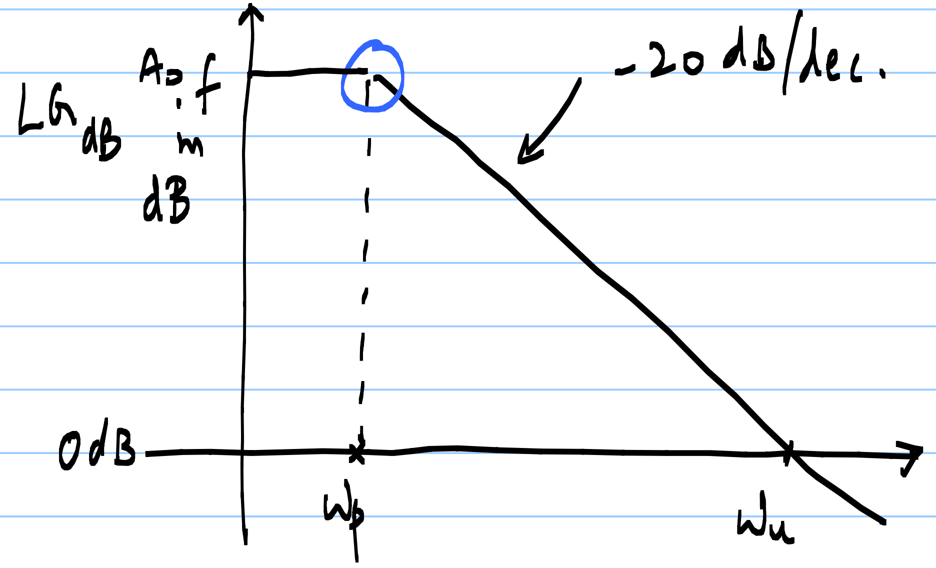
$$= \frac{1}{f} \frac{\frac{A_0 f}{1 + s/\omega_p}}{\left(1 + \frac{A_0 f}{1 + s/\omega_p}\right)}$$

$$= \frac{1}{f} \frac{A_0 f}{\left(1 + A_0 f + s/\omega_p\right)}$$

$$= \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \frac{s}{\omega_p (1 + A_0 f)}}$$

CLG pole is @ $\omega_p (1 + A_0 f)$

$$\text{CLG BW} = \omega_p(1 + A_0 f) \approx \omega_p A_0 f$$



$= A_0 f \cdot \omega_p \approx$ CLG BW

<u>Note:</u>

1) Single-pole system in closed loop has LHP poles

2) Unconditionally stable

2) $\underline{2^{nd} \text{ order system}}$: $A(s) = \dfrac{A_o}{\left(1 + \dfrac{s}{\omega_p}\right)^2}$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_o f}{1 + A_o f} \cdot \frac{1}{\underbrace{1 + \dfrac{2s}{A_o f \omega_p} + \dfrac{s^2}{A_o f \omega_p^2}}}$$

* LHP poles
* Unconditionally stable

general 2nd order system has

$$D(s) = 1 + \frac{s}{Q \cdot \omega_o} + \frac{s^2}{\omega_o^2}$$

compare

Quality factor → Q

(or) $D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

ζ ← damping factor

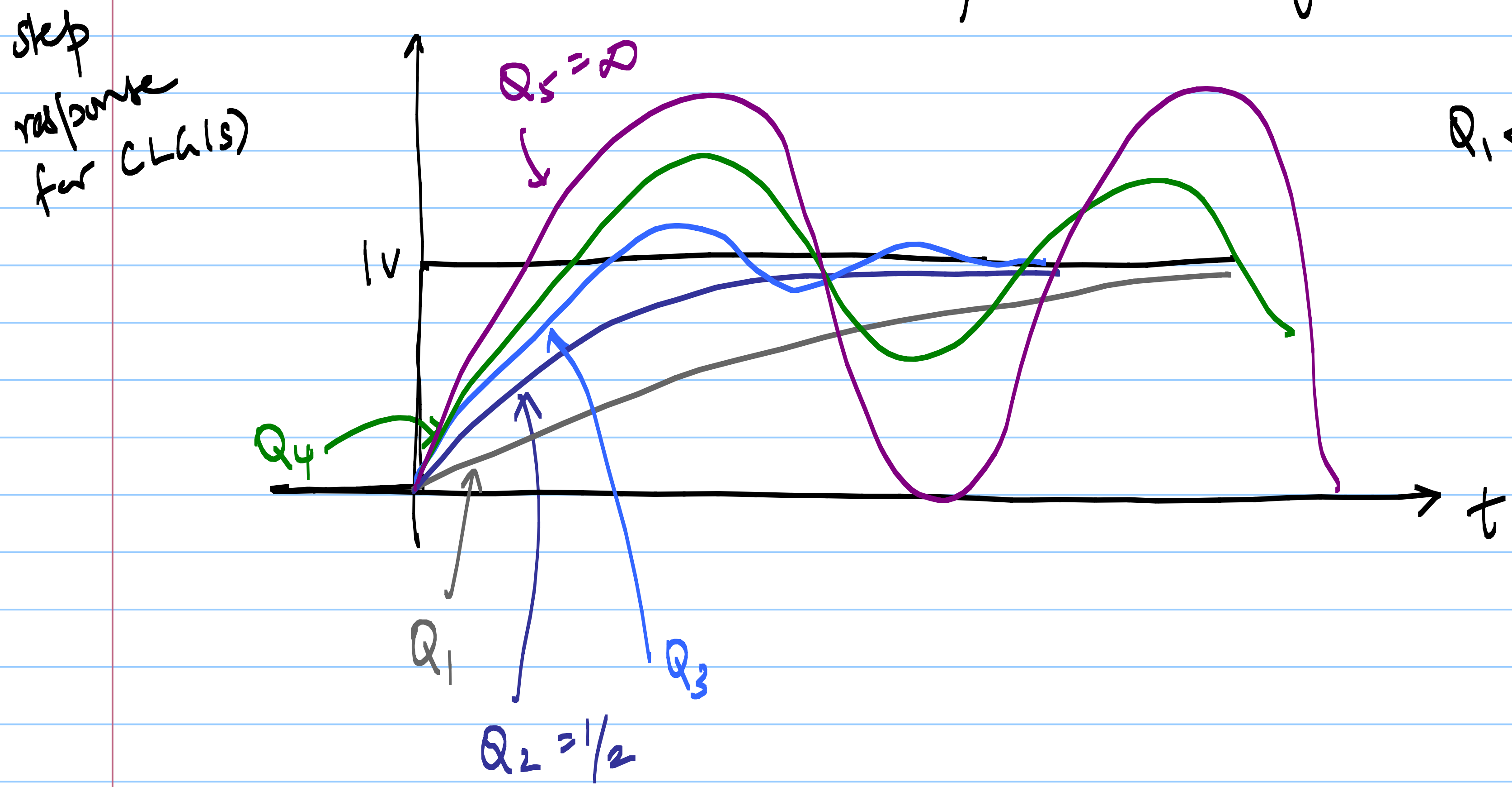
roots are $\dfrac{s}{\omega_o} = -\dfrac{1}{2Q} \pm j\underbrace{\sqrt{1 - \dfrac{1}{4Q^2}}}$

small Q → 2 real LHP poles

$Q = 1/2$ → 2 equal poles

$Q > 1/2$ → pair of complex conjugate poles

$Q = \infty$ → poles on $j\omega$ axis



$Q_1 < Q_2 < Q_3 < Q_4 < Q_5$

Here : $\omega_0 = \omega_p \sqrt{A_0 f}$

$$Q = \frac{\sqrt{A_0 f}}{2}$$

No ringing $\Rightarrow$ $Q \leq 1/2$

But : $A_0 f$ = large due to large LA requirement

3) <u>3rd order system</u> : $A(s) = \dfrac{A_0}{\left(1 + s/\omega_p\right)^3}$

$$= \frac{A_0}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLA(s) = \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \left(\frac{1}{1+A_0 f}\right)\left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right]} \leftarrow D(s)$$

$$D(s) = 1 + \frac{3s}{\omega_p(1+A_0f)} + \frac{3s^2}{\omega_p^2(1+A_0f)} + \frac{s^3}{\omega_p^3(1+A_0f)}$$

$$x = \frac{s}{\omega_p}$$

$$D(x) = 1 + \frac{3x}{1+A_0f} + \frac{3x^2}{1+A_0f} + \frac{x^3}{1+A_0f}$$

$$= \left(\frac{1}{1+A_0f}\right)\left[\underbrace{(1+A_0f) + 3x + 3x^2 + x^3}_{\text{we want roots of}}\right]$$

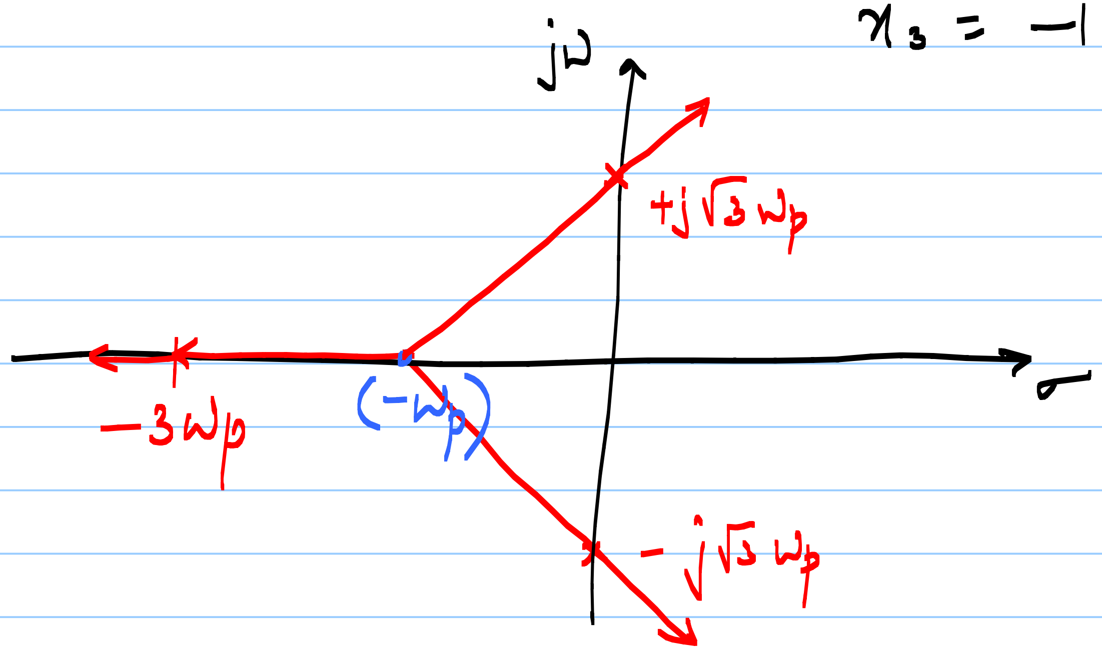
$$(1+x)^3 = -A_0f$$

$$x = -1 + \underbrace{(-A_0f)^{1/3}}_{\text{3 roots}}$$

e.g. 1) $A_{of} = 0 \rightarrow$ all 3 roots @ -1

2) $A_{of} = 8 \rightarrow$

$$\left.\begin{aligned} x_1 &= -1 - 2 = -3 \\ x_2 &= -1 - 2e^{-j2\pi/3} \\ x_3 &= -1 - 2e^{+j2\pi/3} \end{aligned}\right) \quad \begin{aligned} s_1 &= -3\omega_p \\ s_2 &= +j\sqrt{3}\,\omega_p \\ s_3 &= -j\sqrt{3}\,\omega_p \end{aligned}$$



If $A_{of} > 8$

↳ complex conjugate roots move into RHP

Unstable for $A_{of} > 8$