19/01/2022

EE2019 Analog Systems & Lab
Jan-May 2022

Note Title 18-01-2022

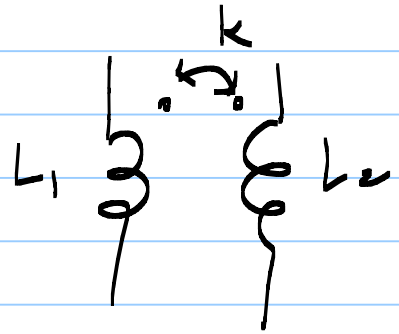
Lecture 2

Passive elements:

R

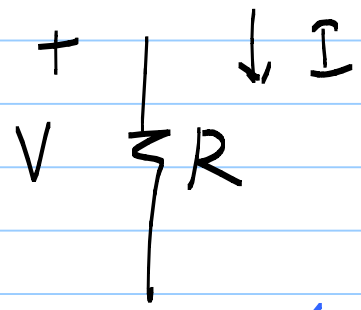
C

L



Controlled elements: Active?

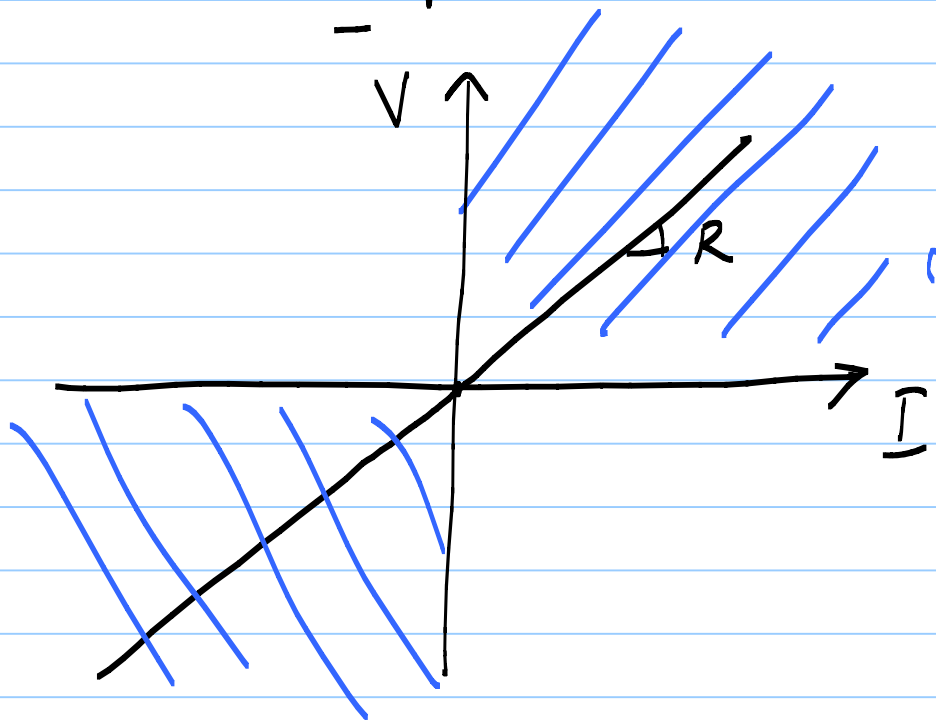
because graph doesn't pass through origin? ✗



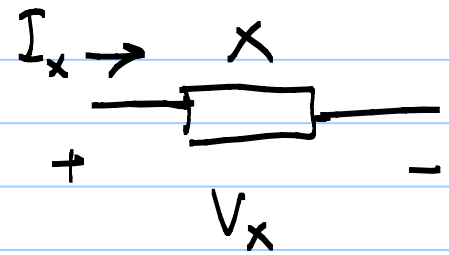
$V = IR$

passivity?

linearity?



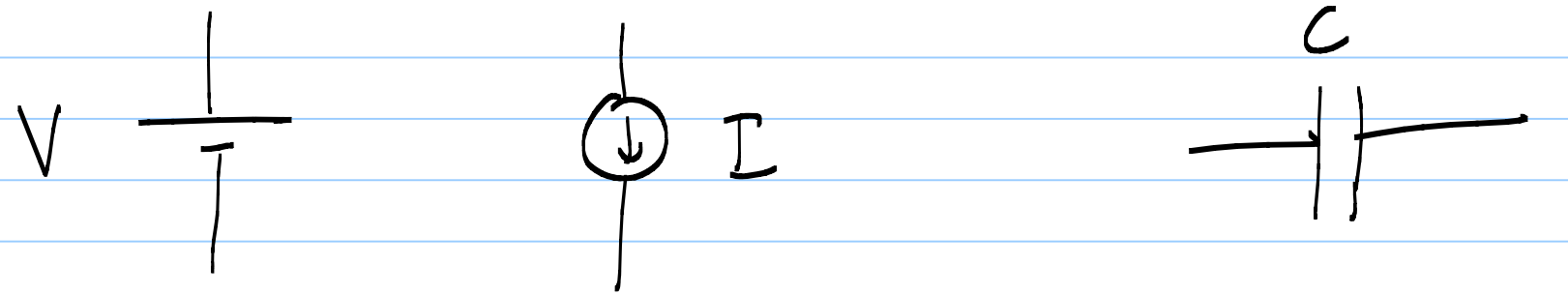
When is an element passive? $V_x I_x \geq 0$

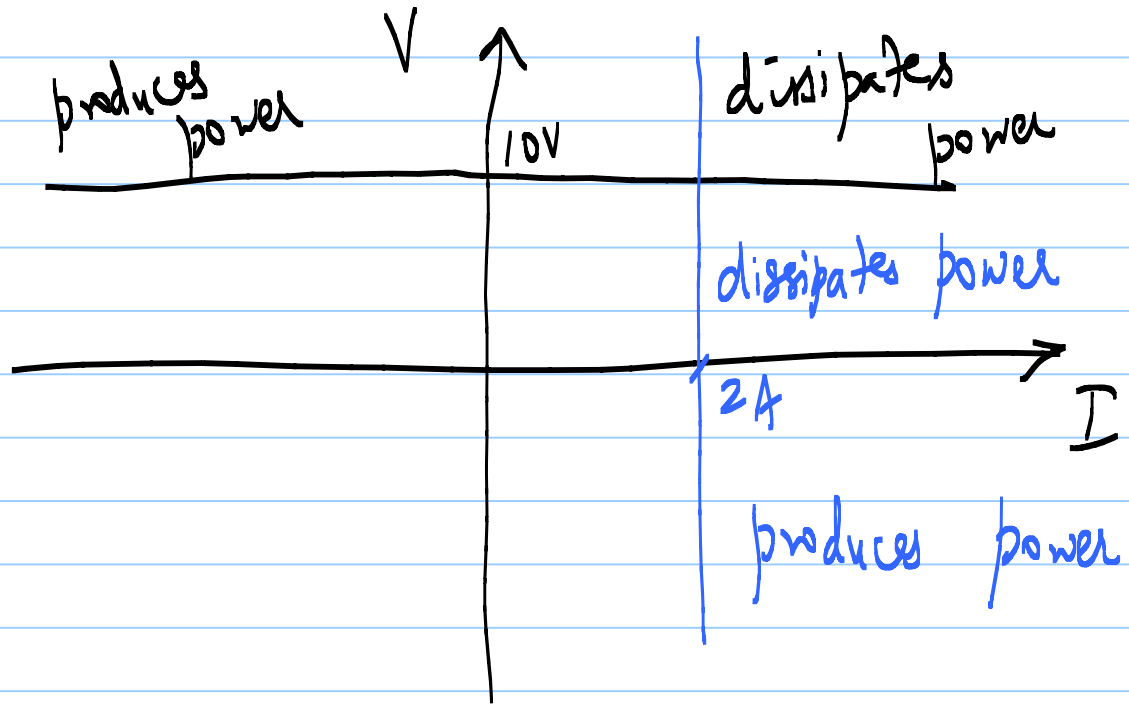


If X is passive

$V = f(I)$

occupies 1st & 3rd quadrants



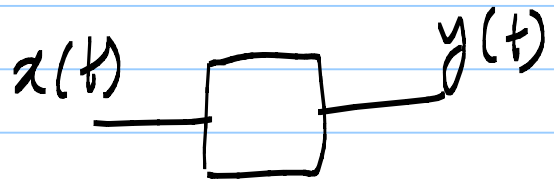


<u>V - I characteristics</u> : If they pass through the origin $\Rightarrow$ the device is <u>linear</u> ?

* Additivity & Homogenity

$x_1 + x_2$ $\qquad$ $a \cdot x_1$

"Superposition"



$$I = c\frac{dV}{dt} \ ?$$

I-V have to pass through the origin?

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

If $A x_1(t) + B x_2(t) \rightarrow A y_1(t) + B y_2(t)$

$\forall\ A, B, t$

$\Rightarrow$ Superposition is valid

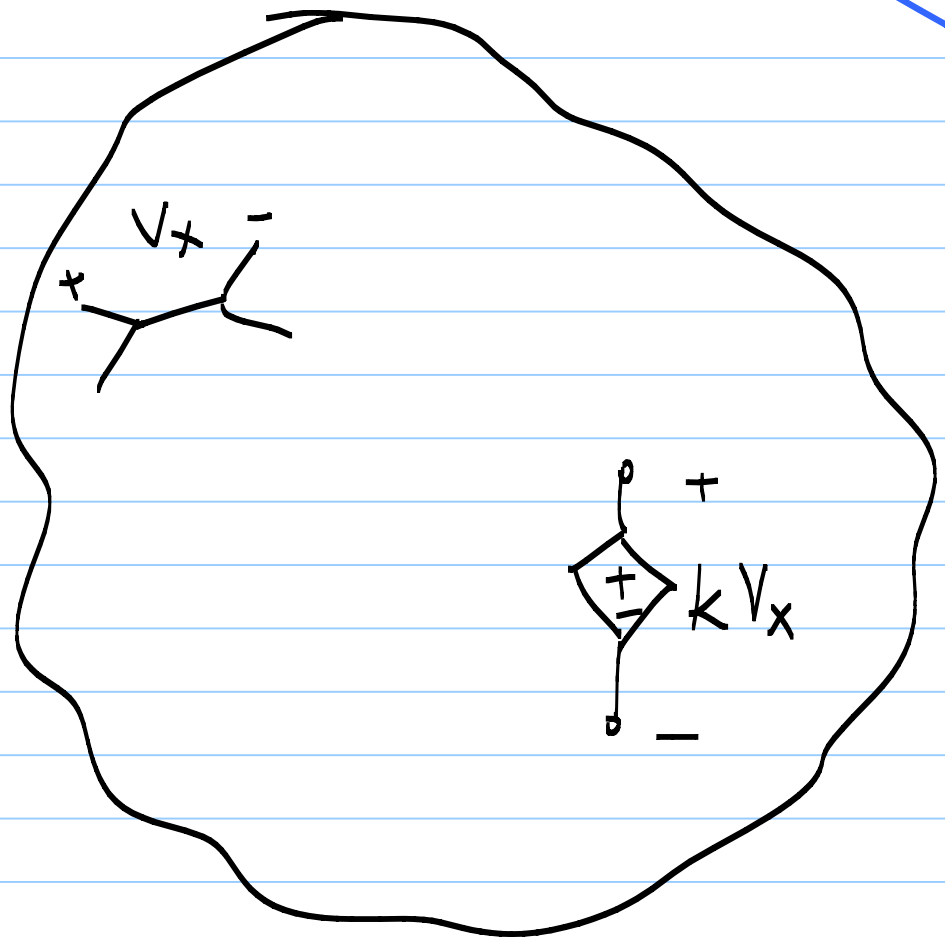
$\rightarrow$ linear system

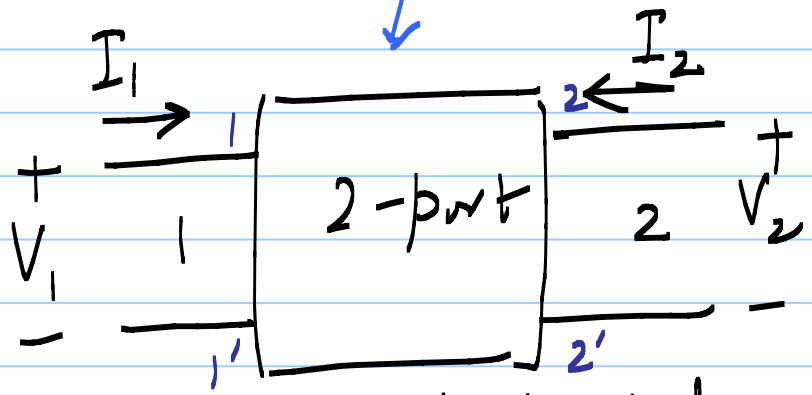
Controlled elements:

VCVS , VCCS

CCVS , CCCS

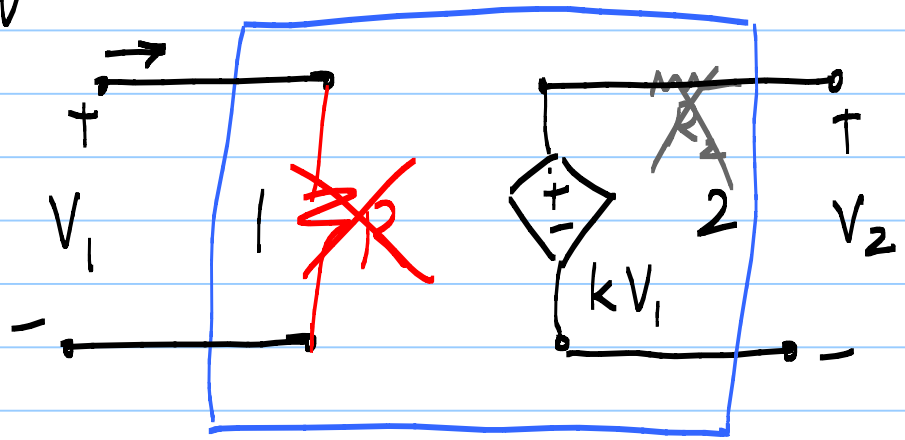
ECN

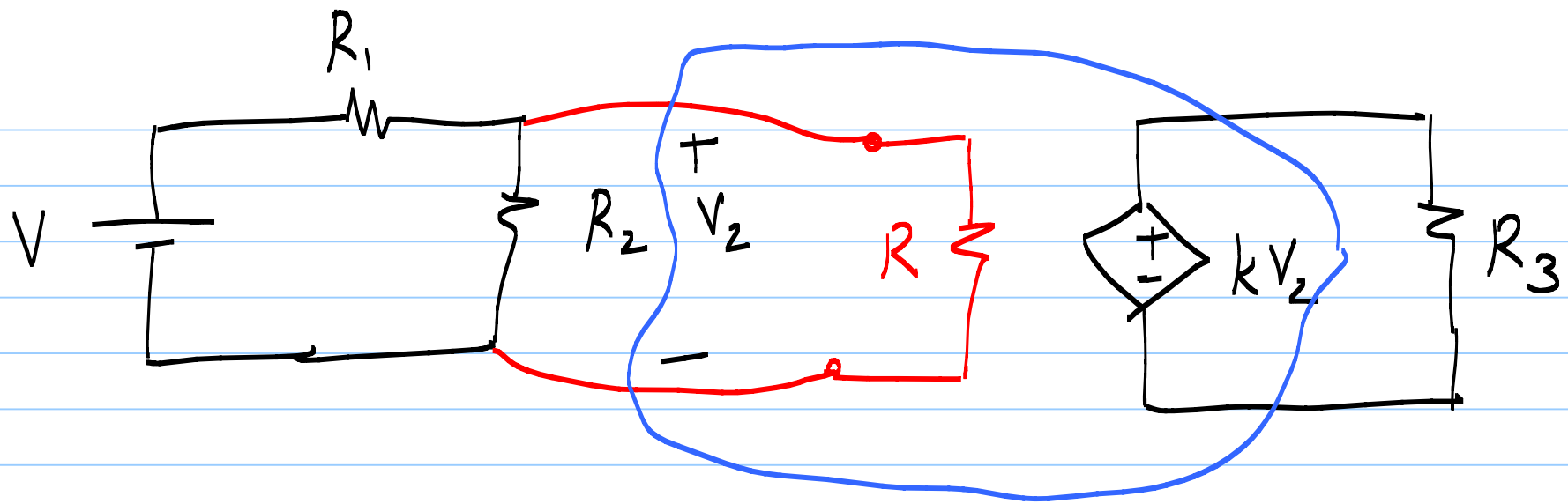




4 terminal 2-port

e.g. VCVS



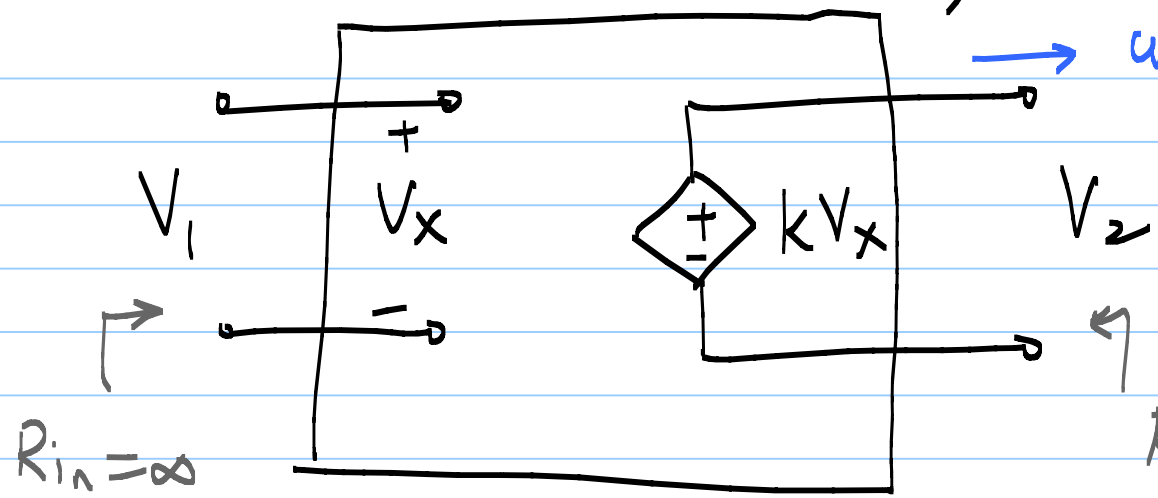


Before connecting VCVS : $V_2 = \dfrac{R_2}{R_1+R_2} \cdot V$

after conn. VCVS : $\dfrac{R_2 \| R}{R_1 + R_2 \| R} \cdot V = V_2$ (changed)

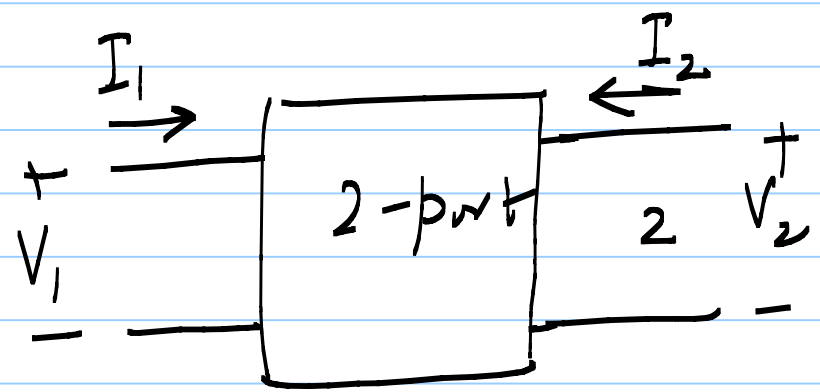
V_2 is the same only if $R = \infty$

VCVS (ideal)



current flow in this direction is possible

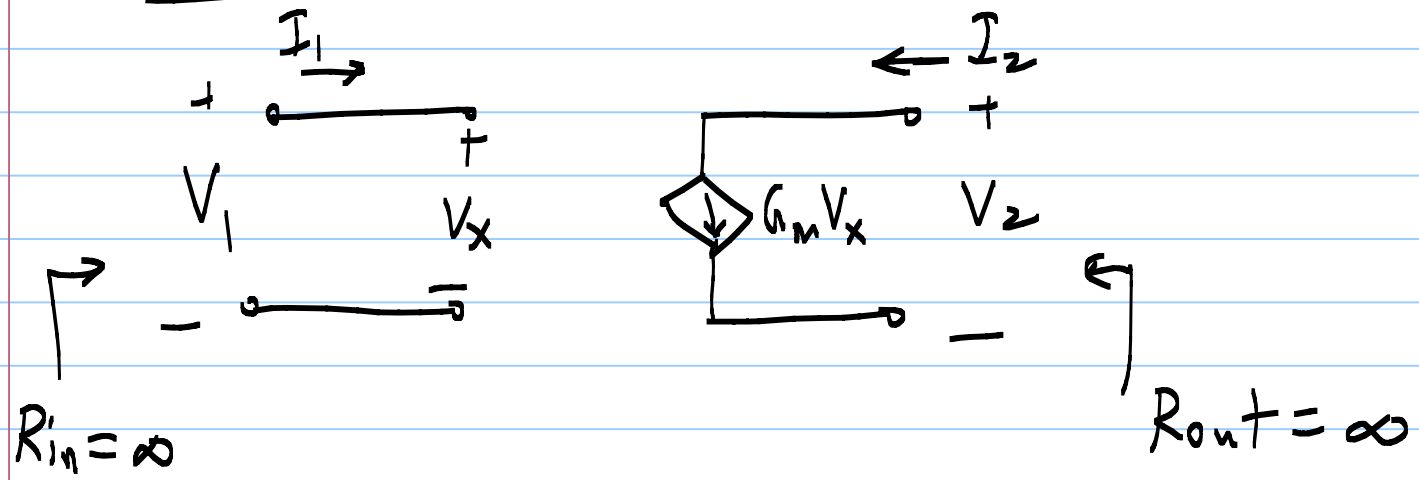
VCVS is active



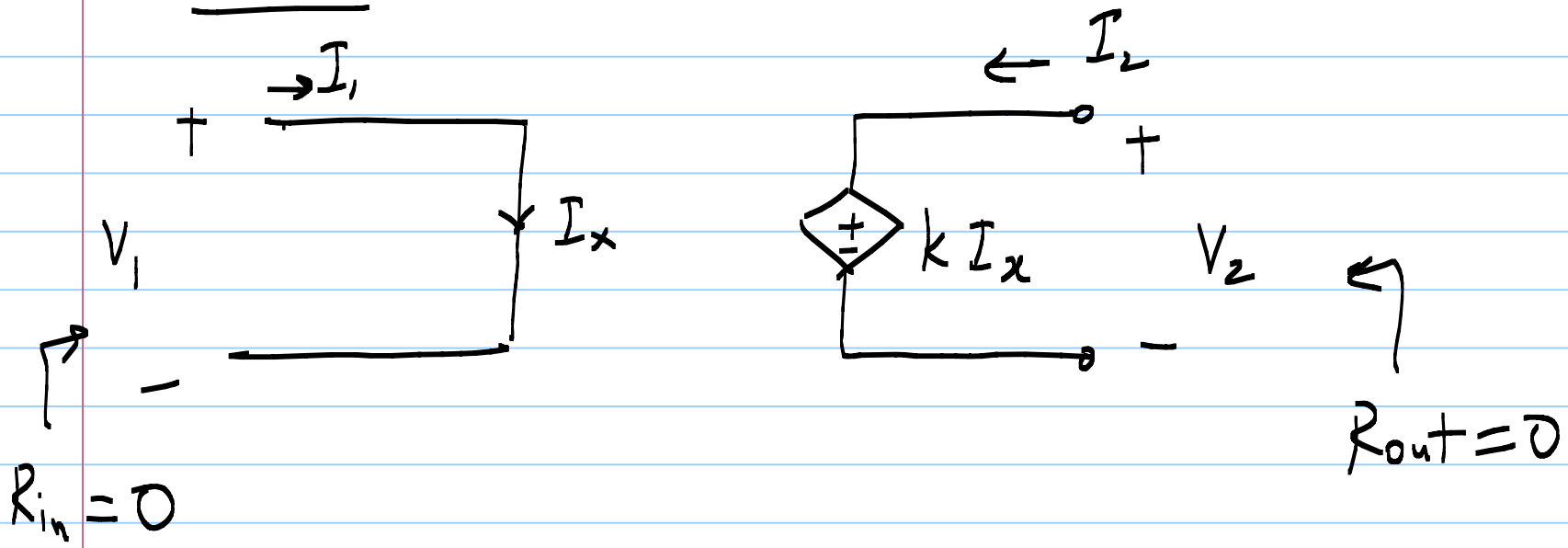
Passive if

$$\underbrace{V_1 I_1}_{0 \text{ for VCVS}} + \underbrace{V_2 I_2}_{\text{can be } < 0} \geq 0$$

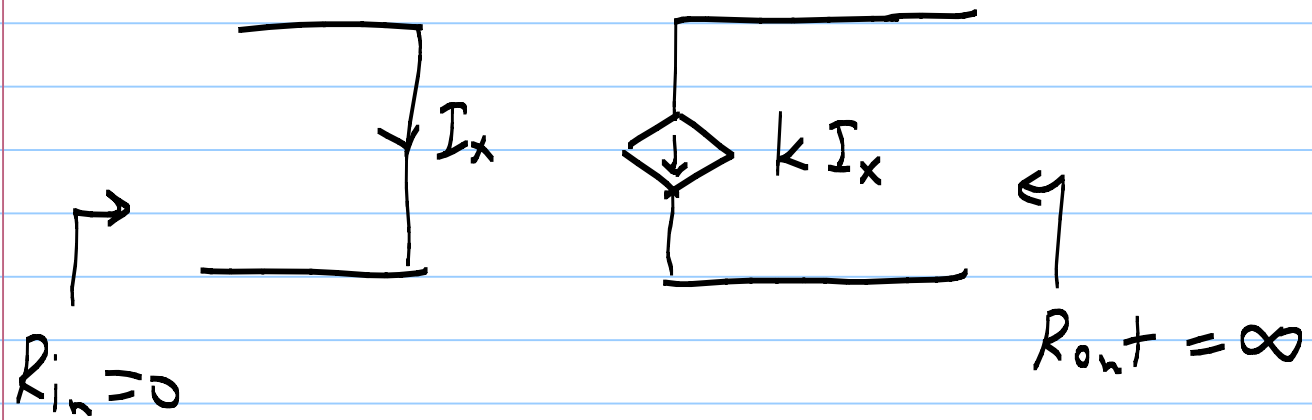
VCCS


I_1
+
V_1
V_x
−
$R_{in} = \infty$
I_2
+
$G_m V_x$
V_2
−
$R_{out} = \infty$

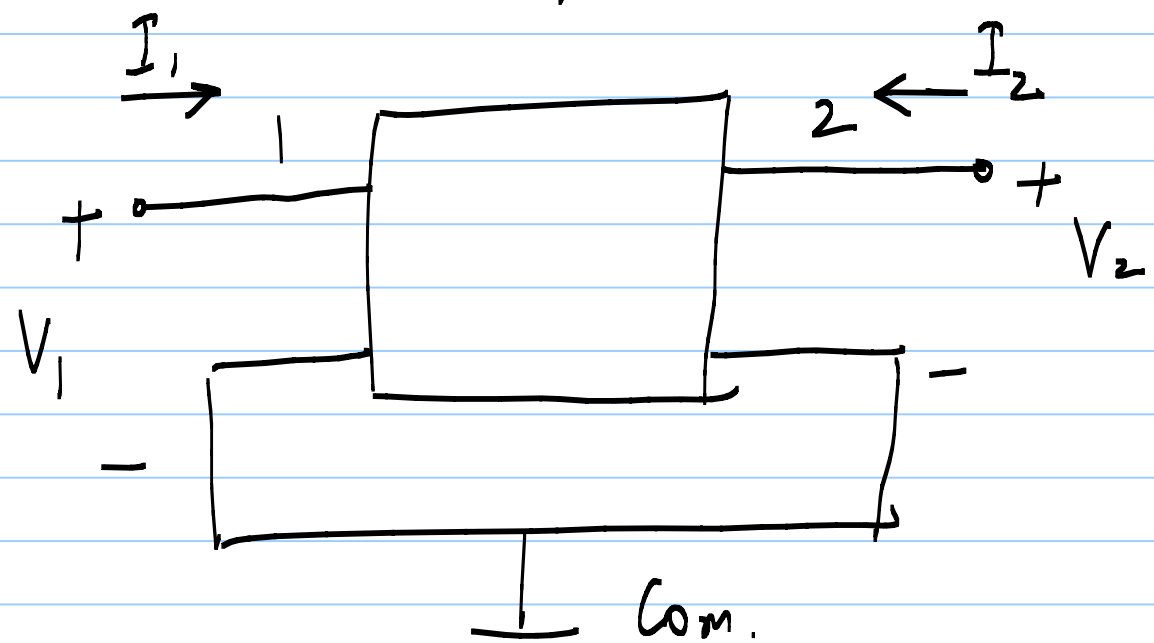
CCVS


I_1
+
V_1
I_x
−
$R_{in} = 0$
I_2
+
$k I_x$
V_2
−
$R_{out} = 0$

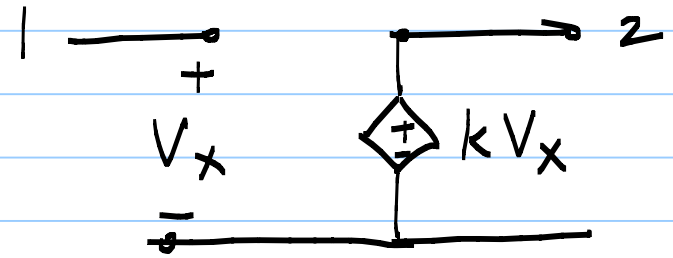
CCCS

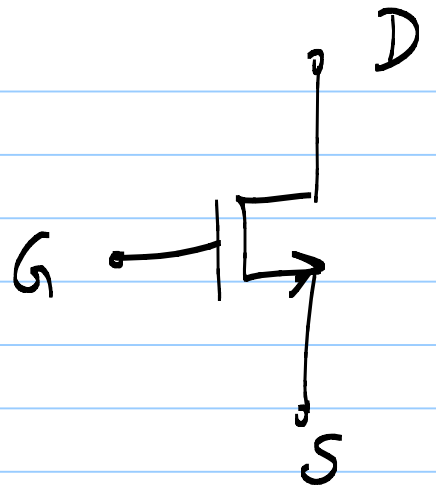


3 terminal 2-ports

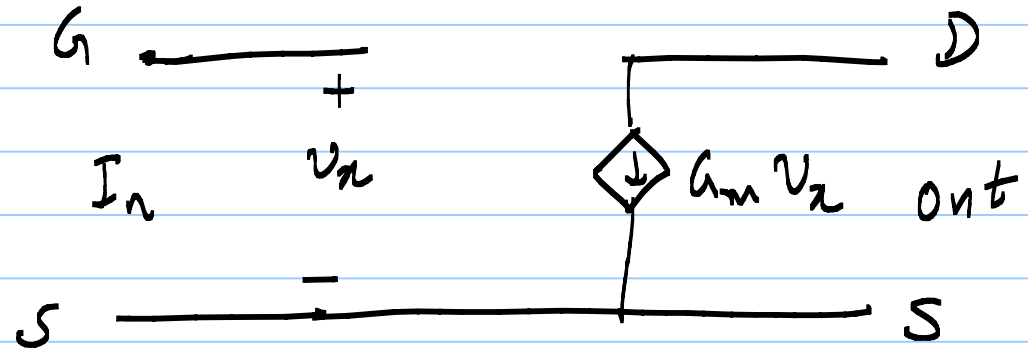


VCVS example





MOSFET



<u>Tomorrow</u>:

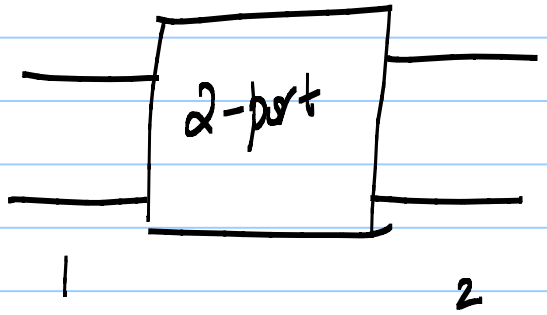
1) What are R_{in} & R_{out} of a 2 port?

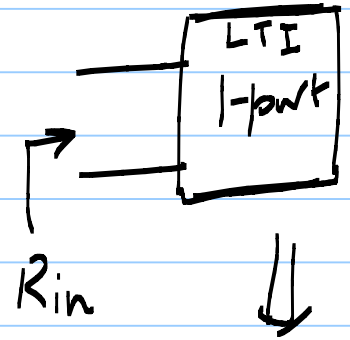
2) Voltage or power gain possible?

3) Negative feedback

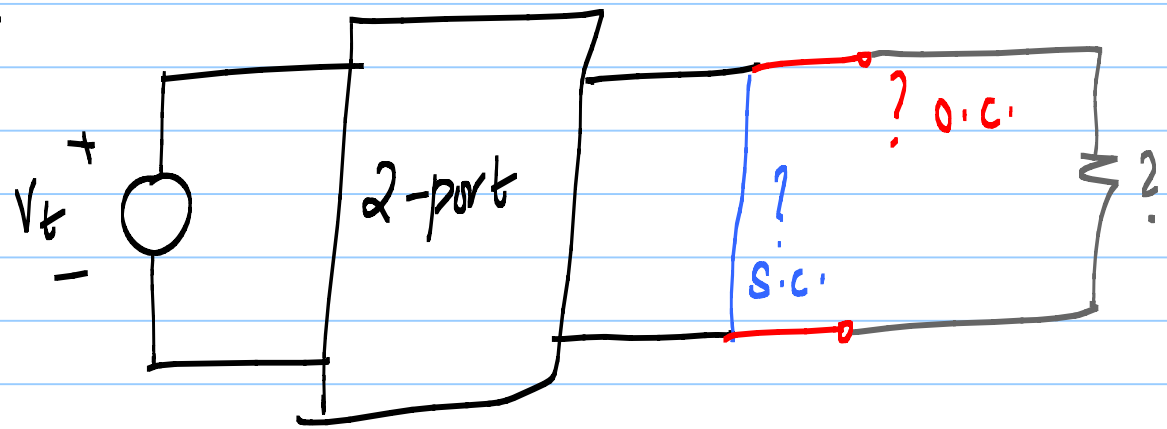
20/01/2022

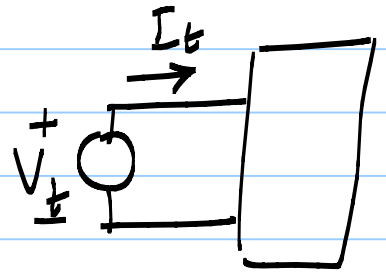
Lecture 3



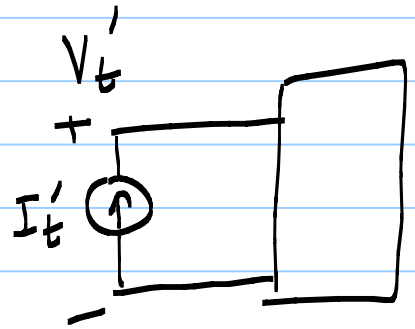


R_{in}

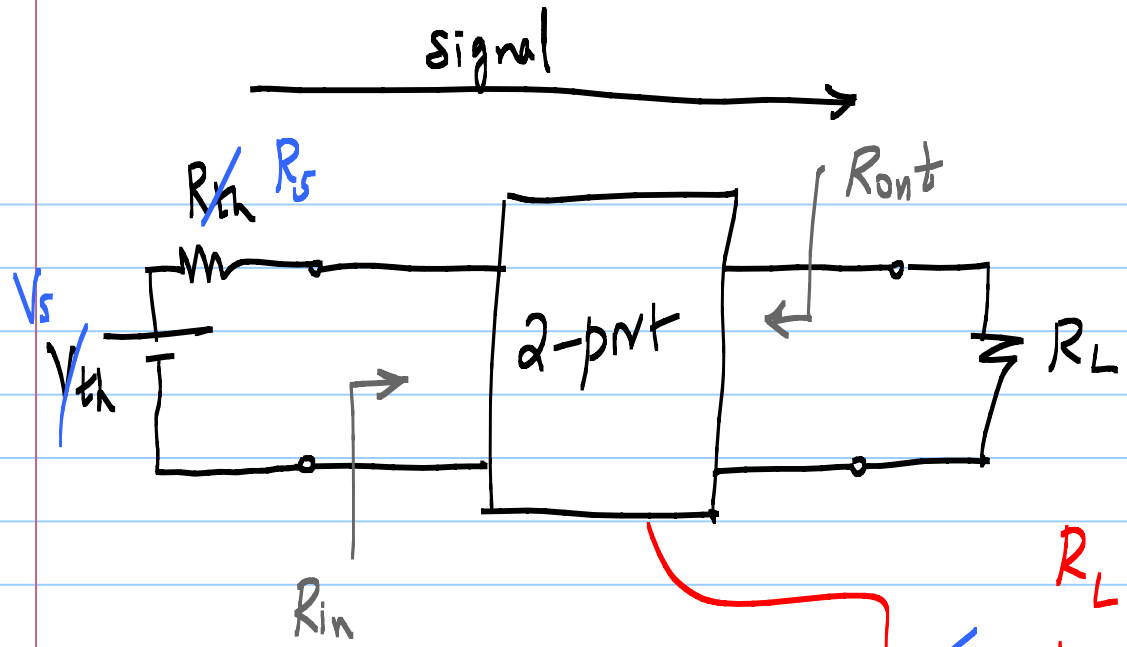




$$R_{in} = \frac{V_t}{I_t}$$

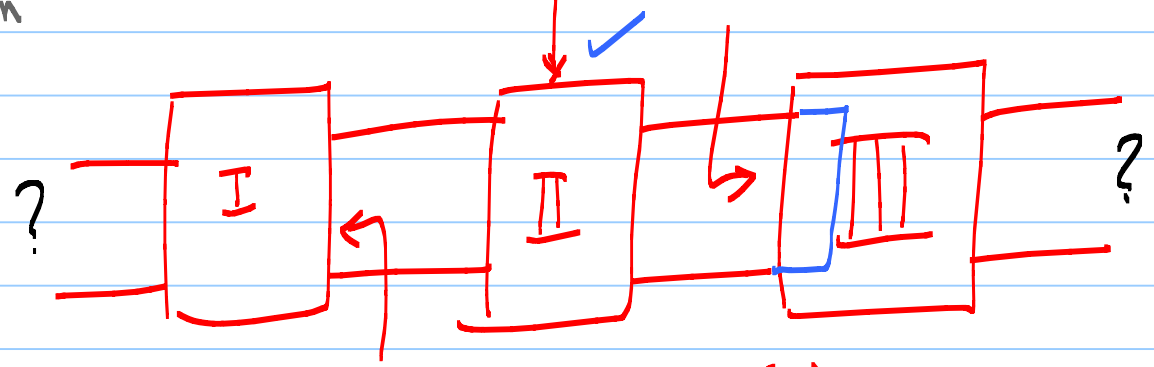


$$R_{in} = V_t'/I_t'$$



* R_{in} : after (right of) V_s & R_s
* R_{out} : before (left of) R_L

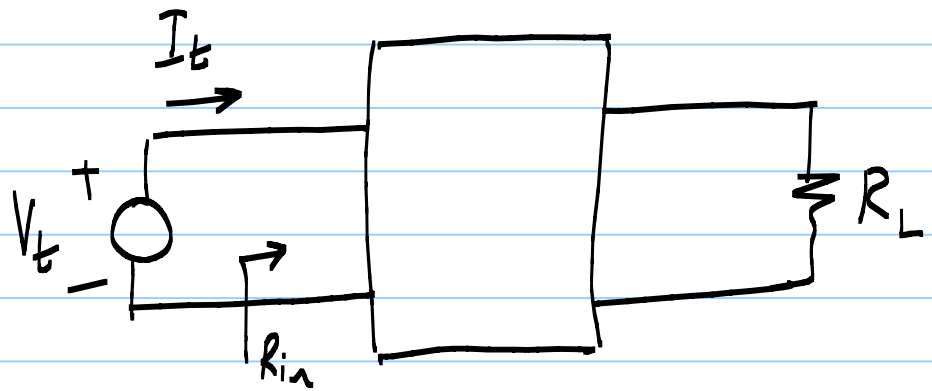
$R_L(\mathrm{II}) = R_{in}(\mathrm{III})$



$R_s(\mathrm{II}) = R_{out}(\mathrm{I})$

HW

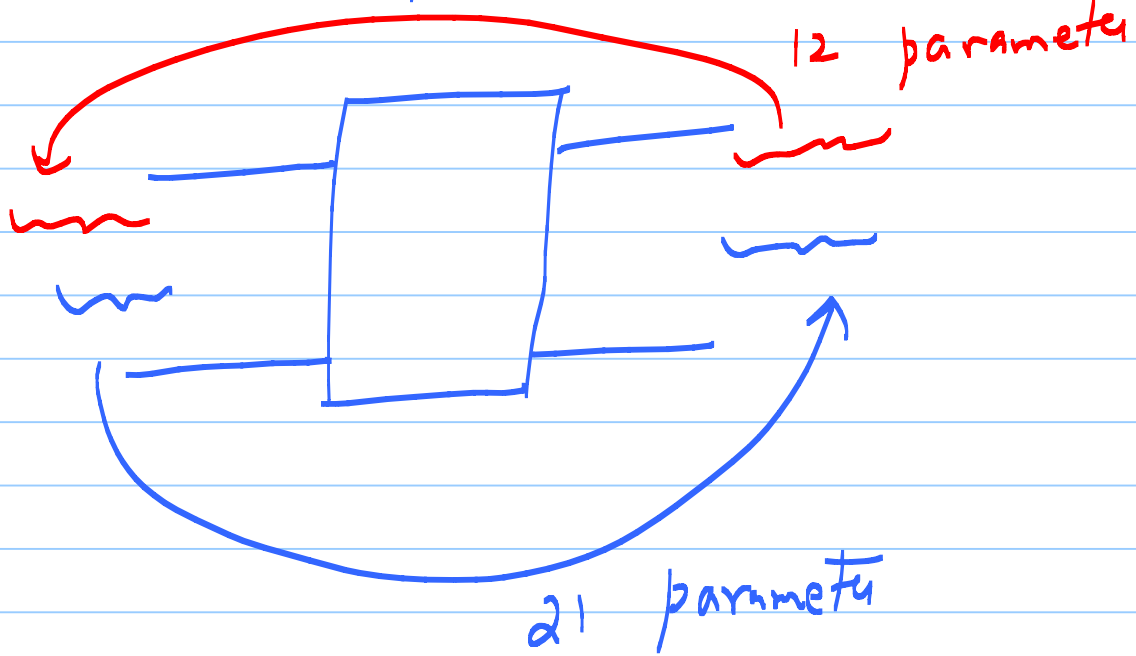
What property (of 2-port) decides if $R_{in} = f(R_L)$ or not?



$$R_{in} = \frac{V_t}{I_t}$$

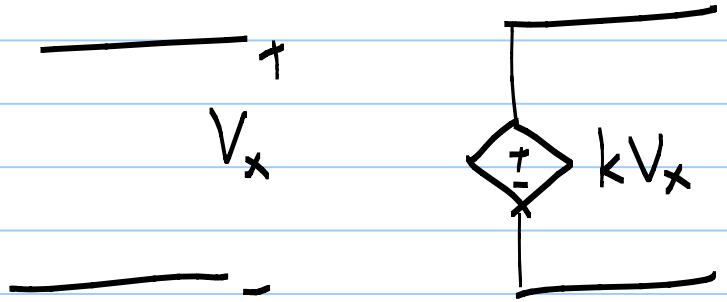
In general, $R_{in} = f(R_L)$

When 12 parameter is 'zero'



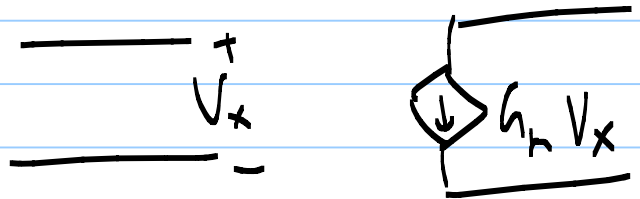
HW

VCVS



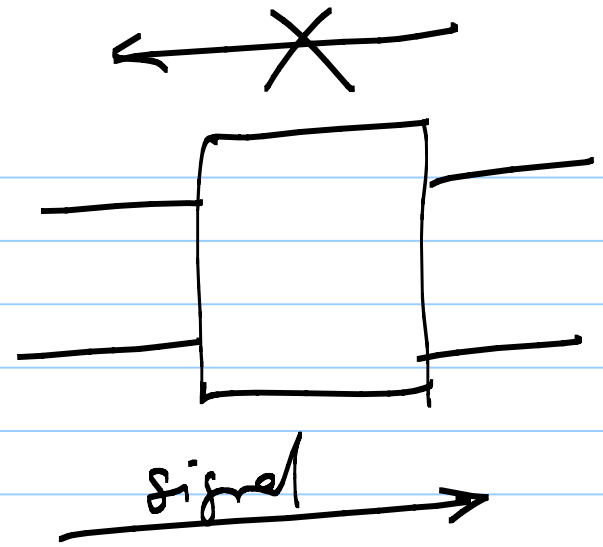
$$[Z] = \begin{bmatrix} & \\ & \end{bmatrix}$$

VCCS

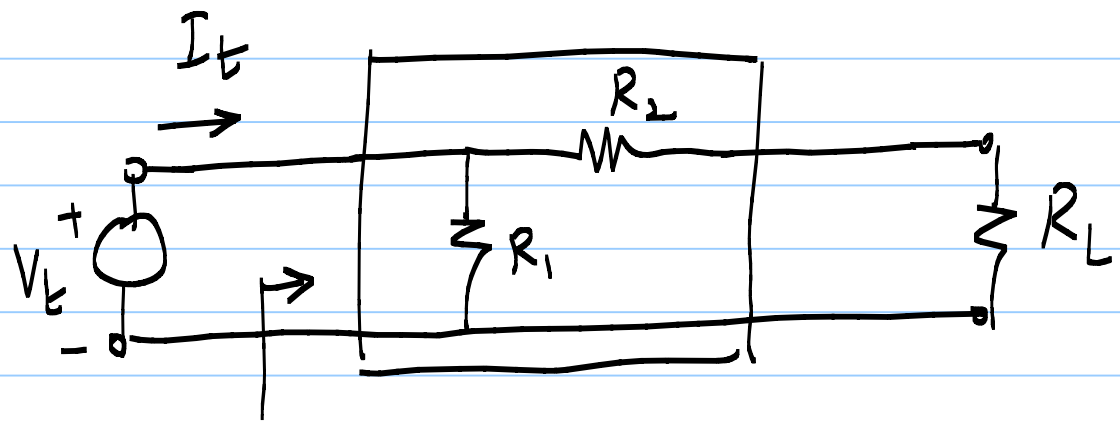


$$[Y] = \begin{bmatrix} & \\ & \end{bmatrix}$$

"Unilateral" 2-port

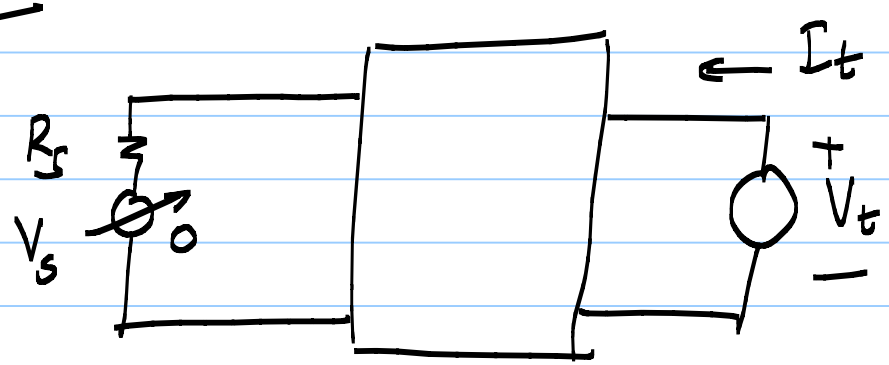


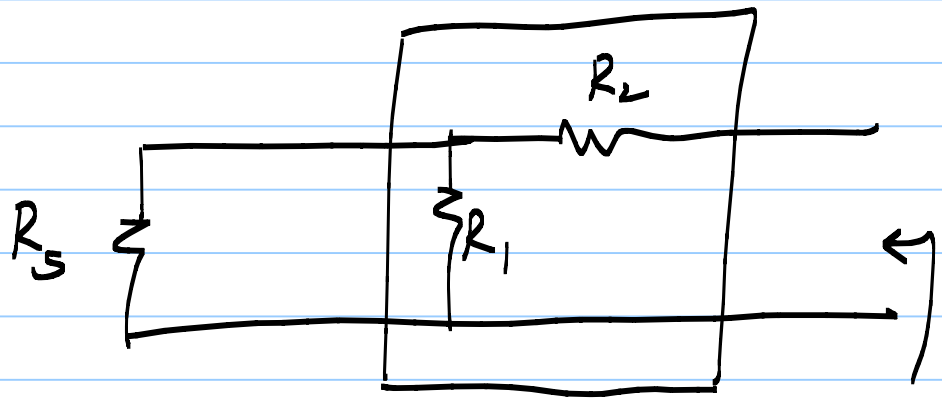
Bilateral 2-port → Xfmr, Resistive 2-ports ...



$$R_{in} = \frac{V_t}{I_t} = R_1 \| (R_2 + R_L)$$

R_{out}

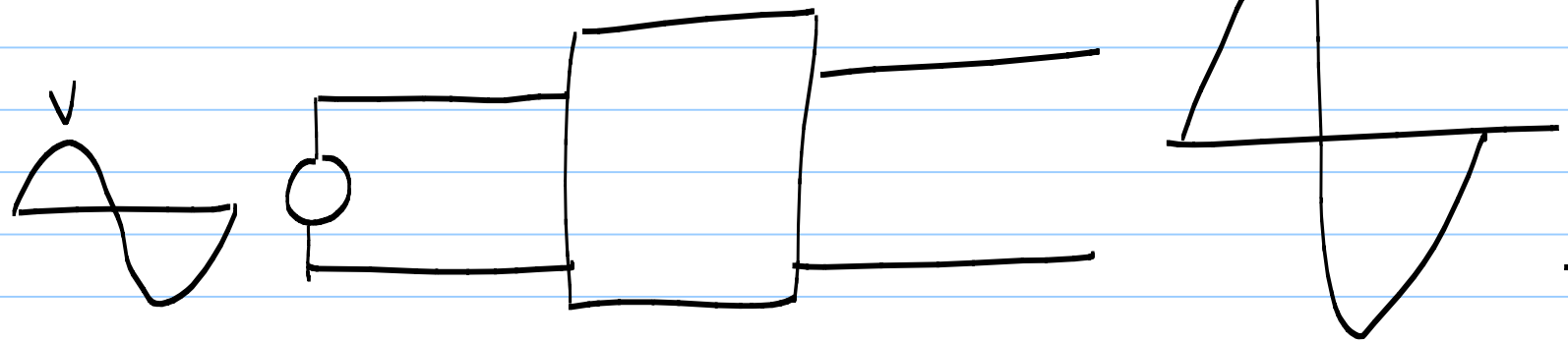




$$R_{out} = R_2 + (R_s \| R_1)$$

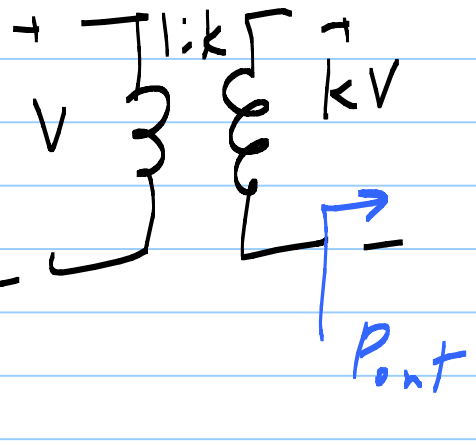
Passive 2-port

Is voltage gain possible?



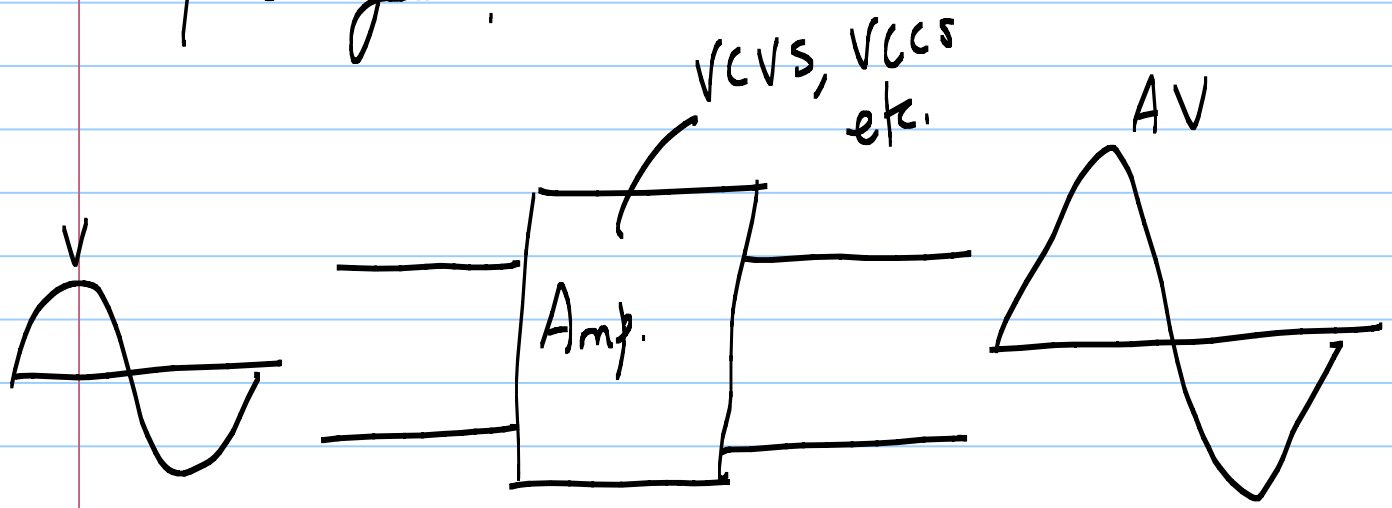
Yes

e.g. X_{fmr}

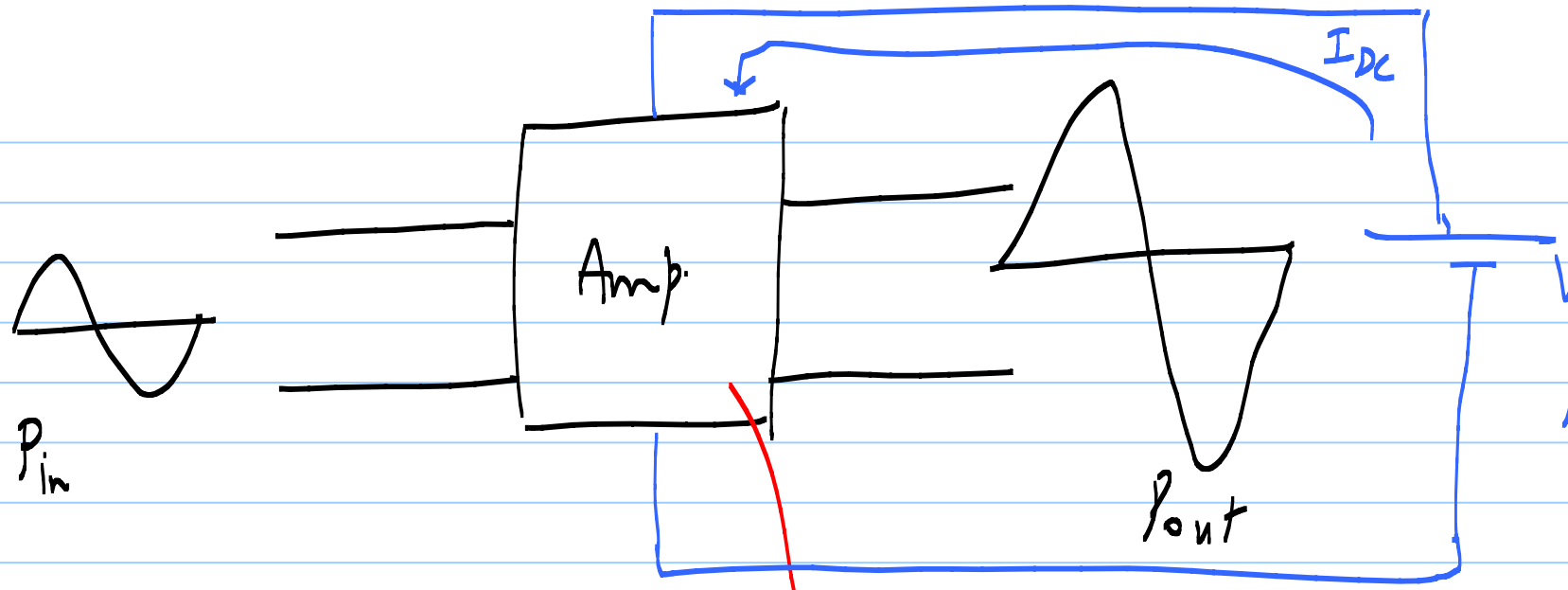


$P_{out} = P_{in}$

power gain?



We want voltage & power gain from amplifier



$P_{out} > P_{in}$

$P_{DC} = V_{DC} \cdot I_{DC}$

$P_{out} < P_{in} + P_{DC}$

R, L, C, M, transistors, diodes ...

Negative Feedback

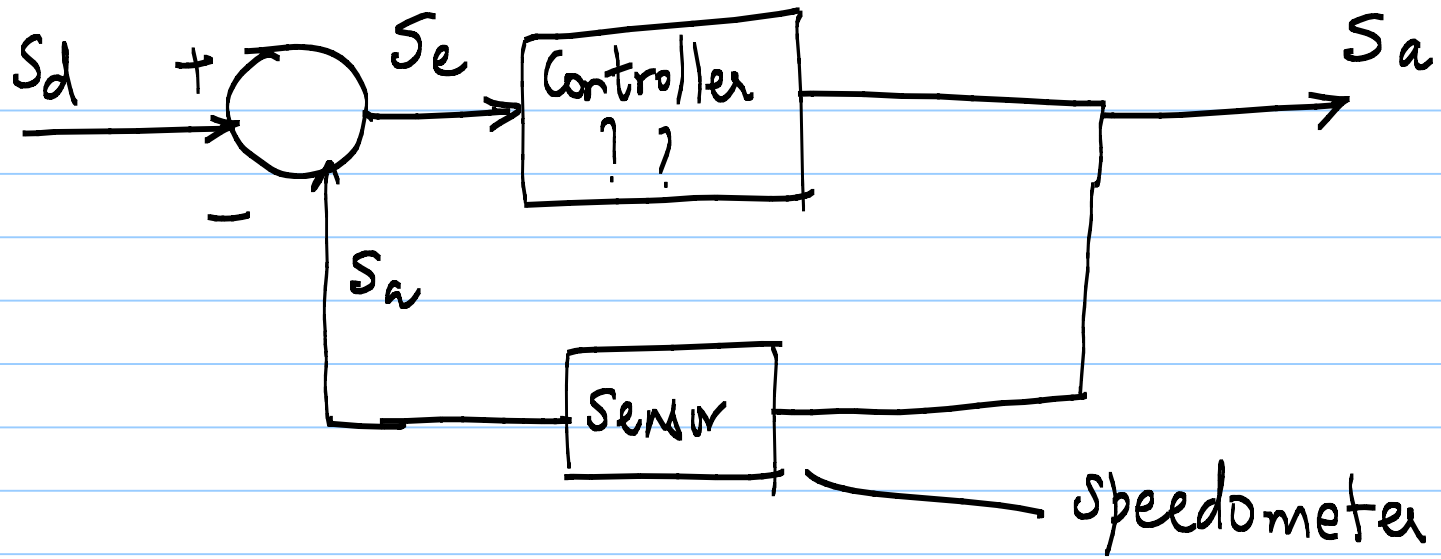
* General engineering principle
* Example : car driving
* In IITM – drive at 35 km/h

* Accelerate if speed < 35 kmph
decelerate if speed < 35 kmph

} Regulate speed.

1) Measure actual speed (speedometer)
2) Compare with desired speed
3) ↑ or ↓ speed depending on whether actual speed is lower or higher than desired speed
4) Comfortable ↑ or ↓

S_a = actual speed ; desired speed = S_d



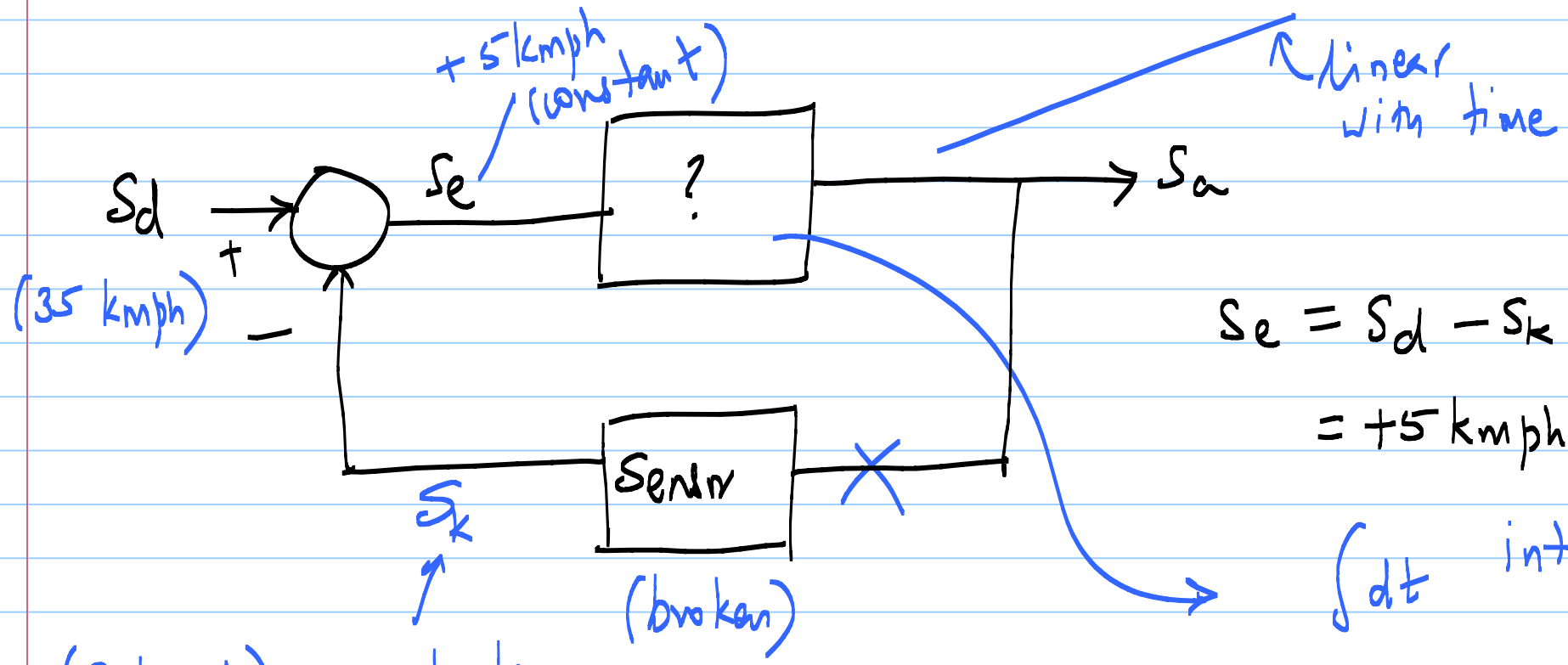
<u>Comparison</u>: $(S_d - S_a) = S_e$

Sign of S_e $\begin{cases} S_e > 0 \Rightarrow \uparrow S_a & (S_d - S_a) > 0 \Rightarrow \uparrow S_a \\ S_e < 0 \Rightarrow \downarrow S_a & (S_d - S_a) < 0 \Rightarrow \downarrow S_a \end{cases}$

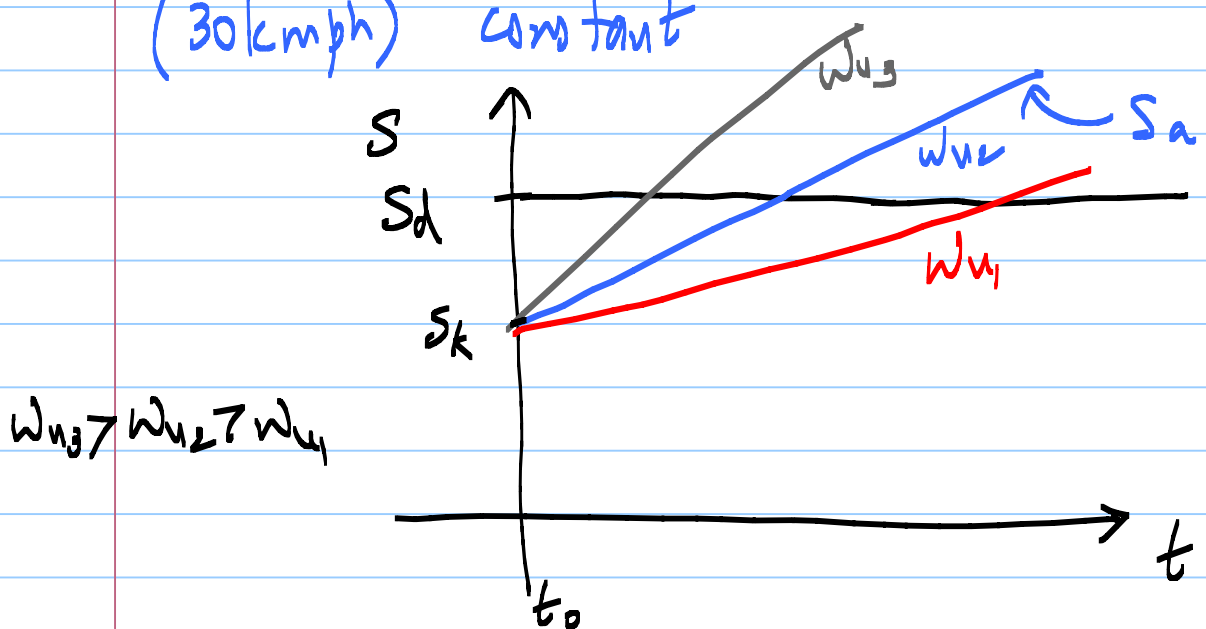
$|S_e| \Rightarrow$ how far S_a is from S_d

21/01/2022

Lecture 4



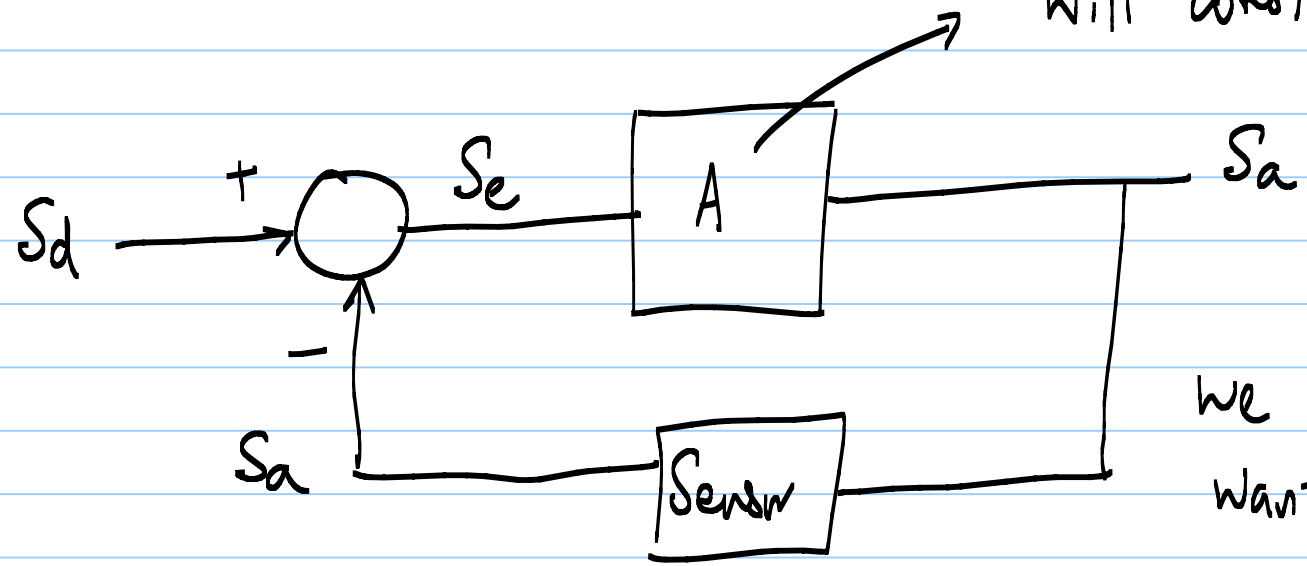
$$S_e = S_d - S_k$$
$$= +5 \text{ kmph}$$



(constant acceleration)

Controller:

$$S_a = k \int S_e \, dt$$
$$= \omega_n \int S_e \, dt$$



Will constant gain block work?

(Sensor not broken)

Steady state:

we want $S_a = S_d \quad \{S_e = 0\}$

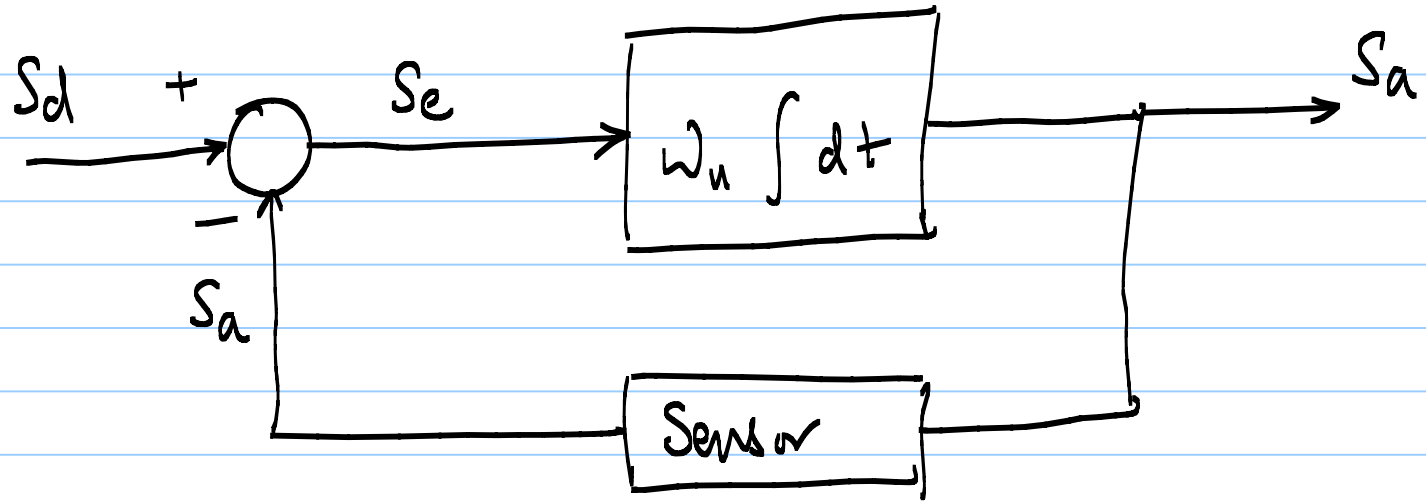
$$S_e \cdot A = S_a$$

$$(S_d - S_a) = \frac{1}{A} \cdot S_a$$

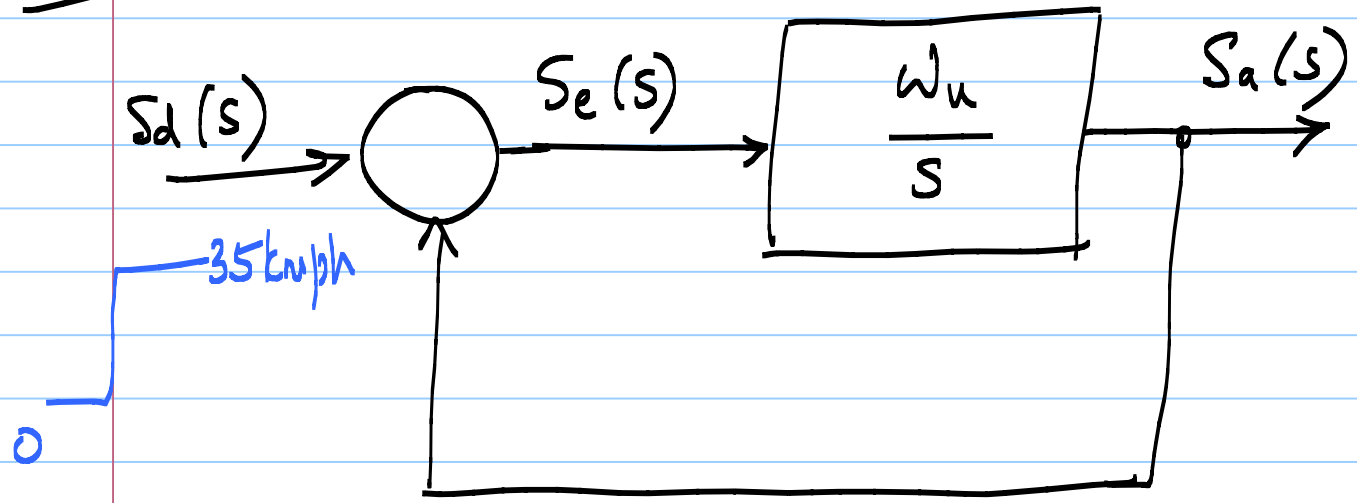
$$S_d = \left(1 + \frac{1}{A}\right) \cdot S_a$$

$$S_a = \left(\frac{A}{A+1}\right) \cdot S_d$$

only if $A = \infty$

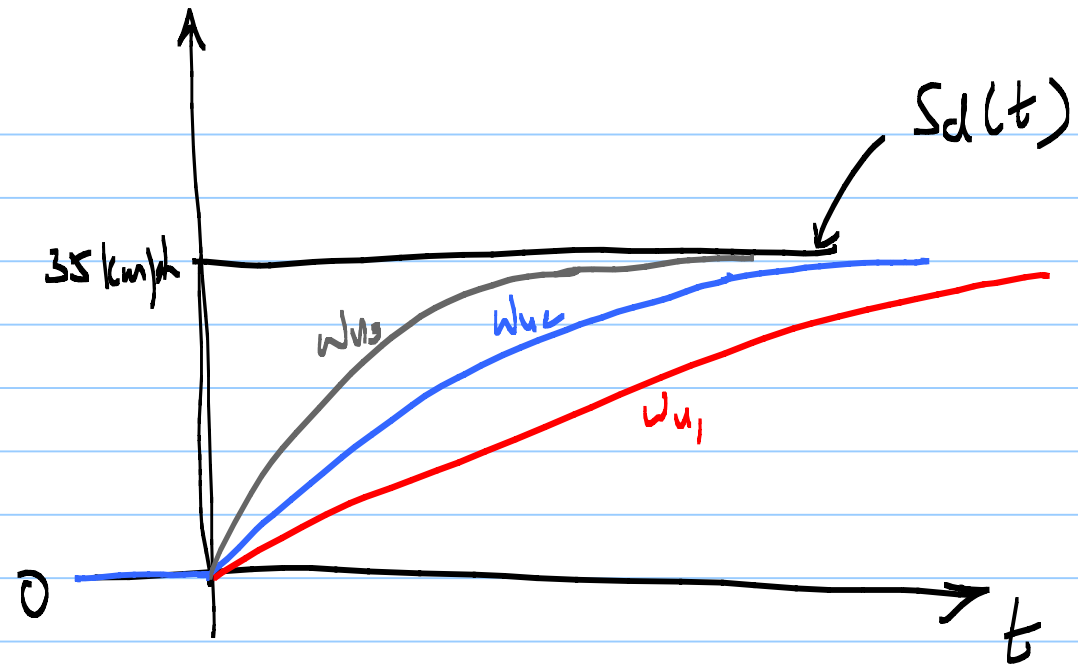


Laplace Domain



$$\frac{S_a(s)}{S_d(s)} = \frac{\omega_n}{\omega_n + s} = \frac{1}{1 + s/\omega_n}$$

$S_d(t) = 35u(t)$; $S_a(t=\infty) = ?$ HW



$$S_d(t) = 35\,u(t)$$

$$S_a(t) = 35\,u(t)\left[1 - e^{-\omega_n t}\right]$$

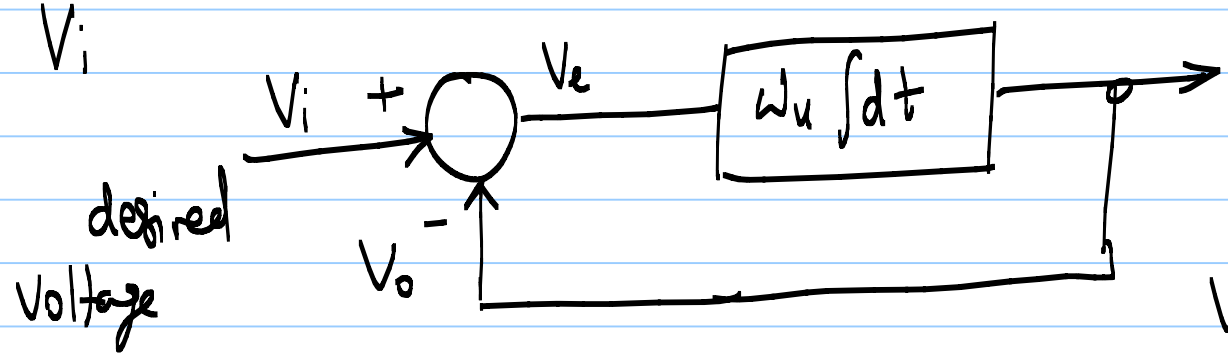
@ $t = \infty$ $\quad S_a(\infty) = 35$ kmph

$$\omega_{u3} > \omega_{u2} > \omega_{u1}$$

REFRESH Signals & Systems

Negative Feedback amplifier ?

First : $V_o = V_i$



$$V_o = \omega_u \int V_e\,dt$$

$$V_e = V_i - V_o$$

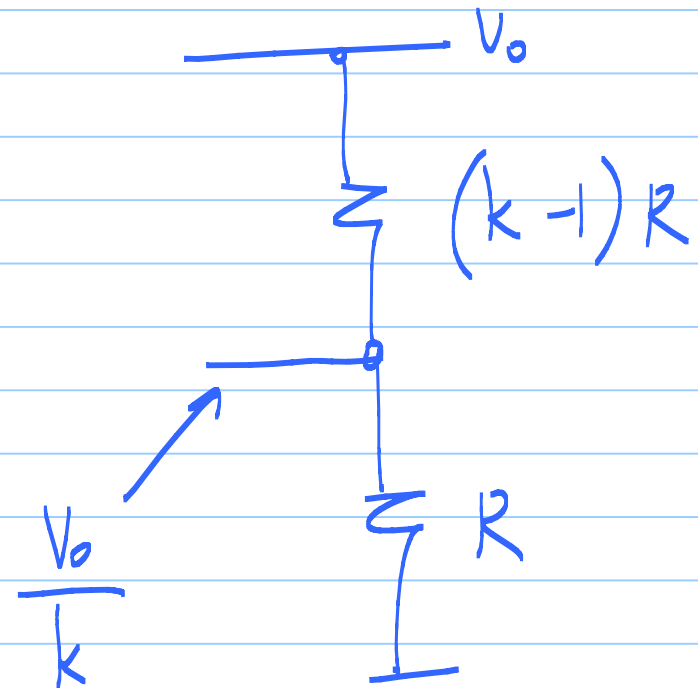
"Amplifier" :

$$V_o = k V_i$$

gain of amplifier

$$V_i = \frac{V_o}{k}$$

compare

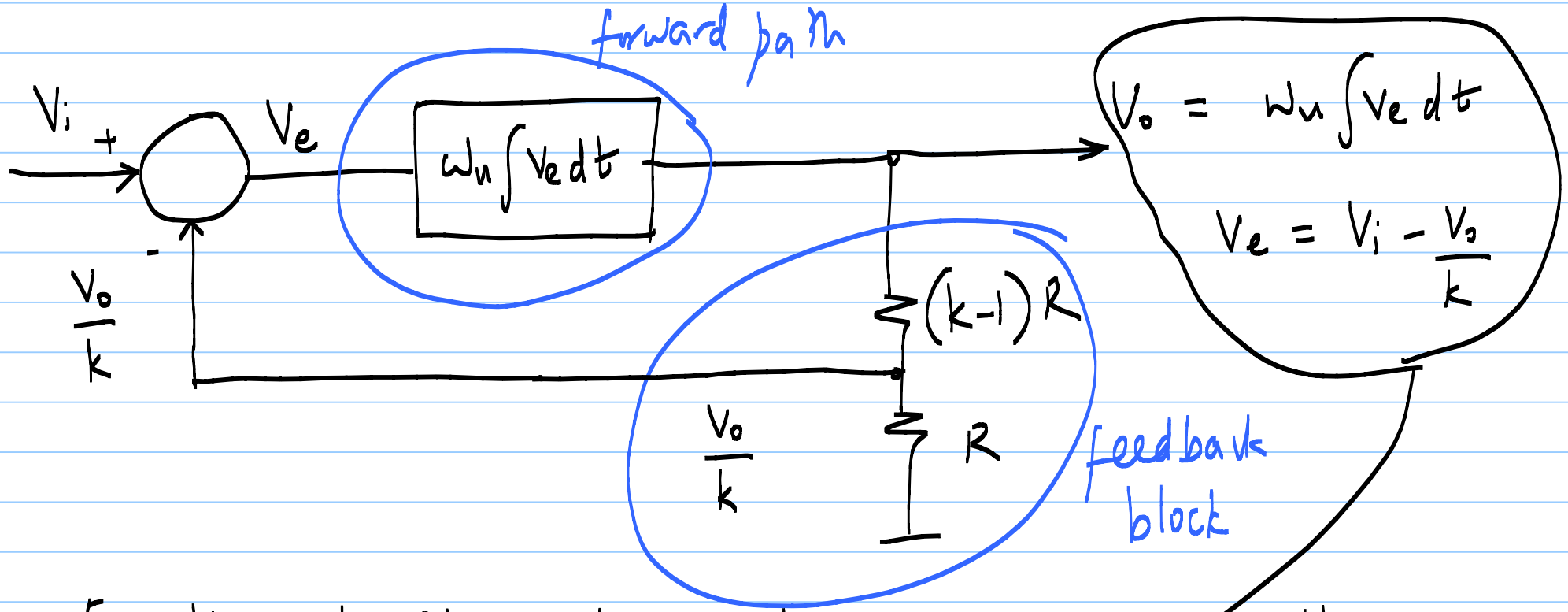


25/01/2022

Lecture 5

Voltage Amplifier

$V_o = k V_i \quad (k > 1)$

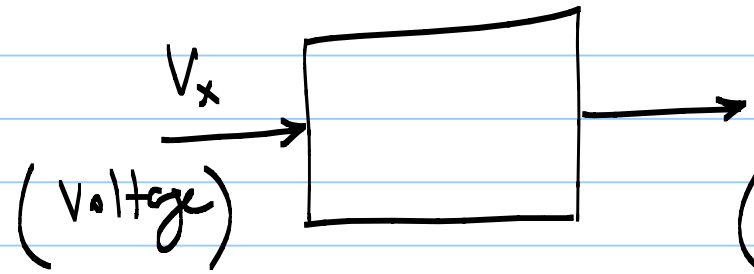


Diff. Equation describing the system:

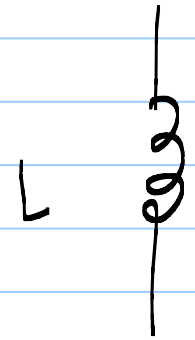
$$\boxed{\frac{dV_o}{dt} + \frac{\omega_n}{k} V_o = \omega_n V_i}$$

HW

solve in time domain & Laplace domain

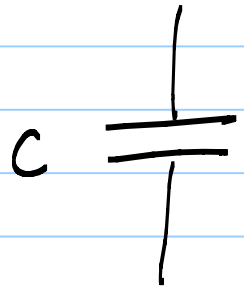


$$V_y = \omega_u \int V_x \, dt$$



$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int V dt$$

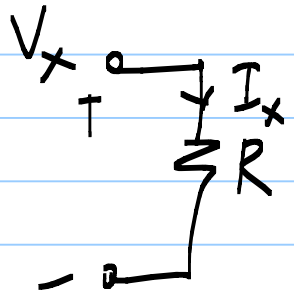


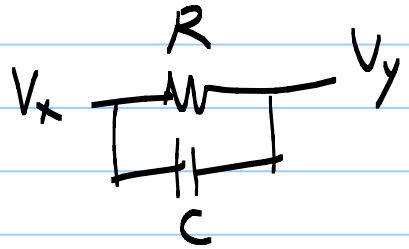
$$i = C\frac{dV}{dt}$$

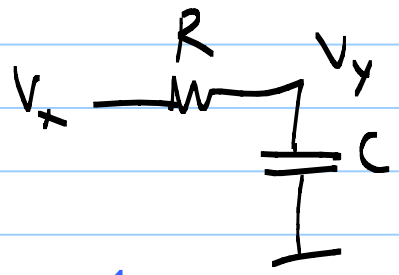
$$V = \frac{1}{C}\int i \, dt$$

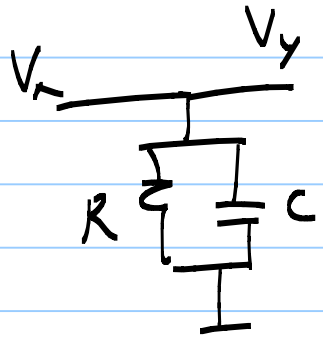
Avoid L as much as possible

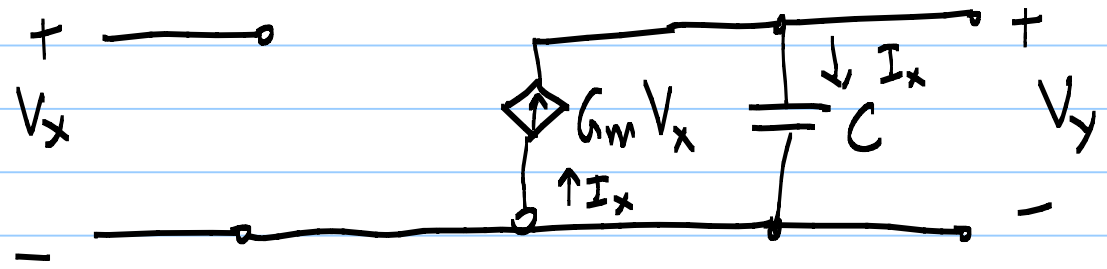
* integration
* $V \rightarrow I$ (R ?)

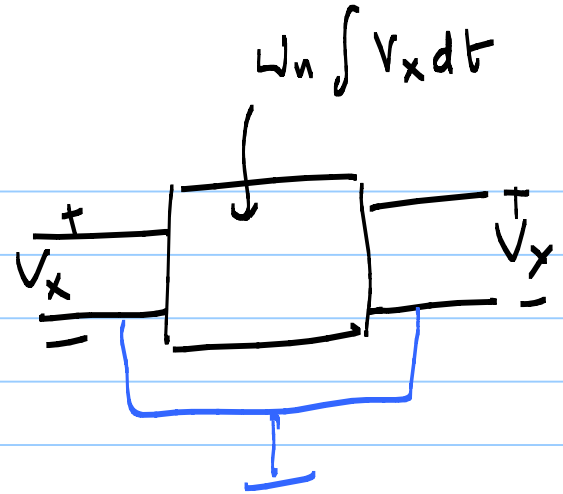










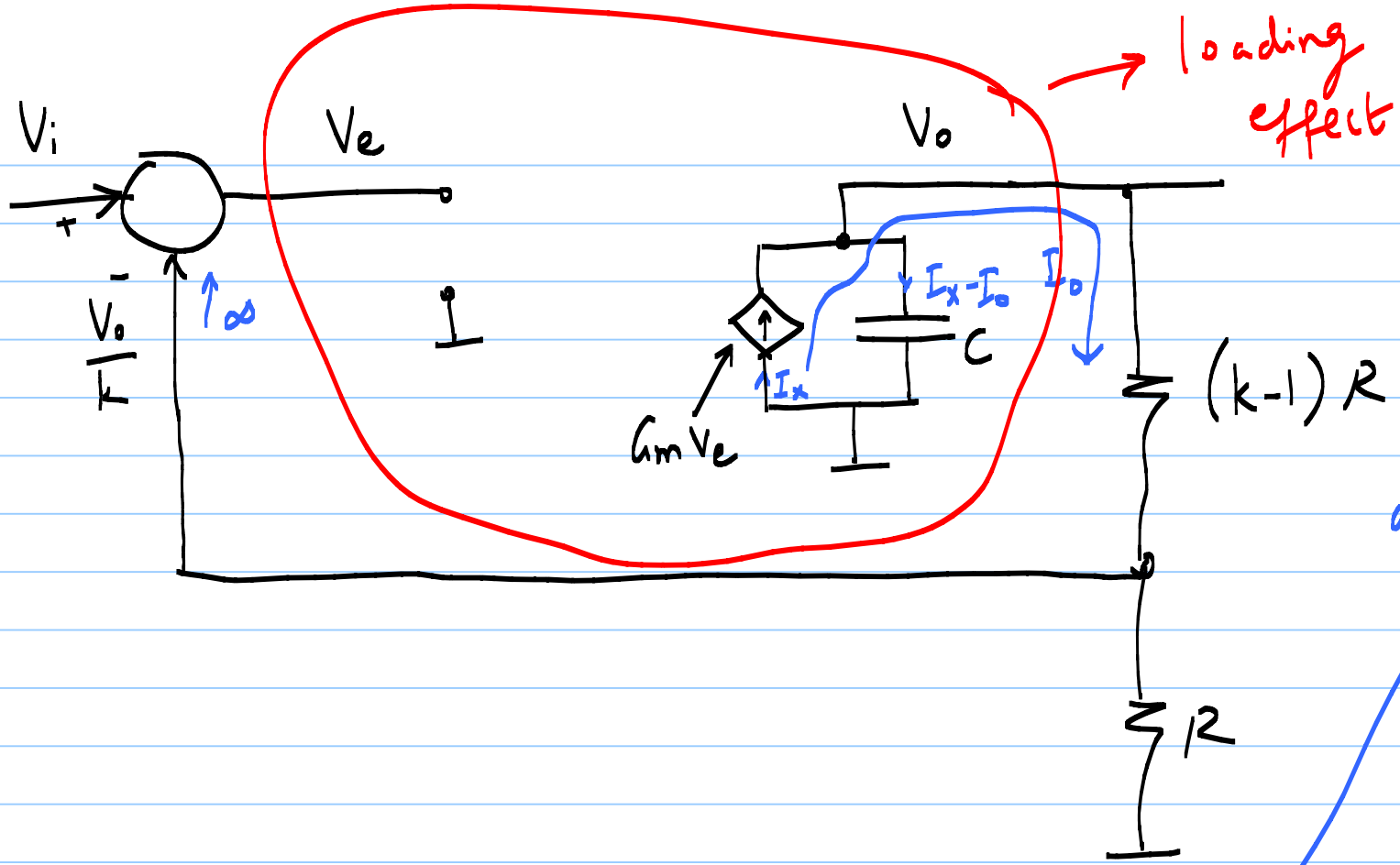


VCCS : $I_x = G_m V_x$

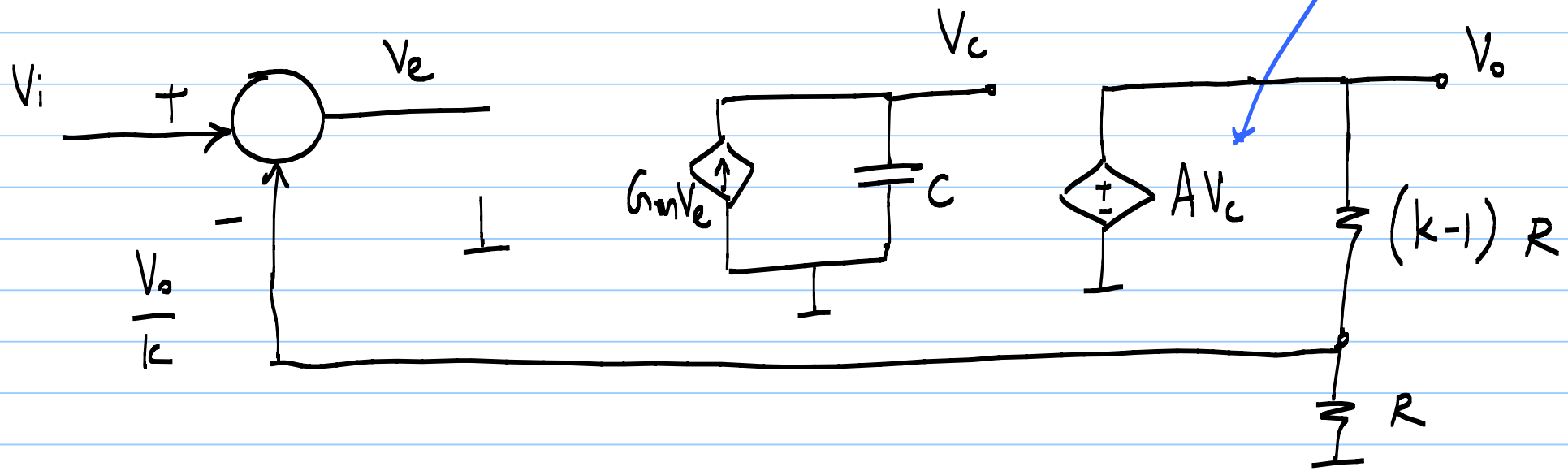
$$I_x = C \frac{dV_y}{dt}$$

$$V_y = \frac{1}{C} \int I_x \, dt = \frac{G_m}{C} \int V_x \, dt$$

$$\boxed{\omega_n = \frac{G_m}{C}}$$



added VCVS to remove effect of loading.

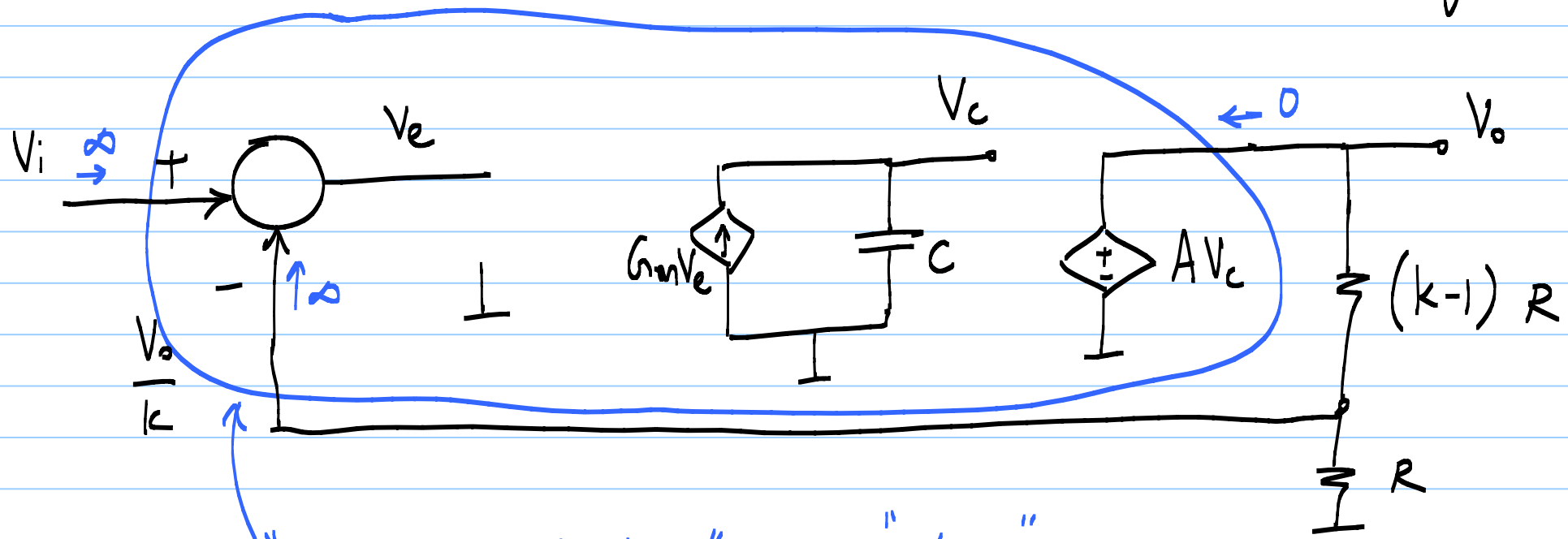


$$V_o = A V_c$$

$$V_c = \frac{G_m}{C} \int V_e \, dt$$

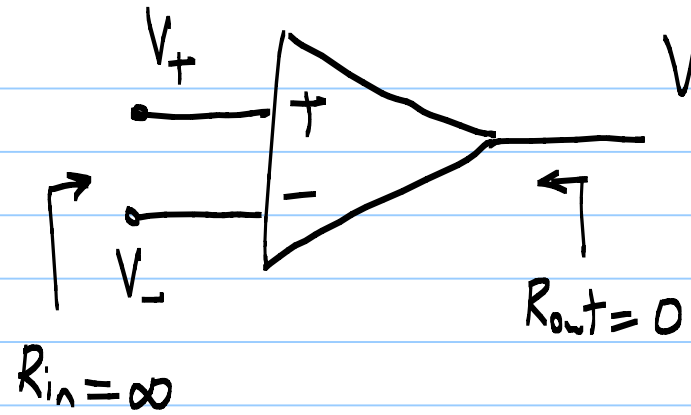
$$V_o = \underbrace{\frac{A G_m}{C}}_{\omega_u} \int V_e \, dt$$

* One possibility : $A = 1$, $\omega_u = \frac{G_m}{C}$ (We will assume this in general)



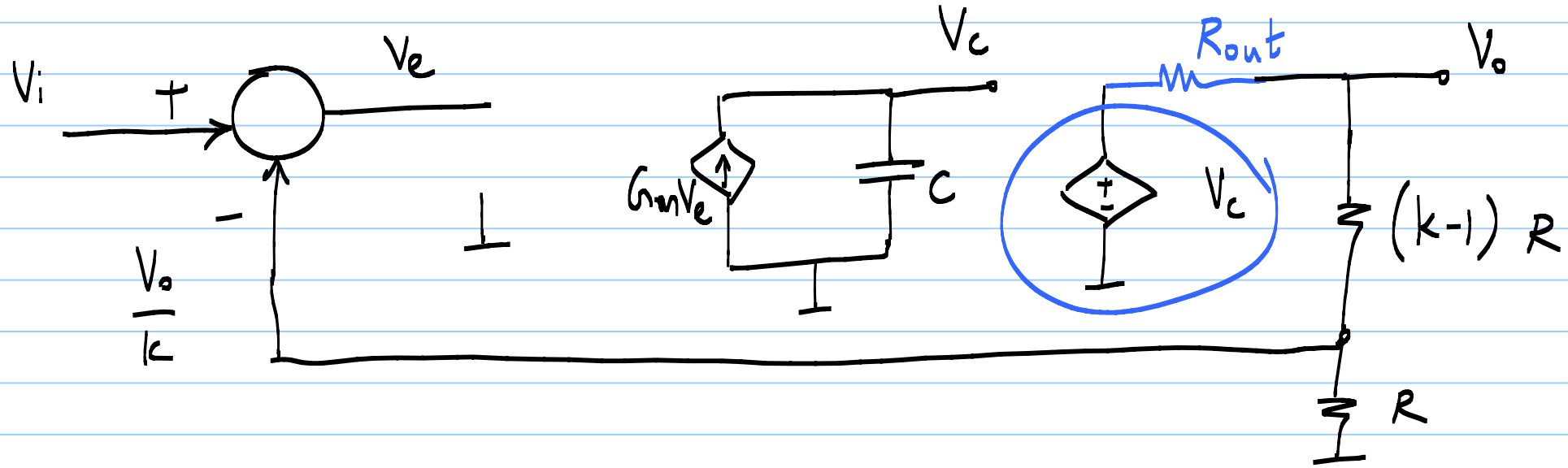
"Operational Amplifier" or "Opamp"

Ideal Opamp :

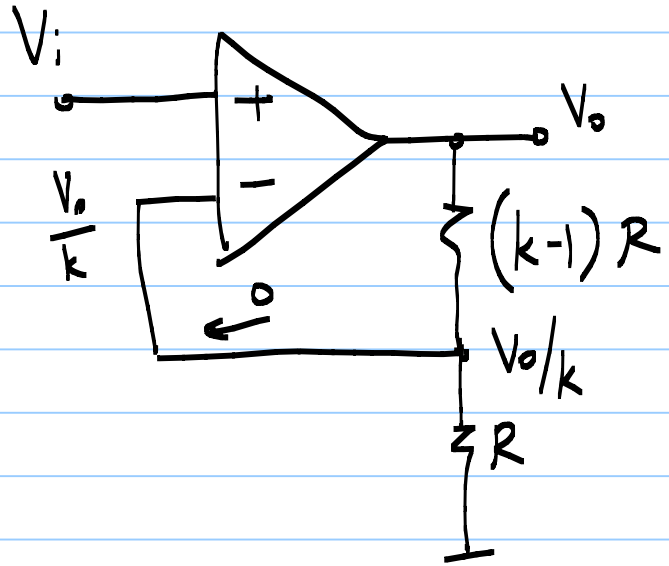


$$V_o = \frac{\omega_u}{s}\left(V_+ - V_-\right)$$

$$\omega_u = \frac{G_m}{C}$$



$\left(\frac{G_m}{C}\right) \cdot \frac{1}{s}$



In steady state

$V_e = 0$

$V_i = \frac{V_o}{k}$

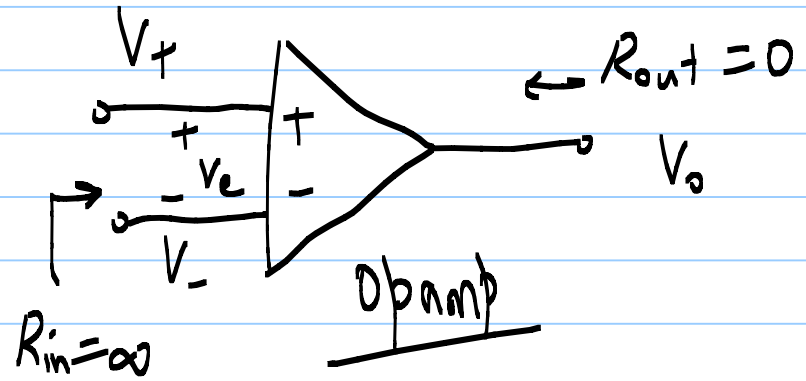
$$\boxed{V_o = kV_i}$$

mag. plot of integrator

DC gain = ∞ of integrator

27/01/2022

Lecture 6



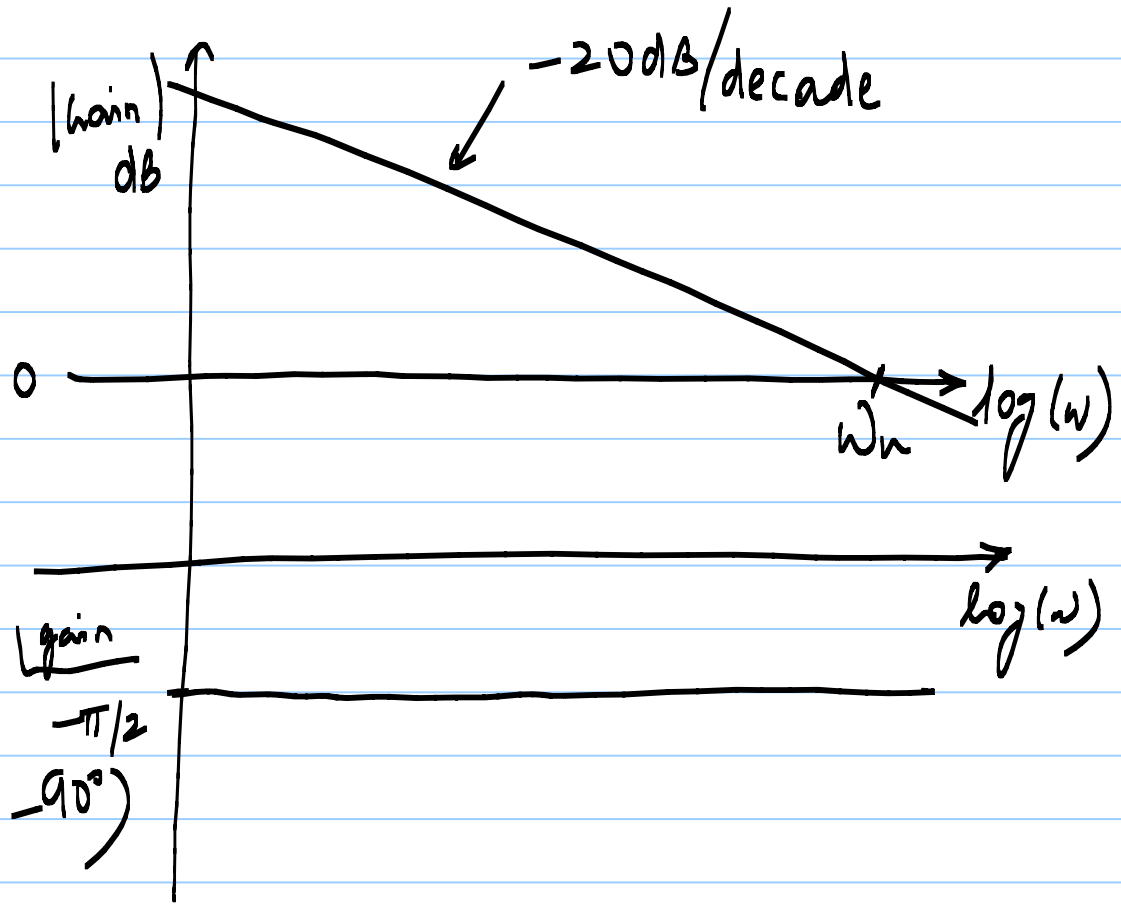
Opamp

$$V_o(s) = \frac{\omega_u}{s}\left(V_+(s) - V_-(s)\right)$$

$$V_o(s) = \frac{G_m}{sC}\left(V_+(s) - V_-(s)\right)$$

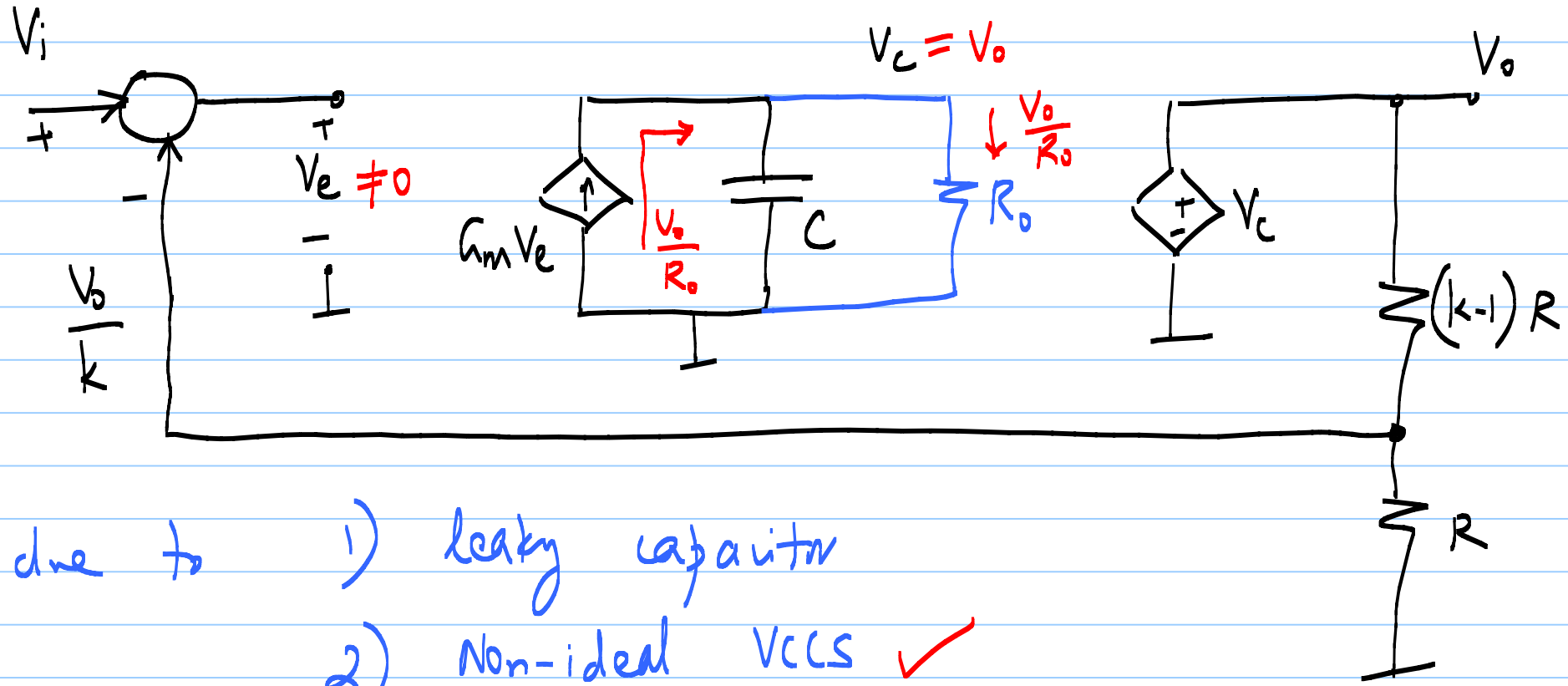
$$\frac{V_o}{V_e}(s) = \frac{G_m}{sC}$$

$$\text{Gain} = \frac{V_o}{V_e}(j\omega) = \frac{G_m}{j\omega C}$$



Gain = 0dB @ $\omega = \omega_u = \frac{G_m}{C}$

ω_u = unity gain freq.



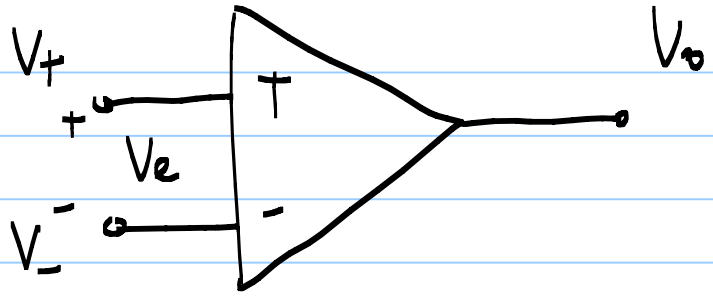
R_o due to 1) leaky capacitor

2) Non-ideal VCCS ✓
(Norton resistance)

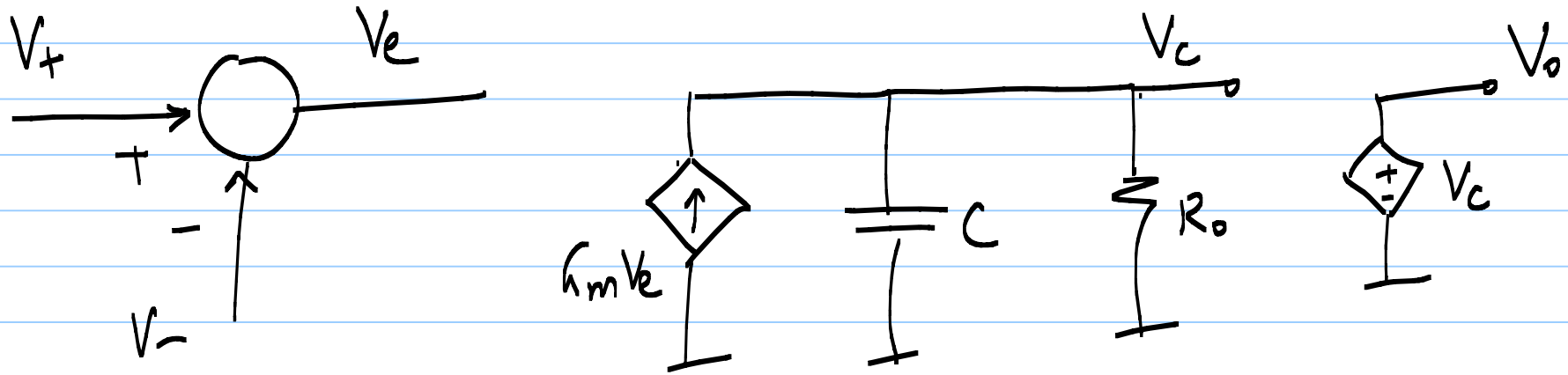
In steady state

$$G_m V_e = \frac{V_o}{R_o} \Rightarrow V_e = \frac{V_o}{G_m R_o} = V_i - \frac{V_o}{k}$$

$$\frac{V_o}{V_i} = \frac{1}{\left(\frac{1}{k} + \frac{1}{G_m R_o}\right)} = k \frac{G_m R_o}{k + G_m R_o}$$



$$\frac{V_o}{V_e}(s) = ?$$



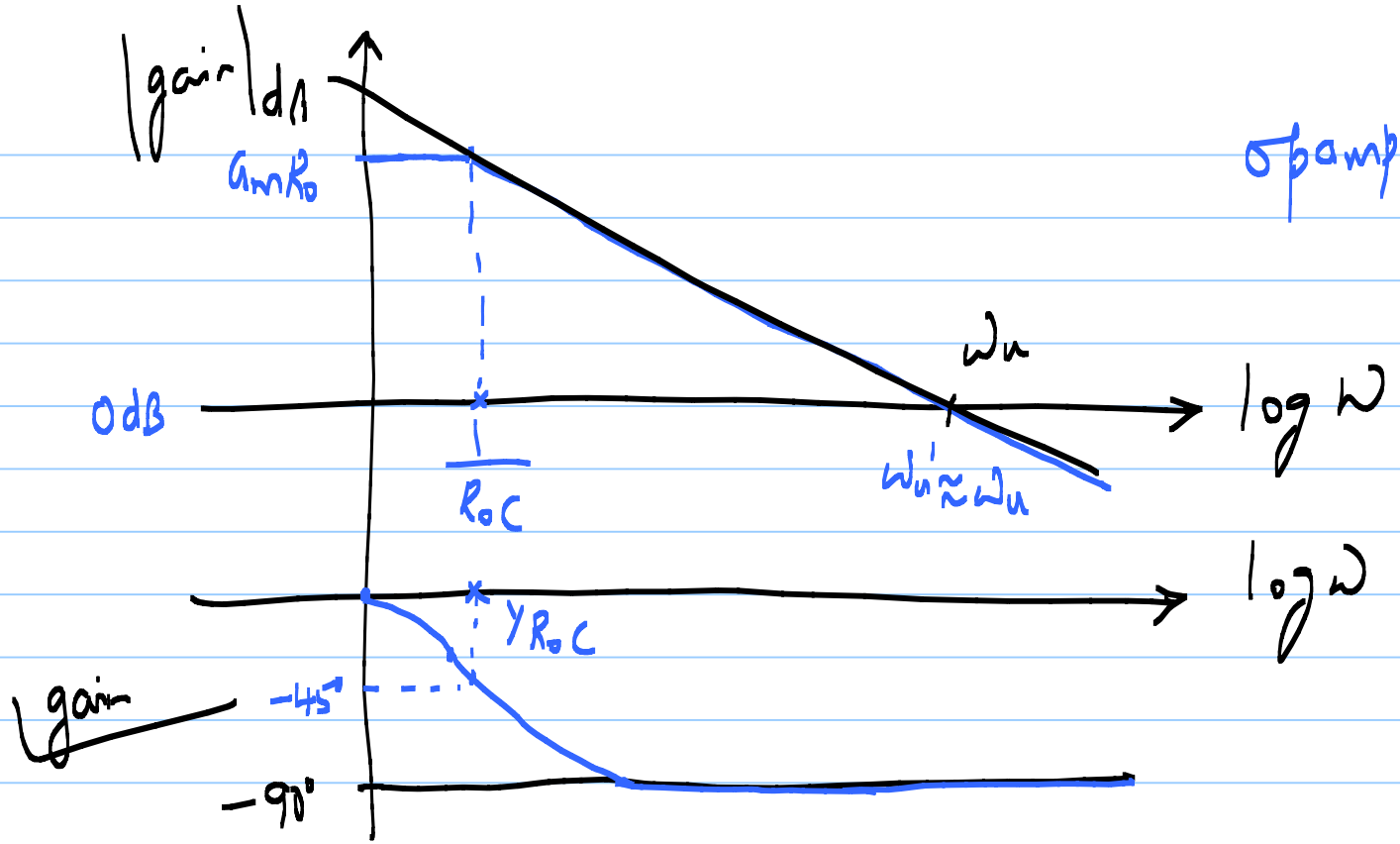
$$\boxed{\frac{V_o}{V_e}(s) = \frac{G_m R_o}{1 + sCR_o}}$$

small if $\omega > \omega_{-3db}$

DC gain $= G_m R_o$

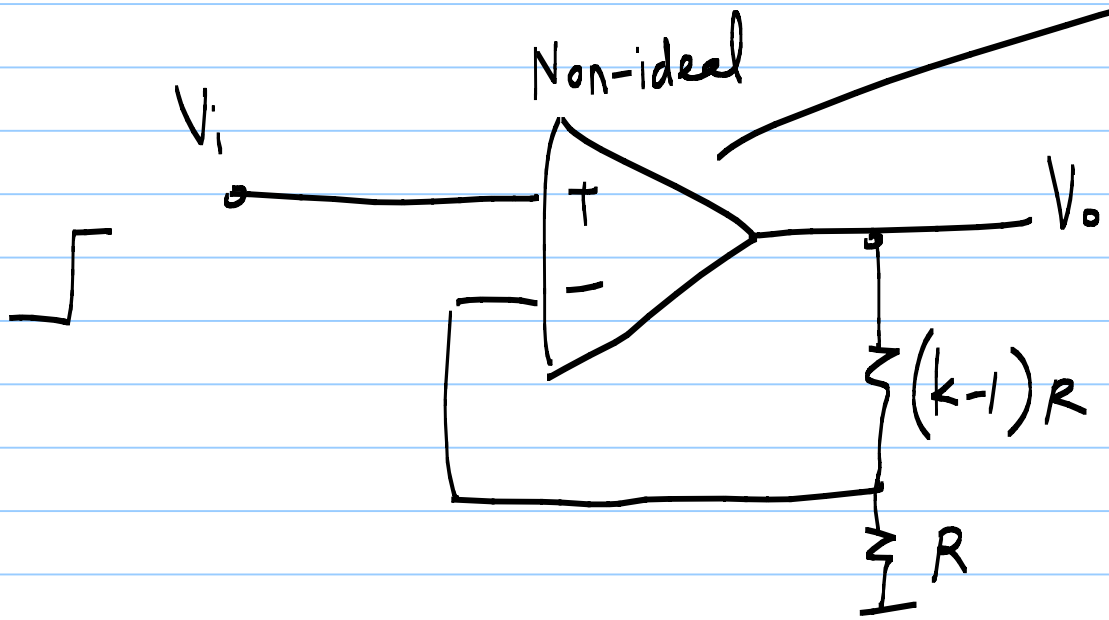
$-3db$ freq. $= \dfrac{1}{R_o C}$

new UGF $\omega_u' \approx \dfrac{G_m}{C}$



opamp gain
"forward gain"

$$\frac{G_m R_o}{1 + sCR_o}$$



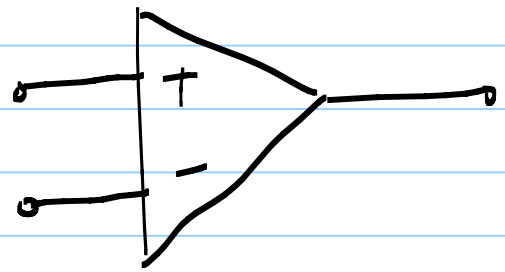
HW

$\frac{V_o}{V_i}(s)$ for the negative feedback amplifier

28/01/2022

Lecture 7

Ideal opamp

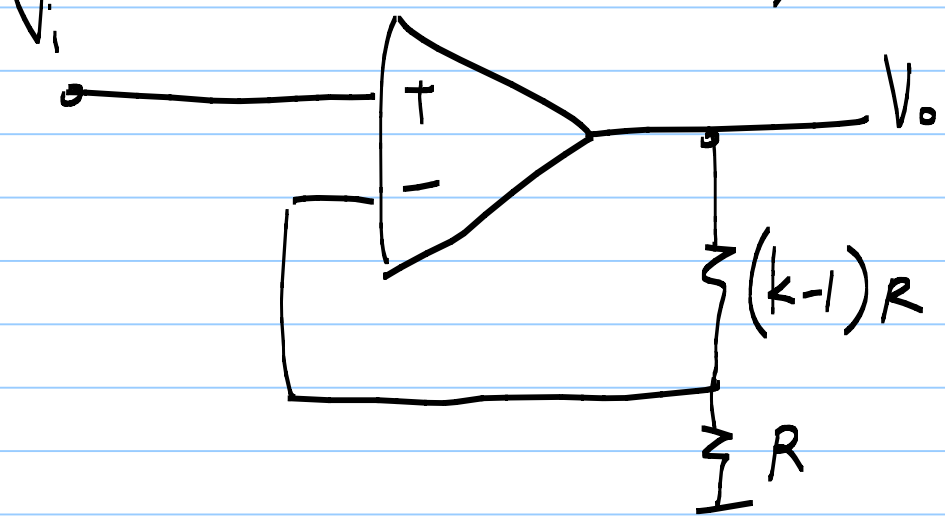


$R_{in} = \infty$

$R_{out} = 0$

$\omega_u = \infty \Rightarrow$ gain of opamp at any freq. is ∞

Non-ideal $(G_m R_o)$



$$\frac{V_o}{V_i}(s) = \frac{1}{\frac{1}{k} + \frac{1+sCR_o}{G_m R_o}}$$

closed loop gain

$$= k \cdot \frac{G_m R_o}{G_m R_o + k(1+sCR_o)}$$

$$\frac{V_o}{V_i}(s) = k \frac{1}{1 + \frac{k}{G_m R_o} + \frac{k s C}{G_m}} = k \left(\quad \right) \underbrace{\frac{1}{1 + s(\quad)}}_{\sqrt{2}}$$

DC error

What is $-3dB$ BW for $\frac{V_o}{V_i}(s)$

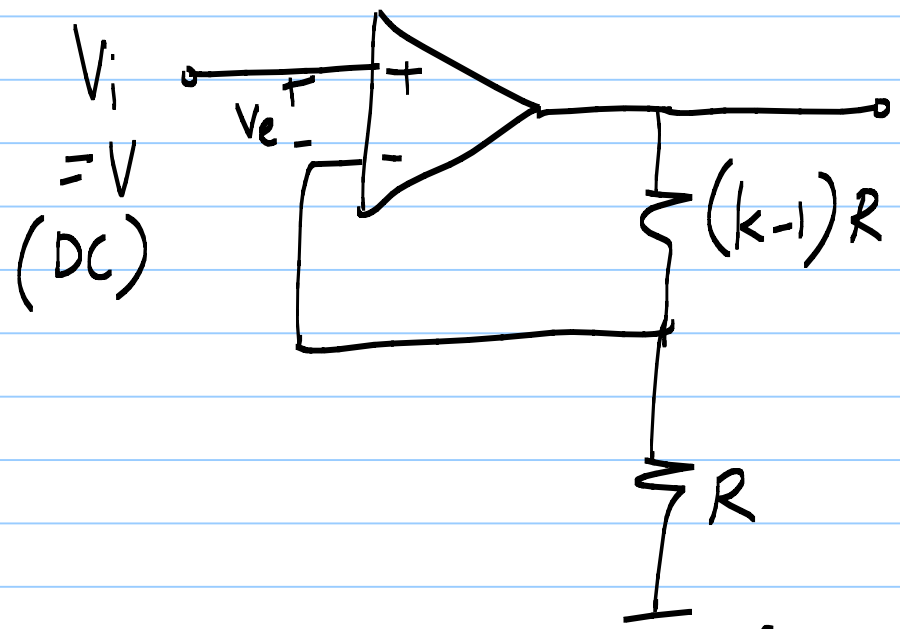
$$\frac{V_o}{V_i}(s) = k \cdot \frac{1}{1 + \frac{k}{G_m R_o}} \cdot \frac{1}{1 + s \cdot \left(\frac{kC/G_m}{1 + k/G_m R_o} \right)}$$

$$= k \cdot \left(\frac{1}{1 + \frac{k}{G_m R_o}} \right) \cdot \underbrace{\left[\frac{1}{1 + s \cdot \left(\frac{k C R_o}{k + G_m R_o} \right)} \right]}_{\text{freq. resp.}}$$

DC error

$$\omega_{-3dB} = \frac{1}{CR_o} + \frac{G_m}{kC} \checkmark$$

I DC input



$$V_o = kV \quad \text{(ideal)}$$

if $R_o = \infty$

$$= k\left(\frac{1}{1 + \frac{k}{G_m R_o}}\right)$$

DC error $= f(G_m R_o$ and $k)$

lower $G_m R_o \rightarrow$ larger DC error

higher k for given $G_m R_o \rightarrow$ larger DC error

e.g. 1) $V = 1V$

$G_m R_o = 1000$

$k = 2$

$$\text{DC error} = \left(\frac{1}{1 + \frac{2}{1000}}\right)$$

$$\approx 0.998$$

$$V_o = 2 \times 0.998 = 1.996\ V$$

(4mV DC error in output)

$$V_e = V_i - \frac{V_o}{k} = 1 - 0.998 = 0.002\ V$$

2) $V = 1V$, $G_m R_o = 100$, $k = 2$

$$\text{DC error} = \left(\frac{1}{1 + 2/100}\right) = 0.98$$

$V_o = 1.96V$ (40mV error); $V_e = 0.02V$

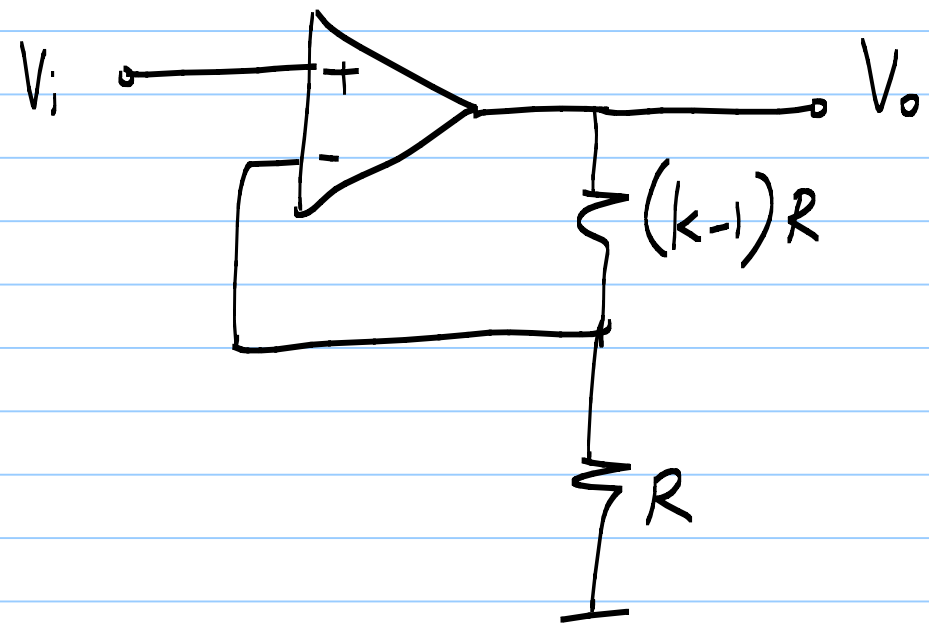
3) $V = 1V$, $G_m R_o = 100$, $k = 4$

$$\text{DC error} = \left(\frac{1}{1 + 4/100} \right) = 0.96$$

$V_o = 3.85V$

$V_e = 0.04V$

II AC input



$V_i = V \cos \omega t$

$V_o = kV \cos \omega t$

(desired) (ideal)

$$H(s) = k \cdot \left(\frac{1}{1 + \frac{k}{G_m R_o}} \right) \cdot \left[\frac{1}{1 + s \cdot \left(\frac{k C R_o}{k + G_m R_o} \right)} \right]$$

$$\downarrow$$

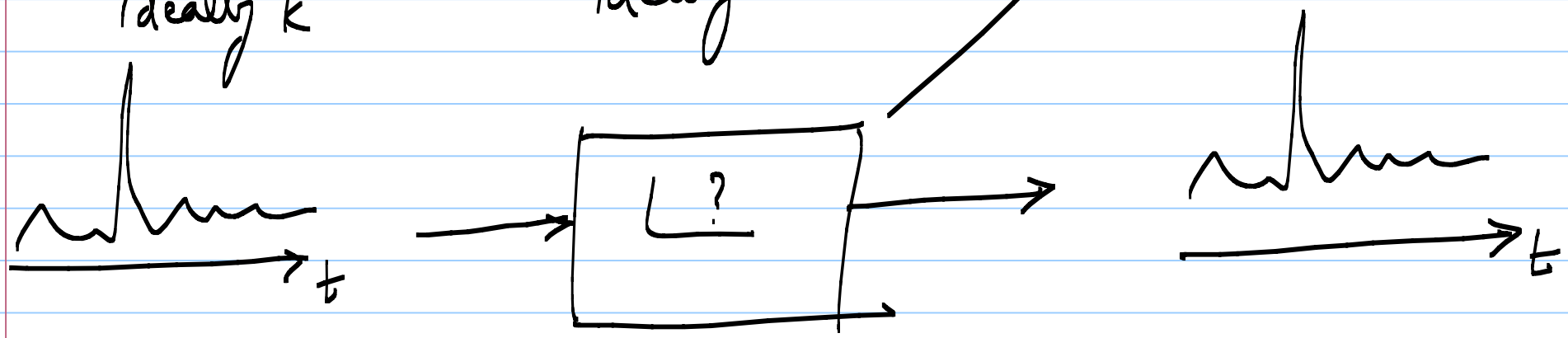
$$H(j\omega)$$

$$|H(j\omega)| \qquad \angle H(j\omega)$$

$$\Downarrow \qquad\qquad \Downarrow$$

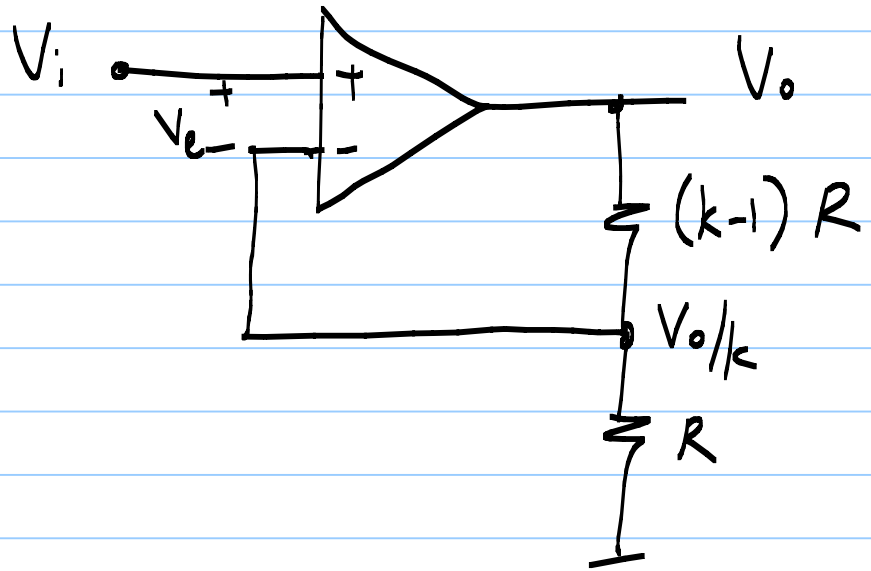
ideally k ideally 0

"constant group delay"



01/02/2022

Lecture 8



$$V_i = V\cos\omega t$$

$$V_o = kV\cos\omega t$$

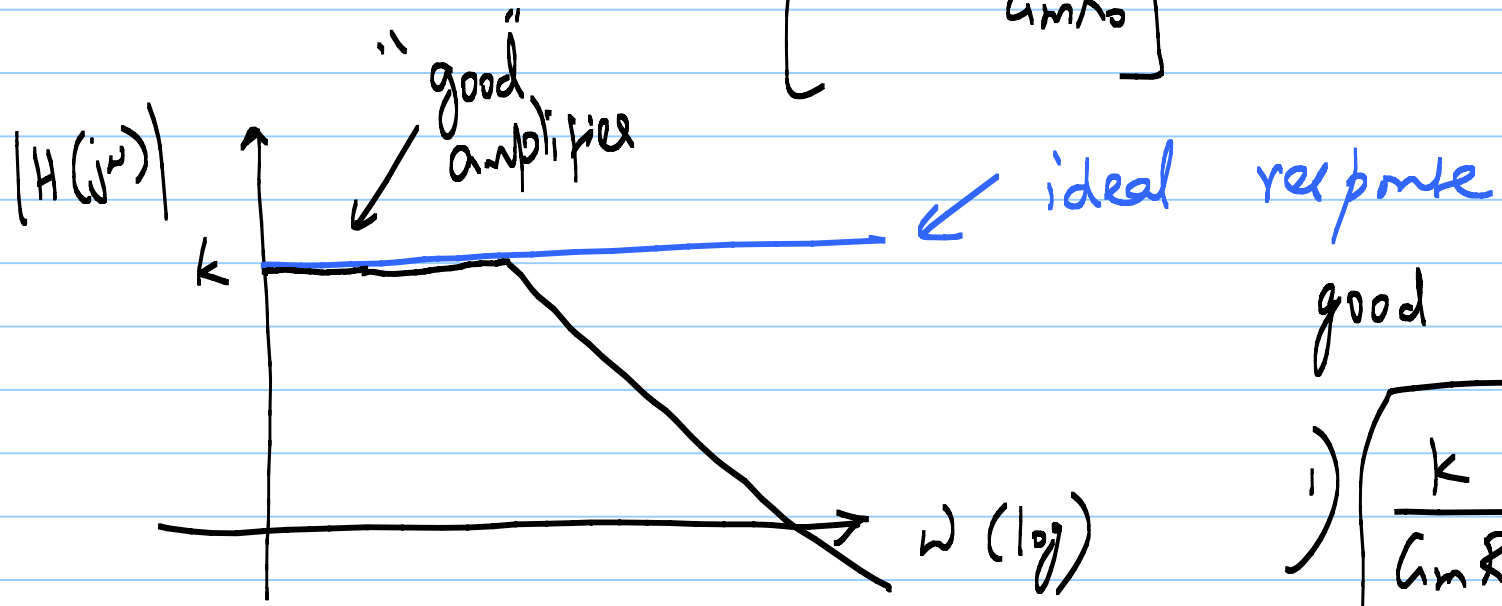
(desired) (ideal)

closed loop $$\frac{V_o}{V_i}(s) = H(s) = \frac{k}{1 + \frac{k}{G_m R_o} + \frac{skC}{G_m}}$$

$$V_o = V \cdot |H(j\omega)| \cdot \cos\left[\omega t + \angle H(j\omega)\right]$$

$$|H(j\omega)| = k \cdot \frac{1}{\left[\left(1 + k/G_m R_o\right)^2 + k^2\omega^2 C^2/G_m^2\right]^{1/2}}$$

$$\underline{H(j\omega)} = -\tan^{-1}\left[\frac{\frac{\omega c k}{G_m}}{1+\frac{k}{G_m R_o}}\right]$$



$$\boxed{\omega \ll \frac{\omega_u}{k}}$$

good amplifier if

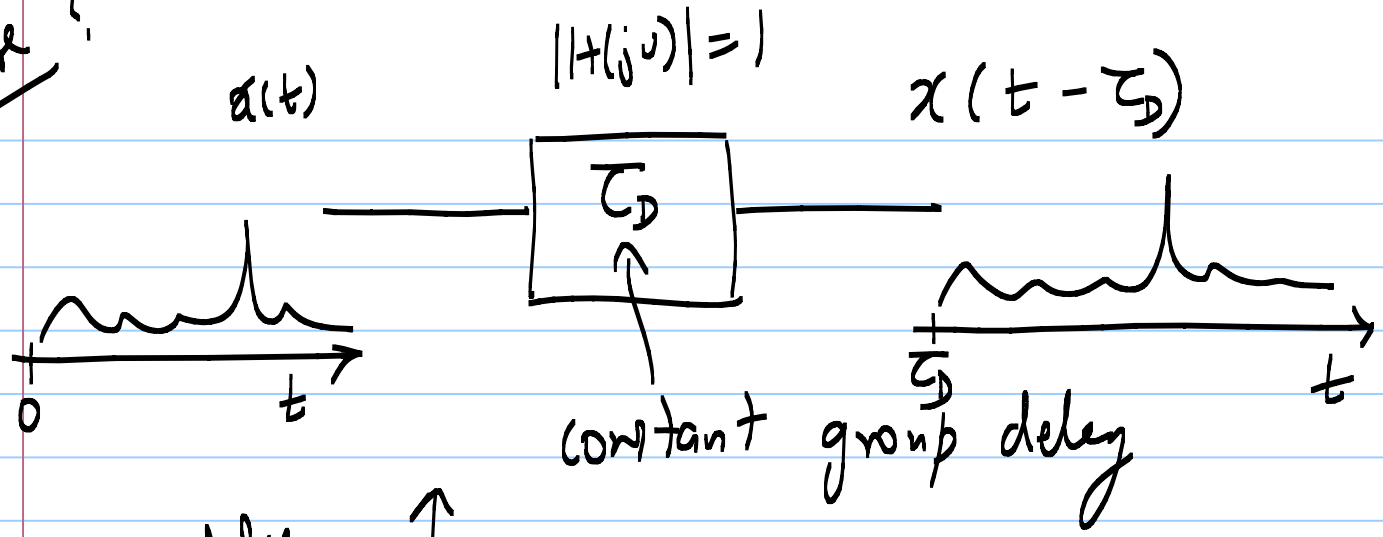
1) $\boxed{\frac{k}{G_m R_o} \ll 1}$

and

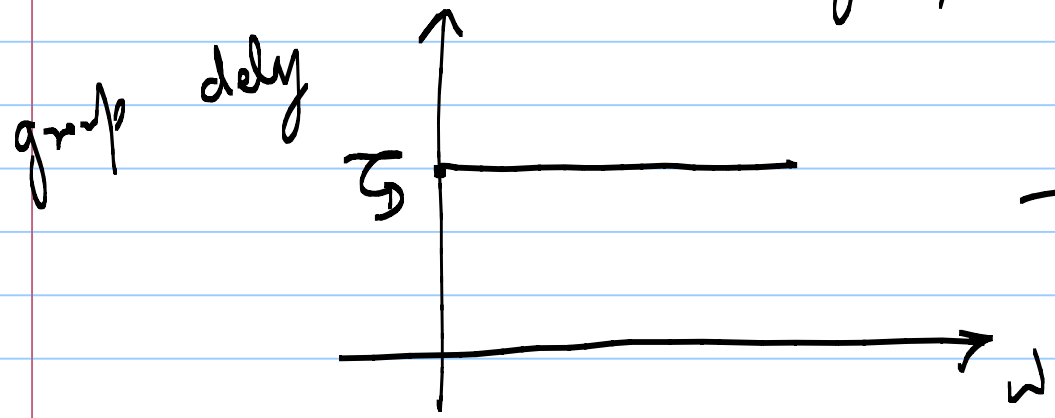
2) $\frac{k^2 \omega^2 c^2}{G_m^2} \ll 1$

$$\Rightarrow \frac{\omega^2}{(\omega_u/k)^2} \ll 1$$

phase ?

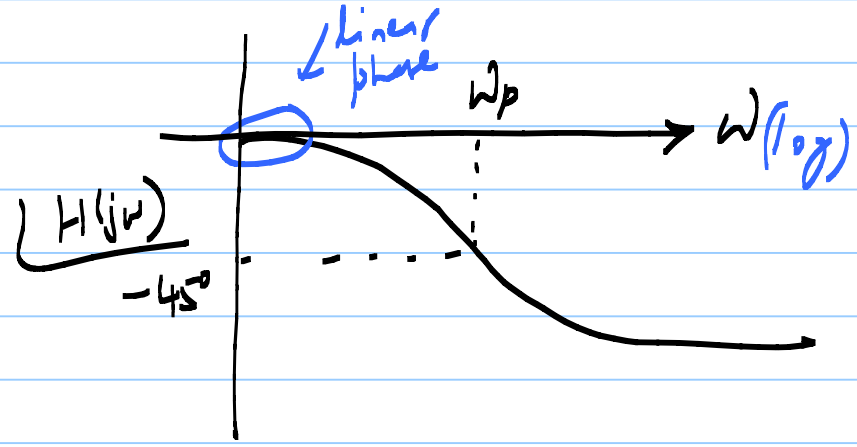


$$H(j\omega) \downarrow \tau(\omega) = -\frac{d}{d\omega}\left(\angle H(j\omega)\right)$$



$$\rightarrow \underbrace{\tau(\omega) = \tau_D}$$

$\angle H(j\omega)$ = linear w.r.t. ω



$$\underline{\angle H(j\omega)} = -\tan^{-1}\left[\frac{\dfrac{\omega c k}{G_m}}{1+\dfrac{k}{G_m R_o}}\right]$$

we want linear with ω

$$\Rightarrow \frac{\omega c k / G_m}{1 + k/G_m R_o} \text{ is small}$$

$$\Rightarrow \boxed{\omega \ll \omega_n / k} \quad \text{and} \quad \boxed{k / G_m R_o \ll 1}$$

* Closed loop response relatively insensitive to small variations in $G_m R_o$ & ω_u

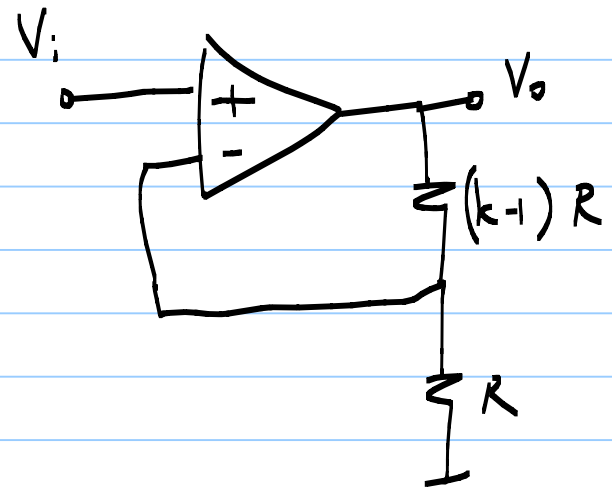
$$\frac{k}{G_m R_o} \ll 1 \quad \& \quad \omega \ll \frac{\omega_u}{k}$$

* Circuit so far: valid for $k > 1$ (+ve gain)

* We may also want $k < -1$

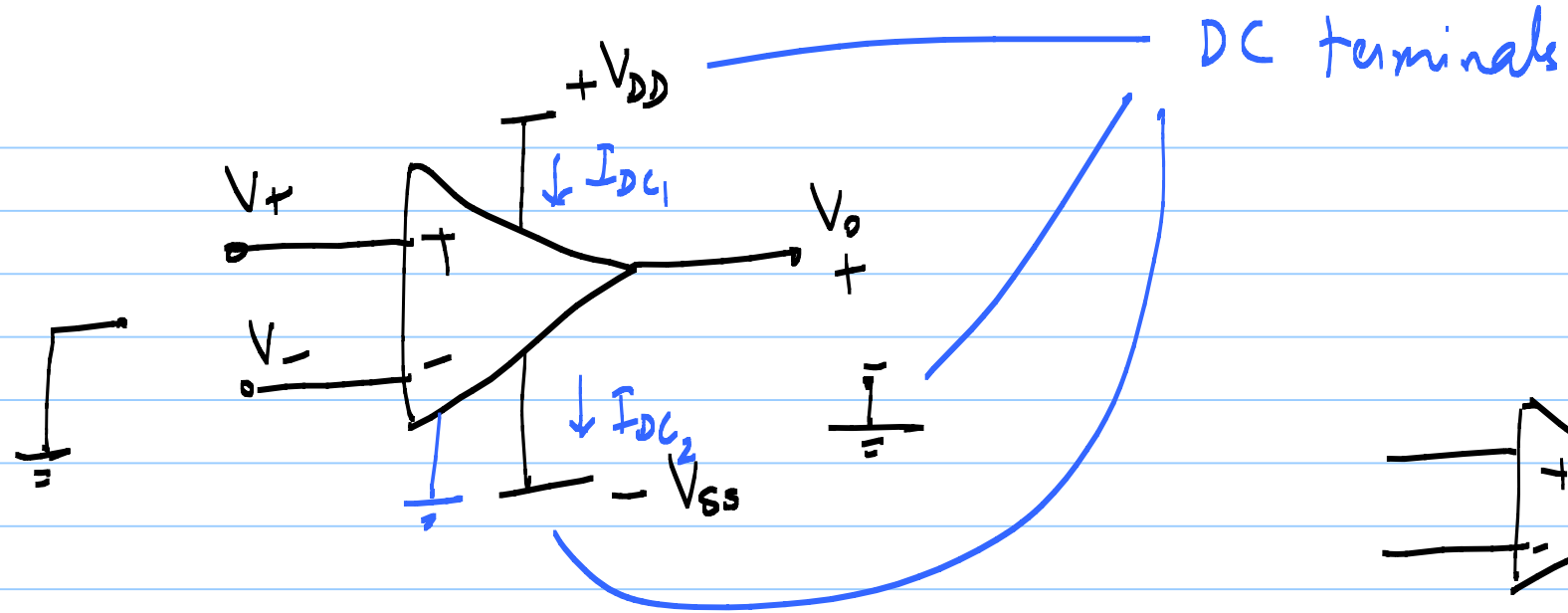
$\Rightarrow$ inverting amplifier

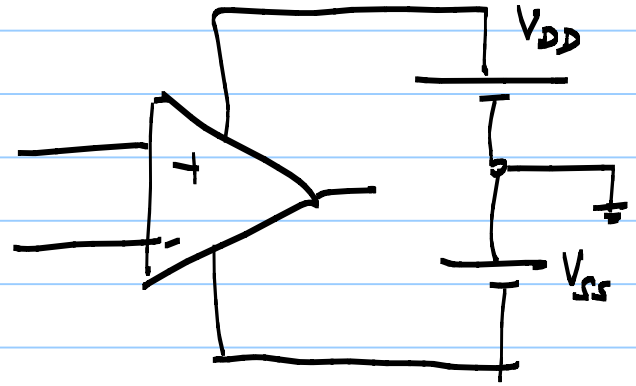
* So far - Non-inverting amplifier $\longrightarrow$



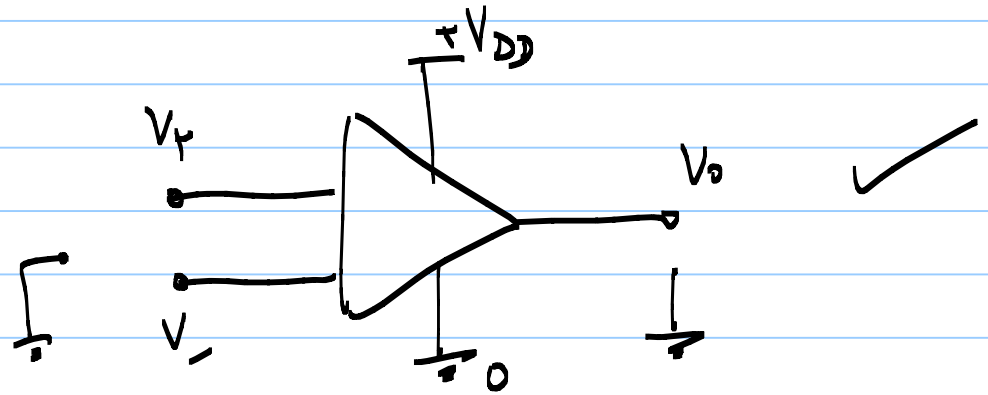
* Opamp has hidden terminals

1)

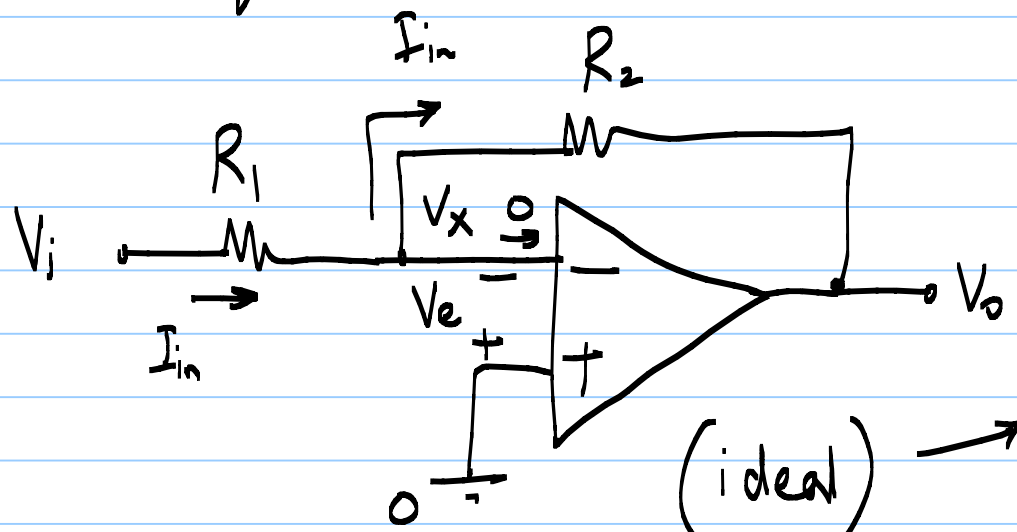




2)



Inverting Amplifier



(ideal) $\rightarrow R_{in} = \infty$

$\hookrightarrow G_m R_o = \infty \rightarrow V_e = 0$ in steady state

$$V_x = 0 \Rightarrow I_{in} = \frac{V_i}{R_1} \Rightarrow \text{flows through } R_2$$

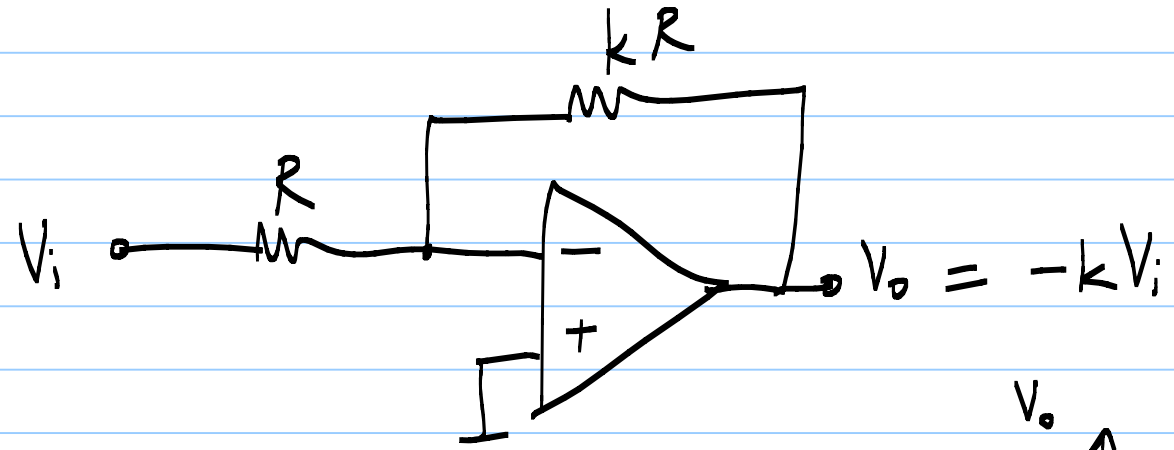
$$V_x - V_o = I_{in} R_2$$

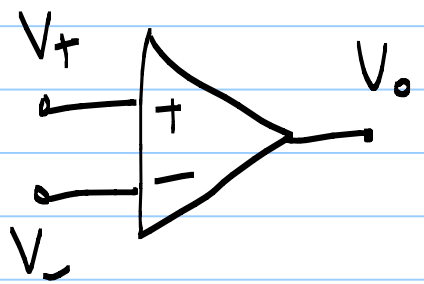
$$0 - V_o = \frac{V_i}{R_1} \cdot R_2 \Rightarrow \boxed{\frac{V_o}{V_i} = \frac{-R_2}{R_1}}$$

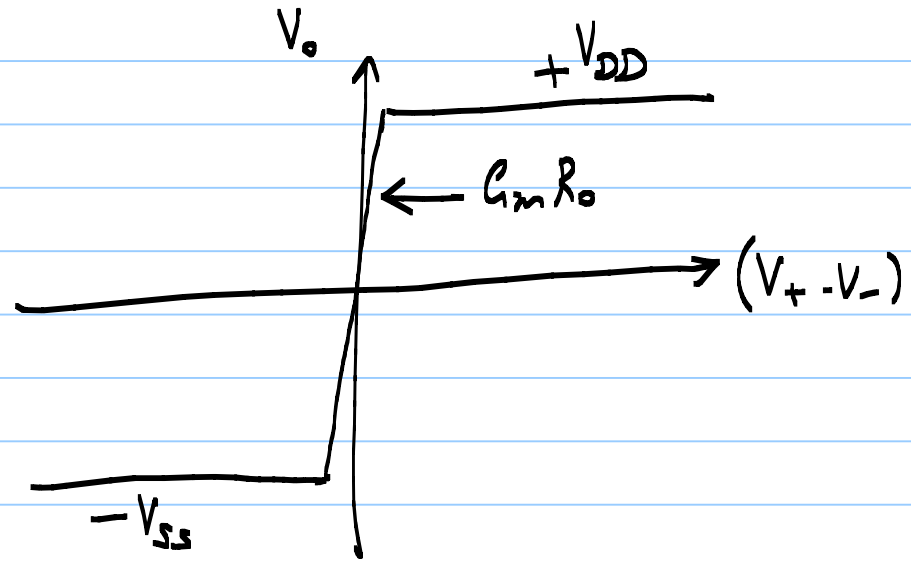
* Gain is inverting

* Choose $R_2 > R_1 \Rightarrow |k| > 1$ — CL gain

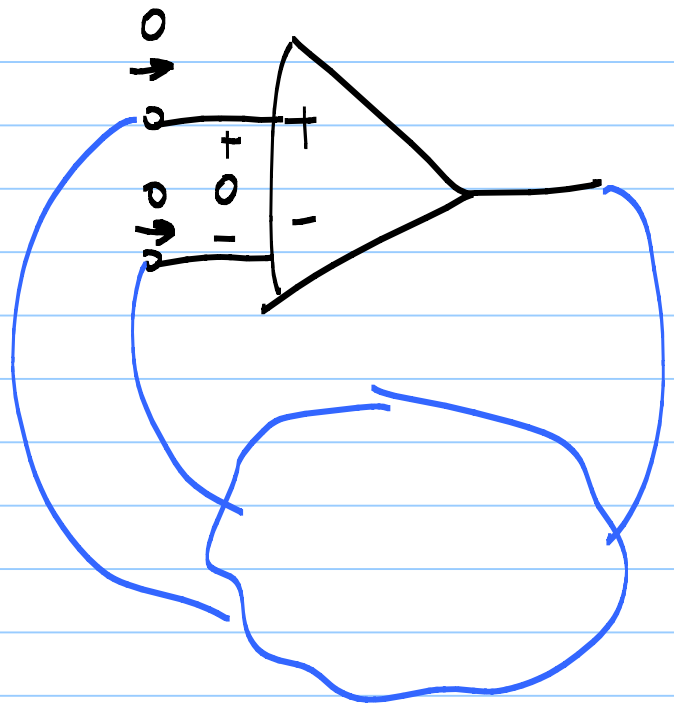
* Gain $= -k$

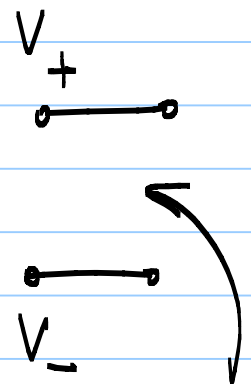






If opamp is placed in negative feedback:





creates

simultaneous

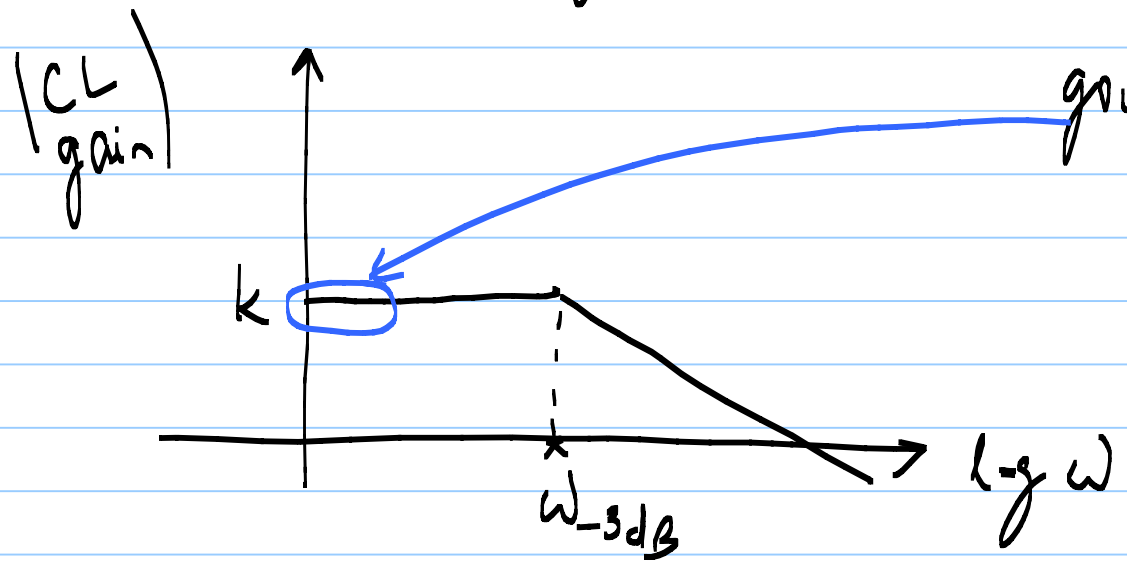
open & short circuit

02/02/2022

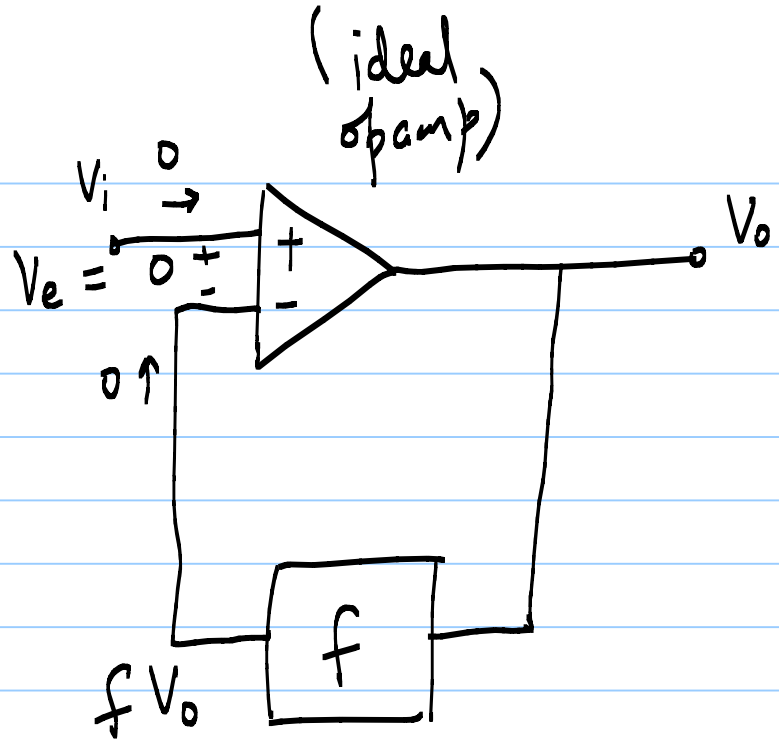
Lecture 9

$$\left(\begin{matrix}\text{closed}\\ \text{loop}\end{matrix}\right) \quad \omega_{-3dB} = \frac{1}{R_o C} + \frac{G_m}{kC} \approx \frac{G_m}{kC} \approx \frac{\omega_u}{k}$$

(R_o as large as possible)



good amplifier for $\omega \ll \omega_{-3dB}$



$$\frac{V_o}{V_i} = \frac{1}{f} \quad (-ve\ f.b.)$$

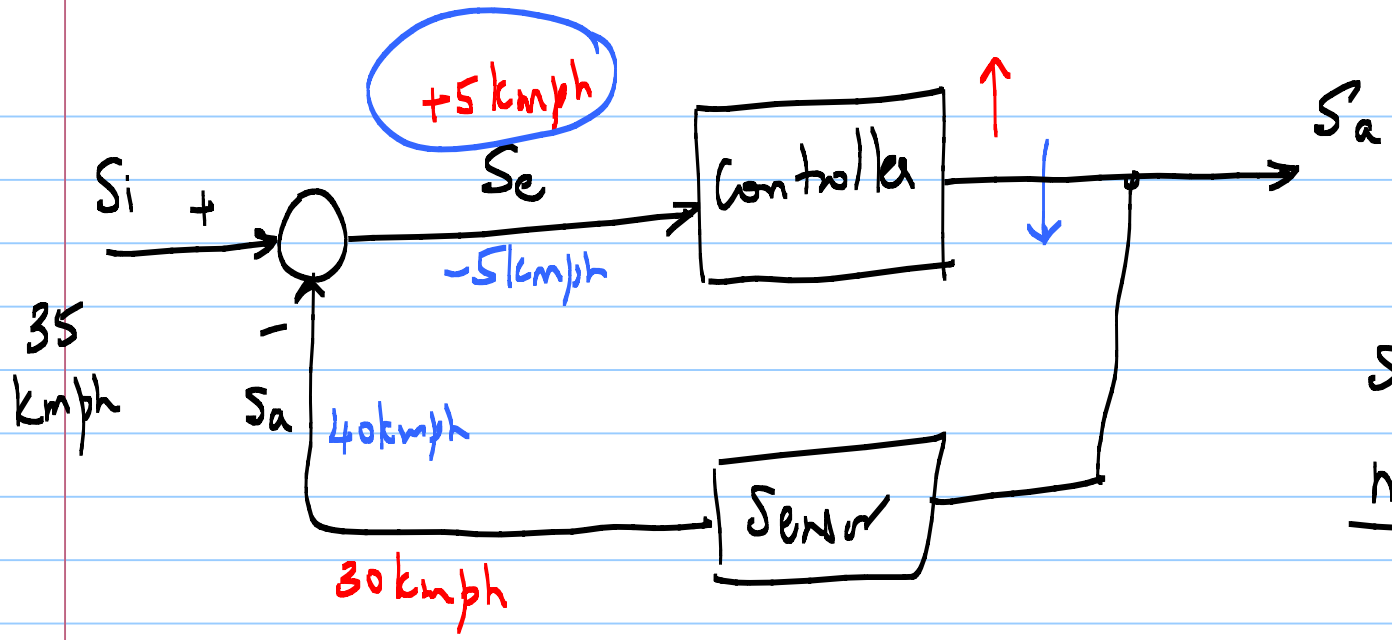
1) Is feedback positive or negative?

2) If feedback is negative:
set $(V_+ - V_-) = 0$
"Virtual short"

3) Do (1) & (2) for every feedback network & every opamp

4) Calculate ideal TFs

5) Put in non-ideal effects for opamps.

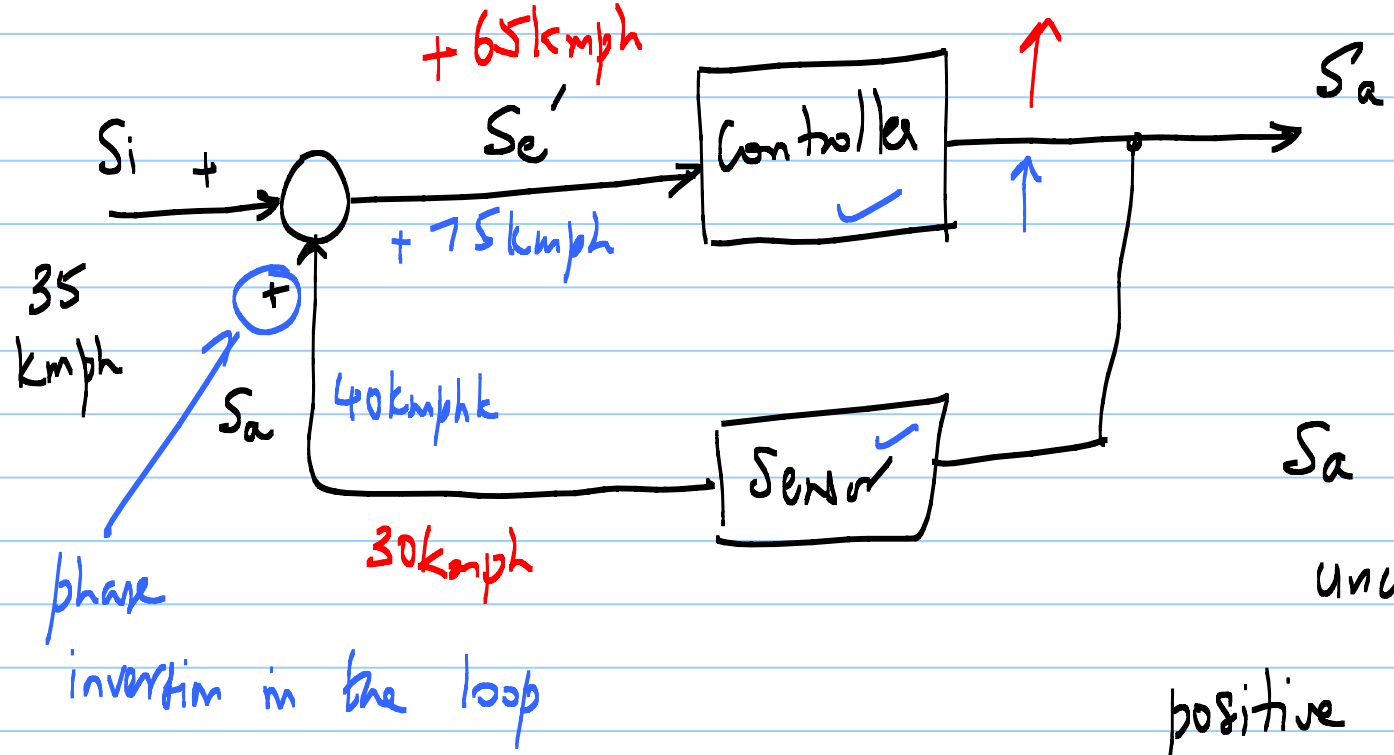


1) $S_a = 40$ kmph

2) $S_a = 30$ kmph

$S_e = S_i - S_a$

negative feedback



1) $S_a = 40$ kmph

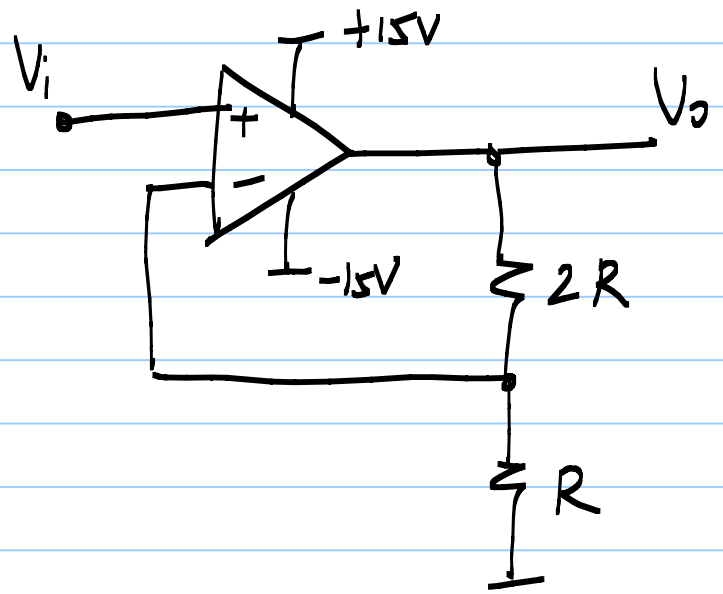
2) $S_a = 30$ kmph

$S_e = S_i + S_a$

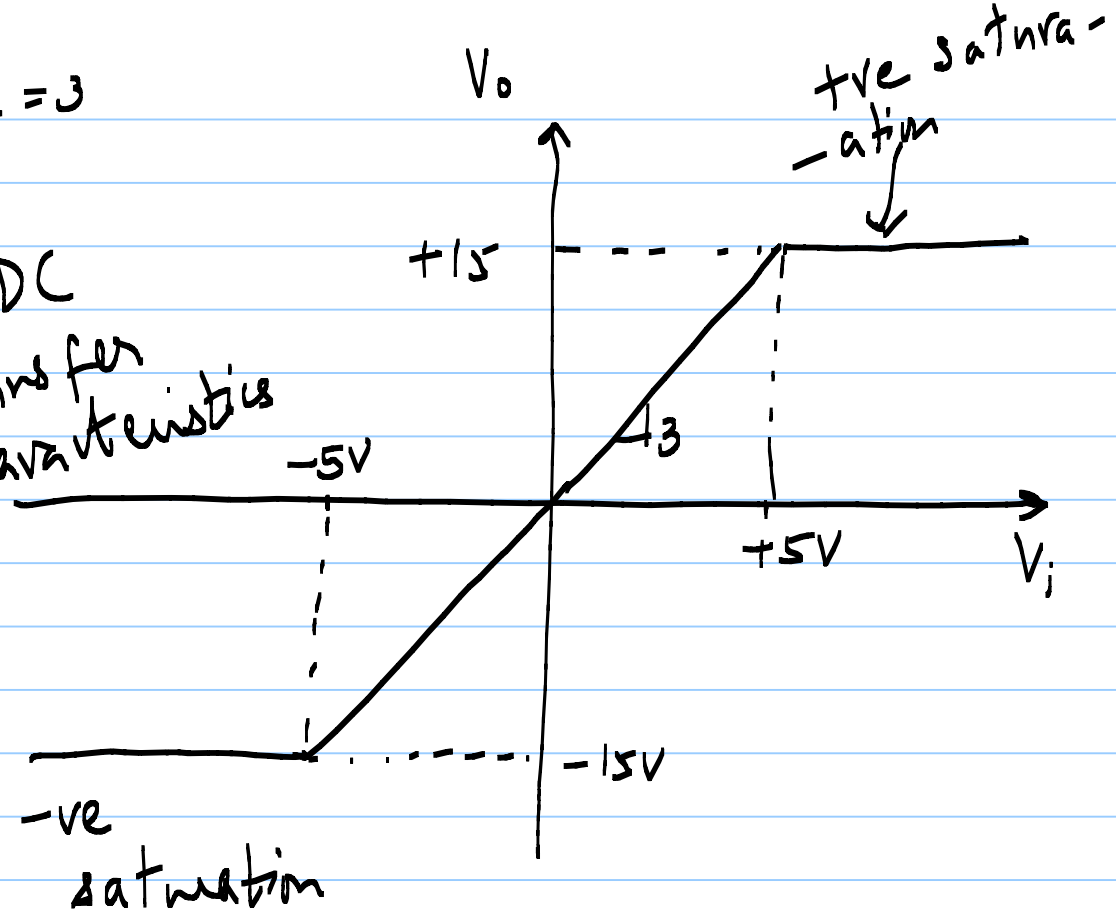
S_a keep increasing uncontrolled

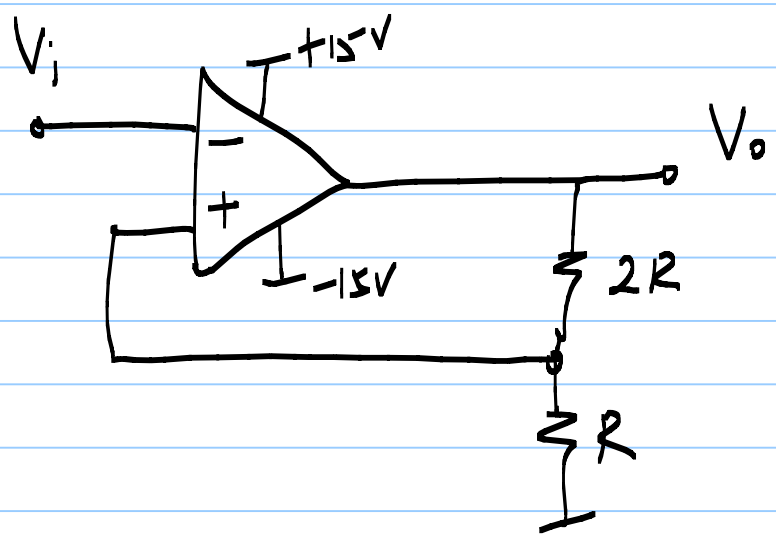
positive feedback

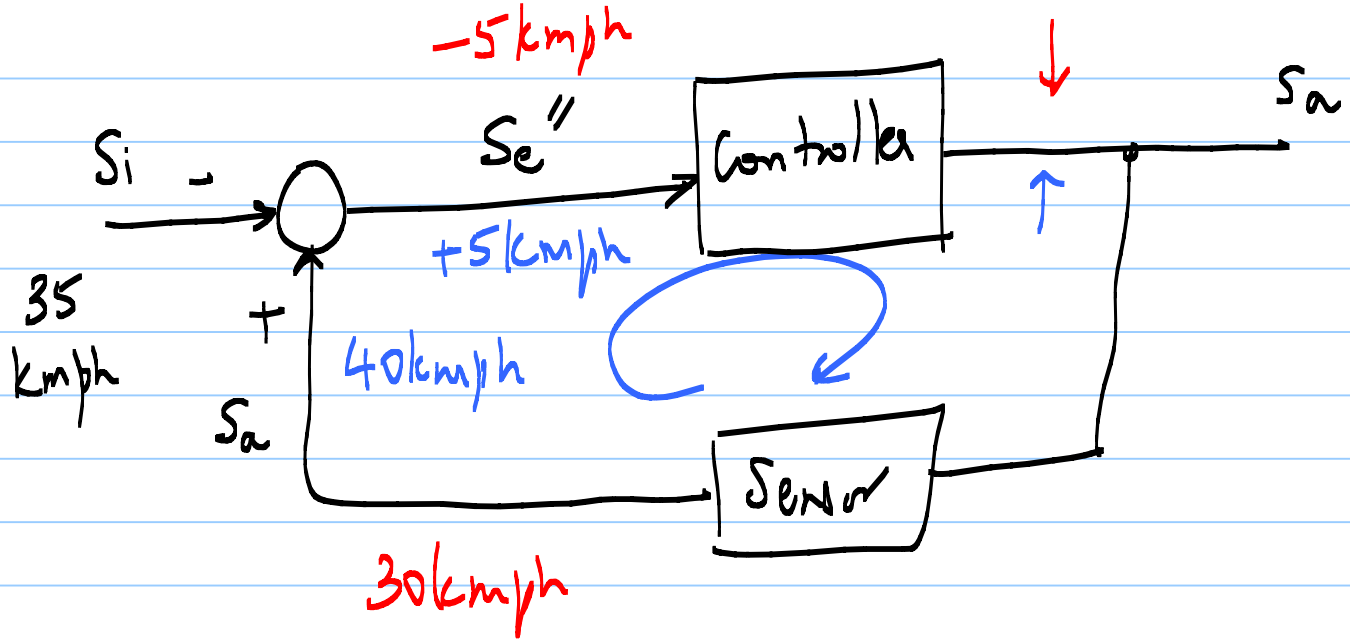
Non-inverting Amplifier , k = 3



DC transfer characteristics



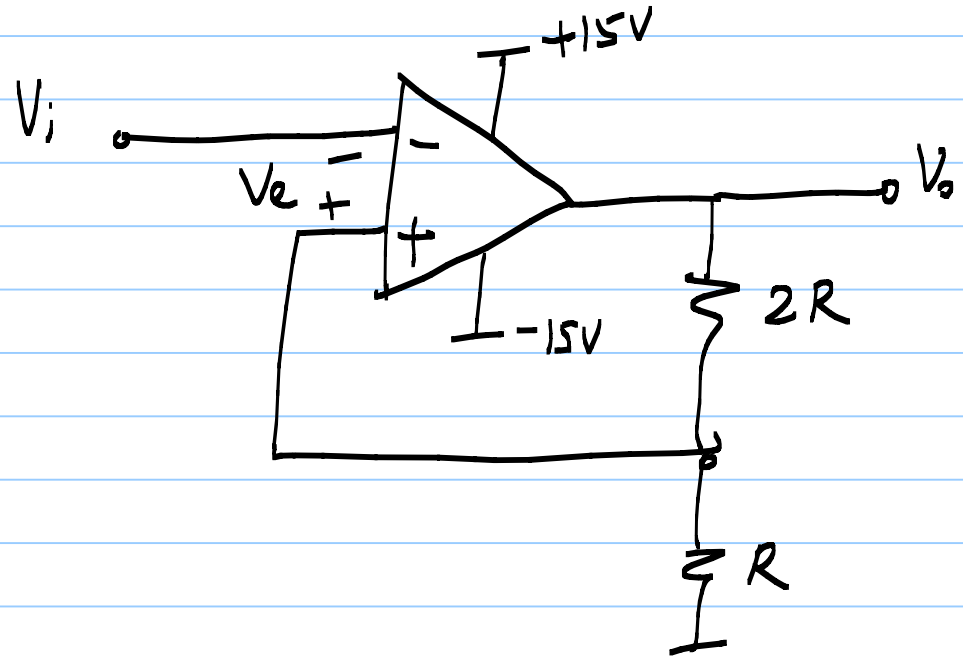




1) $S_a = 40$ kmph

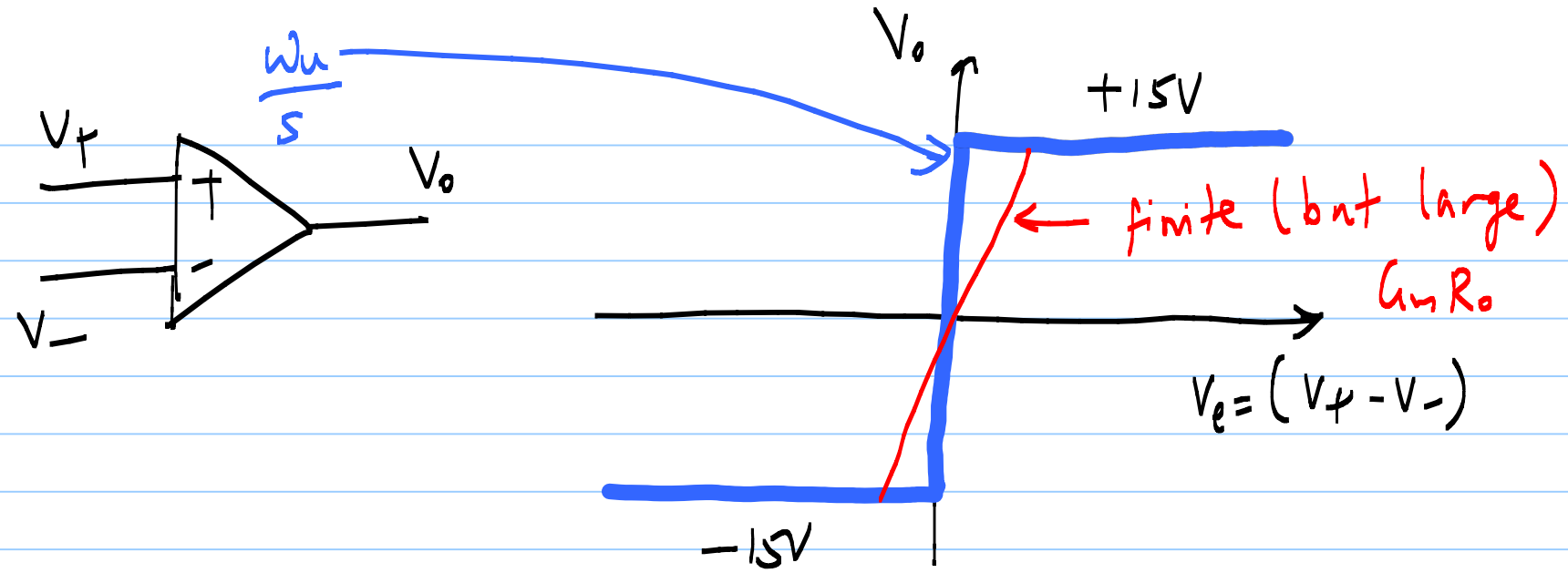
2) $S_a = 30$ kmph

positive feedback



1) f.b. is +ve

2) Can not apply "virtual short" concept

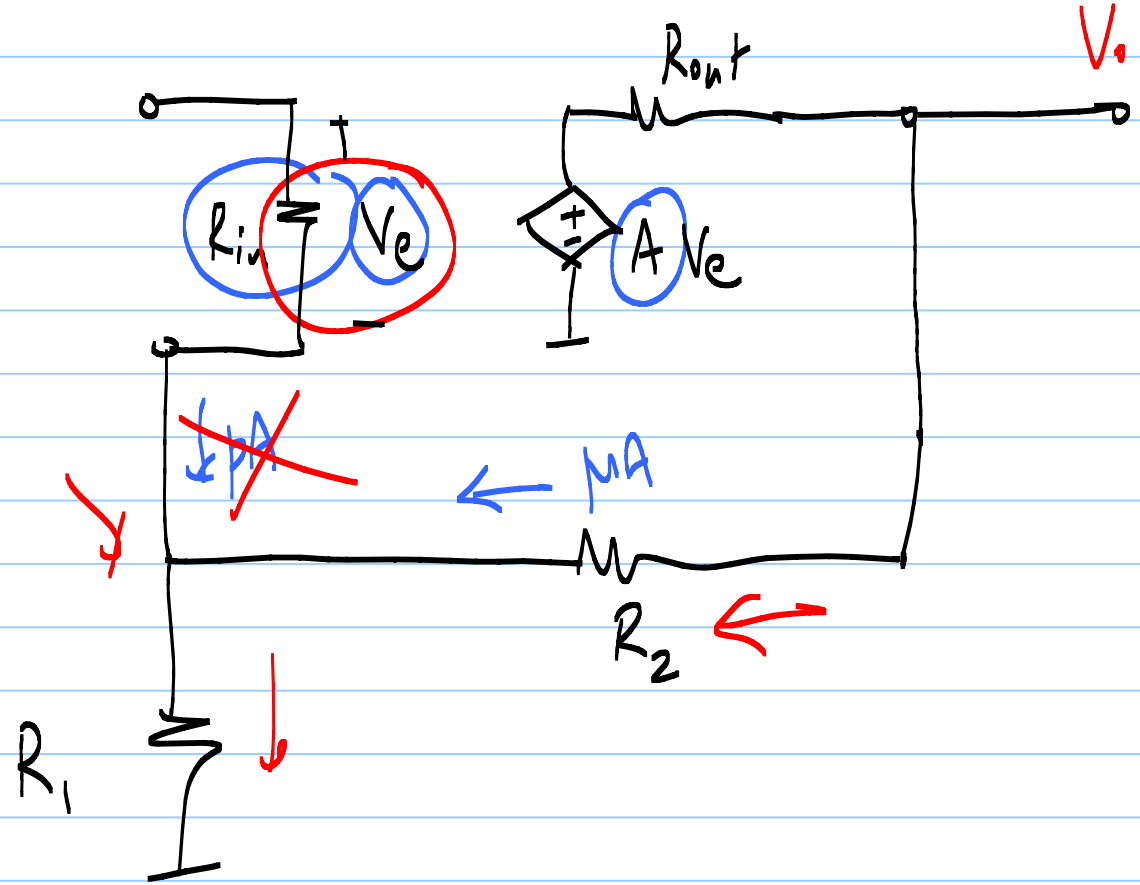


1) $V_e = V_+ - V_- > 0 \Rightarrow V_o = +15V$

2) $V_e = V_+ - V_- < 0 \Rightarrow V_o = -15V$

04/02/2022

Lecture 10



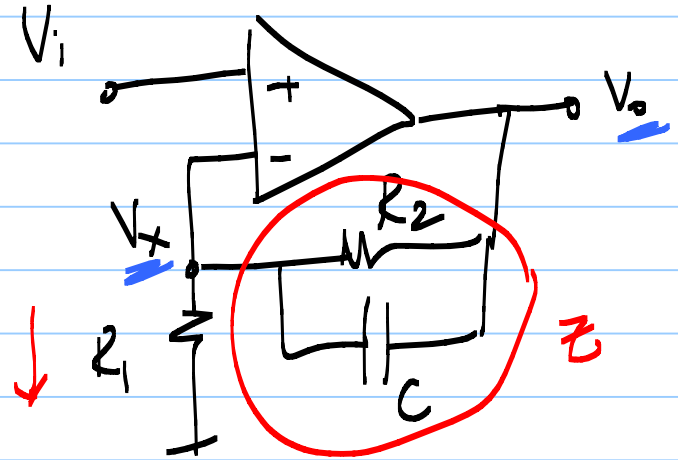
V_o ( )

4V → 5.9996

V_o

3.99959999

Quiz 1 discussion



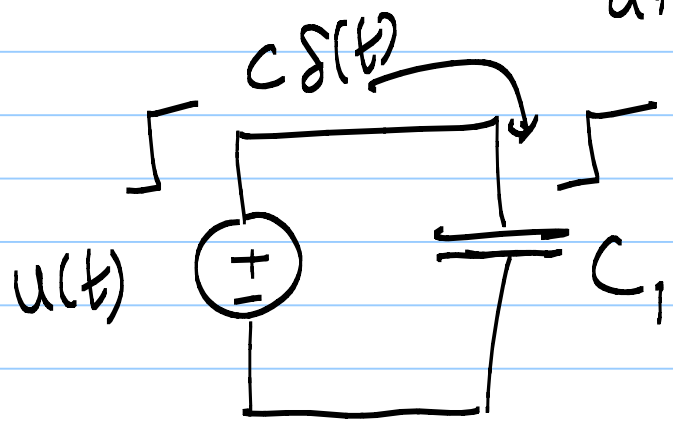
1) $V_o(0^+) = ?$

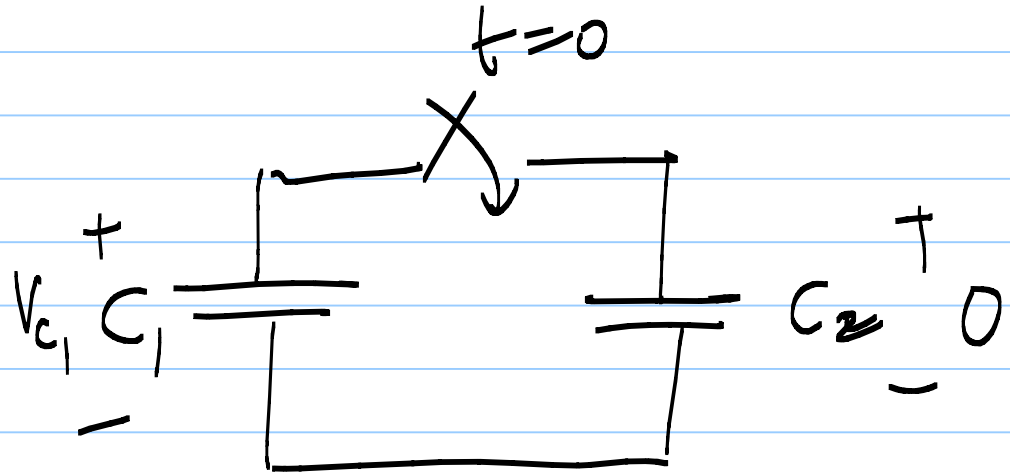
$A \to \infty \Rightarrow V_e = 0$

$\Rightarrow V_i = V_x$

short cap @ 0^+ $\quad V_c(0^-) = 0$

$$i_c = C\frac{dV_c}{dt}$$

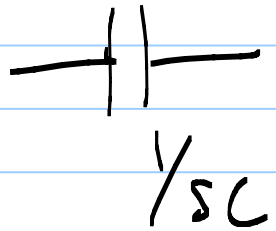




$V_o(0^+) = V_x(0^+) = V_i(0^+) = 1V$

2) $V_o(t)$

$\frac{V_o}{V_i}(s)$?

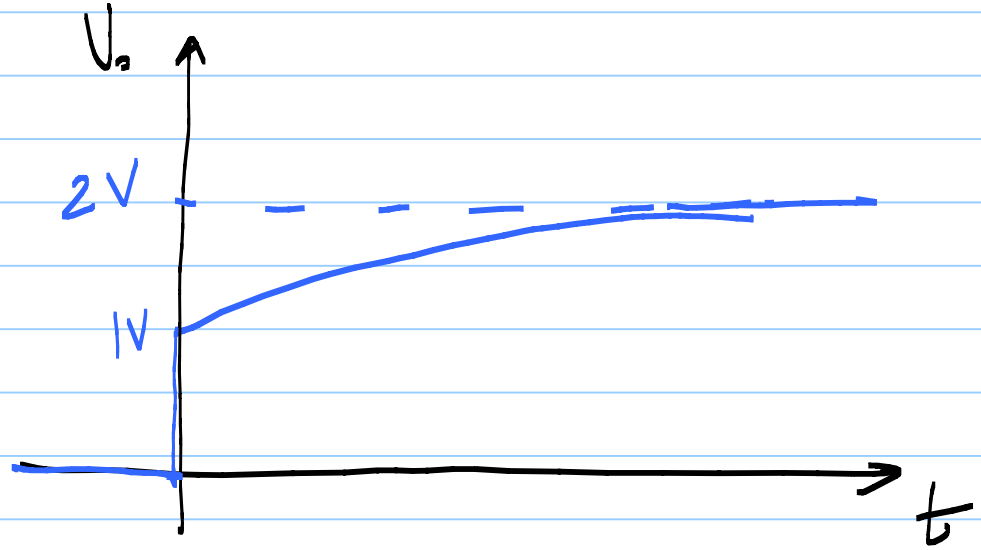


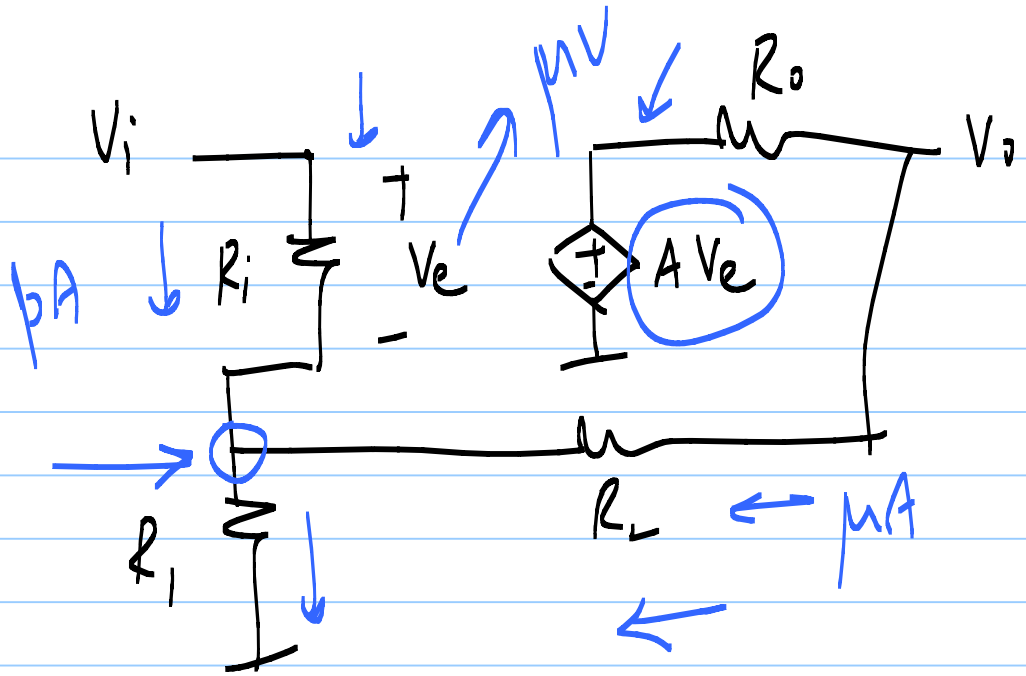
* Laplace ✓
* Differential eq. ✓

$V_x = V_i$

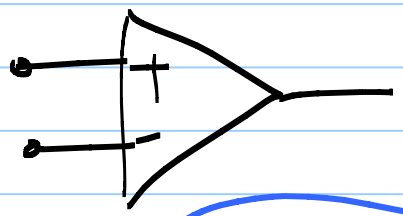
$$\frac{V_x}{R_1} = \frac{(V_o - V_x)}{R_2} + \frac{C\, d(V_o - V_x)}{dt}$$

3)



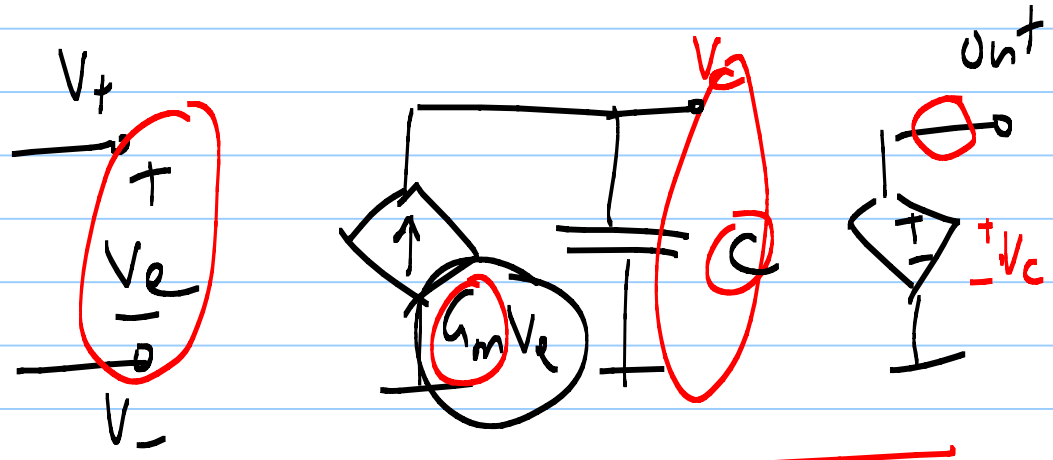


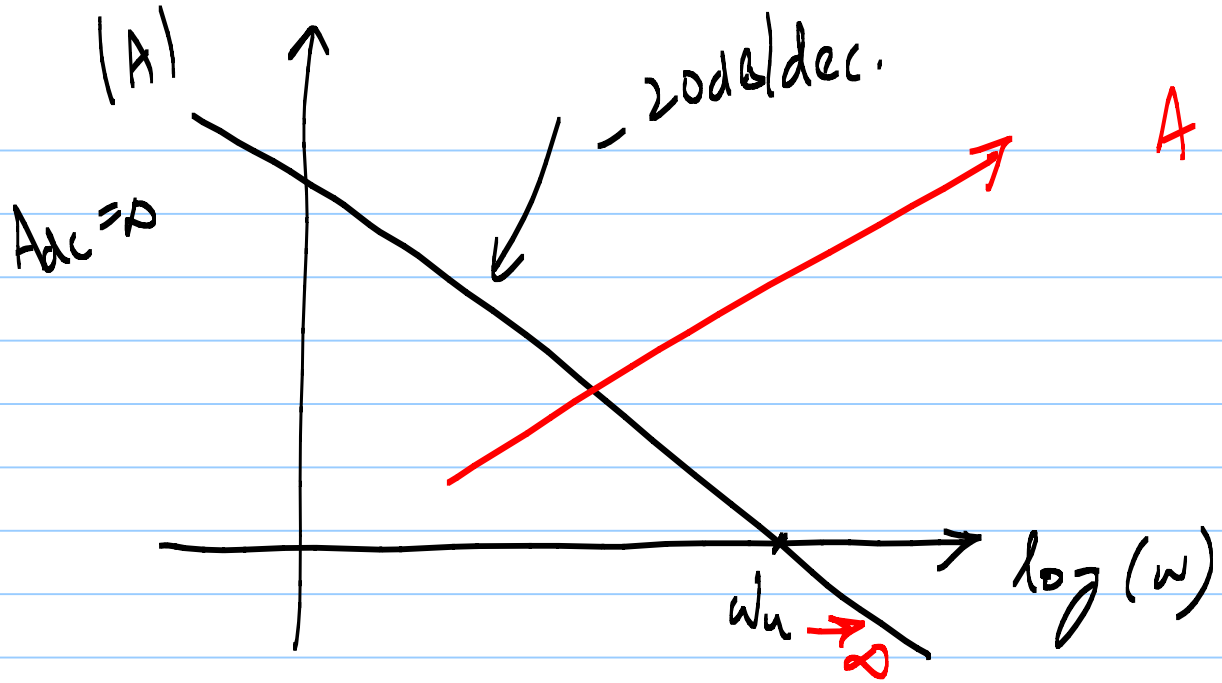
$V_{o\,ideal} - (\ )\,\mu V$



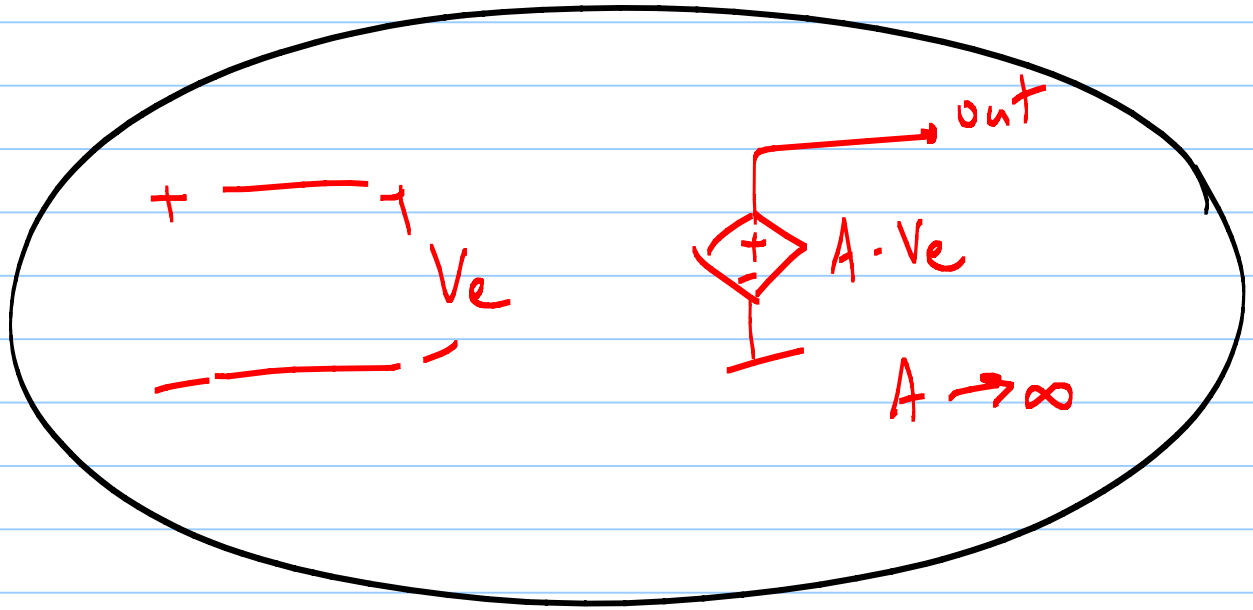
$\cancel{R_{in}}$, $\cancel{R_{out}}$, A, freq. resp.

$A(DC) = \infty$

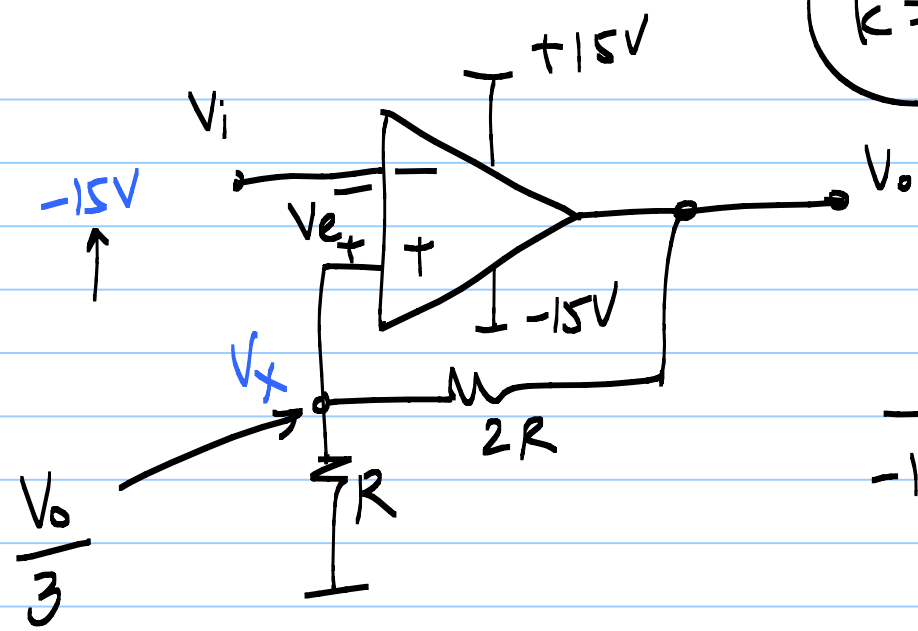


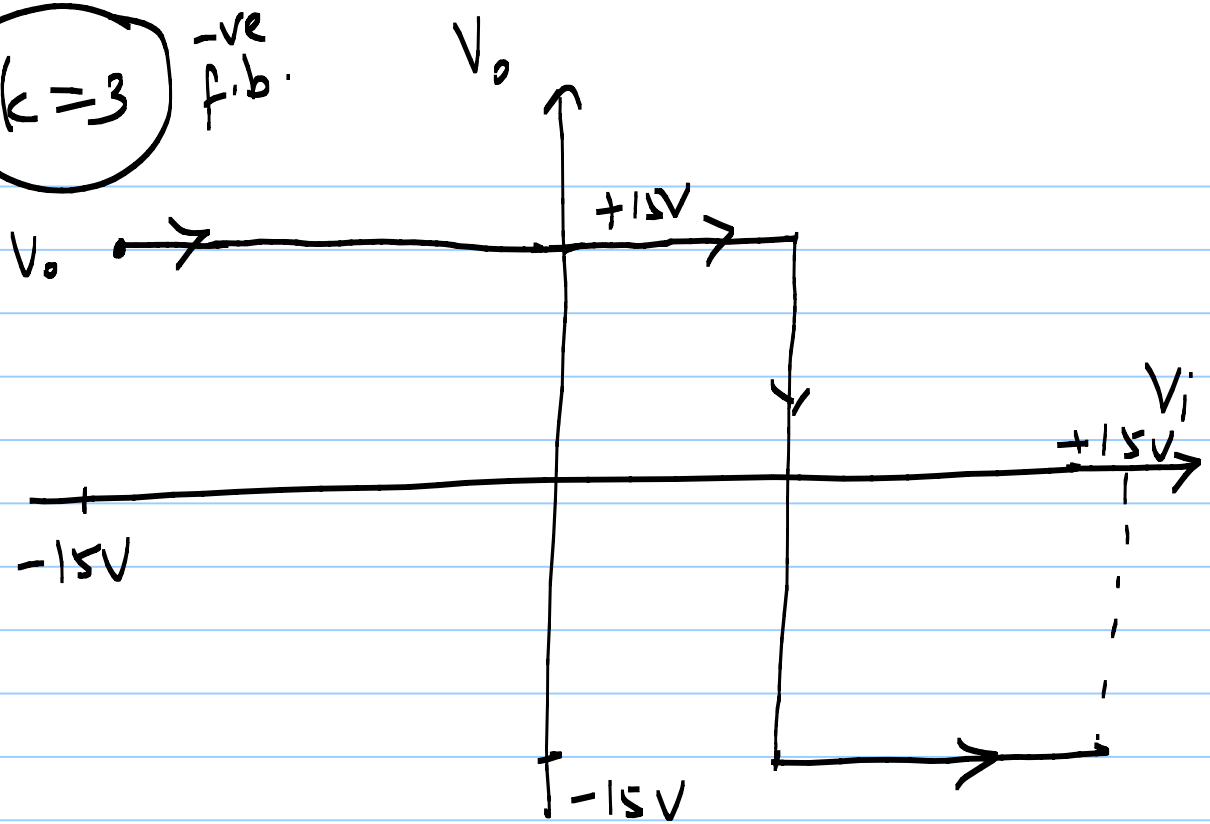


A = ∞ for any freq.



$k = 3$ -ve f.b.





* ideal opamp

* $I_{in} = 0$, $R_{in} = \infty$, $R_o = 0$

1) Assume $V_o = +15V$

$V_x = +5V$

$V_e = 20V$

$V_o = +15V$ ✓

2) Assume $V_o = -15V$

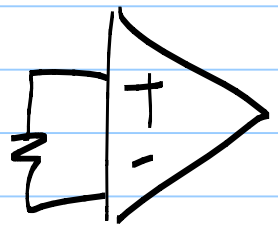
$V_x = -5V$

$V_e = +10V \Rightarrow V_o = +15V$

3) If $V_i = -14V \Rightarrow V_e = 19V$, $V_o = +15V$

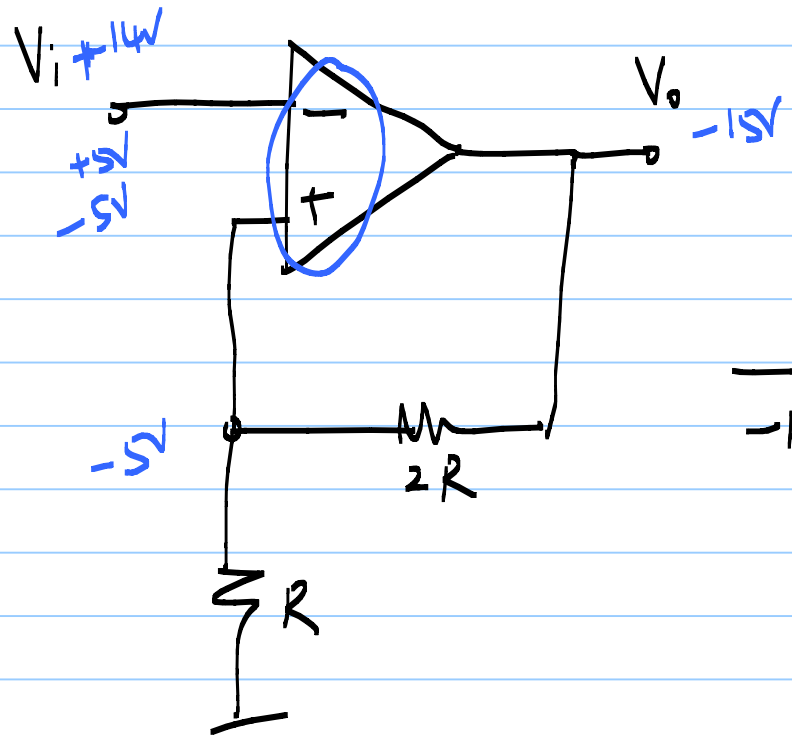
4) $V_i = -5V \Rightarrow V_e = +10V$, $V_o = +15V$

5) $V_i = +5V \Rightarrow V_e = 0 \Rightarrow V_o$ goes from $+15V$ to $-15V$

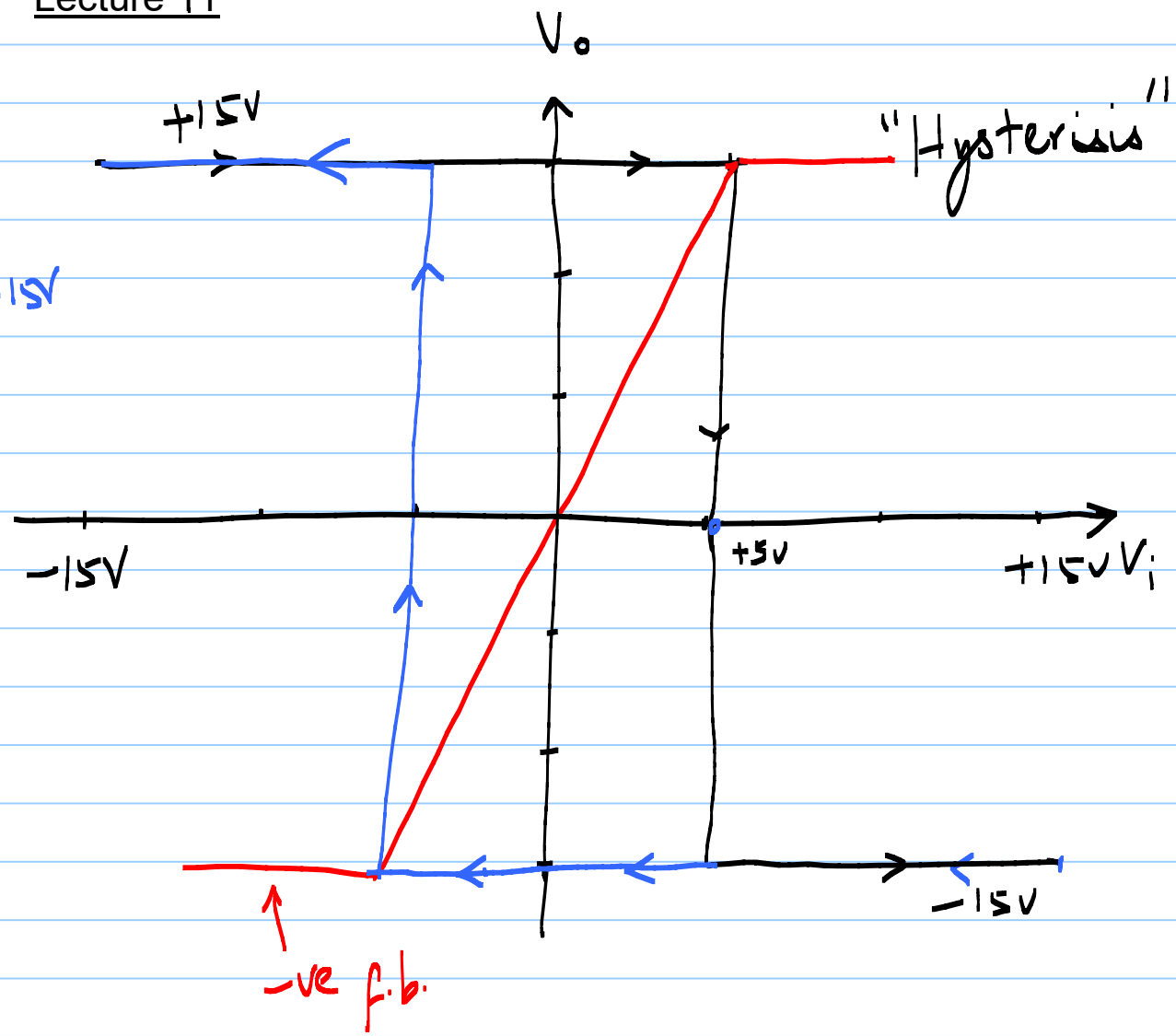


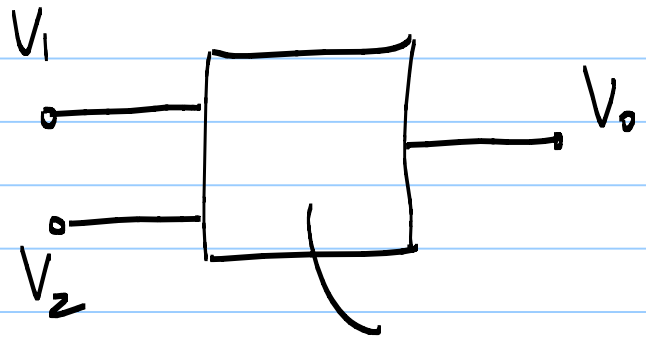
08/02/2022

Lecture 11



"Schmitt Trigger"

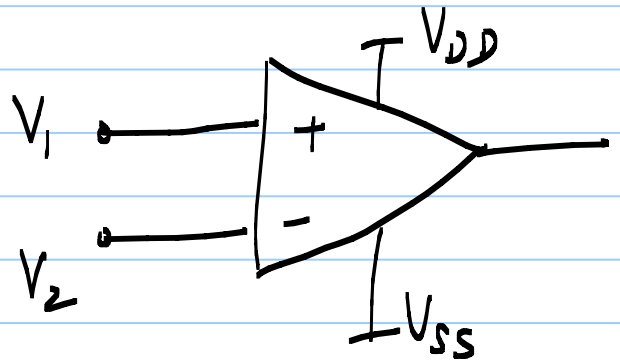


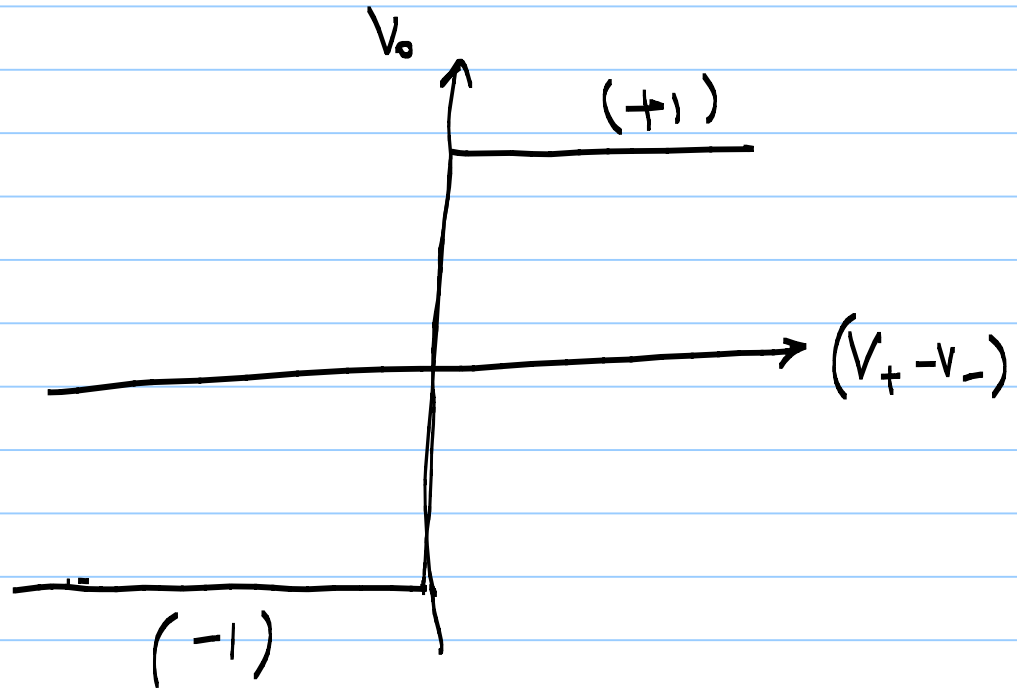


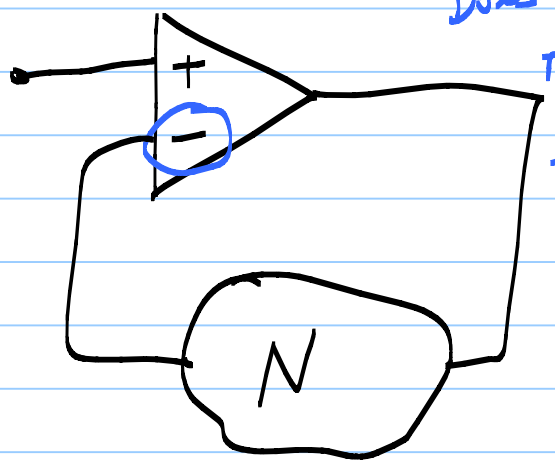
Comparator

If $V_1 > V_2 \Rightarrow V_o \rightarrow$ "+1" state

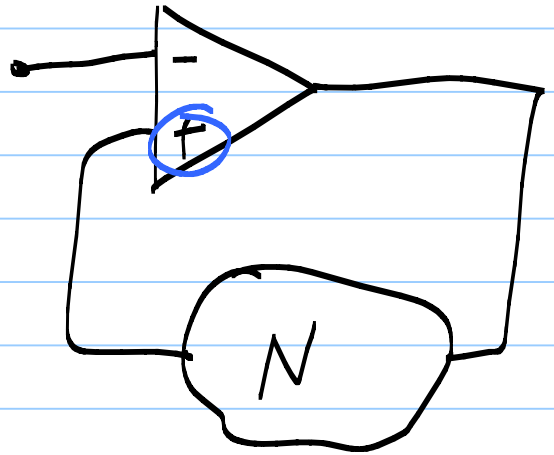
If $V_1 < V_2 \Rightarrow V_o \rightarrow$ "−1" or "0" state





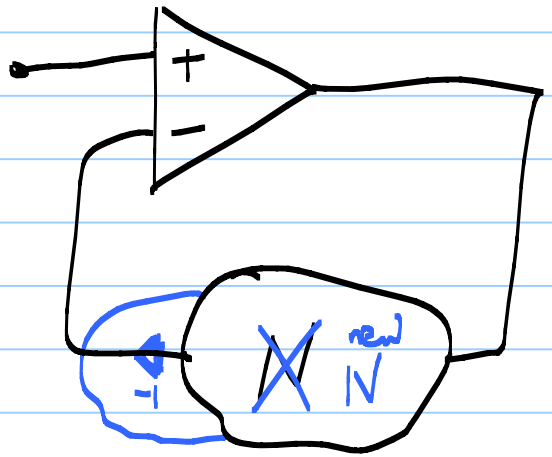


Does not mean −ve f.b.

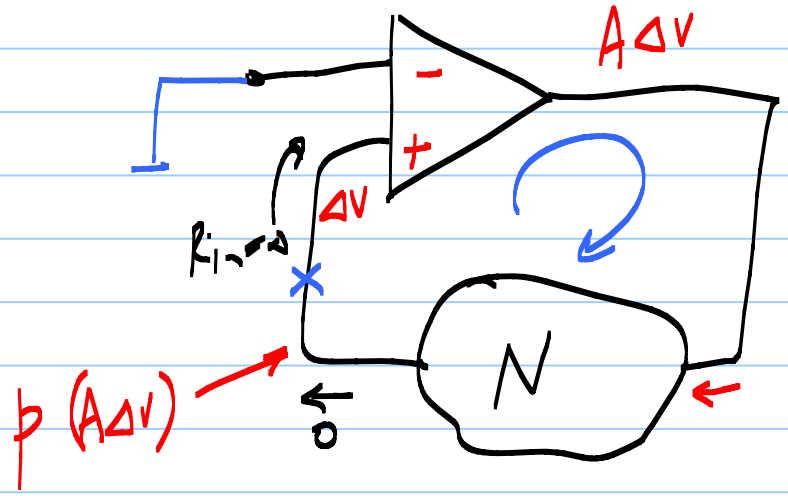


Does not mean ve f.b.

* which one of these is correct? (for negative f.b.)
{for arbitrary N}



Find signs of opamp for -ve f.b:



1) Assume opamp is ideal (large A, W_n, $R_{in}=\infty$, $R_{out}=0$)

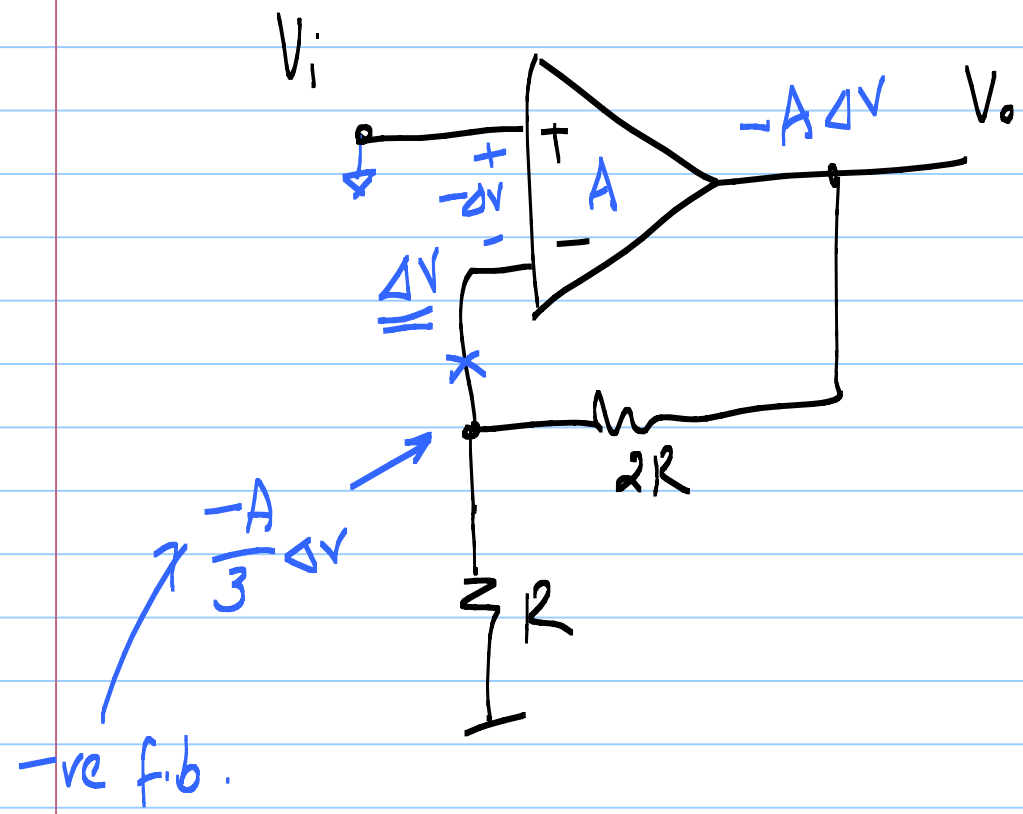
2) Any externally applied inputs that are not part of the feedback loop can be tied to a convenient value

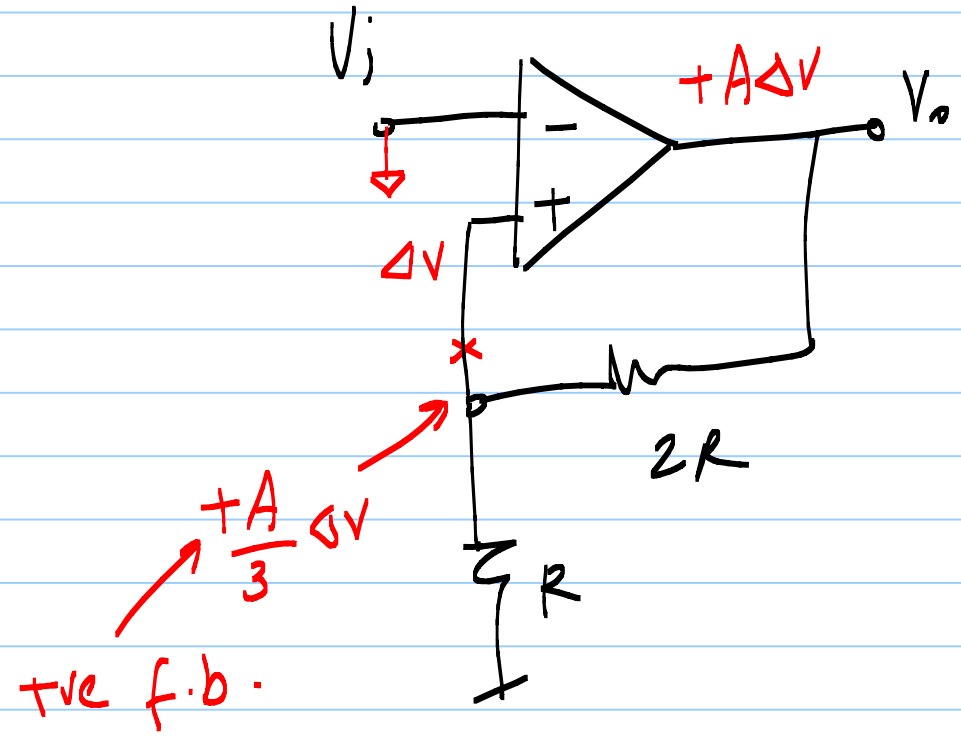
3) Break the loop at a convenient point (typically $R_{in}=\infty$ points)

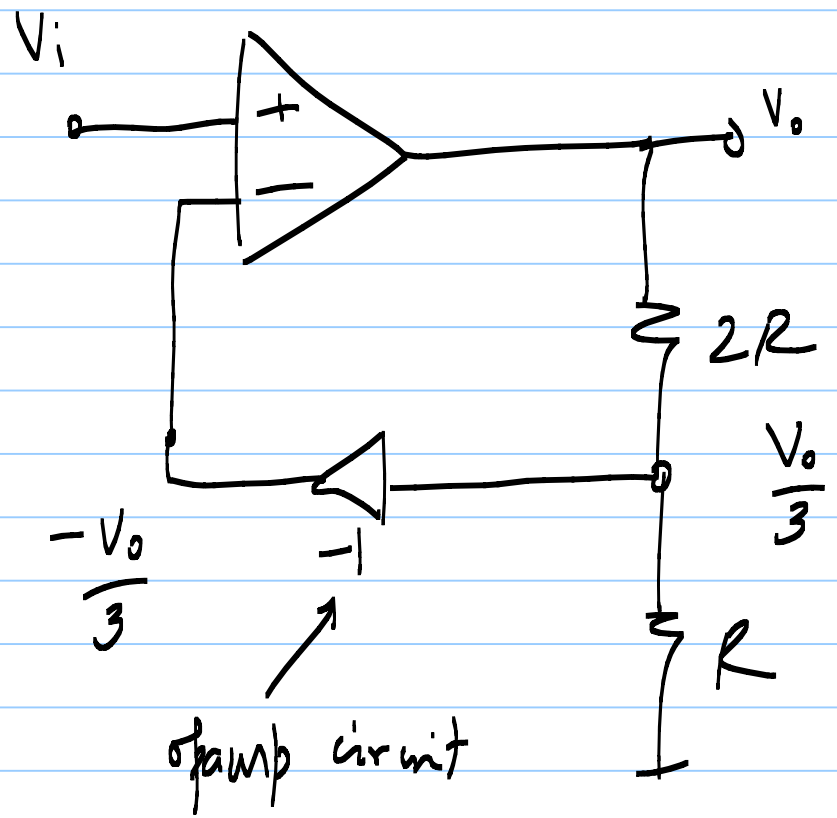
4) Apply ΔV at $R_{in}=\infty$ point and traverse the loop back to the other side of the break. $\{\beta(A\Delta V)\}$

5) Sign of β → −ve → signs of opamp chosen are correct

→ +ve → flip the signs of opamp







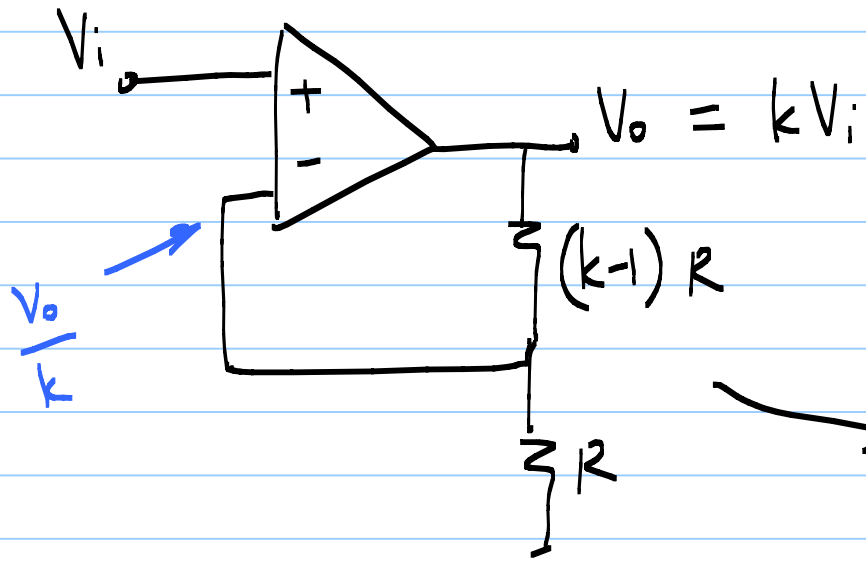
Not −ve f.b.

* Extend to multi-opamp circuits

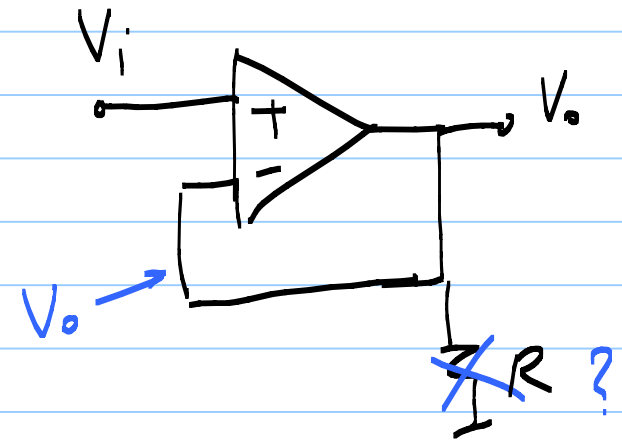
09/02/2022

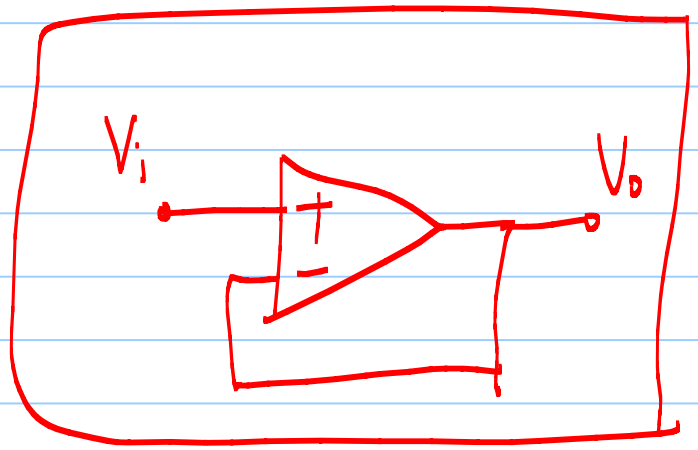
Lecture 12

$V_o = V_i$ – "Buffer" or "voltage follower"

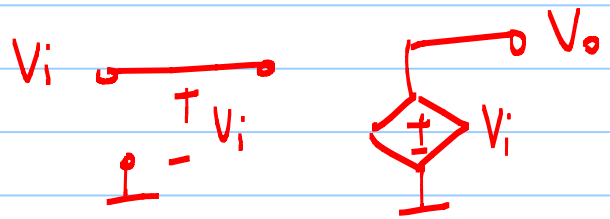


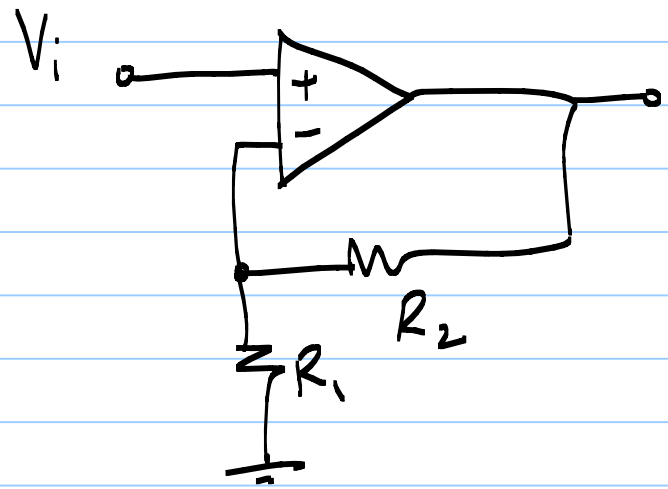
we want $k = 1$





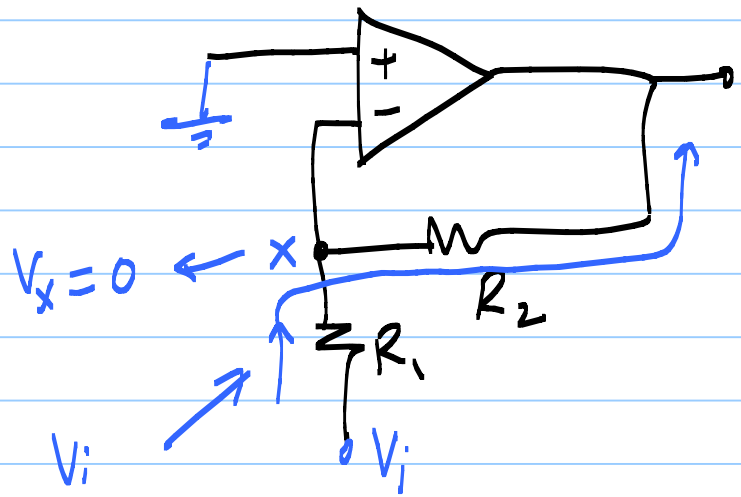
$\equiv$





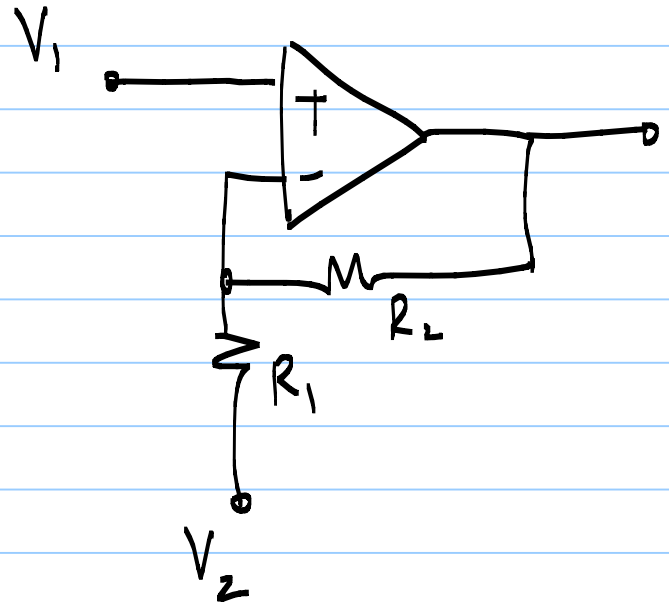
$$V_o = V_i\left(1 + \frac{R_2}{R_1}\right)$$

Non-inv. amp.



$$V_o = -V_i\left(\frac{R_2}{R_1}\right)$$

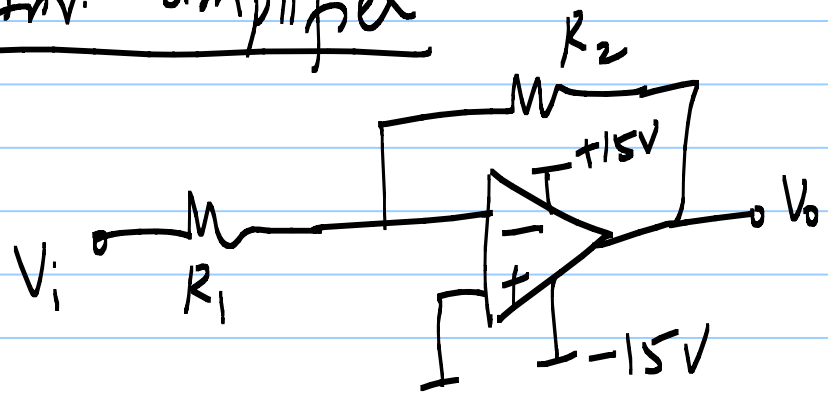
$$\frac{V_i}{R_1}$$



$$V_o = V_1\left(1+\frac{R_2}{R_1}\right) - V_2\left(\frac{R_2}{R_1}\right)$$

Inv. amplifier

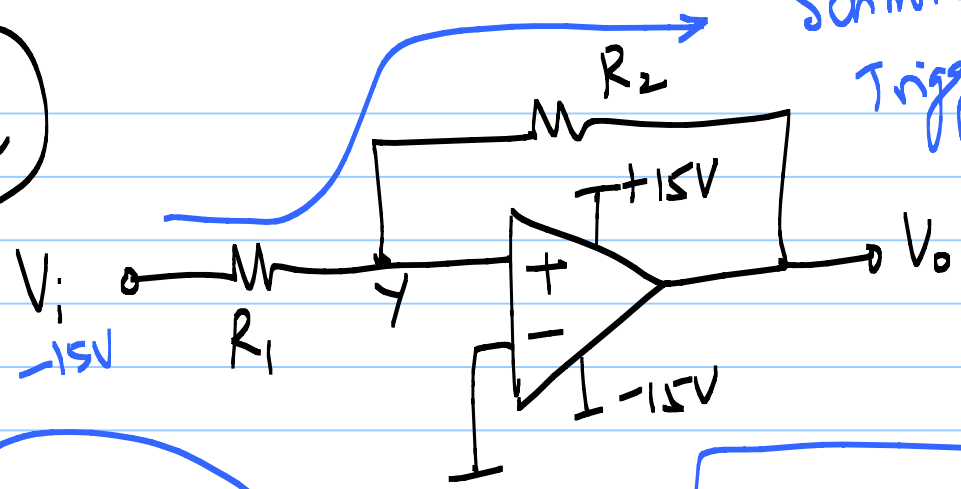
I



$$V_o = \frac{-R_2}{R_1} V_i$$

$$\frac{R_2}{R_1} = 3$$

II



$$\frac{V_i - V_y}{R_1} = \frac{V_y - V_o}{R_2}$$

flip opamp signs

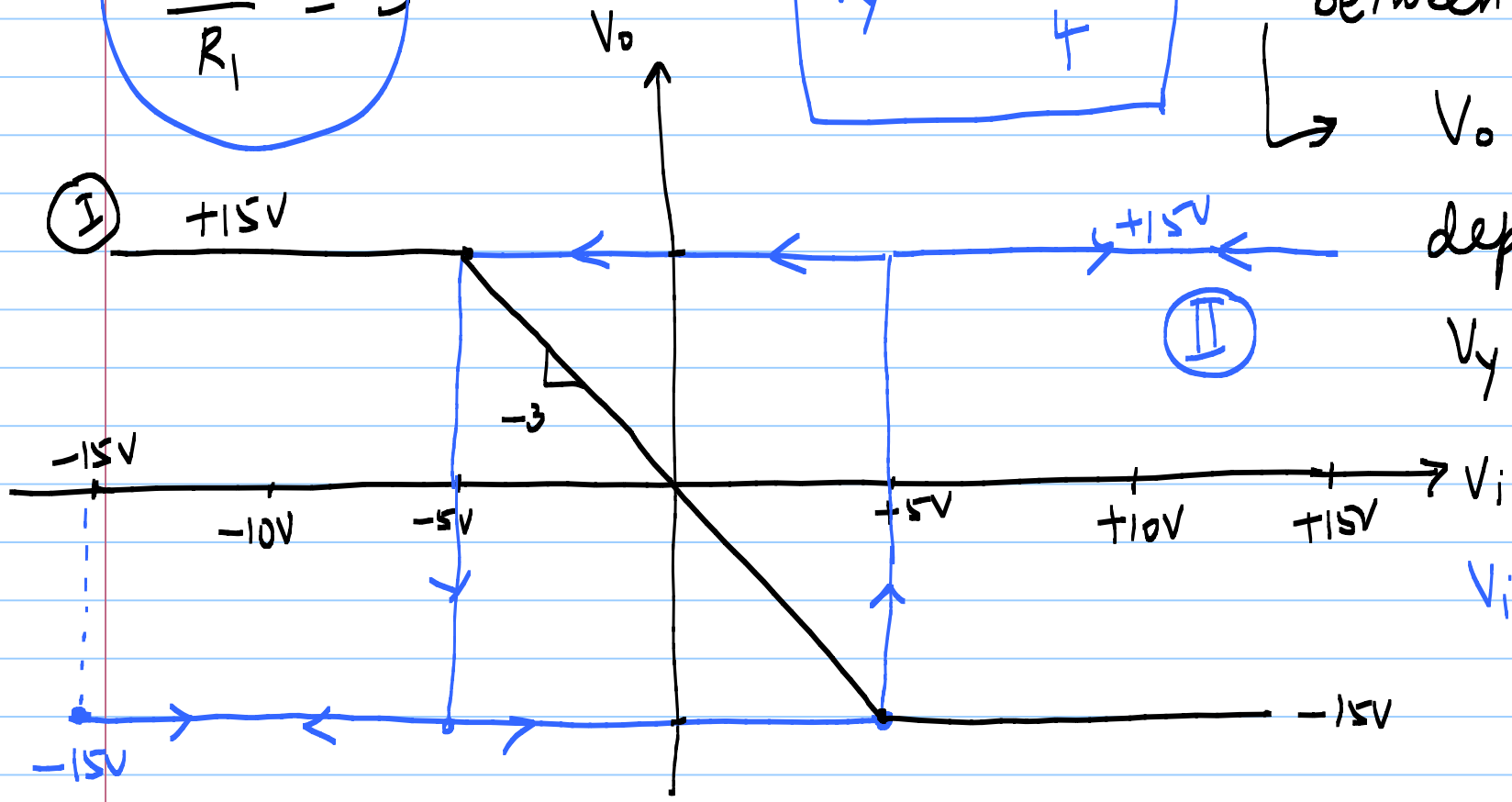
↳ +ve f.b.

↳ No virtual short between Y & gnd

↳ $V_o = +15V$ or $-15V$ depending on whether $V_y > 0$ or < 0

$$\frac{R_2}{R_1} = 3$$

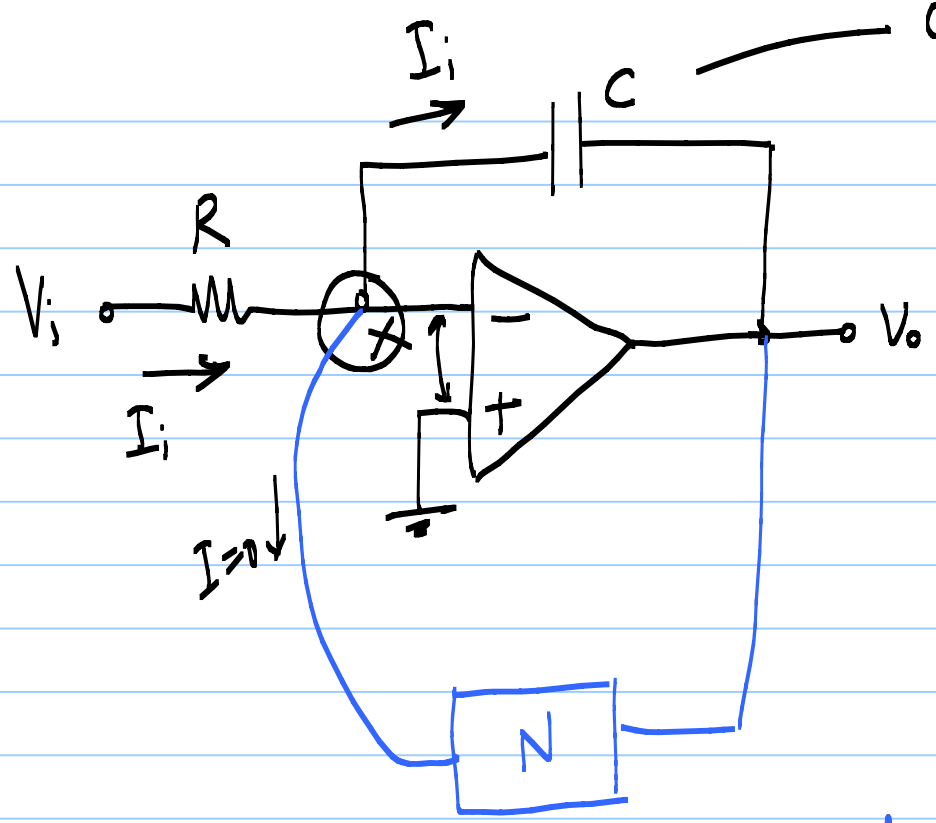
$$\boxed{V_y = \frac{3V_i + V_o}{4}}$$



$V_i = -15V$

$\Rightarrow V_y < 0$

$\Rightarrow V_o = -15V$



DC −ve f.b. loop

open circuit @ DC

* No DC f.b.
* N allows DC −ve f.b.
* X is at virtual gnd

$V_x = 0$

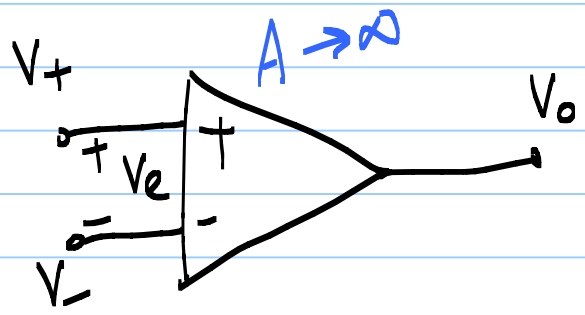
$$I_i = \frac{V_i}{R}$$

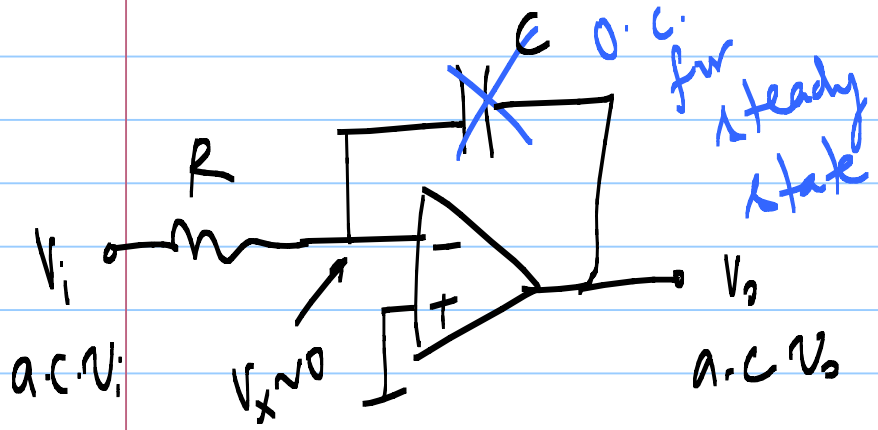
$$V_o(t) = \frac{-1}{RC}\int_0^t V_i(t)\,dt + V_o(0)$$

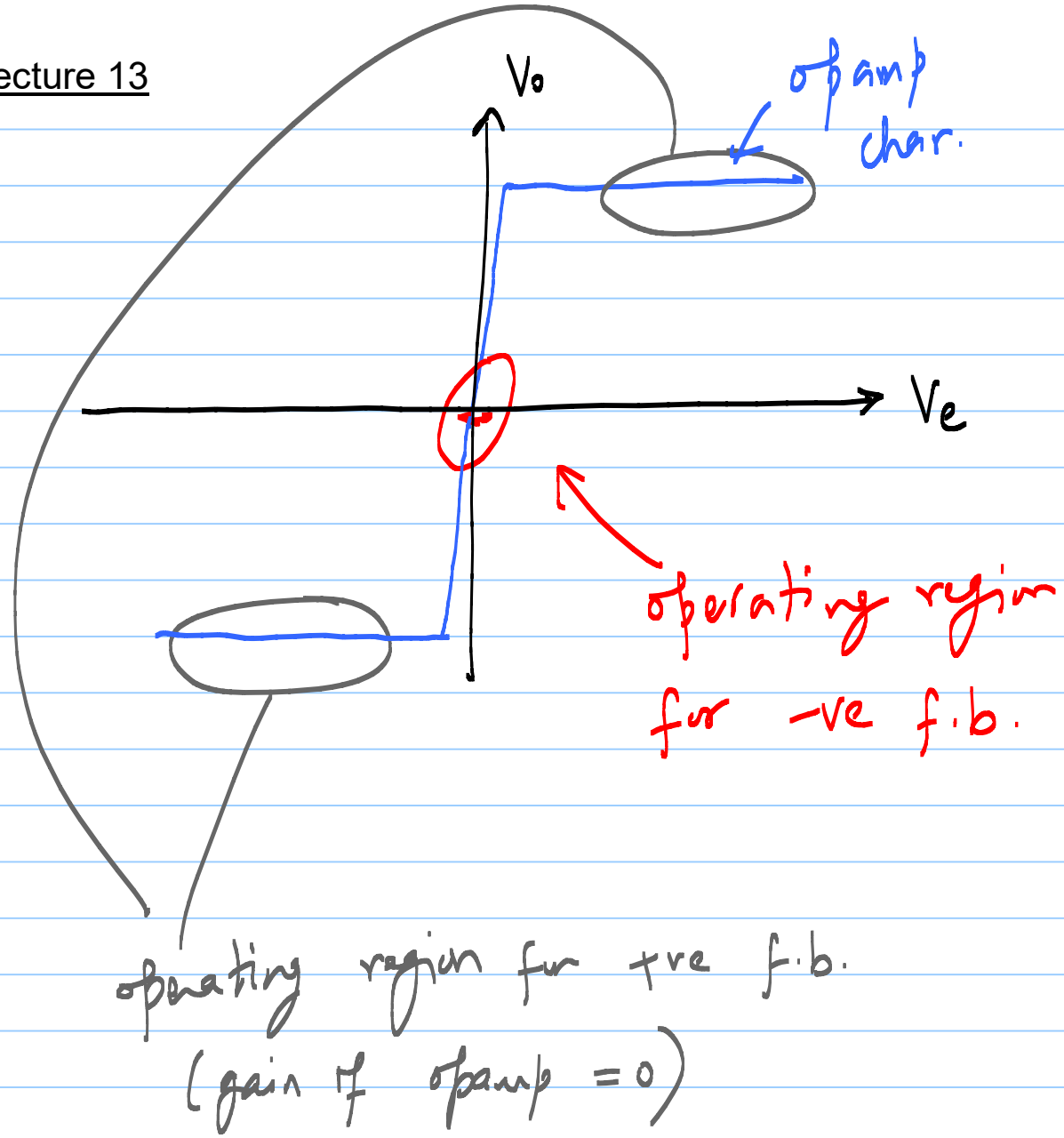
"Integrator"

10/02/2022

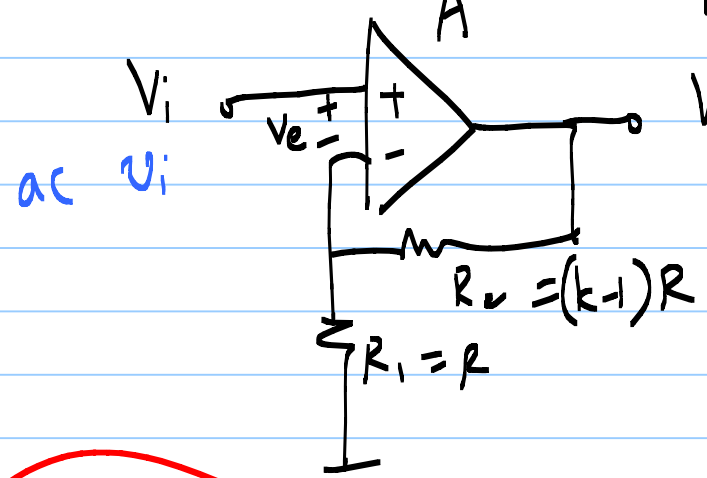
Lecture 13







A was large but finite



$V_o = kV_i$

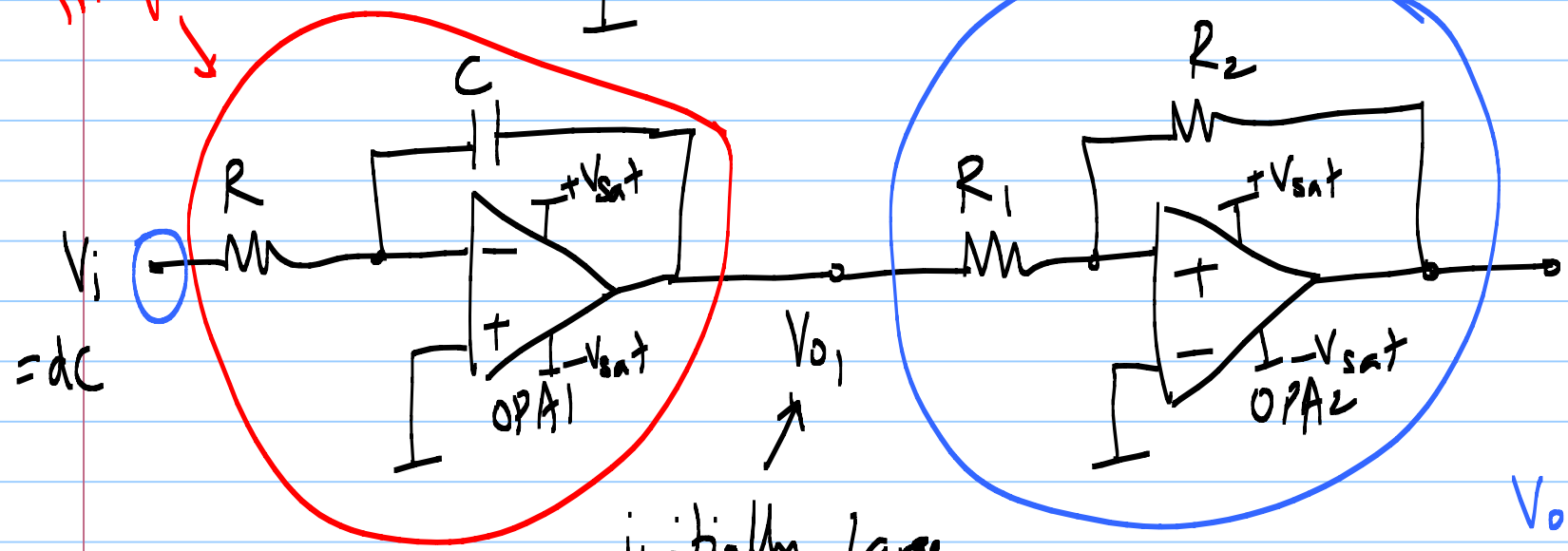
a.c. v_e

ac v_i

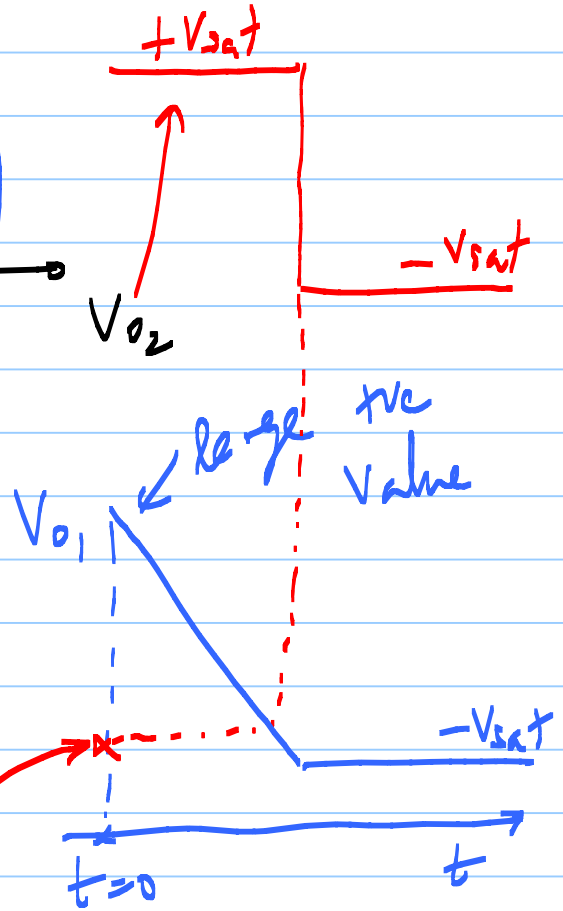
ac $v_o = k v_i$

Schmitt Trigger

integrator

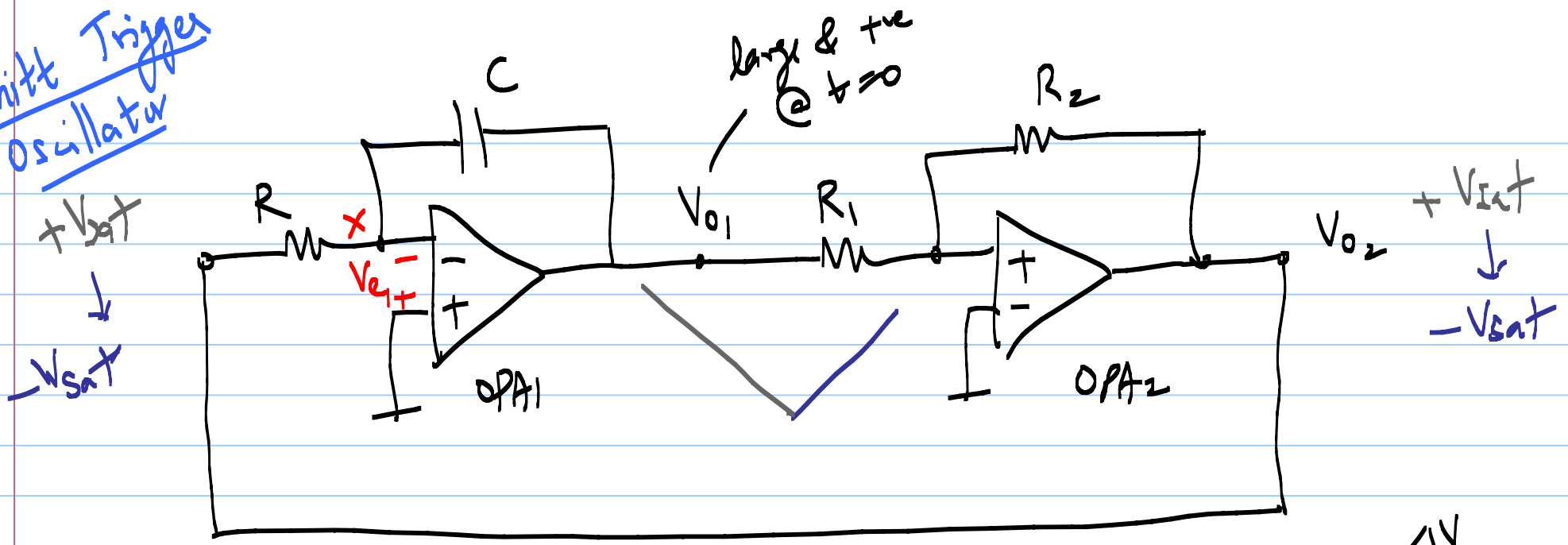


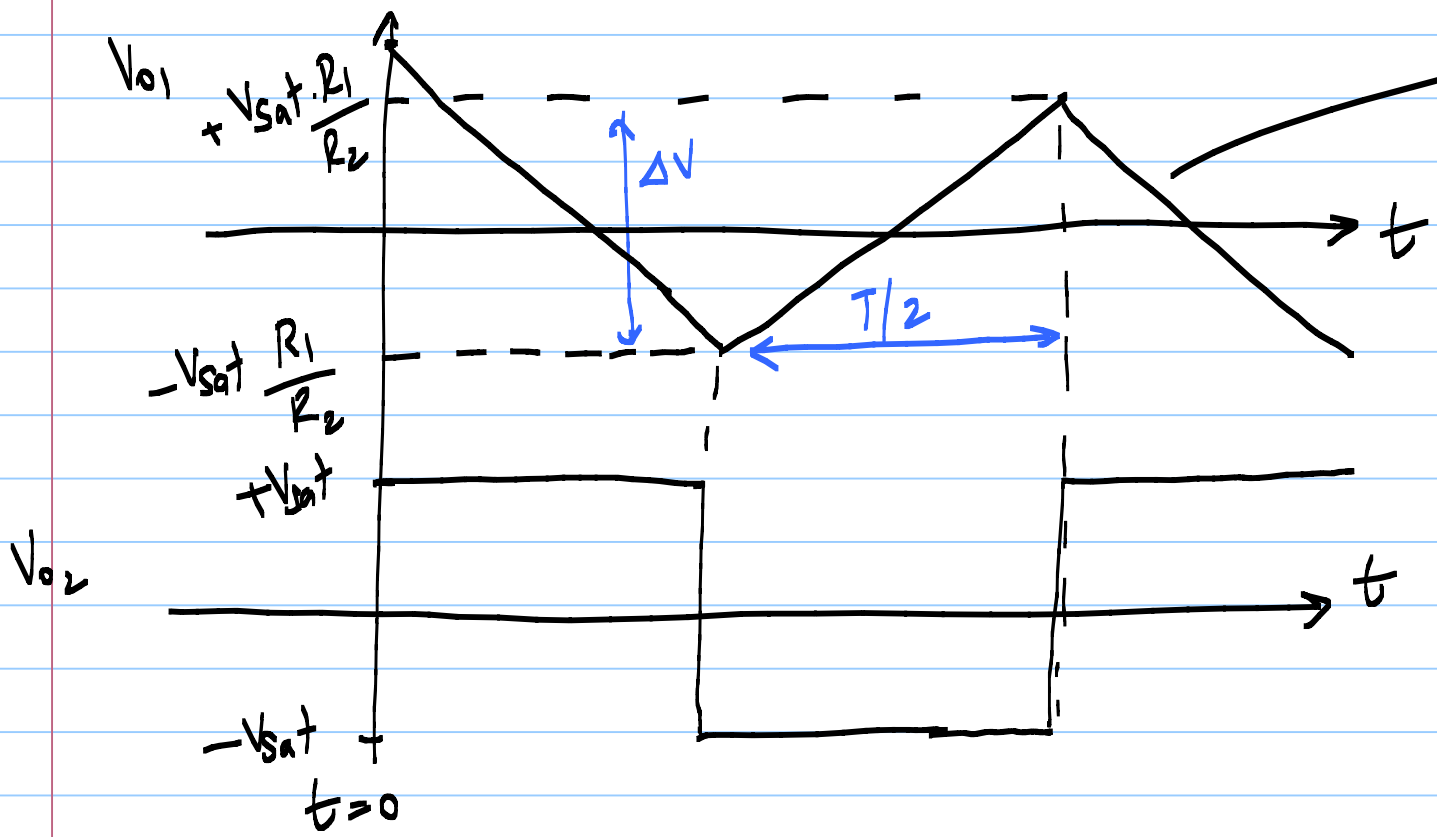
initially large & positive @ $t=0$



V_{o_2} changes state when V_{o_1} crosses $-V_{sat} \cdot \frac{R_1}{R_2}$

Schmitt Trigger Oscillator





$$\text{slope} = \frac{\Delta V}{T/2}$$

$$= \frac{2\,V_{sat}\,R_1/R_2}{T/2}$$

$$\text{slope} = \frac{V_{sat}}{RC}$$

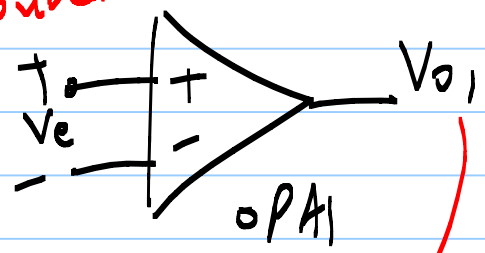
(from integrator char.)

$$\frac{V_{sat}}{RC} = \frac{4 V_{sat}\left(\frac{R_1}{R_2}\right)}{T}$$

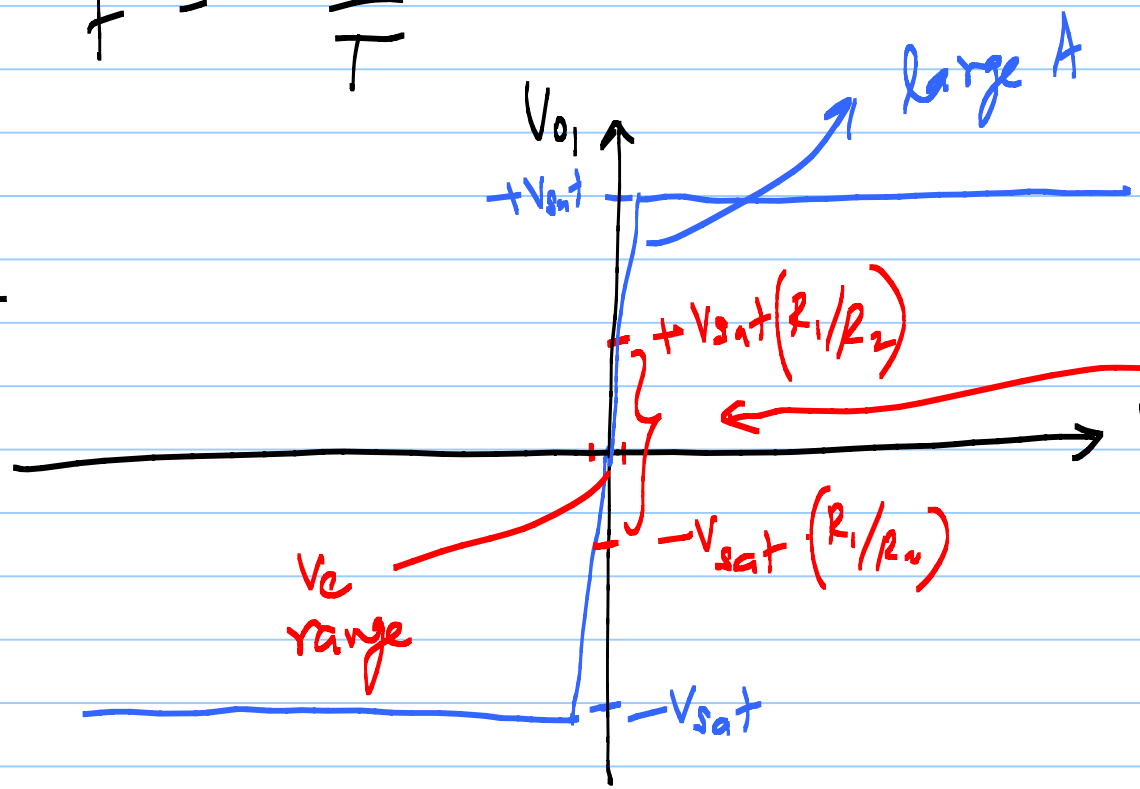
$$T = 4RC\left(\frac{R_1}{R_2}\right)$$

$$f = \frac{1}{T}$$

No -ve f/b.
but V_{e_1} range is very small and around the origin.



OPA1



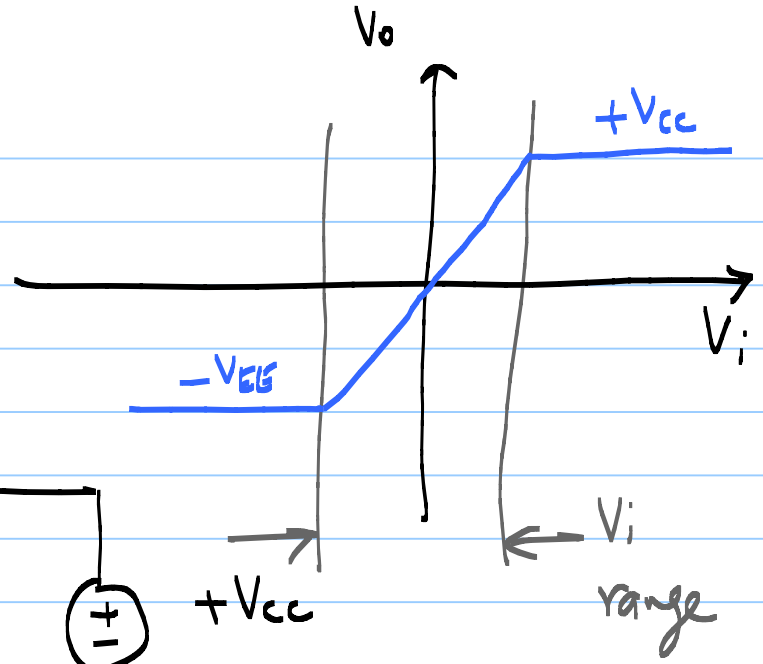
Assumption of $V_x \approx 0$ is valid

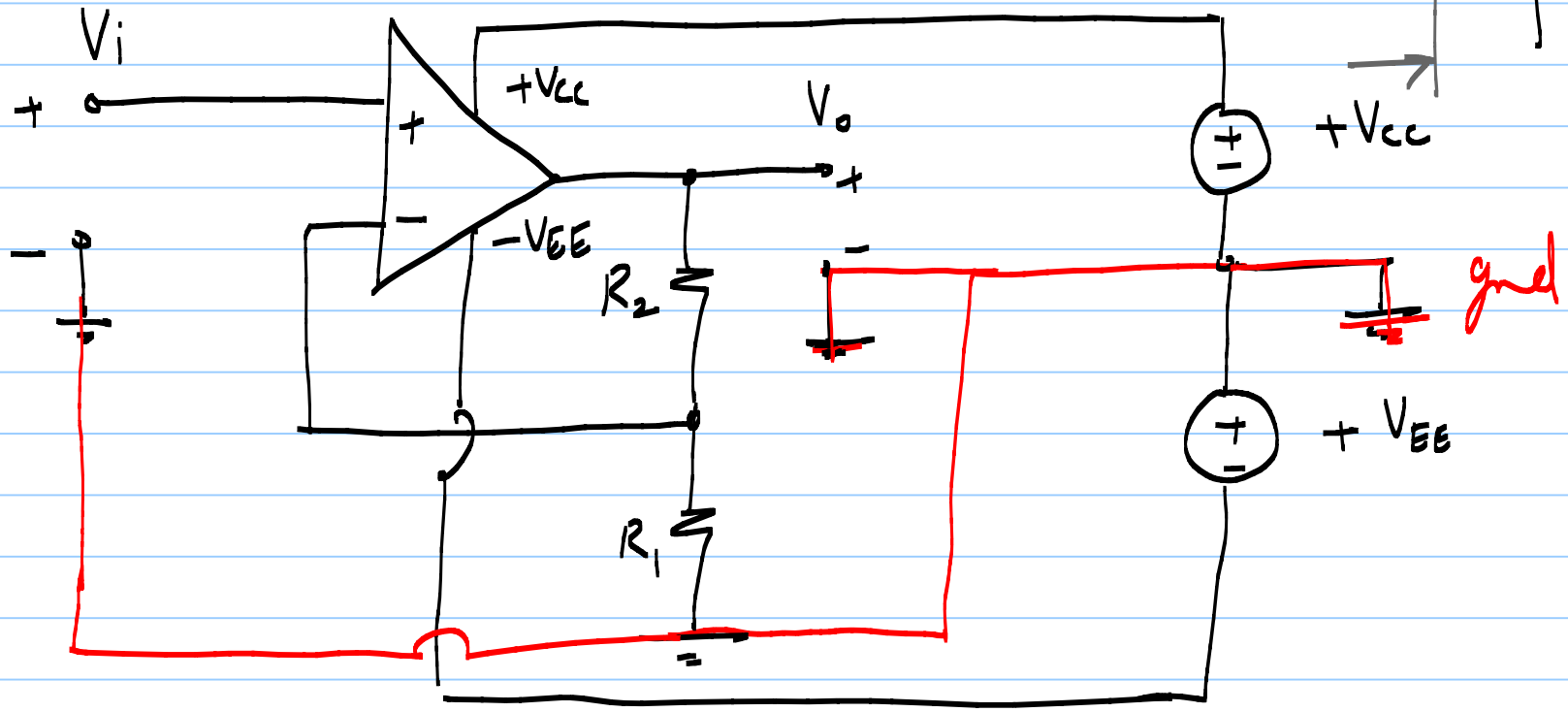
11/02/2022

Lecture 14

1) Bipolar supply

+ Vcc
− VEE

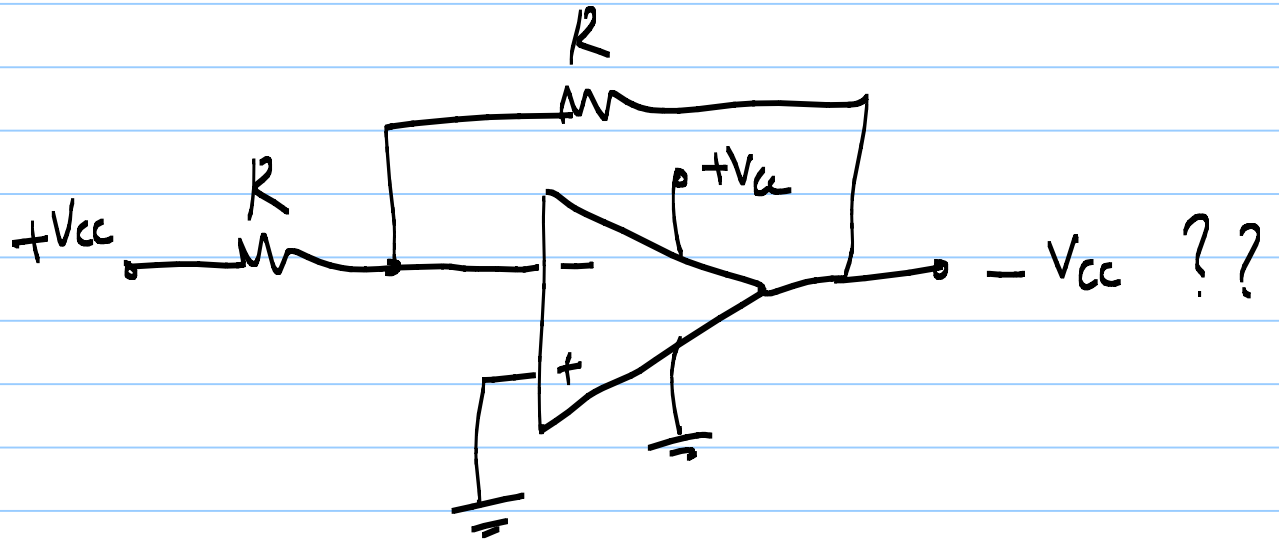




2) Unipolar supply

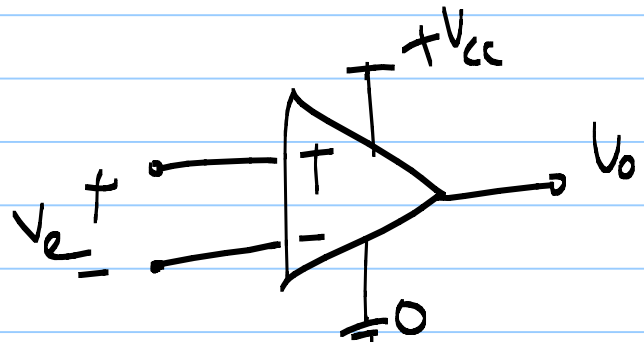
* Suggestion: Use inverting config. to generate $-V_{EE}$ from $+V_{CC}$

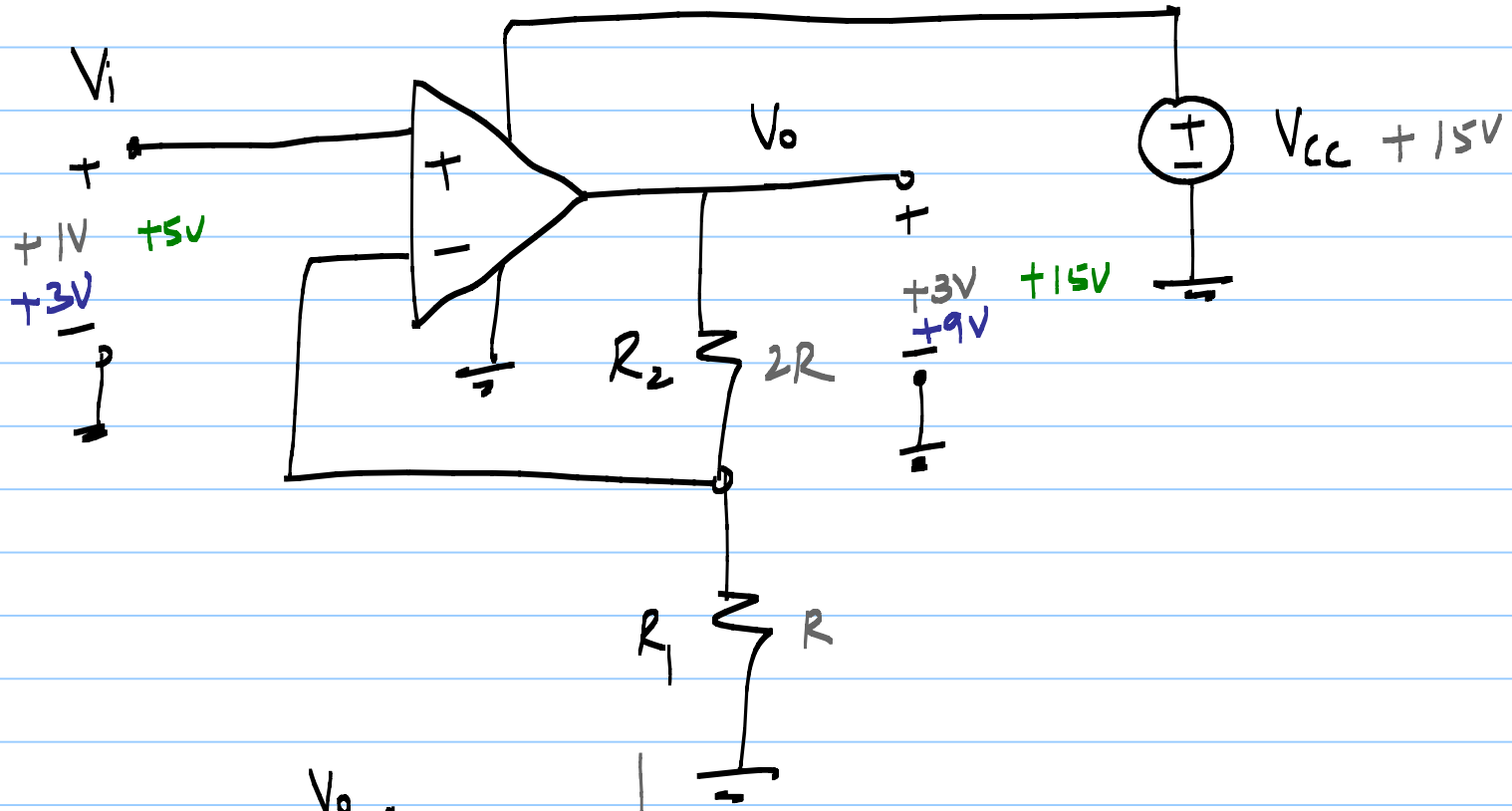
Assume $|V_{EE}| = |V_{CC}|$

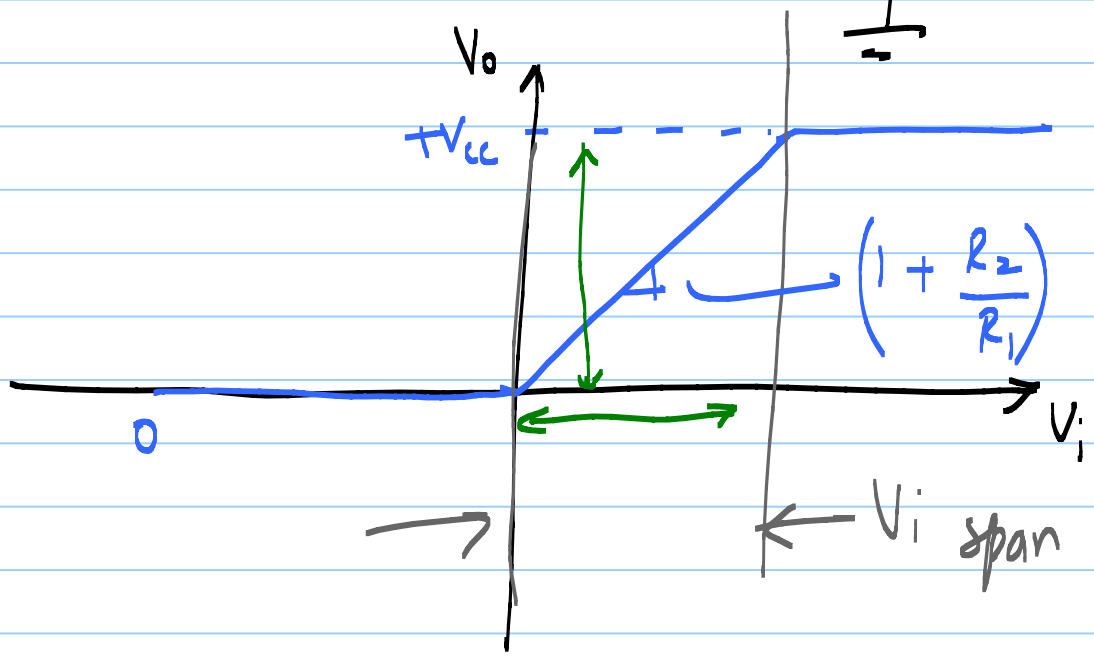


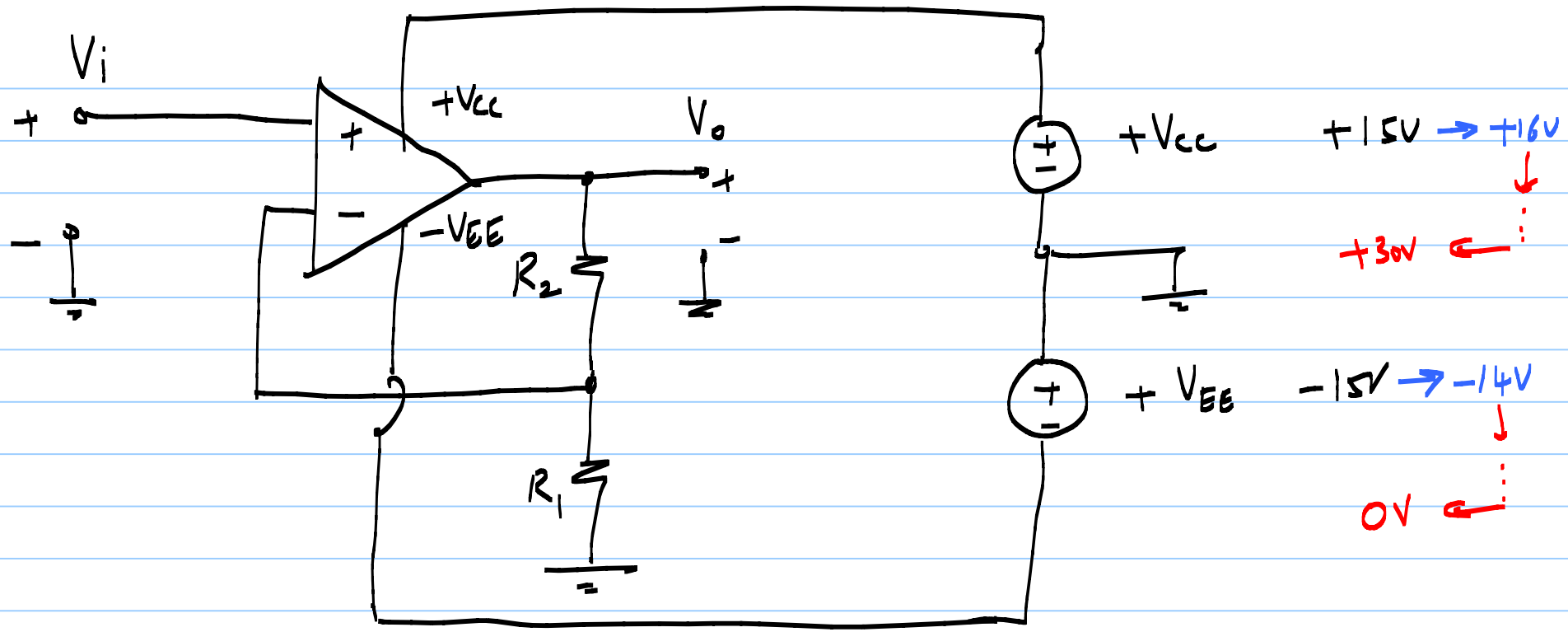
Will not work

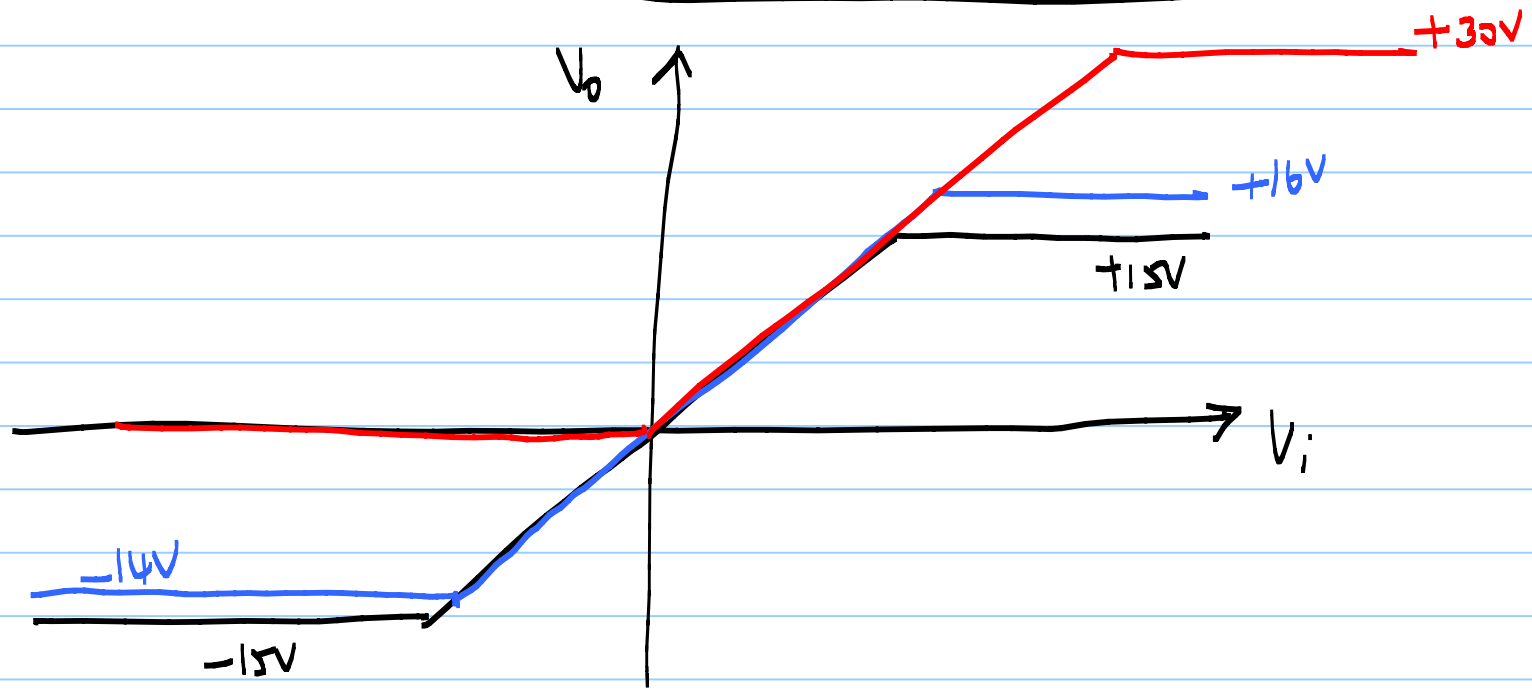
* Can you do away with $-V_{EE}$ & set opamp supply to $+V_{CC}$ & 0V

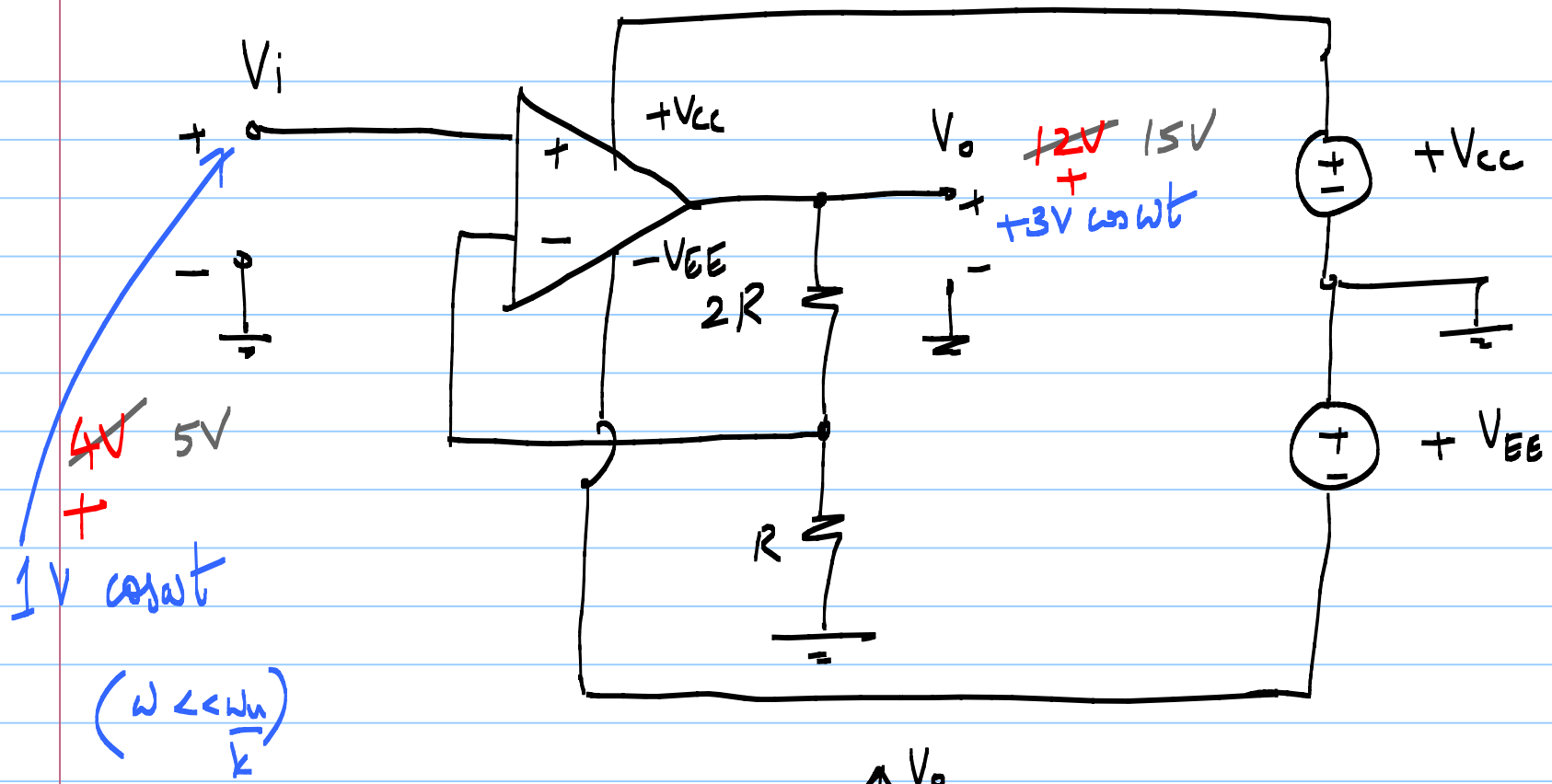


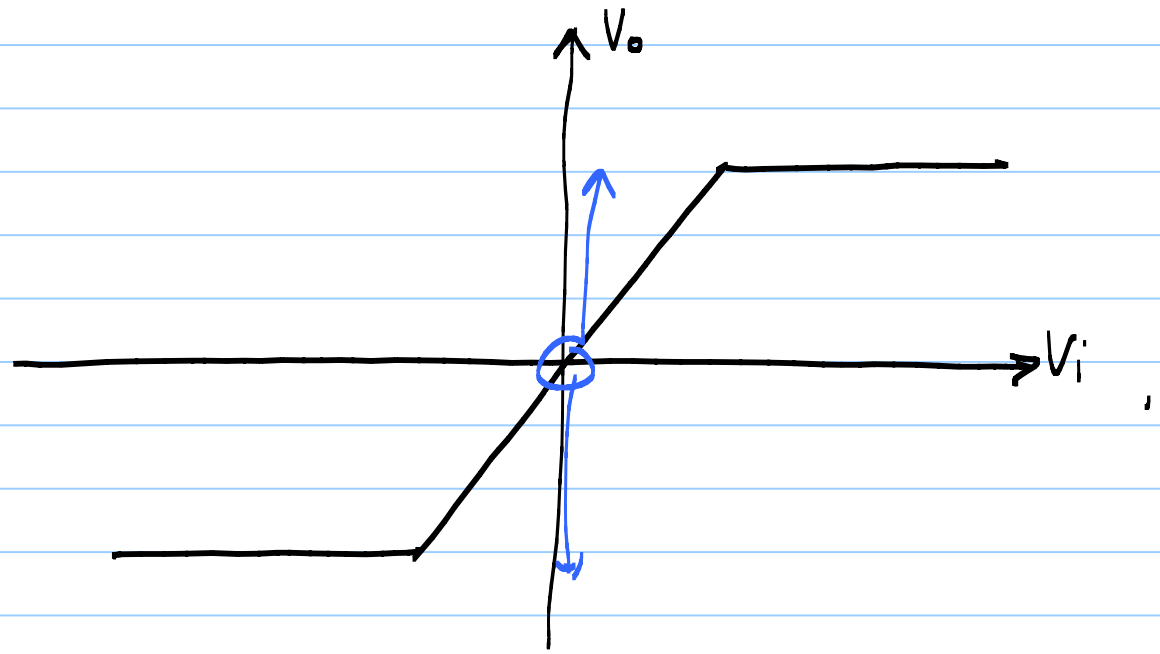

V_i
+1V
+5V
+3V
V_o
+3V
+15V
+9V
V_{cc} +15V
R_2
2R
R_1
R


V_o
$+V_{cc}$
$\left(1 + \frac{R_2}{R_1}\right)$
0
V_i
V_i span


V_i
+
−
+
−
$+V_{CC}$
$-V_{EE}$
V_o
+
−
R_2
R_1
$+V_{CC}$
$+V_{EE}$
+15V → +16V
+30V
−15V → −14V
0V

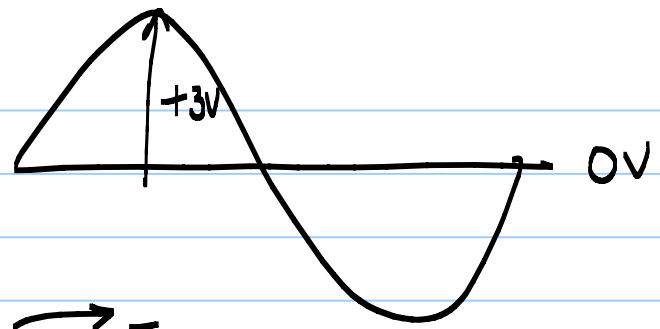

V_o
V_i
+30V
+16V
+15V
−14V
−15V


V_i
+
−
$+V_{CC}$
$-V_{EE}$
V_o
~~12V~~ 15V
+
+3V cos ωt
2R
R
$+V_{CC}$
$+V_{EE}$
~~4V~~ 5V
+
1V cos ωt
$\left(\omega \ll \frac{\omega_n}{k}\right)$


V_o
V_i

V_i V_o

1) 1V cos wt



output DC level $\Big\} = 12V$

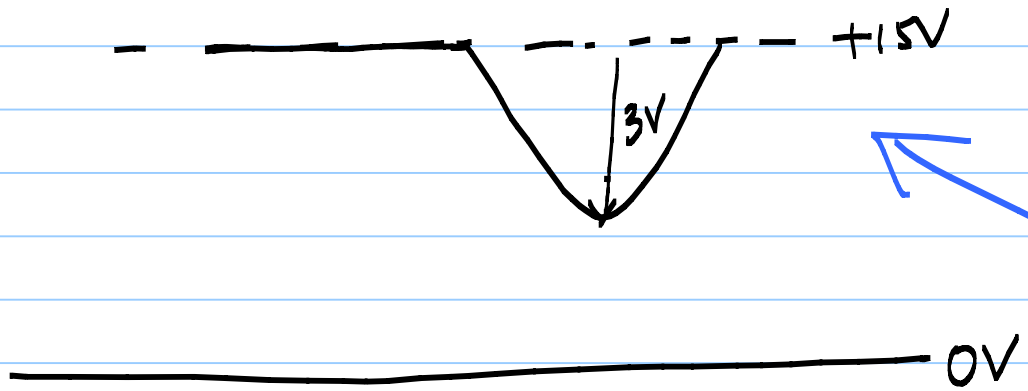
2) 4V + 1V cos wt

after 4V DC

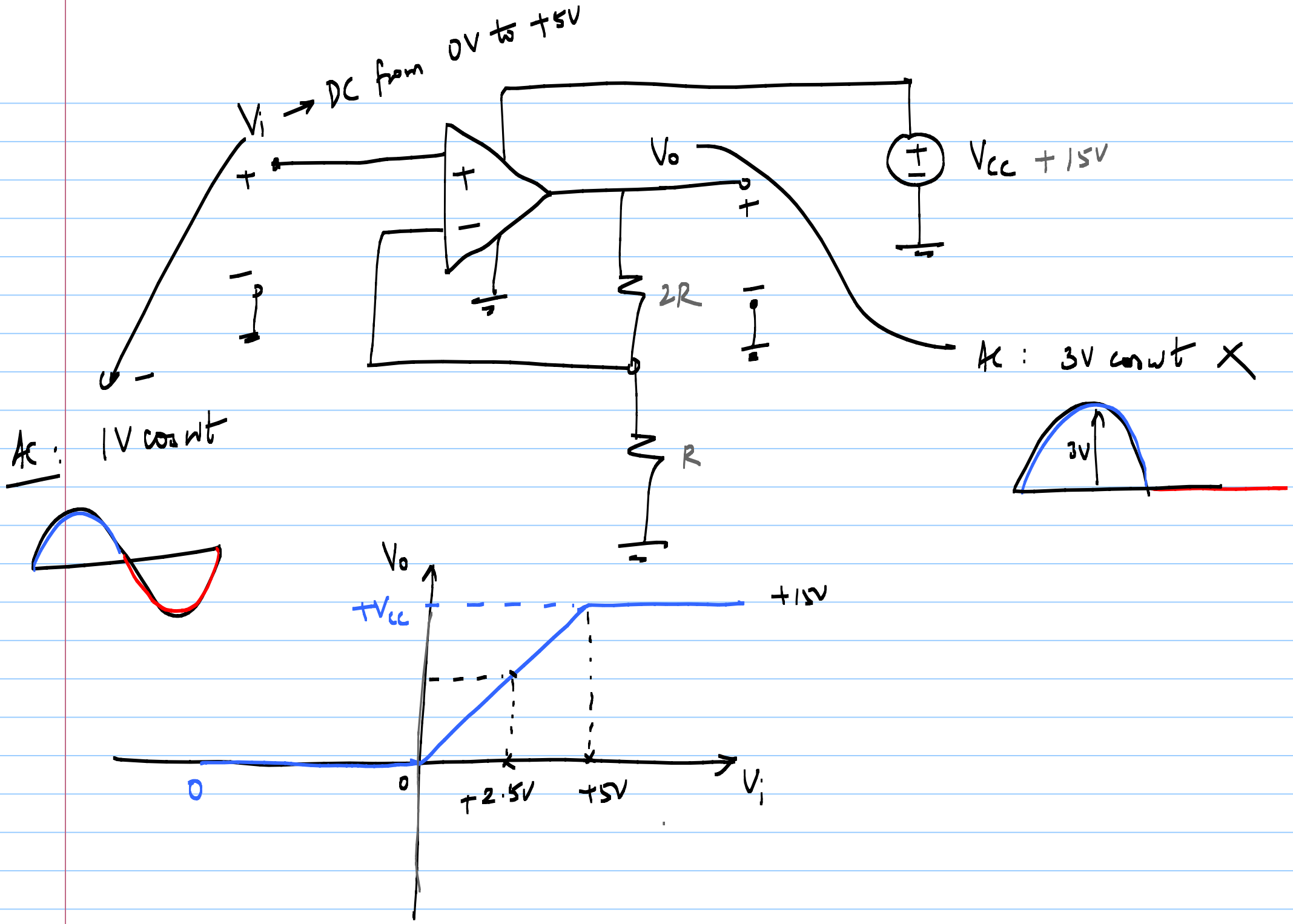
↓

opamp output starts "clipping"

3) 5V + 1 cos wt



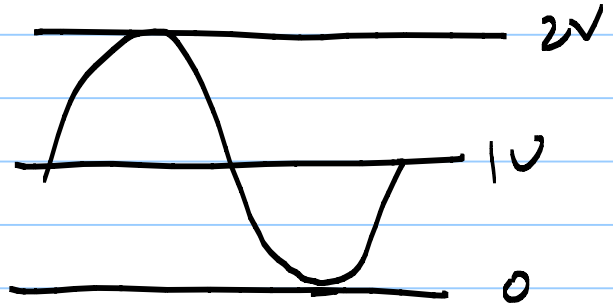
$12V < V_{o_{DC}} < 15V$


Vi → DC from 0V to +5V
+
−
+
−
Vo
+
−
2R
R
Vcc +15V
AC : 3V const ✗
3V
AC : 1V const
Vo
+Vcc
+15V
0
0
+2.5V
+5V
Vi

V_i

1V cos wt

1V + 1V cos wt



2.5V + 2.5V cos wt

V_o

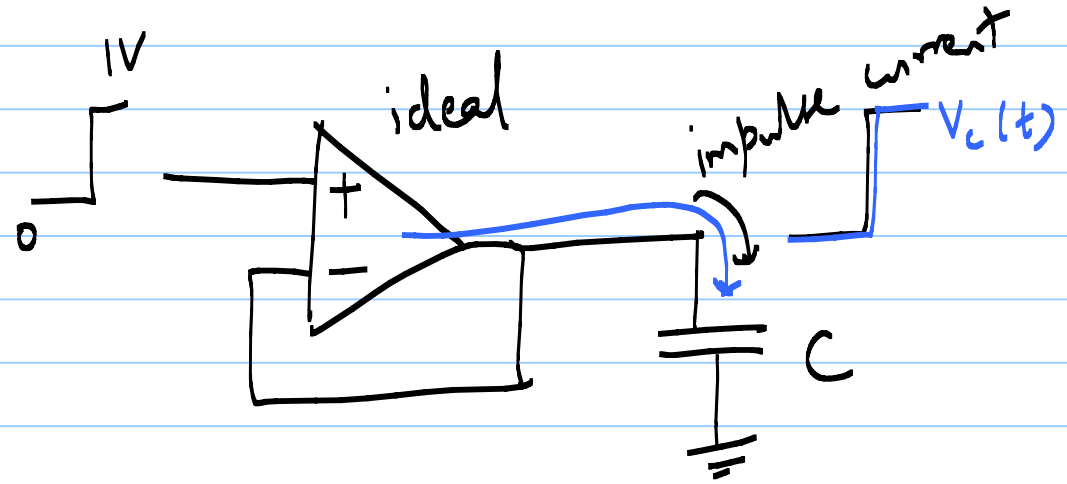
clipped

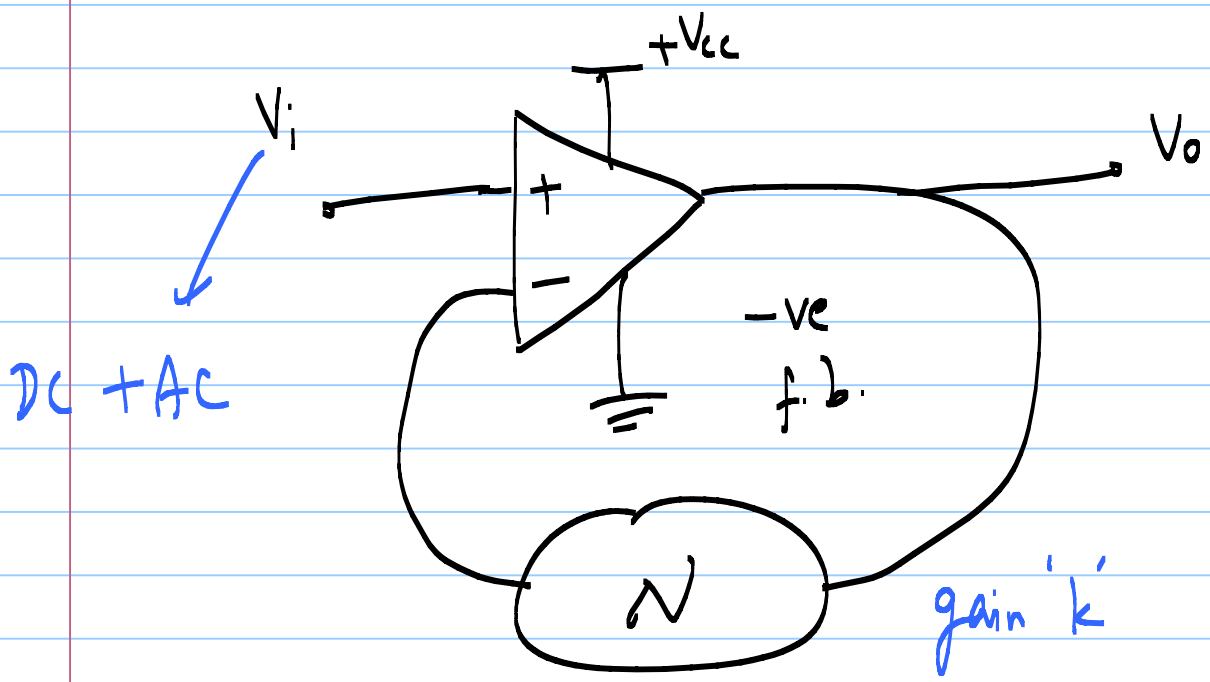
3V + 3V cos wt

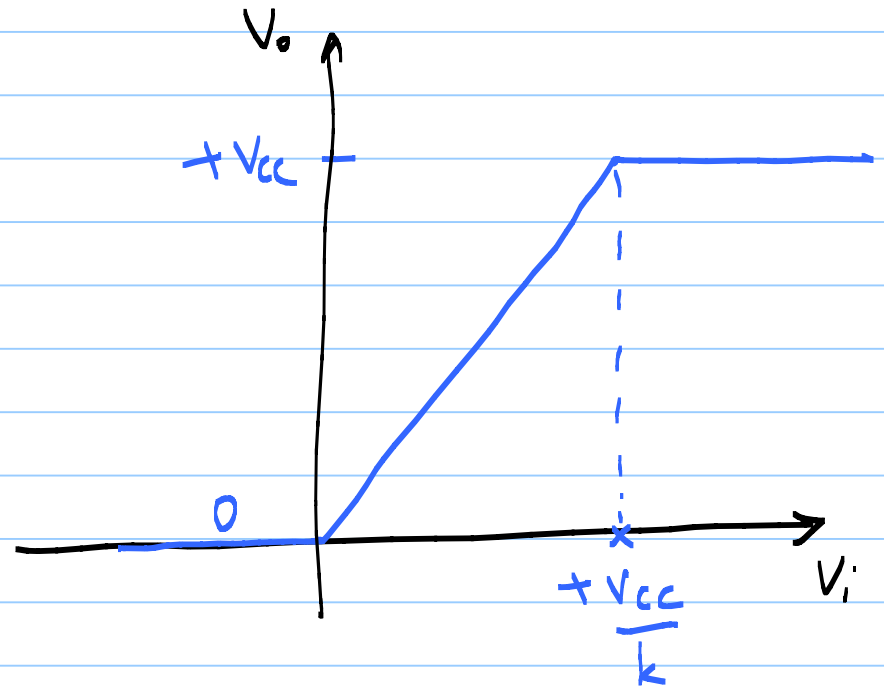
7.5V + 7.5V cos wt

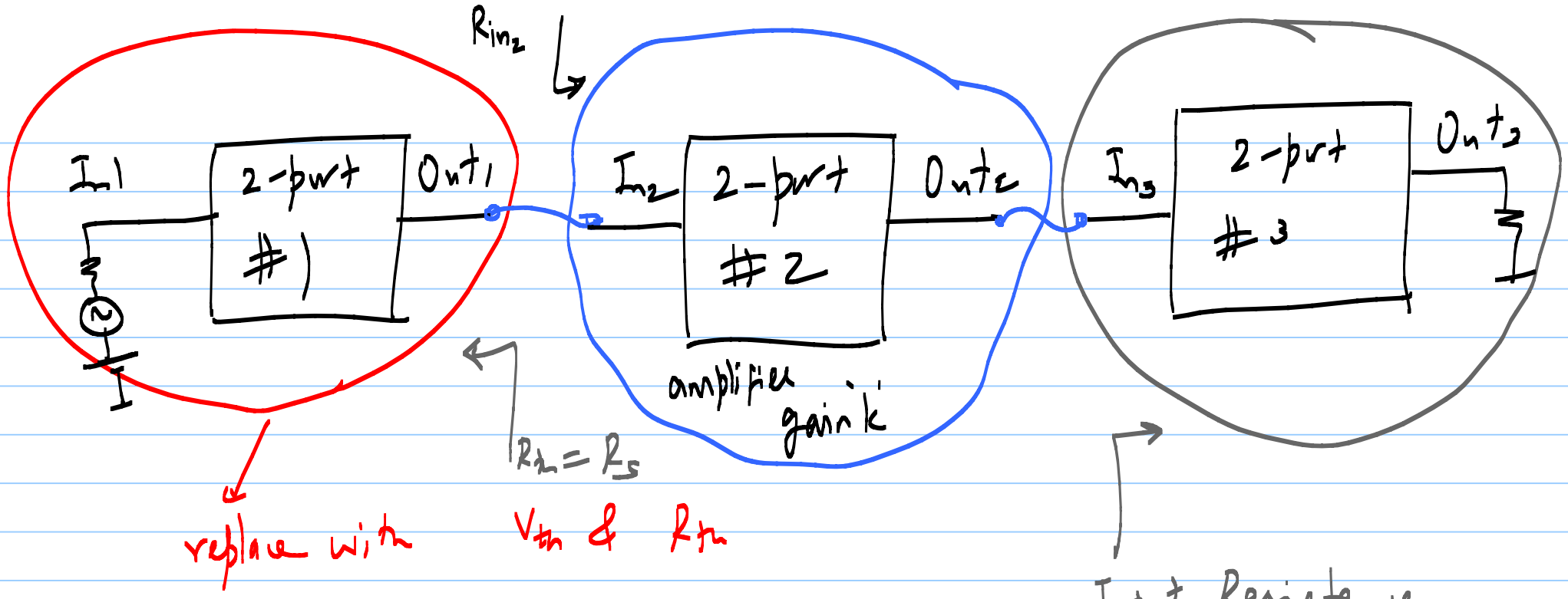
15/02/2022

Lecture 15

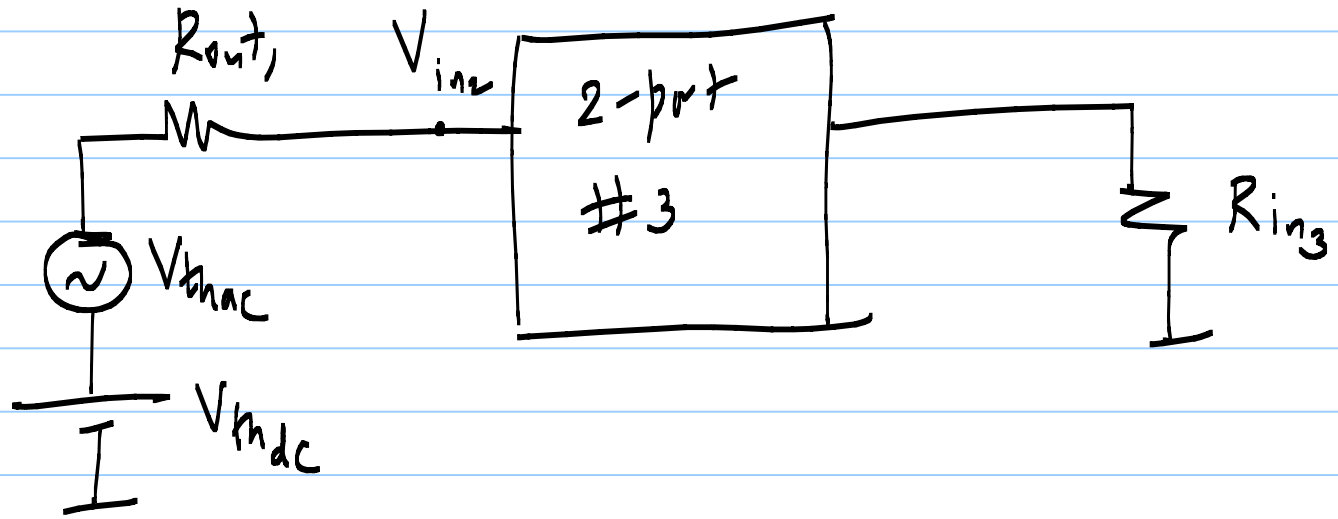


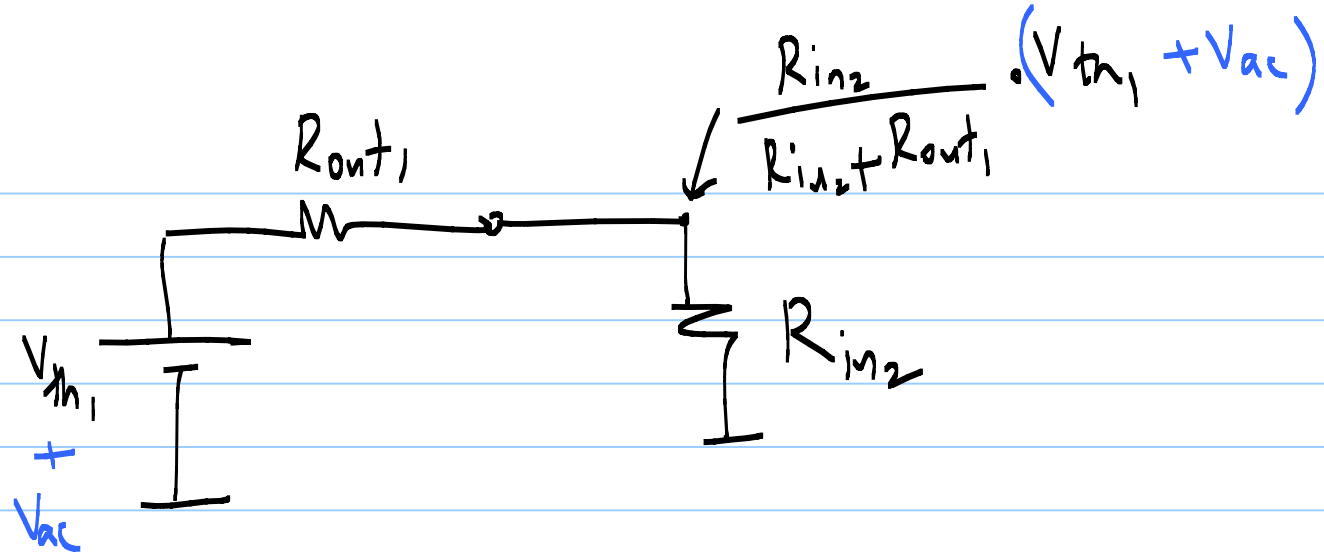


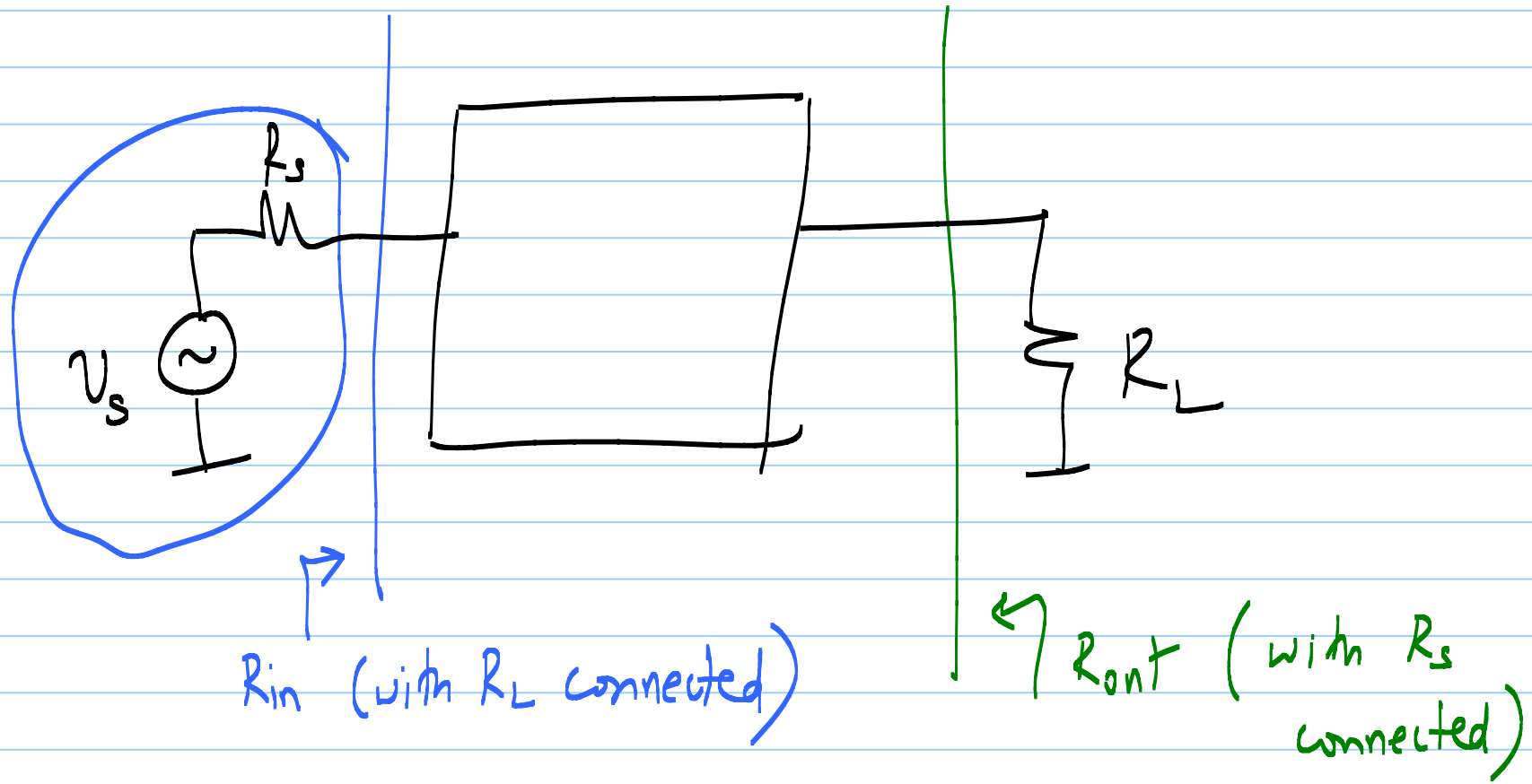


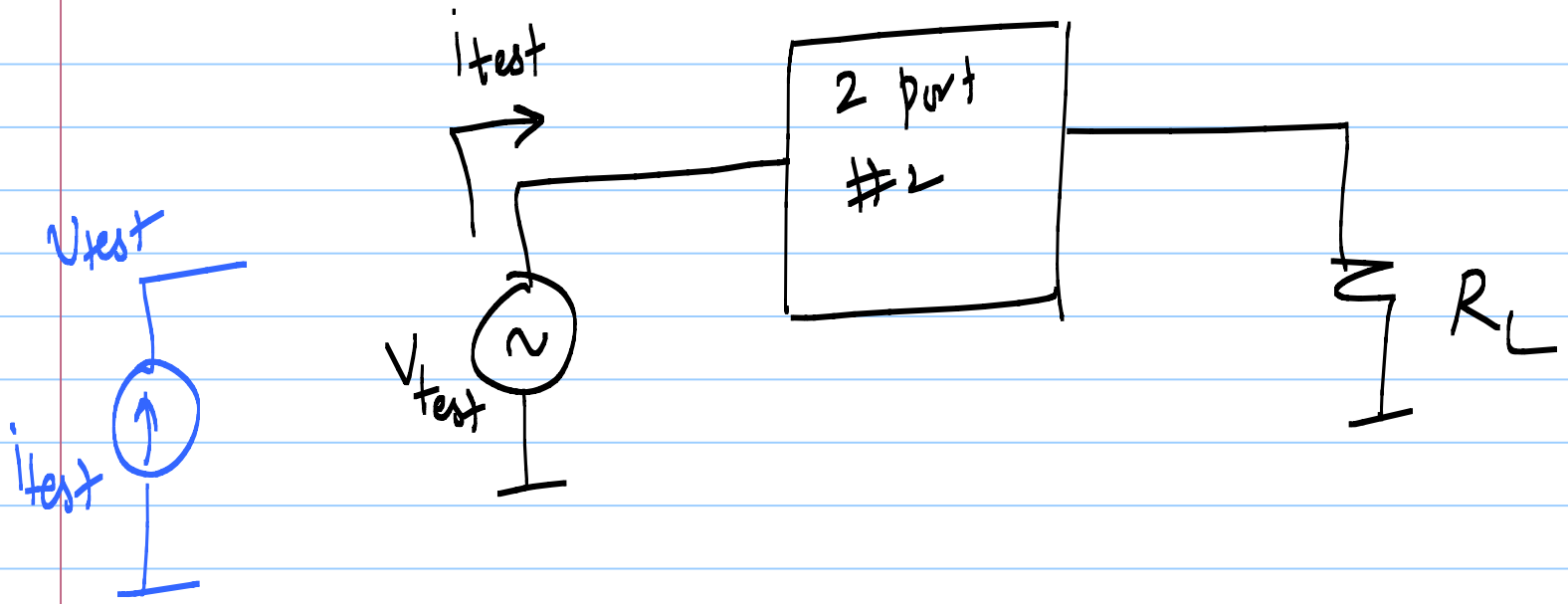


$R_{in_2} \nearrow R_{out_1}$

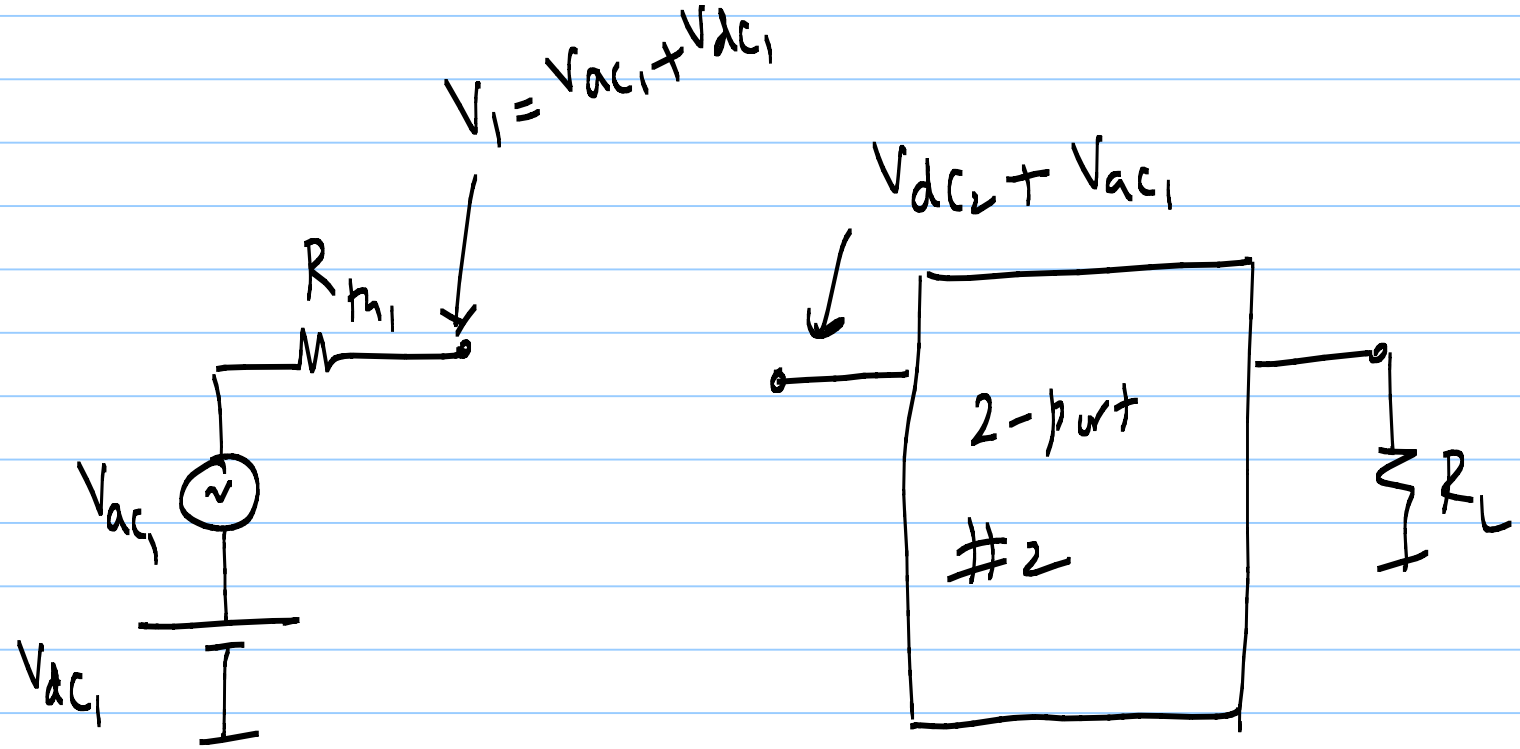



$\frac{R_{in_2}}{R_{in_2}+R_{out_1}} \cdot (V_{th_1} + V_{ac})$
R_{out_1}
R_{in_2}
V_{th_1}
+
V_{ac}


R_s
v_s
R_L
R_{in} (with R_L connected)
R_{out} (with R_s connected)



$$\frac{v_{test}}{i_{test}} = R_{in}$$

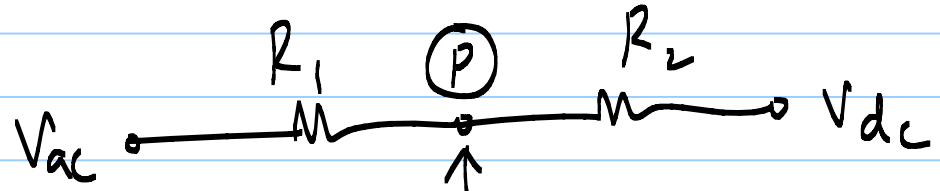


We want to remove V_{dc_1} from V_1 & add V_{dc_2} to it.

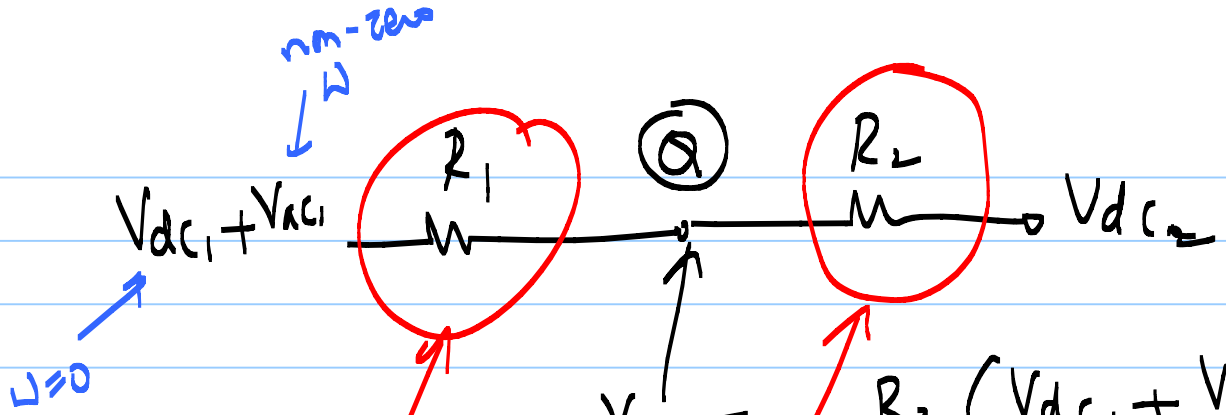
$\Rightarrow \quad V_{ac_1} \quad \oplus ?? \quad V_{dc_2}$

$\Rightarrow \quad V_{dc_x} \quad \oplus ?? \quad V_{dc_y}$

$$V_{dc_z} = \frac{R_1 V_{dc_y} + R_2 V_{dc_x}}{R_1 + R_2}$$

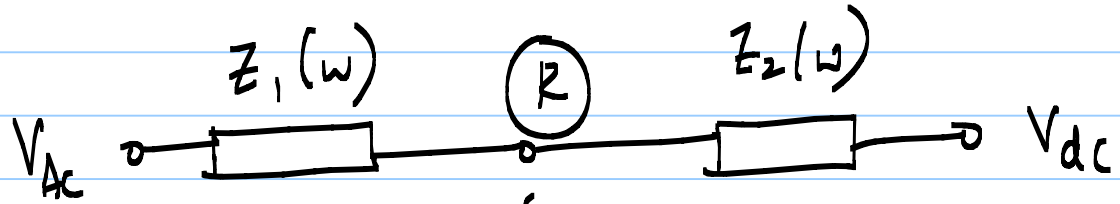


$$V_p = \frac{R_1 V_{dc} + R_2 V_{ac}}{R_1 + R_2}$$

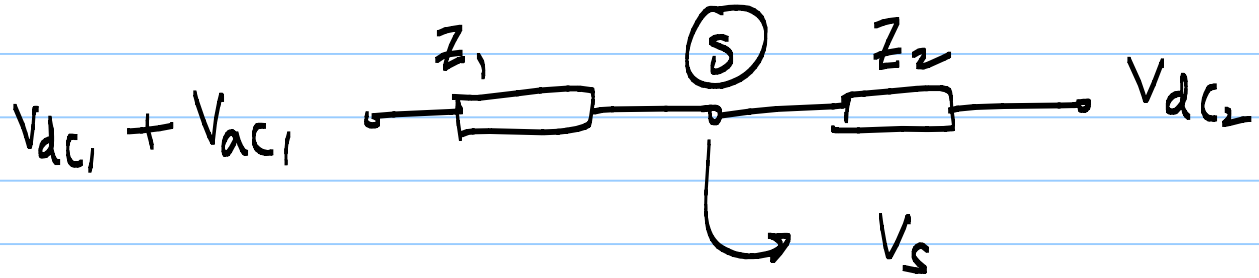


$$V_Q = \frac{R_2(V_{dc_1} + V_{ac_1}) + R_1(V_{dc_2})}{R_1 + R_2}$$

need frequency selectivity



$$V_R = \frac{Z_2(\omega)}{Z_1(\omega) + Z_2(\omega)} V_{Ac} + \frac{Z_1(0)}{Z_1(0) + Z_2(0)} \cdot V_{dc}$$



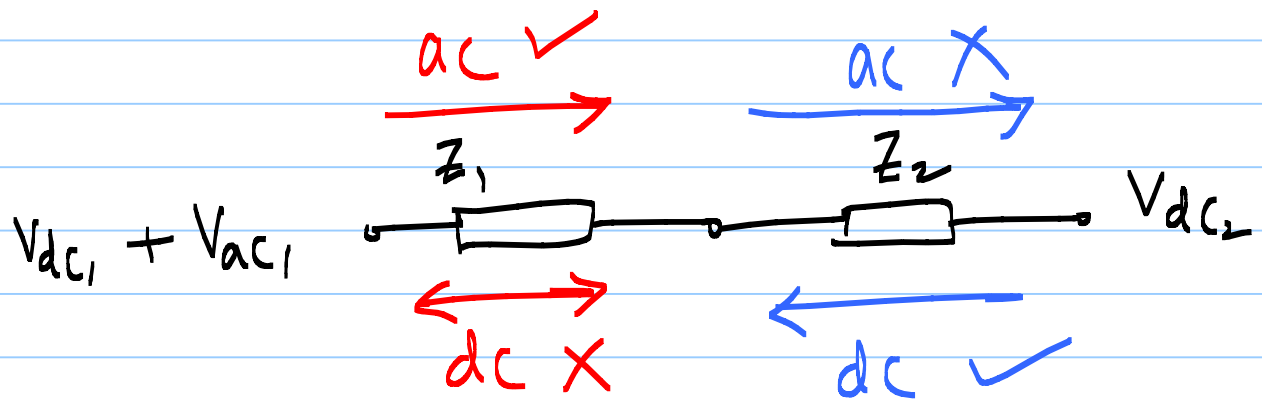
$$V_s = \frac{Z_2(\omega)}{Z_1(\omega)+Z_2(\omega)} \cdot V_{ac_1} + \frac{Z_2(0)}{Z_1(0)+Z_2(0)} \cdot V_{dc_1} + \frac{Z_1(0)}{Z_1(0)+Z_2(0)} \cdot V_{dc_2}$$

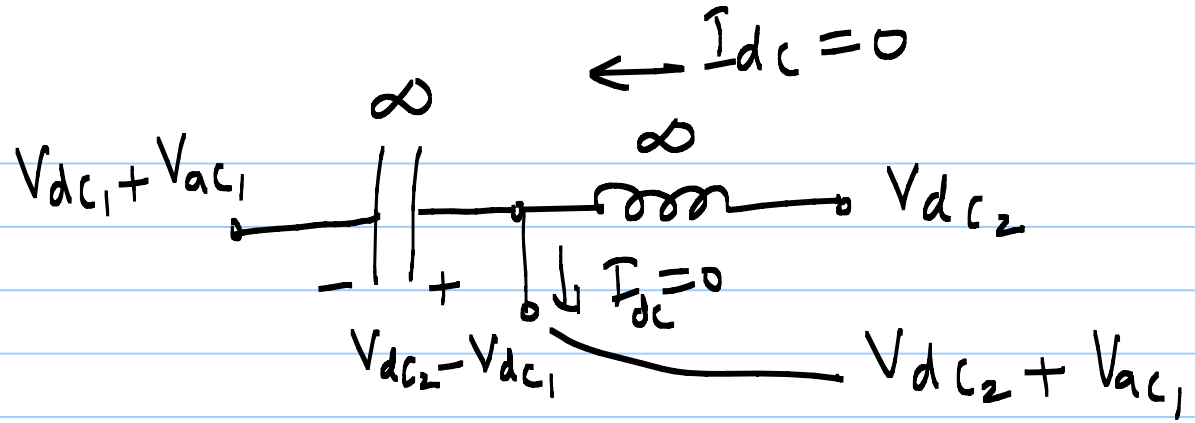
≈ 1 $\qquad$ ≈ 0 $\qquad$ ≈ 1

$Z_2(\omega) \gg Z_1(\omega)$

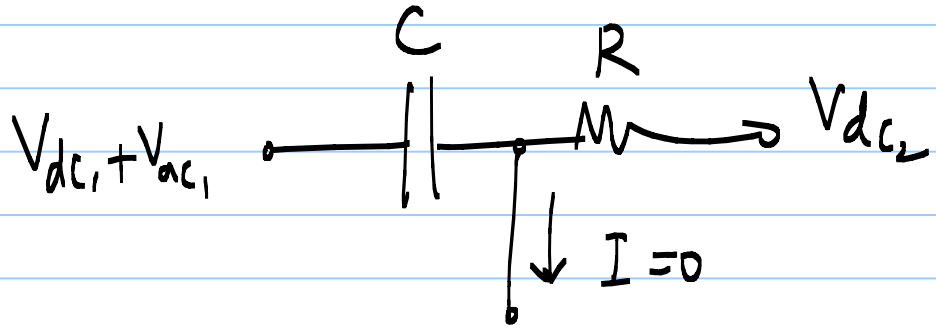
Z_2 should be reactive

$Z_1(0) \gg Z_2(0)$





* Analog circuits paradigm : Avoid inductors!



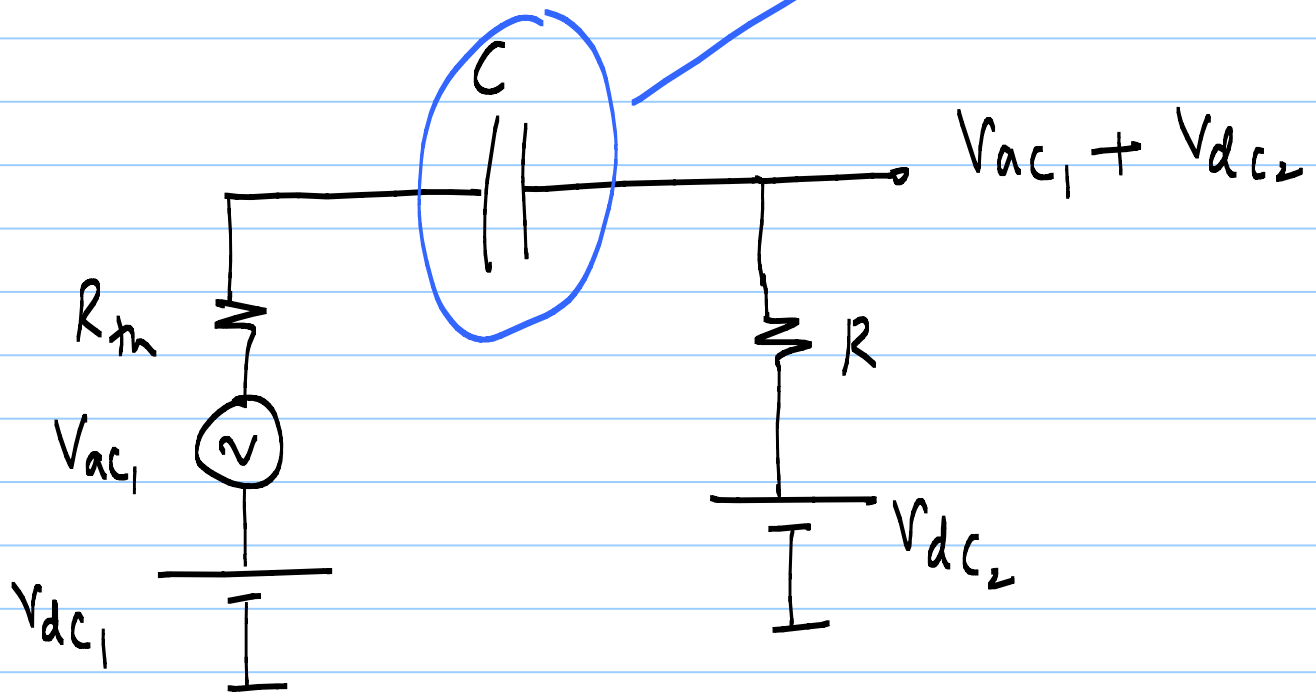
$$V_{dc_2} + \frac{R}{R + \frac{1}{j\omega C}} \cdot V_{ac_1}$$

$\approx 1 \Rightarrow$ $\boxed{|R| \gg \left|\frac{1}{\omega C}\right|}$ over range of desired ω

16/02/2022

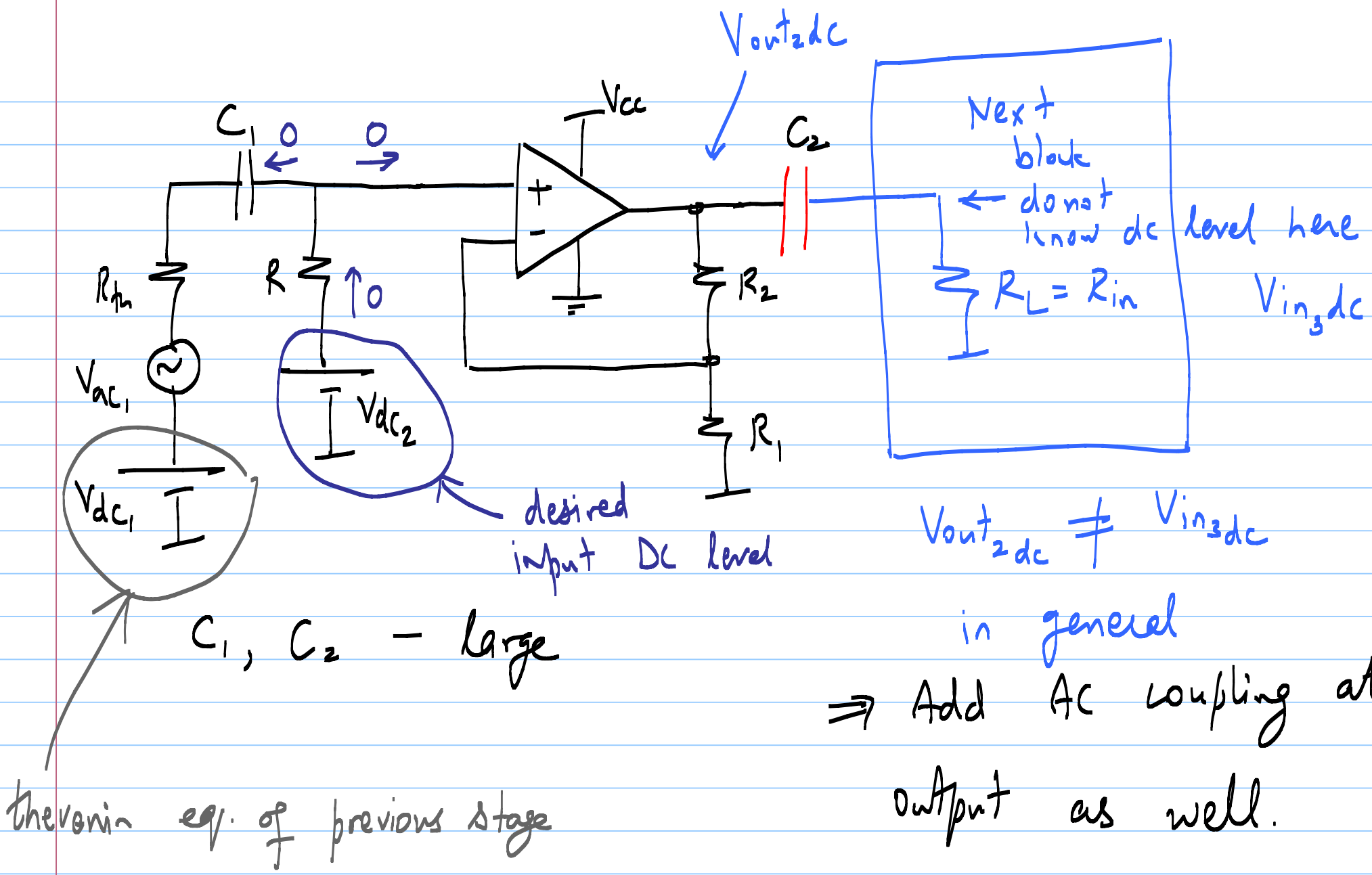
Lecture 16

Re-biasing with AC-coupling capacitor



if

$$|R| >> \left| \frac{1}{\omega C} \right|$$



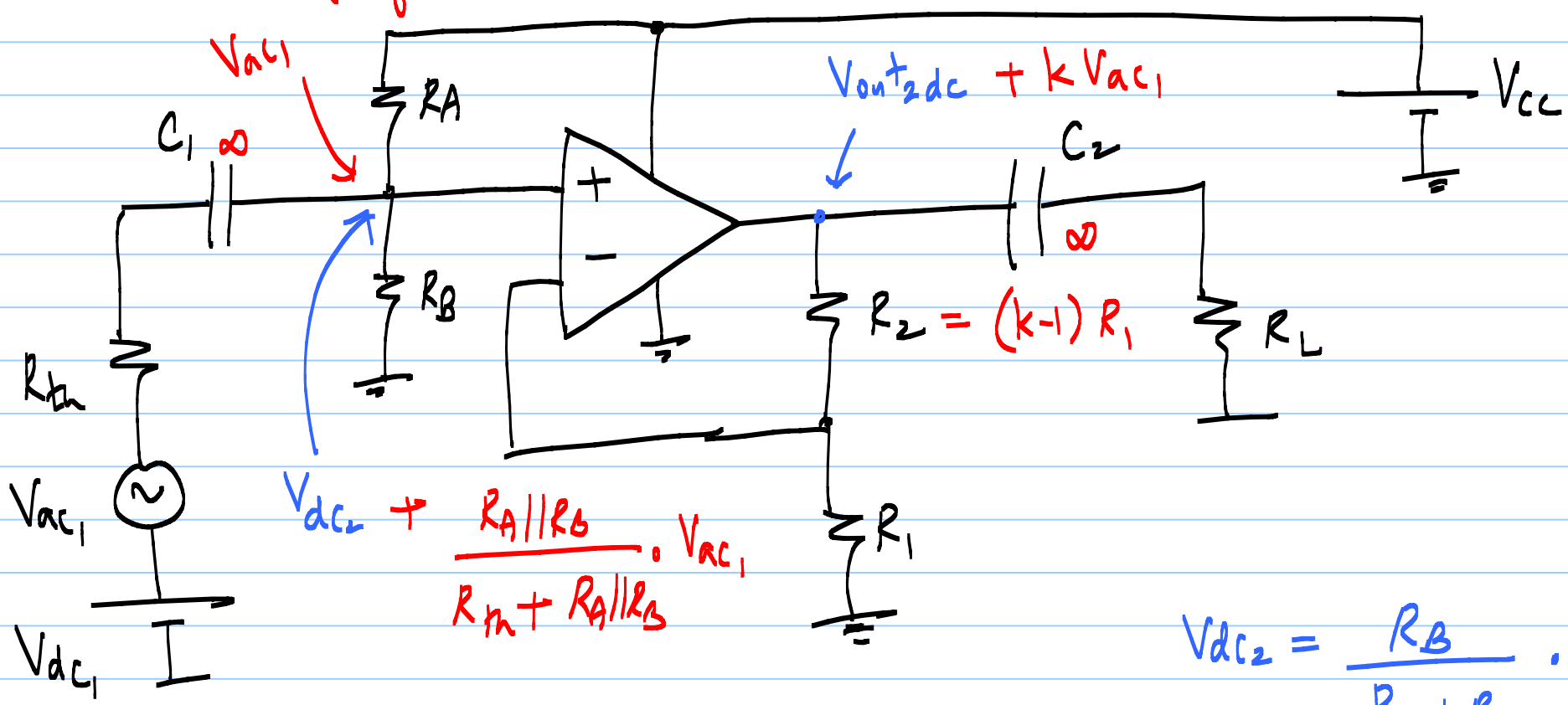
C_1, C_2 – large

$V_{out_{2dc}} \neq V_{in_{3dc}}$ in general

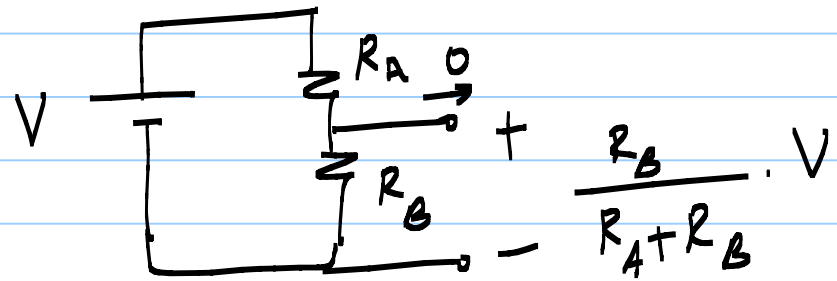
$\Rightarrow$ Add AC coupling at output as well.

$(R_A \| R_B) \gg R_{th}$

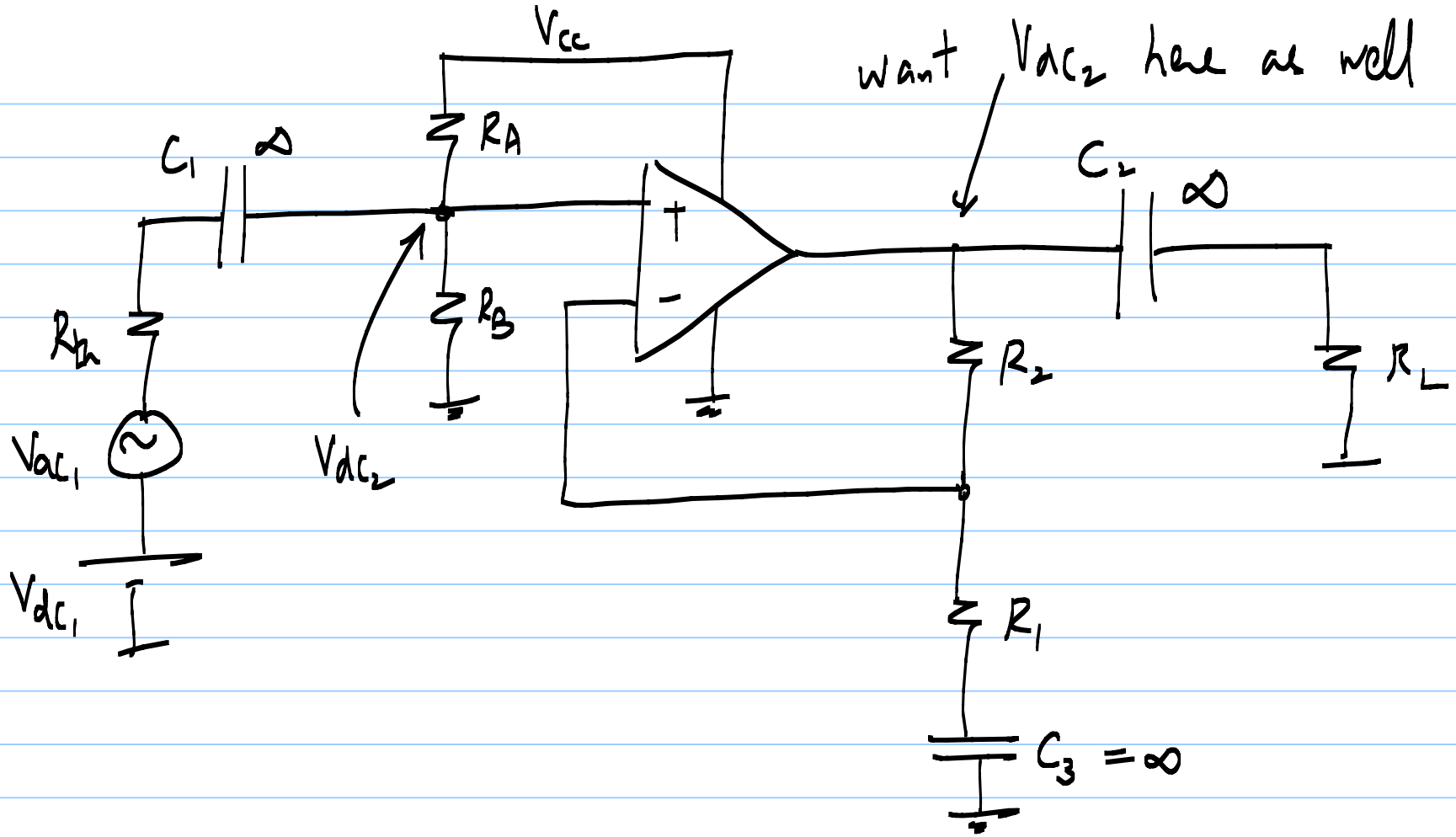
only if

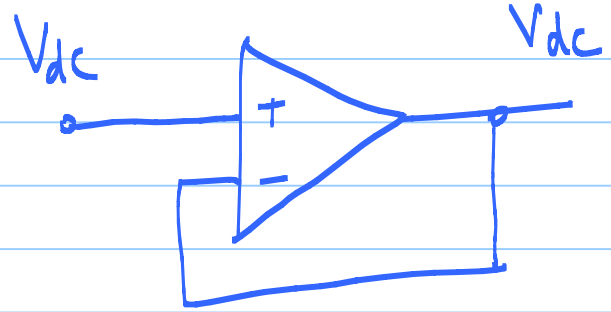


$$V_{dc_2} = \frac{R_B}{R_A + R_B} \cdot V_{cc}$$

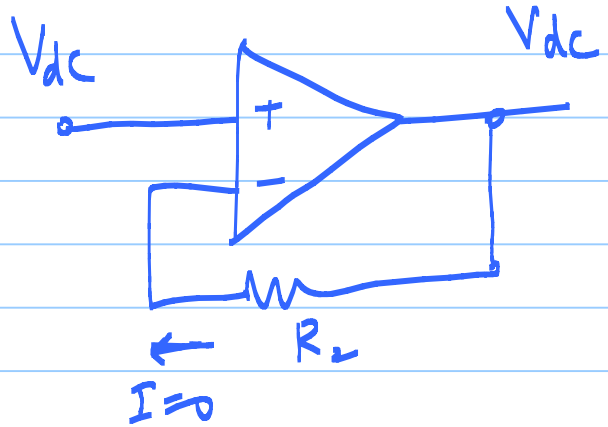


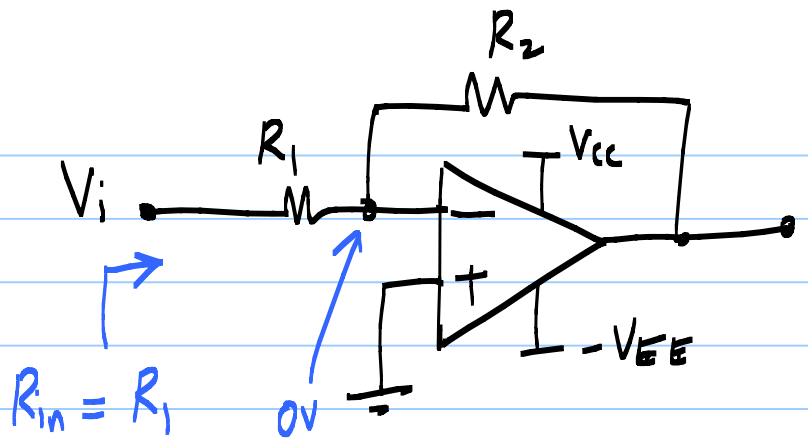
$$V_{out_{2,DC}} = V_{dc_2} \cdot \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{k}$$


Vcc
want Vdc2 here as well
C1
∞
RA
C2
∞
RB
Rth
R2
RL
Vac1
Vdc2
Vdc1
R1
C3 = ∞


Vdc
Vdc

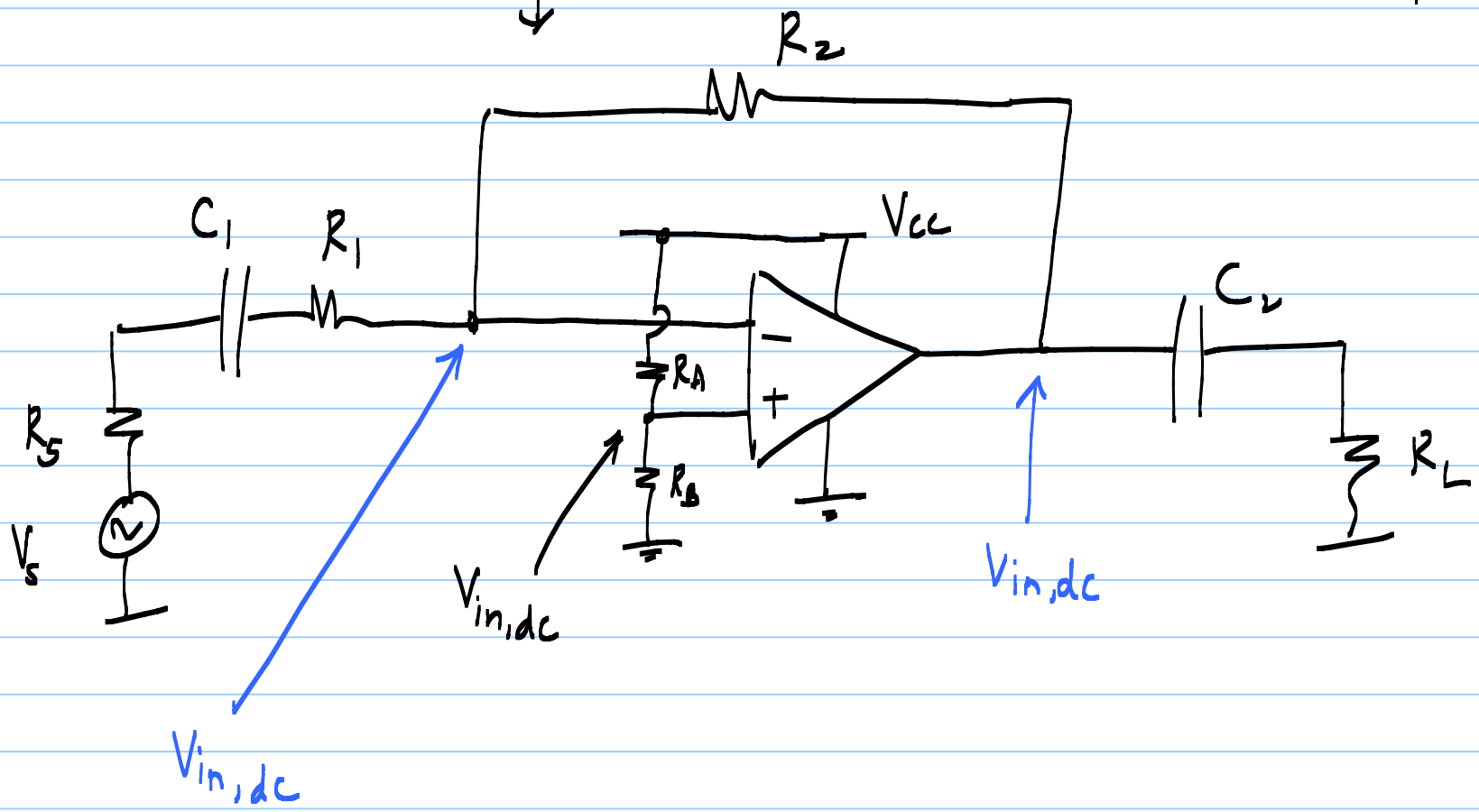
or


Vdc
Vdc
R2
I=0

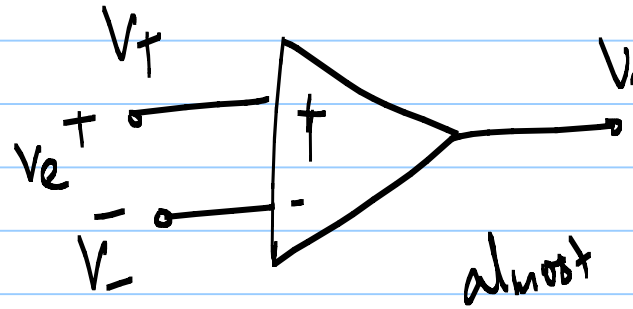


$$V_o = \frac{-R_2}{R_1} \cdot V_i$$

need $R_1 \gg R_s$

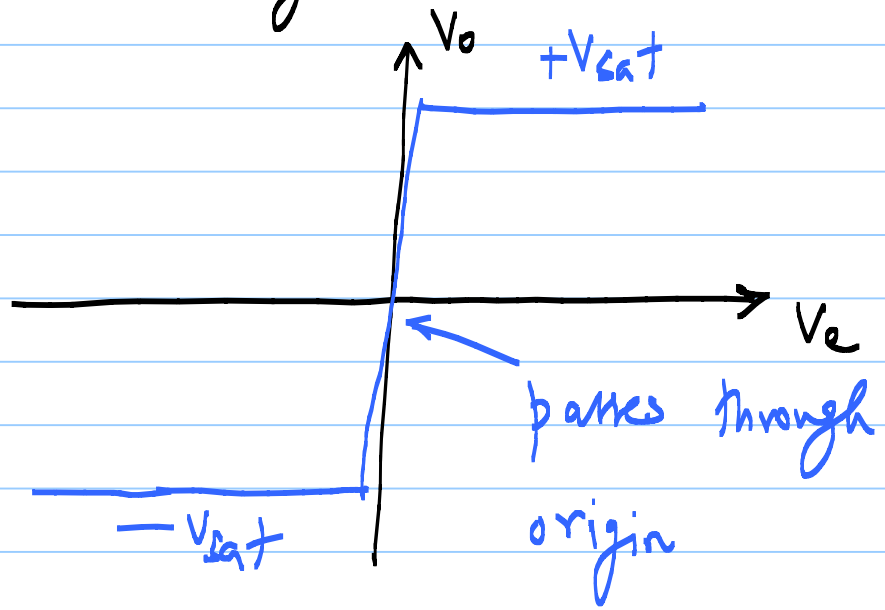


One more non-ideality – Input Offset Voltage

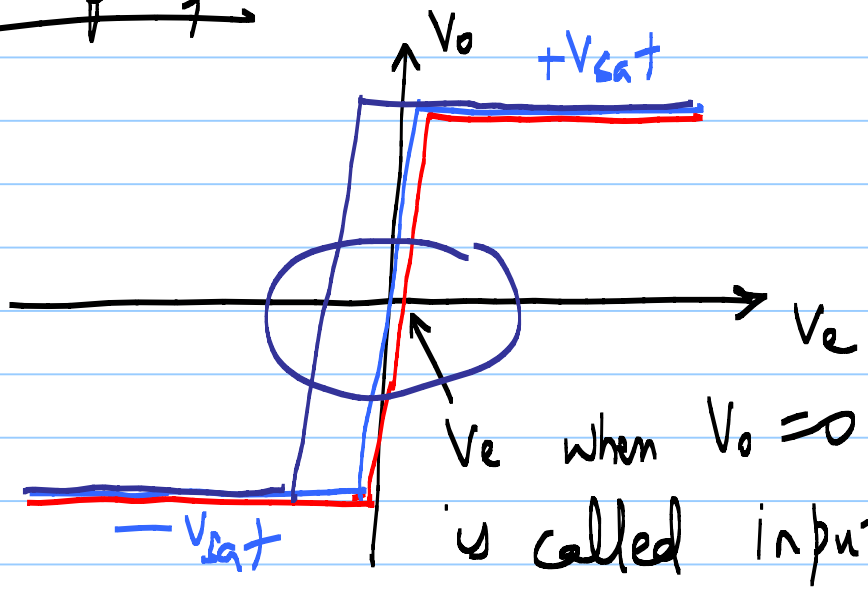


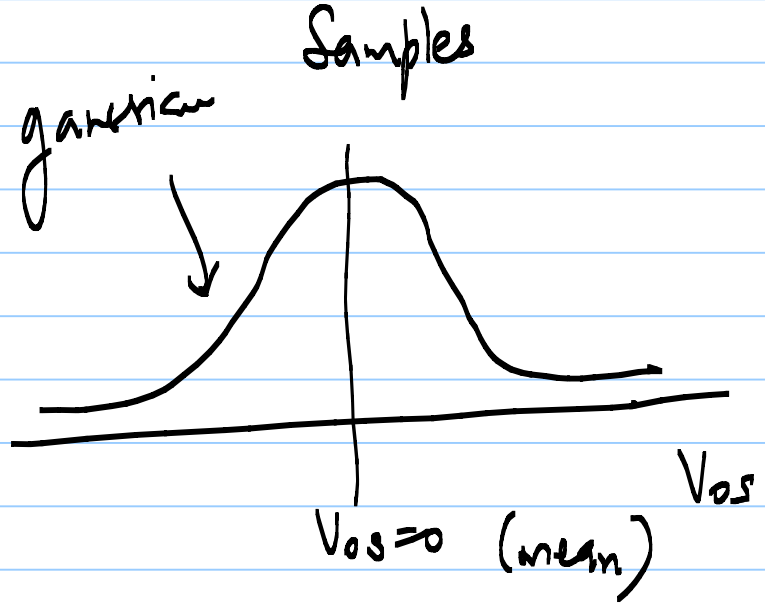
$V_o = A V_e$

almost ideal opamp ($A \to \infty$)

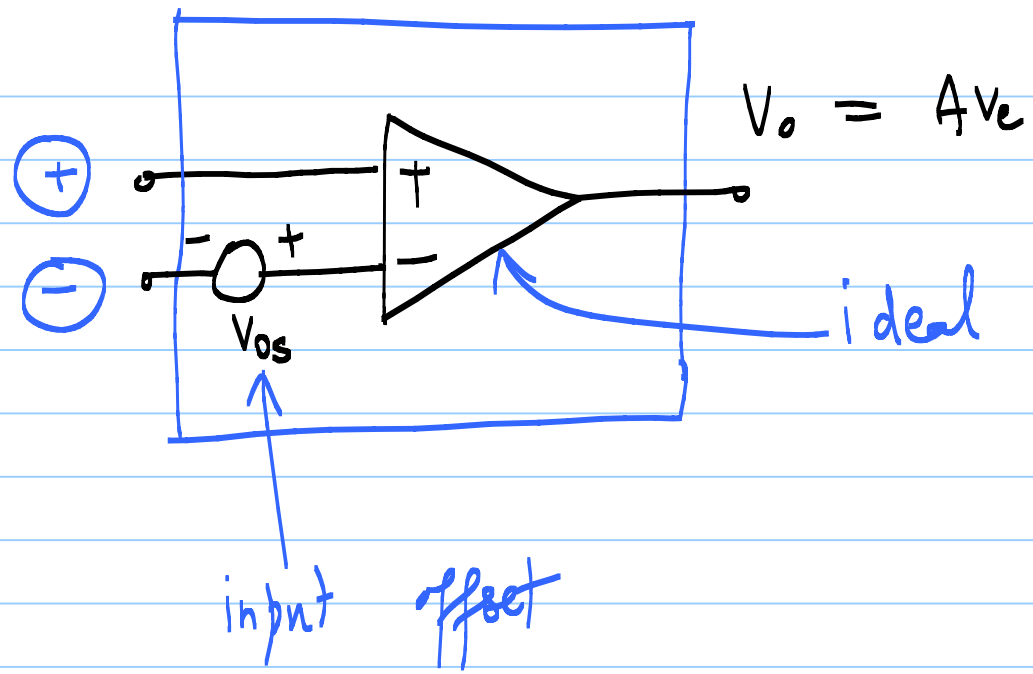


Real opamp





V_e when $V_o = 0$ is called input offset of opamp "V_{os}"



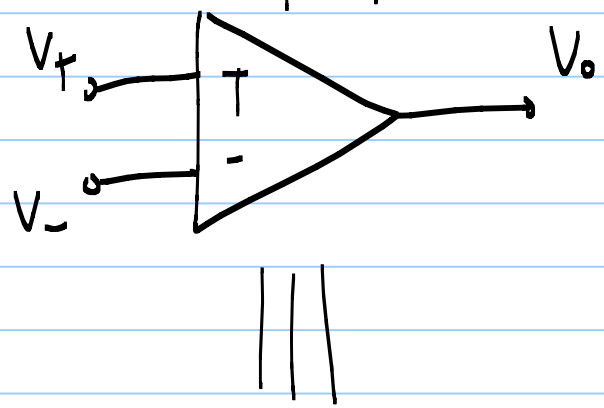
$V_o = A v_e$

Real opamp
(with offset)

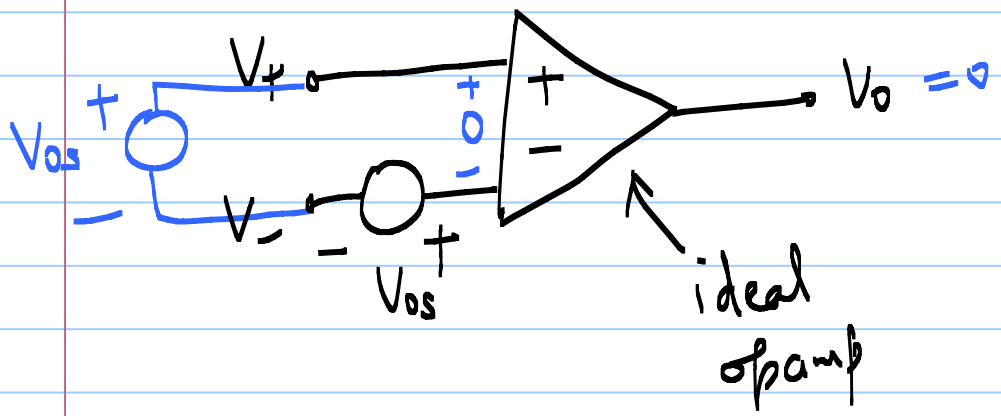
18/02/2022

Lecture 17

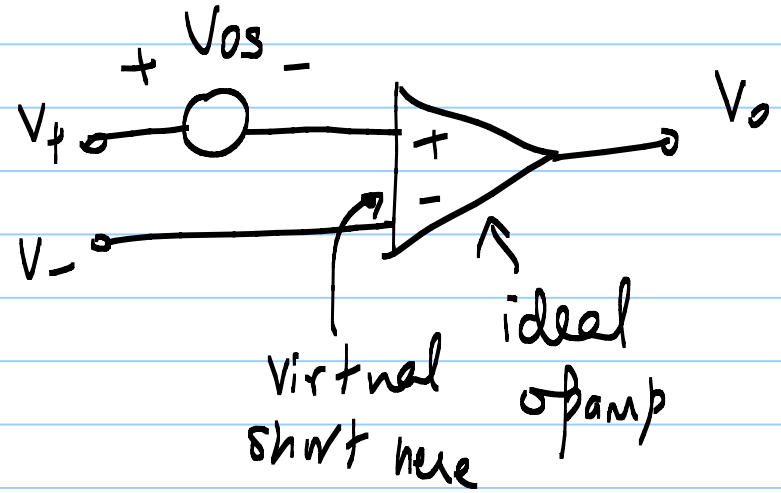
opamp with offset



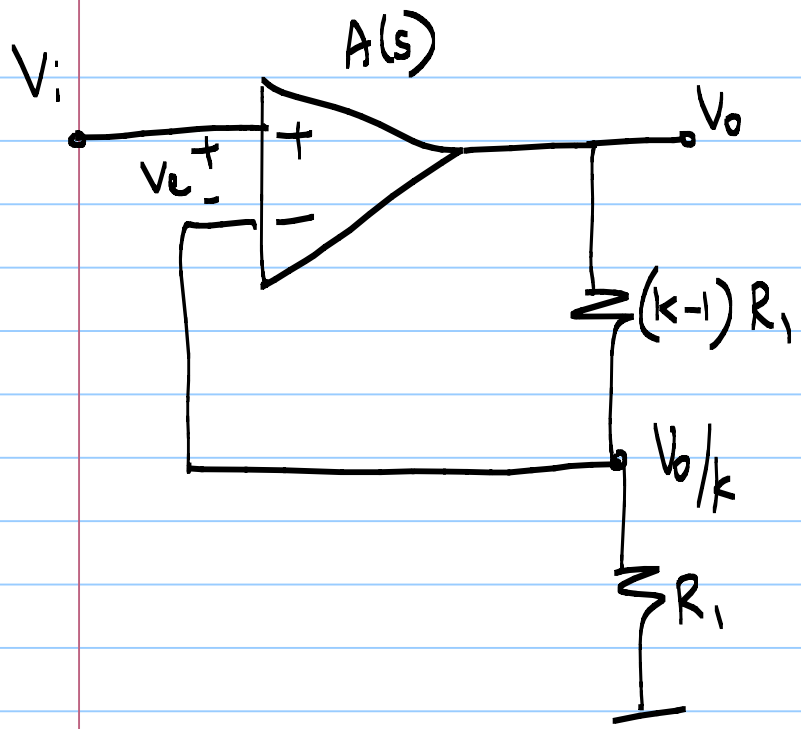
V_{os} = voltage that needs to be applied at input so that $V_o = 0$



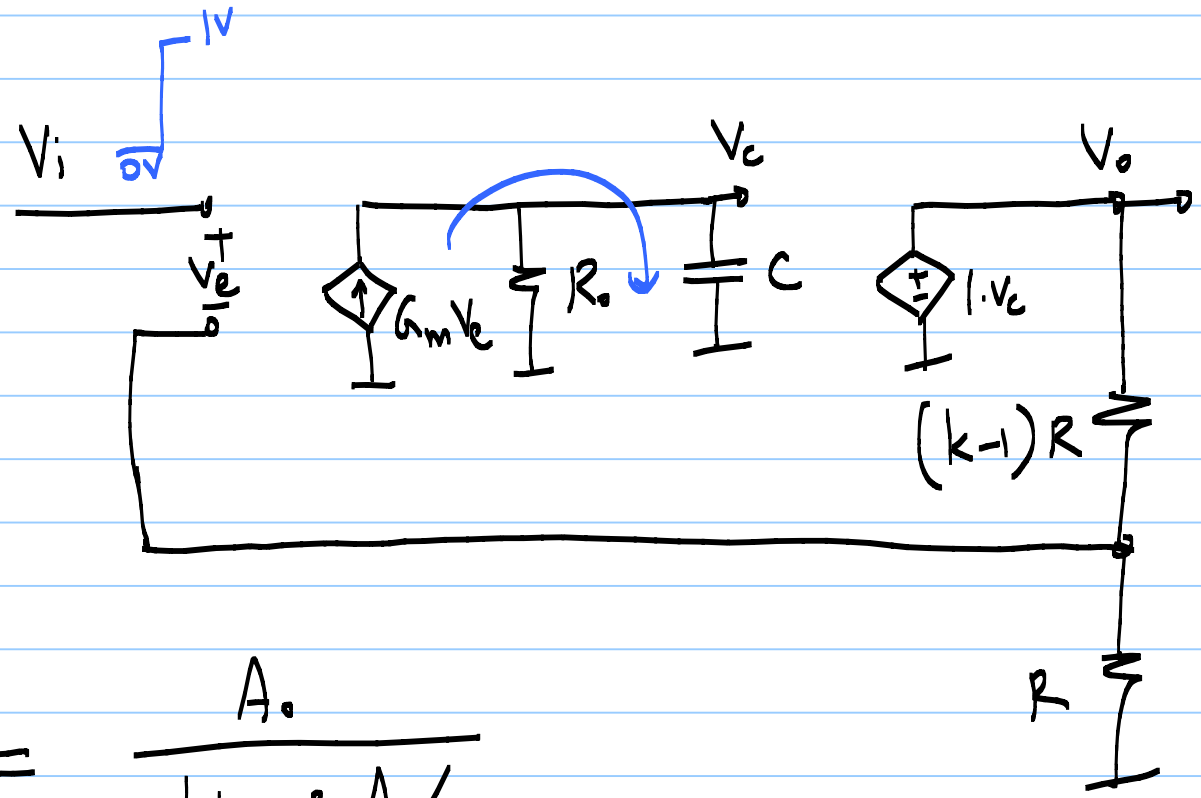
(or)



$$V_e = V_i - V_o/k$$



$\Longrightarrow$



$$\frac{V_o}{V_e}(s) = A(s) = \frac{A_o}{1 + sCR_o} = \frac{A_o}{1 + s\,A_o/\omega_u}$$

$$A_o = G_m R_o \quad ; \quad \omega_u = \frac{G_m}{C}$$

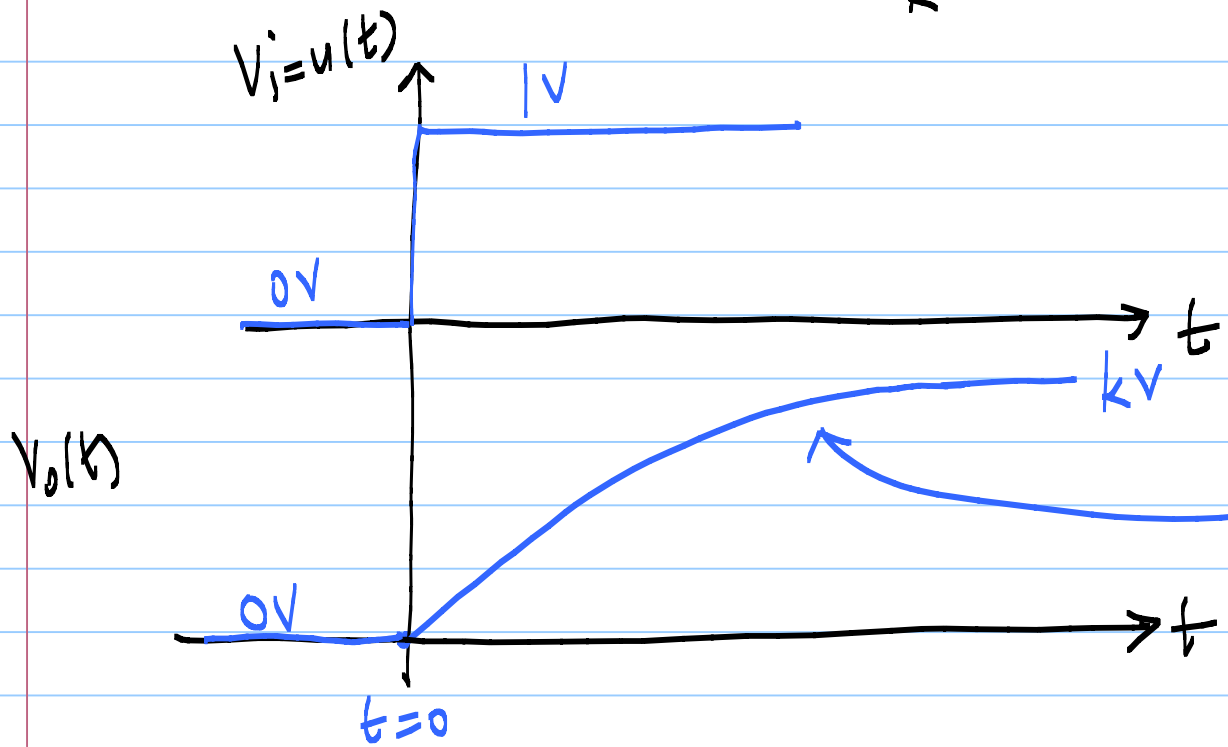
$$\frac{V_o}{V_i}(s) = ?$$

$$V_e(s) = V_i(s) - \frac{V_o(s)}{k}$$

$$V_o(s) = A(s) \cdot V_e(s)$$

$$= A(s) \cdot V_i(s) - \frac{A(s)}{k} V_o(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{A(s)}{1 + \frac{A(s)}{k}}$$



$G_m R_o = \infty$

exponential with time constant $= \frac{k}{\omega_n}$

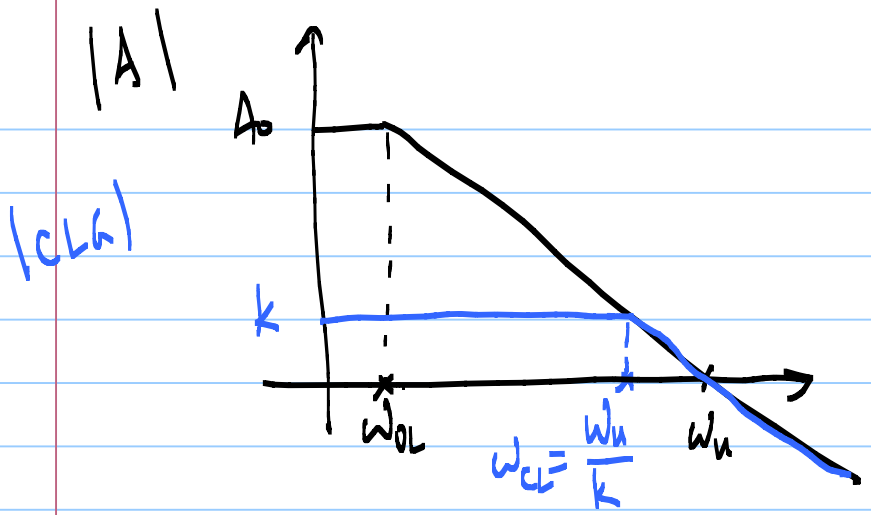
$$\frac{V_o(s)}{V_i(s)} = \frac{A(s)}{1+\frac{A(s)}{k}} = \frac{A_o/(1+sCR_o)}{1+\frac{A_o/(1+sCR_o)}{k}}$$

$$= \frac{k\,A_o}{k(1+sCR_o)+A_o} = \underbrace{(\;k\;)}_{\text{ideal gain}}\underbrace{(\quad)}_{\text{DC error}}\underbrace{(\quad)}_{\text{freq. dep.}}$$

$$= \frac{k\,A_o}{(k+A_o)+sCR_ok}$$

$$= k\cdot\left(\frac{1}{1+\frac{k}{A_o}}\right)\cdot\frac{1}{1+s\cdot\frac{CR_ok}{k+A_o}}$$

$$\omega_{-3dB} \approx \frac{\omega_u}{k}$$



$$\omega_{CL} = \frac{k + A_0}{CR_0 k} \approx \frac{A_0}{CR_0 k} \approx \frac{\omega_u}{k}$$

$$\omega_{OL} = \frac{\omega_u}{A_0}$$

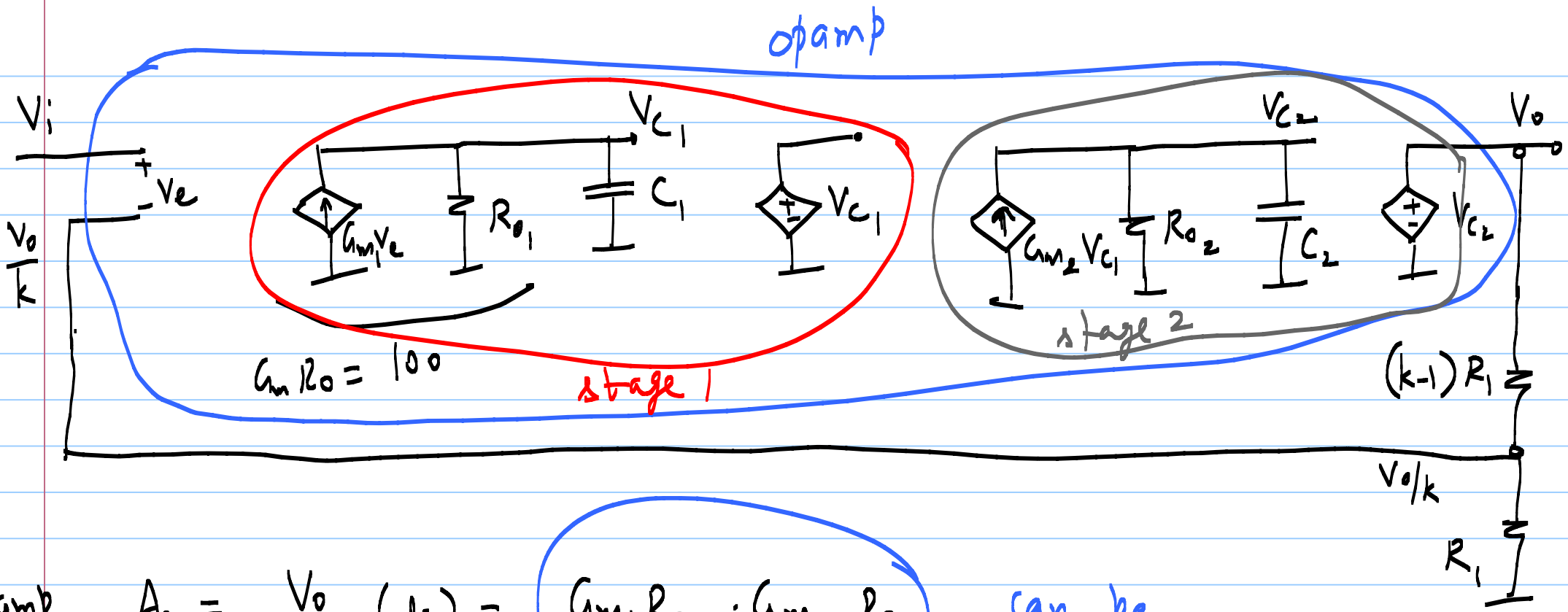
$$\omega_{CL} \approx \frac{A_0}{k} \cdot \omega_{OL}$$

fast response $\rightarrow$ large $\frac{A_0}{k}$, large ω_u $\rightarrow$ large G_m, small C

low DC error $\rightarrow$ large A_0 $\rightarrow$ $G_m R_0$ large

modern technologies: $G_m R_0$ is finite and relatively small (few 10's to few 100's)

tradeoff between A_0 & ω_u



opamp $\quad A_0 = \dfrac{V_o}{V_e}(dc) = \underbrace{G_{m_1}R_{o_1}}_{A_{0_1}} \cdot \underbrace{G_{m_2}R_{o_2}}_{A_{0_2}}$ can be much larger

$$A(s) = A_1(s) \cdot A_2(s) = \frac{V_o}{V_e}(s)$$

$$A_1(s) = \frac{A_{01}}{1 + sC_1R_{o_1}} \quad ; \quad A_2(s) = \frac{A_{02}}{1 + sC_2R_{o_2}}$$

two poles : $p_1 = \dfrac{1}{C_1 R_{o_1}}$; $p_2 = \dfrac{1}{C_2 R_{o_2}}$

$$\frac{V_o(s)}{V_i(s)} = \frac{A(s)}{1 + \dfrac{A(s)}{k}} = \frac{k}{1 + \dfrac{k}{A(s)}}$$

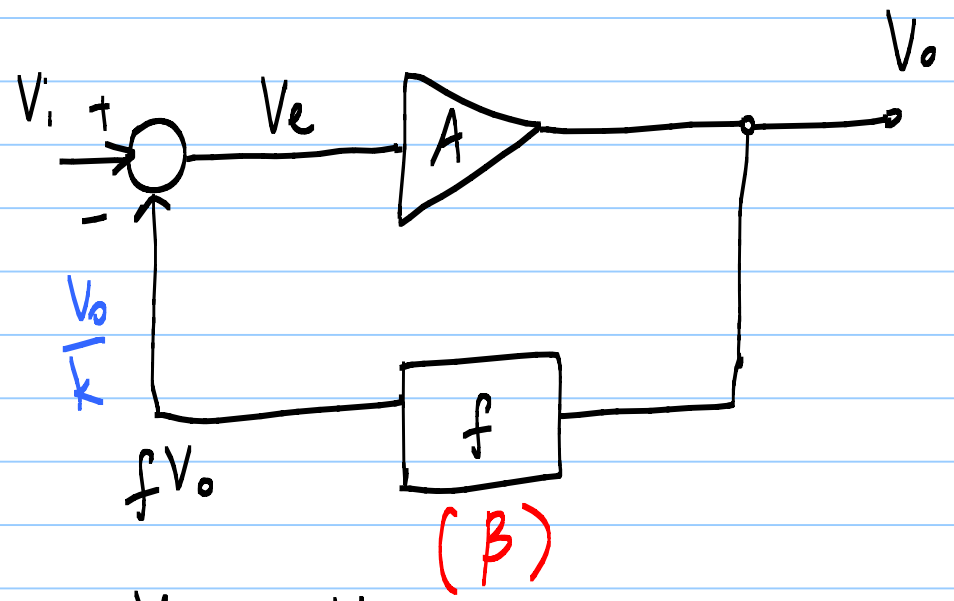
$$A_o = A_{o_1} \cdot A_{o_2}$$

$$A(s) = \left(\frac{A_{o1}}{1 + s/p_1}\right)\left(\frac{A_{o_2}}{1 + s/p_2}\right)$$

$$\frac{V_o}{V_i}(s) = \frac{k}{1 + \dfrac{k}{A_o}(1 + s/p_1)(1 + s/p_2)}$$

$$\frac{V_o}{V_i}(s) = \frac{k}{\left(1 + \frac{k}{A_o}\right) + \frac{sk}{A_o}\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1 p_2} \cdot \frac{k}{A_o}}$$

Feedback Systems



closed loop gain

$$\frac{V_o}{V_i} = \frac{1}{f}\,\frac{Af}{1+Af}$$

$\frac{1}{f} \to k$ (ideal CLG)

$$V_e = V_i - fV_o$$

$$V_o = AV_e = AV_i - AfV_o$$

$$V_o = \frac{A}{1+Af}\cdot V_i \Rightarrow \frac{V_o}{V_i} = \frac{A}{1+Af} = \frac{1}{f}\,\frac{Af}{1+Af}$$

$$\frac{V_o}{V_i} = \frac{1}{f} \quad \text{only if} \quad Af \gg 1 \quad \{Af \to \infty\}$$

$$\left\{ \frac{A}{k} \to \infty \right\}$$

If $A = A(s)$ {assume f is freq. indep.}

$$\frac{V_o}{V_i} = \frac{1}{f} \frac{A(s) \cdot f}{1 + A(s) \cdot f}$$

1) $$A(s) = \frac{A_o}{1 + s/\omega_p} \qquad \omega_p = \frac{\omega_u}{A_o} = \omega_{oL}$$

@ low freq. $|A(j\omega)| \approx A_o \Rightarrow A_o f$ is large

$$\Rightarrow \frac{V_o}{V_i} \approx \frac{1}{f}$$

$$\frac{V_o}{V_i}(s) = \frac{1}{f} \cdot \frac{\frac{A_o}{1+s/\omega_p} \cdot f}{1 + \frac{A_o}{1+\frac{s}{\omega_p}} \cdot f}$$

$$= \frac{1}{f} \cdot \frac{A_o f}{(1+A_o f) + \frac{s}{\omega_p}}$$

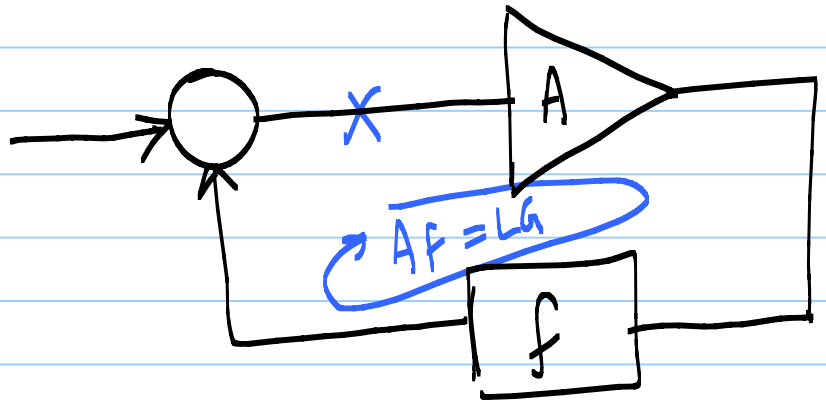
$$= \frac{1}{f} \cdot \frac{A_o f}{1+A_o f} \cdot \frac{1}{1 + \frac{s}{\omega_p (1+A_o f)}}$$

↳ CL pole $\approx \omega_p \cdot A_o f$

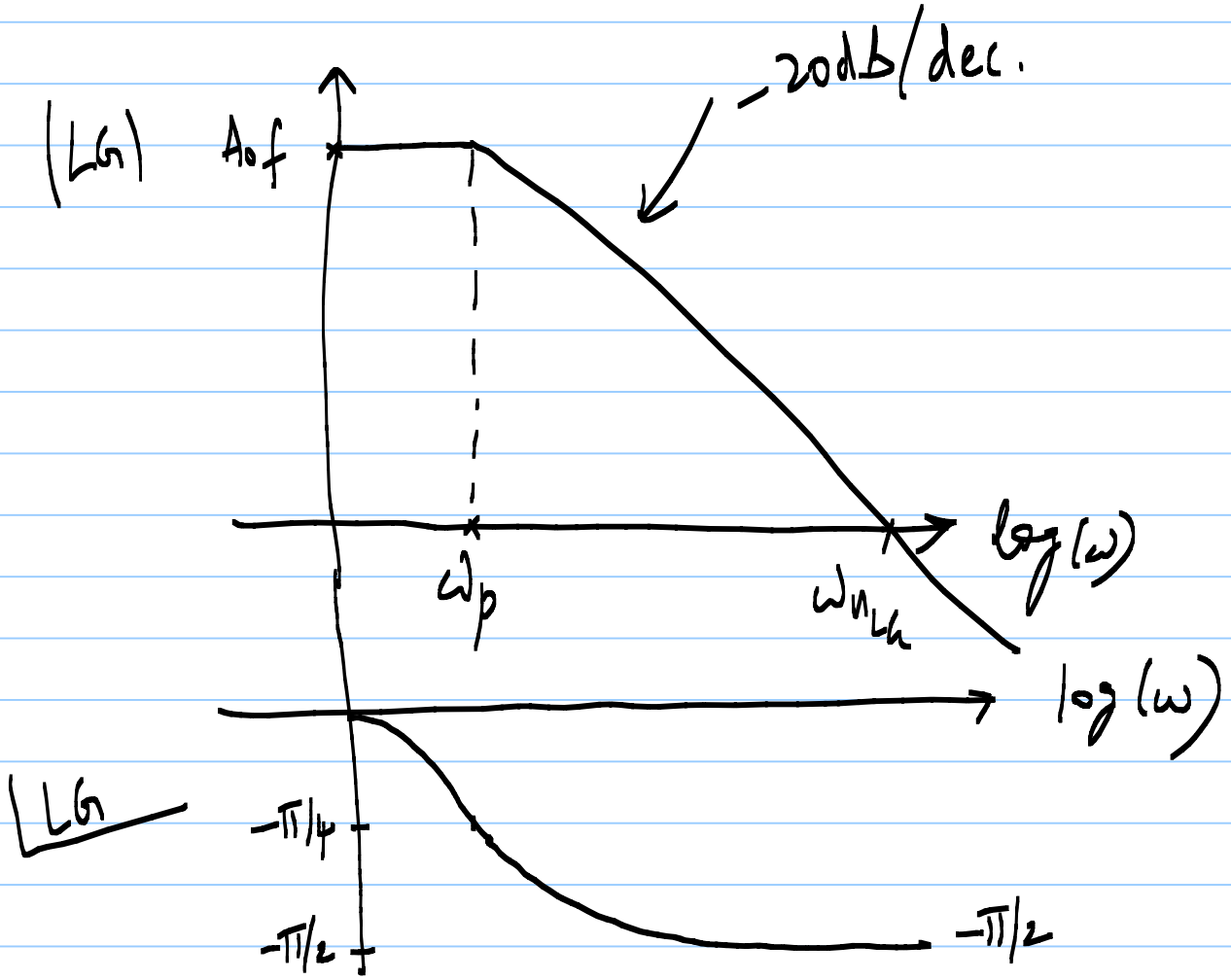
If $A(s) \cdot f \gg 1$

$$\Rightarrow \frac{V_o}{V_i} \approx \frac{1}{f}$$

$A(s) \cdot f \equiv$ Loop Gain (LG)



$$LG = \frac{A_o f}{1 + s/\omega_p}$$



$$\omega_{u_{LG}} \approx \omega_p (A_o f)$$

$\approx$ -3 dB BW of CLG

CL

* Step response = exponential
* CLG has 1 pole (LHP)
* CL system is unconditionally stable

2) <u>Second order system</u>

$$A(s) = \frac{A_0}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

CLG $$\frac{V_o}{V_i} = \frac{1}{f}\,\frac{f\,A(s)}{1+f\,A(s)} = \frac{1}{f}\cdot\frac{f\,A_0/(\,)(\,)}{1+f\,A_0/(\,)(\,)}$$

$$\frac{V_o}{V_i} = \frac{1}{f} \frac{A_o f}{A_o f + (1 + s/p_1)(1 + s/p_2)}$$

$$CLG = \underbrace{\frac{1}{f}}_{\text{ideal gain}} \cdot \underbrace{\frac{A_o f}{1 + A_o f}}_{\text{DC error}} \cdot \underbrace{\frac{1}{1 + \left(\frac{1}{1+A_o f}\right)\left(\frac{1}{p_1} + \frac{1}{p_2}\right) \cdot s + \left(\frac{1}{1+A_o f}\right) \frac{s^2}{p_1 p_2}}}_{\text{freq. response } D(s)}$$

All coefficients are positive $\Rightarrow$ 2 LHP poles

$$D(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$

ξ = damping factor

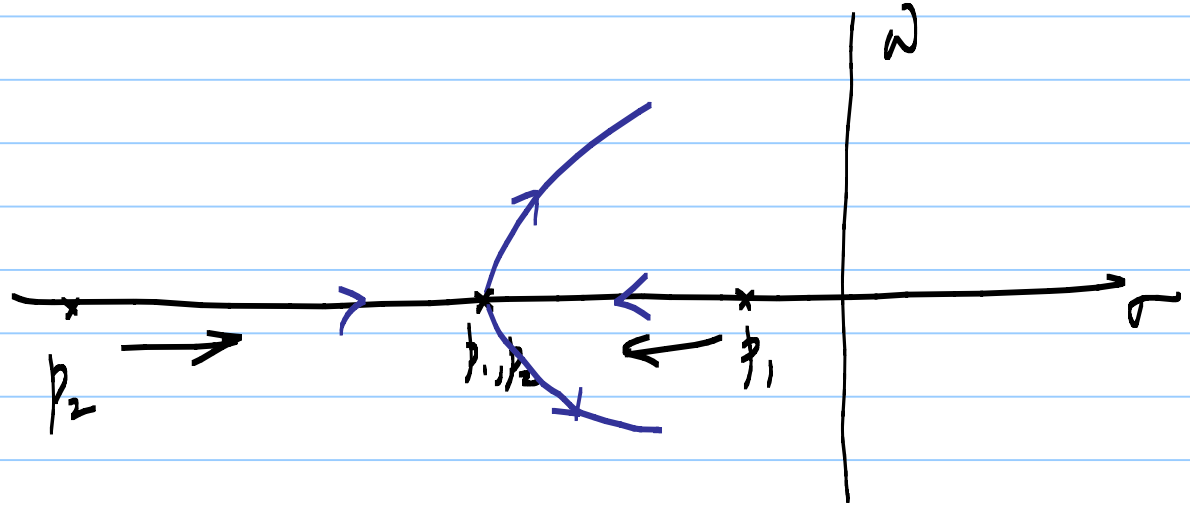
$$D(s) = s^2 + \frac{\omega_n}{Q} \cdot s + \omega_n^2$$

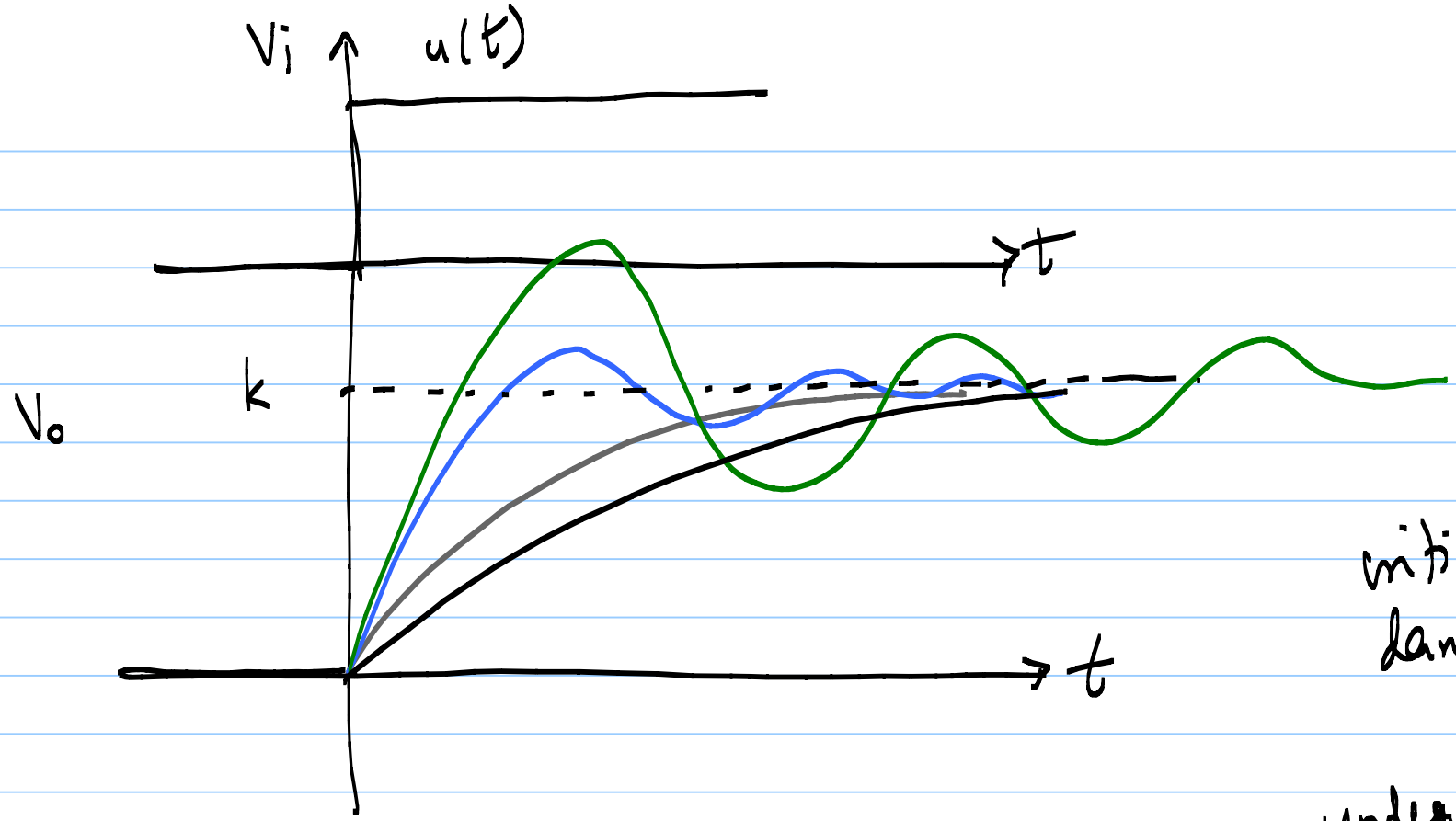
Q = quality factor

$$Q = \frac{1}{2\xi}$$

$V_i = u(t)$; V_o depends on positions of p_1 & p_2

$p_1 << p_2$





overdamped $\{\text{large } \zeta\}$

critically damped $\{\zeta_c\}$

underdamped system $\{\zeta < \zeta_c\}$ $\{\text{very small } \zeta\}$

$$\omega_n = \sqrt{(p_1 p_2)(1 + A_{of})}$$

$$\zeta = \frac{p_1 + p_2}{2\sqrt{1 + A_{of}} \cdot \sqrt{p_1 p_2}} = \underbrace{\frac{1}{2}\left[\sqrt{\frac{p_1}{p_2}} + \sqrt{\frac{p_2}{p_1}}\right] \cdot \frac{1}{\sqrt{1 + A_{of}}}}_{\text{depends on } p_1,\ p_2,\ A_{of}}$$

large A_0 → 2 stages → 2 poles → 3 small

if A_0 is large

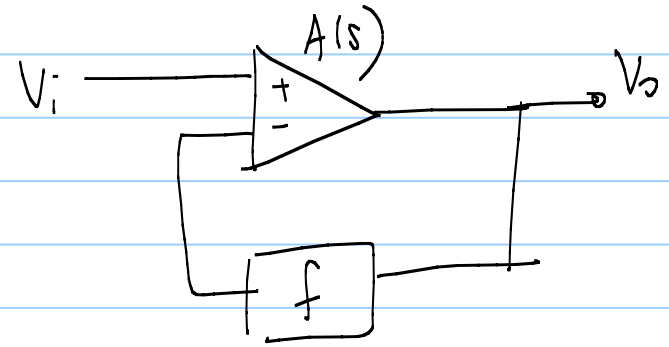
* Technically unconditionally stable
  { CLA has LHP poles }
* possibility of ringing { not good }
* larger $A_0 f$ → smaller } ⇒ more ringing
* space poles apart ⇒ avoid ringing
  $(p_1 << p_2)$

24/02/2022

Lecture 19

II 2-stage opamp ($p_1 = p_2 = \omega_p$)

$$\frac{V_o}{V_i} = \frac{1}{f} \cdot \frac{f\,A(s)}{1 + f\,A(s)}$$



$f > 0$

$$A(s) = \frac{A_o}{(1 + s/\omega_p)^2}$$

$$CLG(s) = \frac{V_o}{V_i}(s) = \frac{1}{f} \cdot \frac{f A_o}{1 + f A_o} \cdot \frac{1}{D(s)}$$

$$CLG = \frac{1}{f} \cdot \frac{f\,A(s)}{1 + f\,A(s)} = \frac{1}{f} \frac{f\,A_o}{f A_o + (1 + s/\omega_p)^2}$$

$$D(s) = \left[(1 + f A_o) + \frac{2s}{\omega_p} + \frac{s^2}{\omega_p^2}\right] \Big/ (1 + f A_o)$$

$$D(s) = 1 + \frac{2s}{(1+fA_0)\omega_p} + \frac{s^2}{\omega_p^2(1+A_0 f)}$$

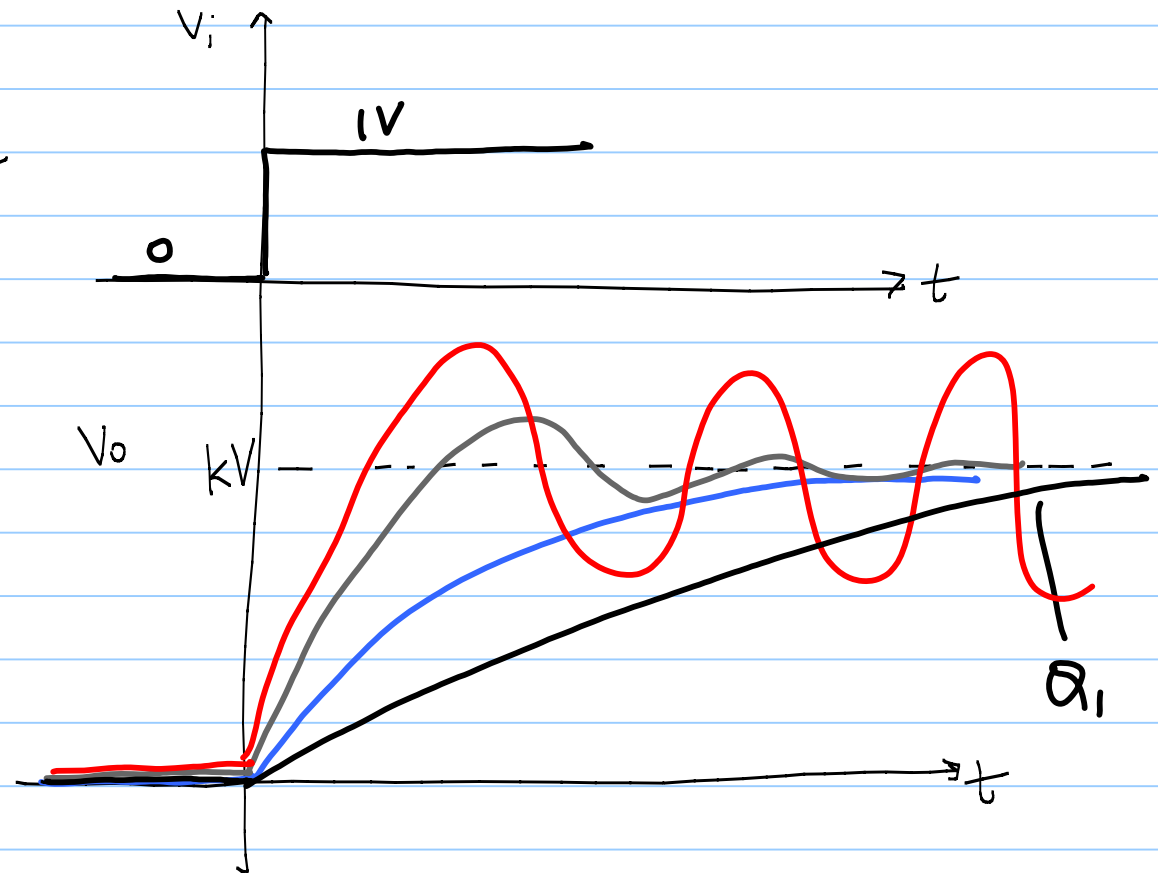
<u>std. form</u>

$$D(s) = s^2 + \frac{\omega_n s}{Q} + \omega_n^2$$

$$\frac{s^2}{\omega_n^2} + \frac{s}{Q\omega_n} + 1$$

$$\omega_n = \omega_p\sqrt{1+A_0 f}$$

$$Q = \frac{\sqrt{1+A_0 f}}{2}$$



$Q_1 << 1/2$ $\quad$ $Q = \infty$

$Q = 1/2$

$Q > 1/2$

$$D(s) = \frac{s^2}{\omega_n^2} + \frac{s}{Q\omega_n} + 1$$

roots are $\frac{s}{\omega_n} = \frac{-1}{2Q} \pm j\sqrt{1 - \frac{1}{4Q^2}}$

$$\left.\begin{aligned} \omega_n &= \omega_p\sqrt{1+A_0 f} \\ Q &= \frac{\sqrt{1+A_0 f}}{2} \end{aligned}\right\}$$

large $A_0 f$ $\Rightarrow$ lots of ringing

* Unconditionally stable
* lot of ringing for large $A_0 f$

III

3-stage opamp $(p_1 = p_2 = p_3 = \omega_p)$

$$A(s) = \frac{A_0}{\left(1 + s/\omega_p\right)^3} = \frac{A_0}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLG(s) = \frac{V_o}{V_i}(s) = \frac{1}{f} \cdot \frac{fA_o}{1+A_o f} \cdot \frac{1}{D(s)}$$

$$D(s) = 1 + \left(\frac{1}{1+fA_o}\right)\left(\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right)$$

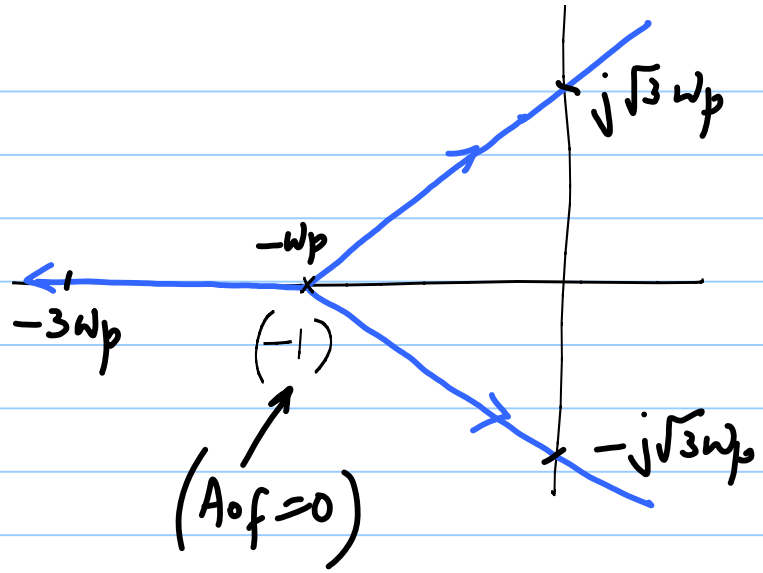
$$= \left(\frac{1}{1+fA_o}\right) \underbrace{\left(1 + A_o f + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right)}_{D'(s)}$$

roots of $D'(s) = 0$

roots of $\left(1 + s/\omega_p\right)^3 + A_o f = 0$

$$\frac{s}{\omega_p} = -1 + \left(-A_o f\right)^{1/3} \longrightarrow 3 \text{ roots}$$

$* \; A_o f = 0 \Rightarrow 3$ roots @ -1



$$A_0 f = 8$$

$$\frac{s}{\omega_p} = -1 - 2 = -3$$

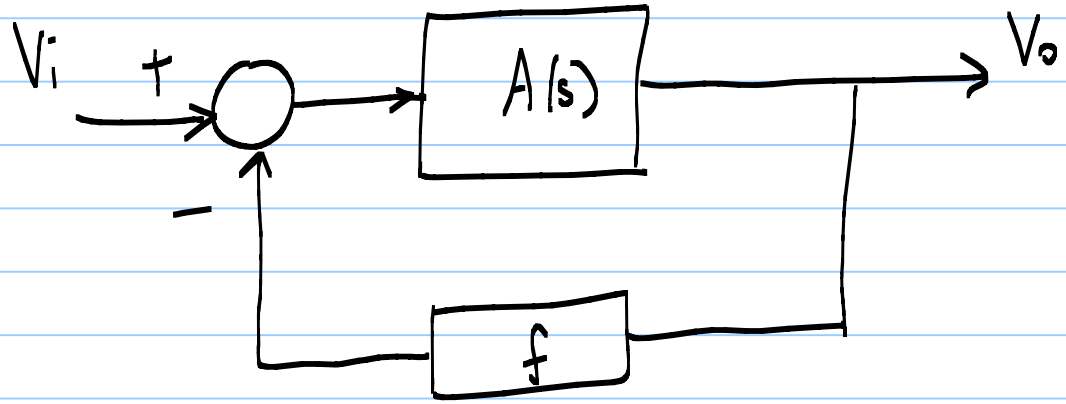
$$= -1 - 2e^{-j2\pi/3} = +j\sqrt{3}$$

$$= -1 - 2e^{+j2\pi/3} = -j\sqrt{3}$$

$A_0 f > 8 \Rightarrow$ unstable

25/02/2022

Lecture 20



$$\frac{V_o}{V_i}(s) = \frac{1}{f}\,\frac{f\,A(s)}{1+f\,A(s)}$$

If $|f\,A(s)| \gg 1 \Rightarrow CLG \approx \frac{1}{f}$

If $1+f\,A(s) = 0$

$|f\,A(j\omega)| = 1$; $\angle f\,A(j\omega) = -180^\circ$ at some ω

$\Rightarrow CLG \sim \infty \longleftarrow$ not detirable

system is <u>unstable</u>

If system is all-pole (no zeroes):

Barkhausen criteria for stability:

@ $|A_f| = 1$: $\angle A_f > -180^\circ$

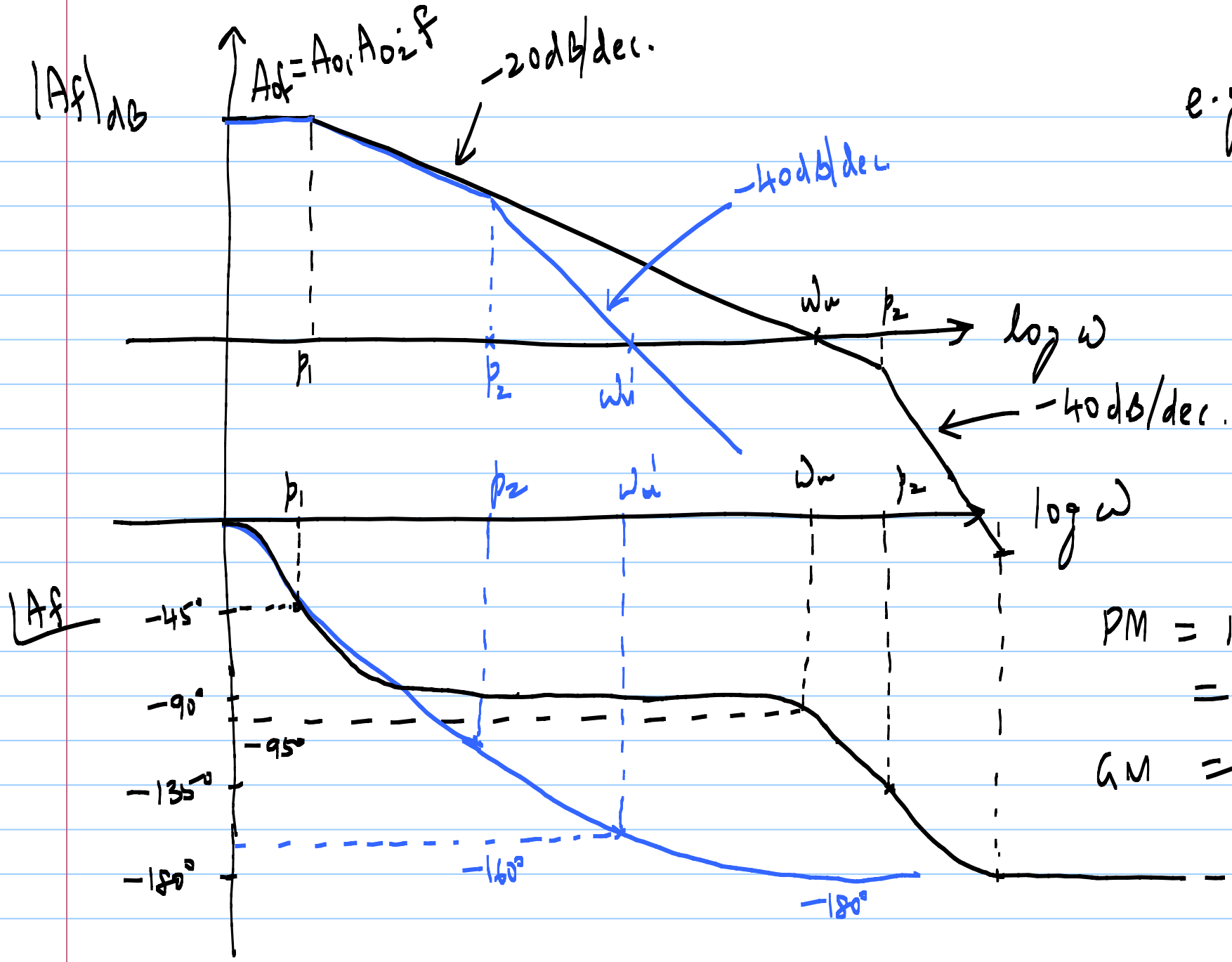
@ $\angle A_f = -180^\circ$: $|A_f| << 1$

System is stable

<u>Quantify stability using:</u>

"Phase margin" $= 180^\circ + \angle A_f$ @ $|A_f| = 1$ frequency

"Gain Margin" $= -|A_f|_{dB}$ @ $\angle A_f = -180^\circ$ frequency



e.g. Bode plots for 2-stage opamp

$p_1 << p_2$

$PM = 180° - 95°$
$= 85°$

$GM = \infty$

$PM = 180° - 160° = 20°$; $GM = \infty$

$p < p_2$
(not much smaller)

HW: 1) Plot for 3rd order system with 3 coincident poles

2) Plot $|CLG(s)|$ for 2nd order system for different α^s

<u>Summary</u>:

order	
1	Unconditionally stable Small A_{of} ✗
2	Unconditionally stable lot of ringing for large A_{of} ✗
3	Stable only if $A_{of} < 8$ ✗

equal poles

✓

* Make $\underbrace{\text{higher order system}}_{\text{due to large } A_{of}}$ look like a first-order system

Original 3rd order system: $A(s) = \dfrac{A_0}{\left(1 + s/\omega_p\right)^3}$

$$f A'(s) = \frac{f A_0}{\left(1 + s/\omega_d\right)\left(1 + s/\omega_p\right)^3}$$

make $\omega_d << \omega_p$

e.g. $\omega_d = \dfrac{\omega_p}{1000}$

$$|A(s) \cdot f| = 1 \quad \& \quad \underbrace{\angle A(s) \cdot f = -180^\circ}$$

$$0 - \tan^{-1}\left(\frac{\omega_0}{\omega_d}\right) - 3\tan^{-1}\left(\frac{\omega_0}{\omega_p}\right) = -\pi$$

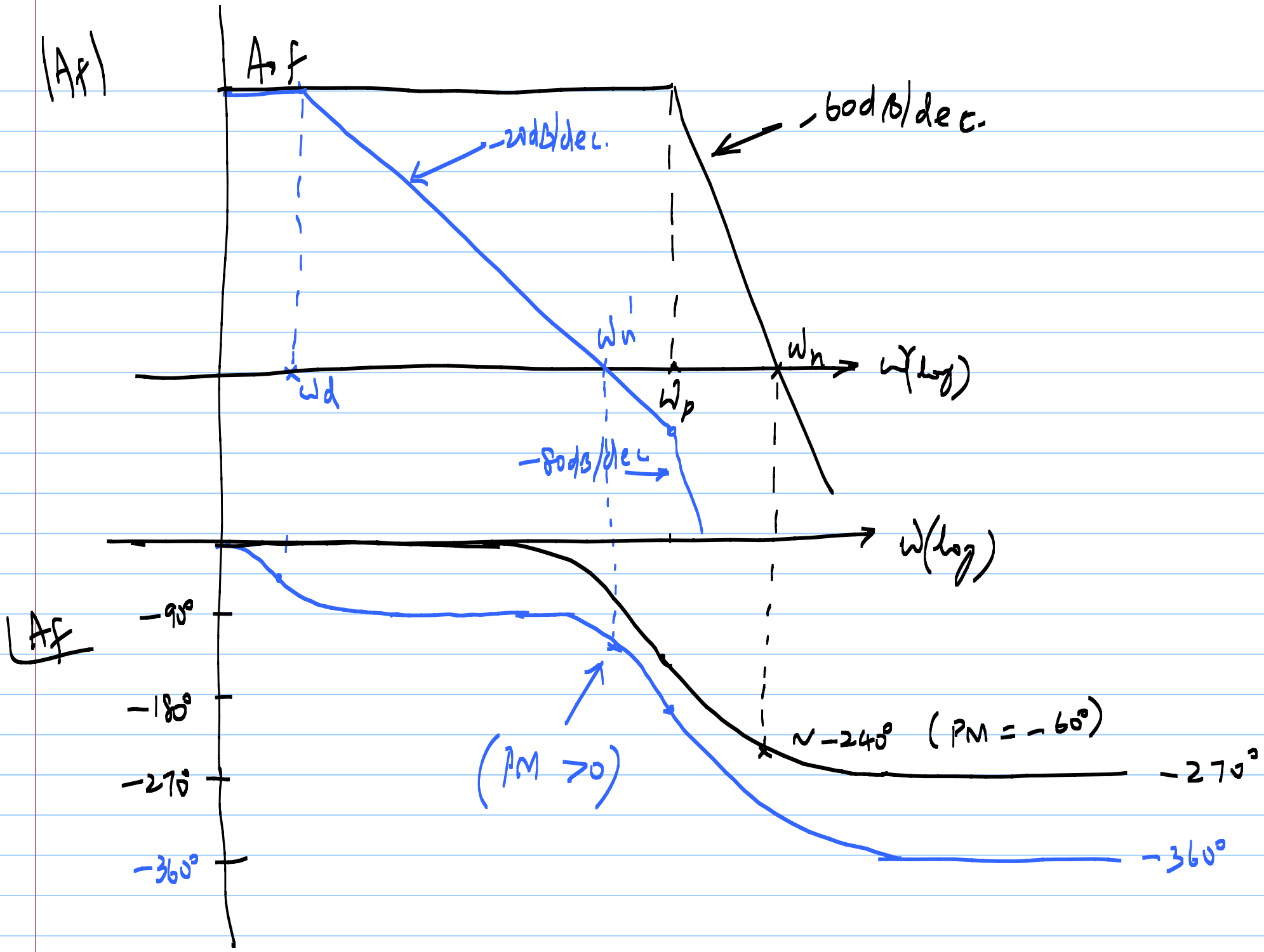
$$\underbrace{-\tan^{-1}\left(\frac{1000\omega_0}{\omega_p}\right)}_{-\pi/2} - 3\tan^{-1}\left(\frac{\omega_0}{\omega_p}\right) = -\pi$$

$$-3\tan^{-1}\left(\omega_0/\omega_p\right) = -\pi/2$$

$$\tan^{-1}\left(\omega_0/\omega_p\right) = \pi/6$$

$$\boxed{\frac{\omega_0}{\omega_p} = \frac{1}{\sqrt{3}}} \qquad \omega_0 = \frac{\omega_p}{\sqrt{3}}$$

Apply $|Af| = 1$ @ $\omega_0 = \omega_p/\sqrt{3}$


|Af|
Af
-20dB/dec.
-60dB/dec.
ωn
ωn
ω(log)
ωd
ωp
-80dB/dec
ω(log)
∠Af
-90°
-180°
-270°
-360°
(PM >0)
~-240° (PM = -60°)
-270°
-360°

$$\left| \frac{A_0 f}{\left(1 + \frac{1000j}{\sqrt{3}}\right)\left(1 + j/\sqrt{3}\right)^3} \right| = 1$$

$\frac{1000}{\sqrt{3}} \gg 1$

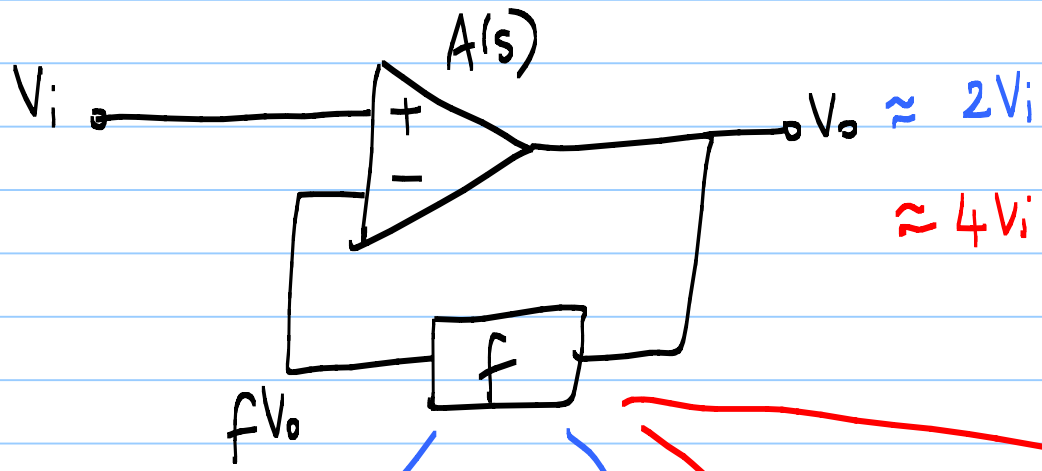
$$\frac{A_0 f}{\left(1 + 1/3\right)^{3/2}\left(\frac{1000}{\sqrt{3}}\right)} = 1 \Rightarrow A_0 f = 888.9$$

much larger than $A_0 f = 8$ limit

* BW is smaller
* Stable for large $A_0 f$
* ω_d = "Dominant pole"
* "Dominant pole compensation"

01/03/2022

Lecture 21



$\approx 2V_i$

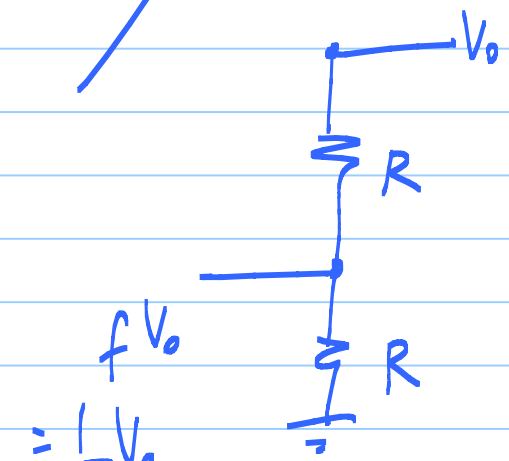
$\approx 4V_i$

For stability

worst-case value of $f = f_{max} = 1$

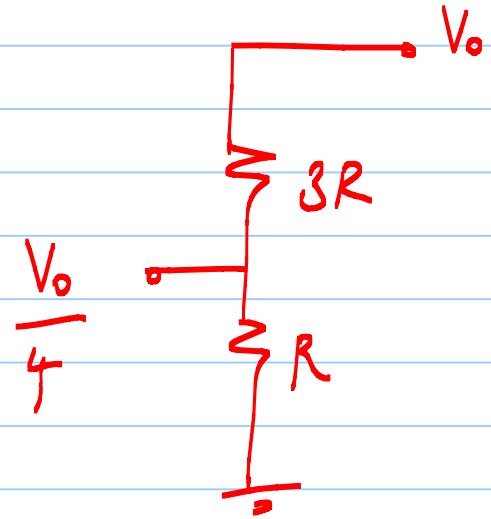
$\updownarrow$

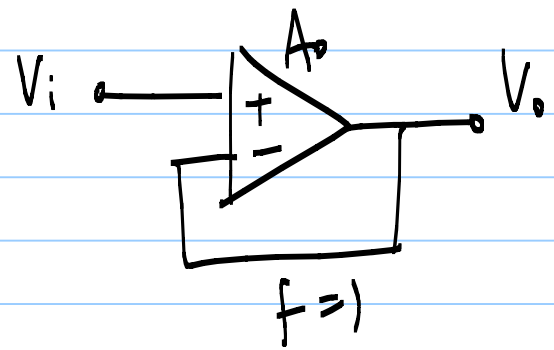
$k_{min} = 1$



$= \frac{1}{k} V_o$

$= \frac{V_o}{2}$



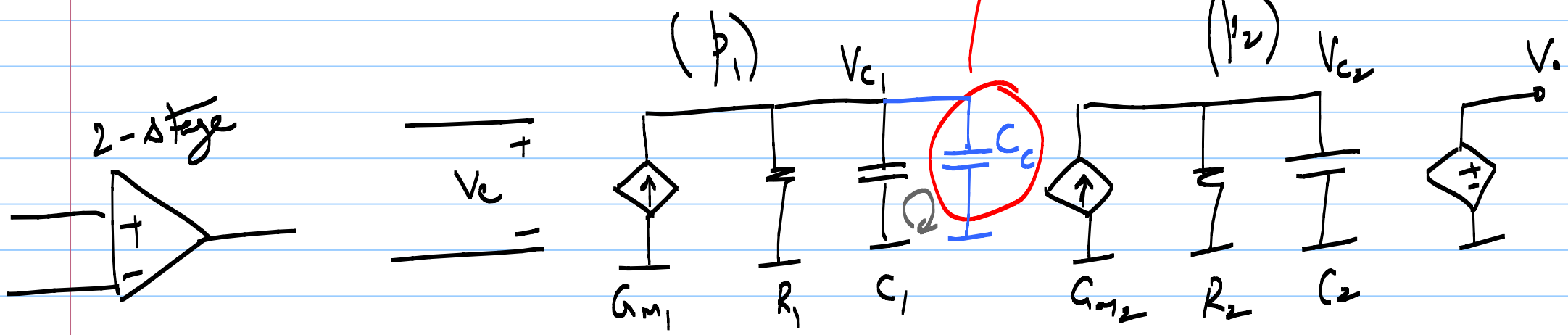


Compensate for $f=1$

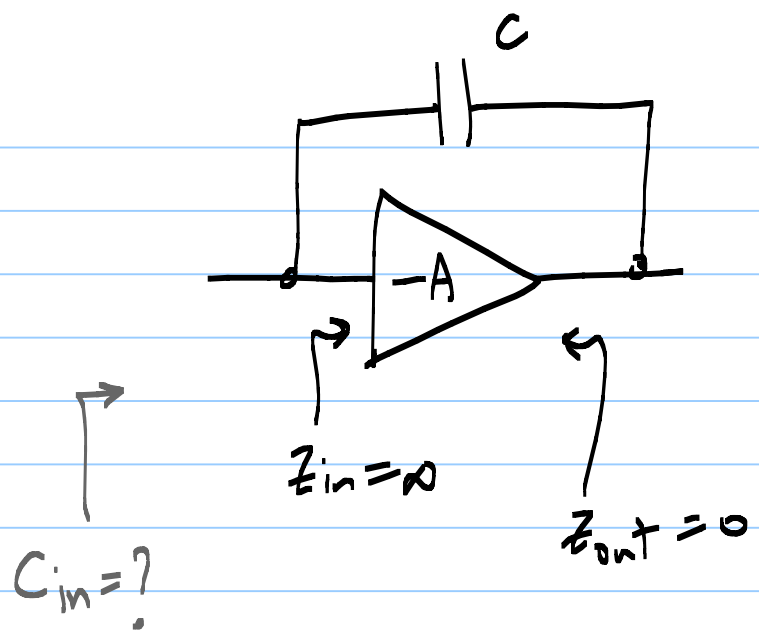
↳ assured of stability even for other feedback values (other gains)

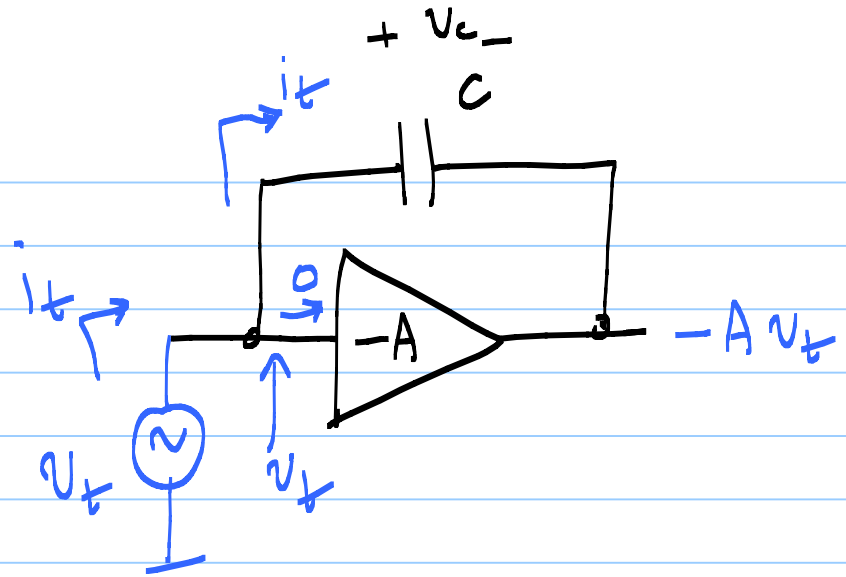
opamp is said to be "unity gain compensated"

compensation capacitor



$p_1 = p_2 \rightarrow$ ringing $\Rightarrow$ make $p_1 << p_2 \Rightarrow$ add C_c





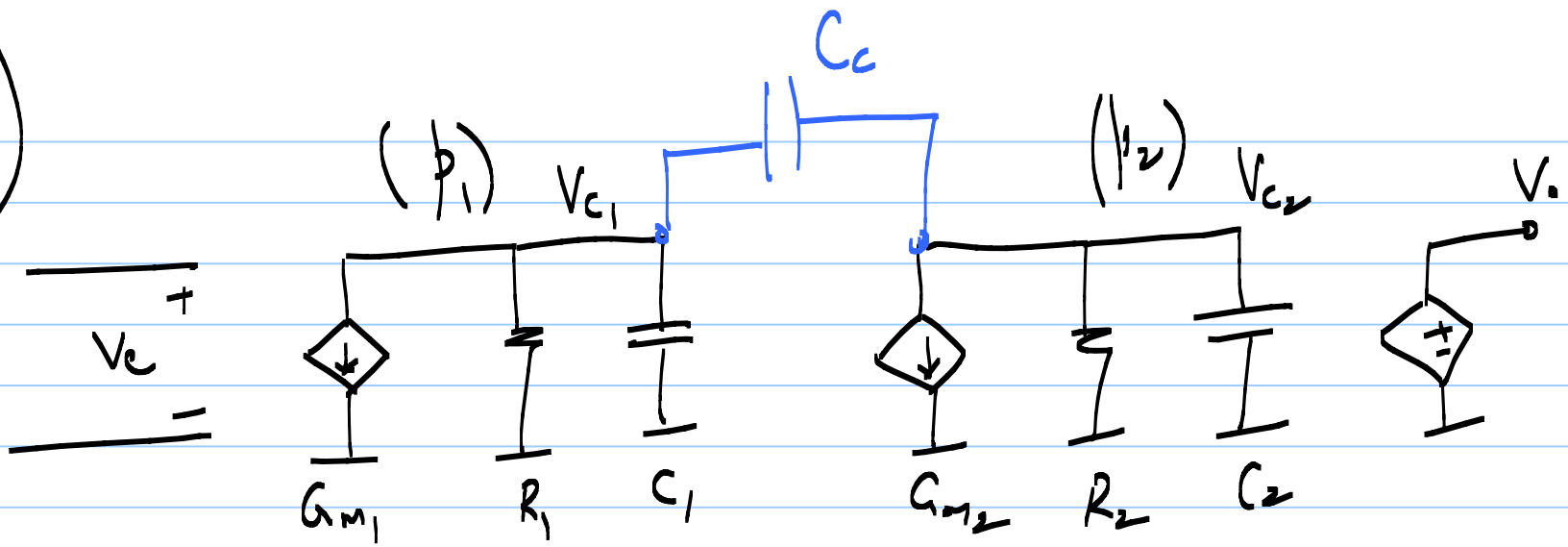
$$i_t = sC\,v_c$$

$$= sC\,(1+A)\,v_t$$

$$\frac{v_t}{i_t} = \frac{1}{sC\,(1+A)} \Rightarrow \boxed{C_{in} = C(1+A)}$$

"Miller Effect" or "Miller multiplication"

HW



$$A_1 = -G_{m_1} R_1 \qquad A_2 = -G_{m_2} R_2$$

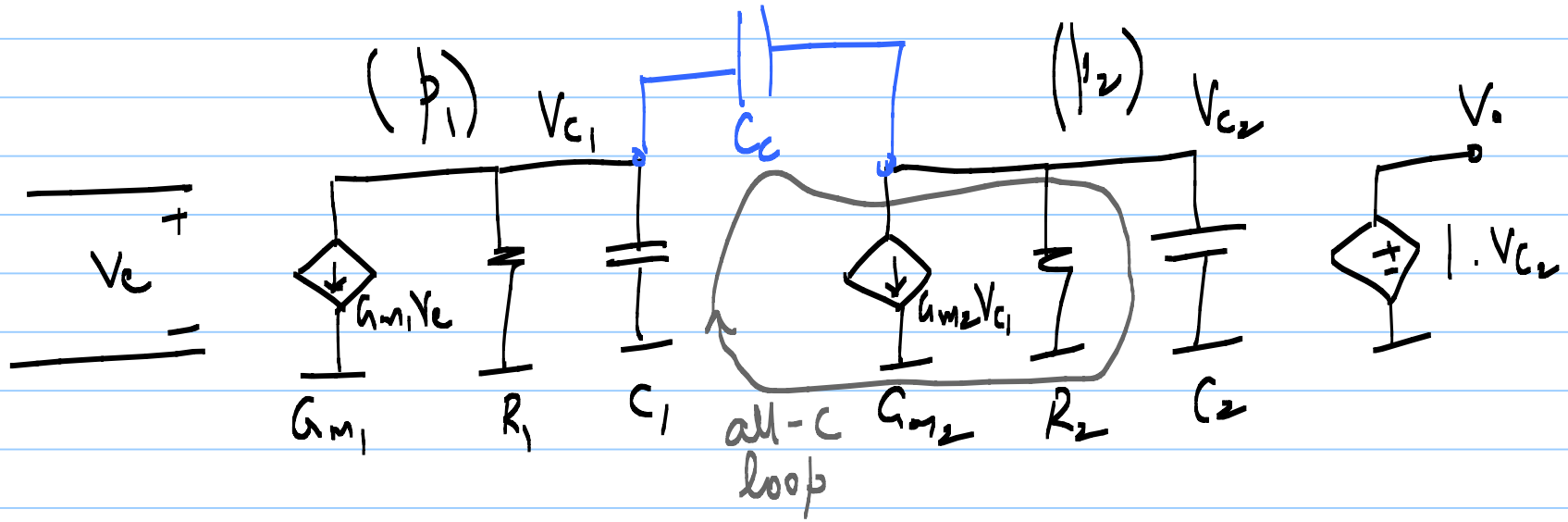
$$A_0 = A_1 A_2$$

$$\frac{V_{c_2}}{V_{c_1}} = G_{m_2} R_2$$

HW : $\dfrac{V_o}{V_e} = ?$

02/03/2022

Lecture 22



$$\frac{V_o}{V_e}(s) = \frac{+G_{m_1} G_{m_2} R_1 R_2 \left(1 - sC_c/G_{m_2}\right)}{\left[s^2 R_1 R_2 \left(C_1 C_c + C_2 C_c + C_1 C_2\right) + s\left(R_1 C_1 + R_2 C_2 + R_1 C_c \left\{1 + \frac{R_2}{R_1} + G_{m_2} R_2\right\}\right) + 1\right]} = \frac{N(s)}{D(s)}$$

HW → check this

without C_c → poles are $p_{1_0} = \frac{1}{R_1 C_1}$

$$p_{2_0} = \frac{1}{R_2 C_2}$$

After adding C_c

$$p_1 = ?$$

$$\boxed{p_1 << p_2}$$

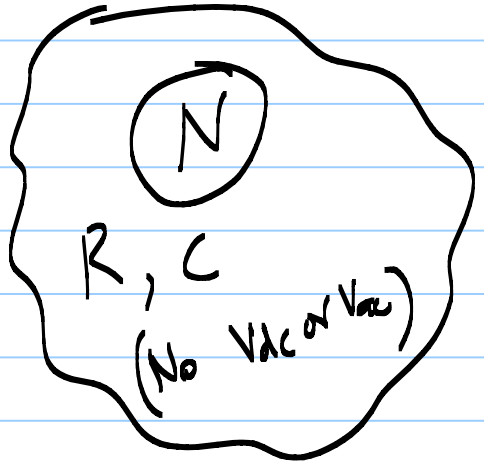
$$p_2 = ?$$

$$z = +\frac{G_{m2}}{C_c} \quad \text{(RHP zero)}$$

$$A_1 = -G_{m1} R_1 = \frac{V_{c1}}{V_e}$$

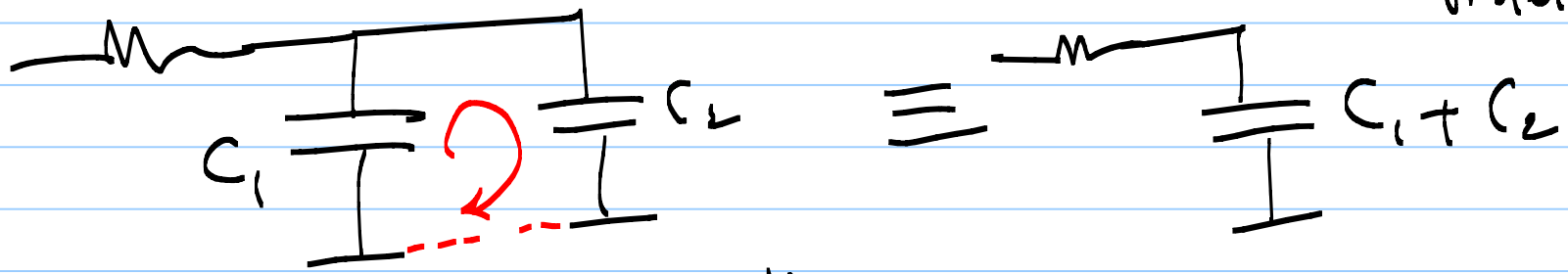
$$A_2 = -G_{m2} R_2 = \frac{V_{c2}}{V_{c1}}$$

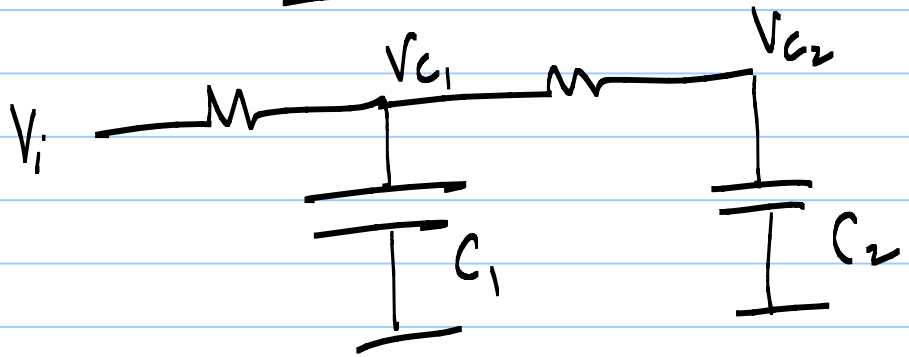
C_c should get multiplied by $\approx |A_2|$



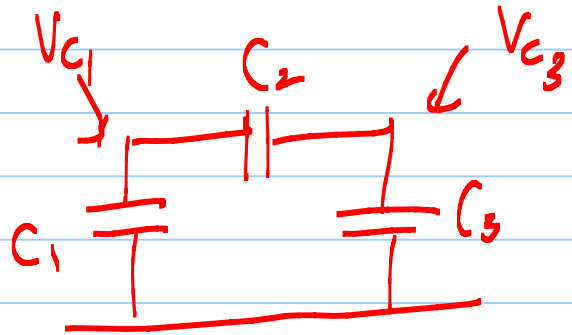
Order = # C's − # all-capacitor loops

order = 1

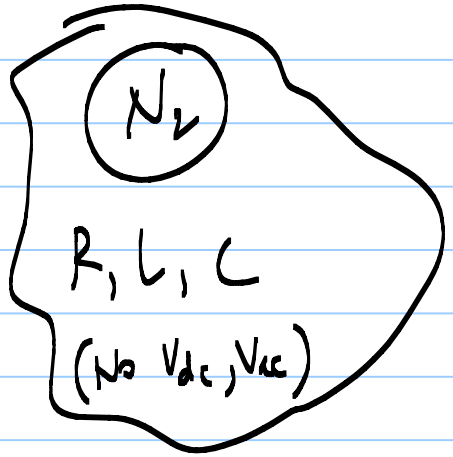




order = 2



$I_c = C\,dV_c/dt$

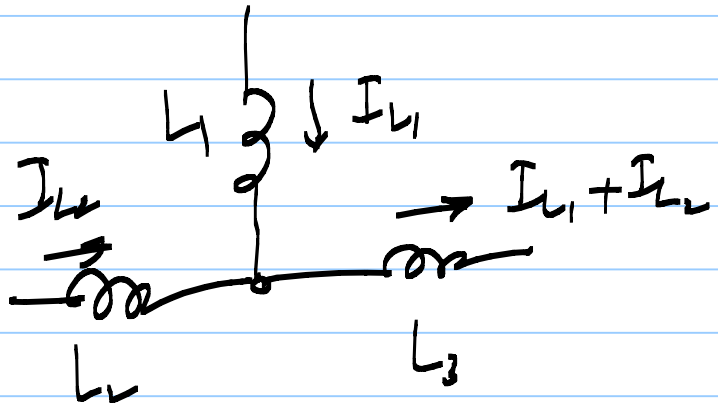


Order = # L's + # C's

− # all-cap loops

− # nodes with only L's connected

$$V_L = \frac{L\,dI_L}{dt}$$



$\underline{p_1}$: $C @ V_{c_1} = C_1 + C_c(1 + A_2)$

$$\approx C_1 + C_c \cdot G_{m_2} R_2$$

$$\approx G_{m_2} R_2 C_c$$

$$p_1 << p_{10}$$

$D(s) = as^2 + bs^2 + c$ & $p_1 << p_2$

$$p_1 \approx -\frac{c}{b}$$

$$p_2 \approx -\frac{b}{a}$$

$$p_1 = \frac{-1}{R_1C_1 + R_2C_2 + R_1C_c\left\{1 + \frac{R_2}{R_1} + G_{m2}R_2\right\}}$$

$$G_{m2}R_2 \gg 1$$

$$\text{"} \quad \gg \frac{R_2}{R_1}$$

$$p_1 \approx \frac{-1}{R_1C_1 + R_2C_2 + (G_{m2}R_2)R_1C_c} \approx \frac{-1}{R_1\underbrace{\left(C_c(G_{m2}R_2)\right)}_{\text{Miller cap. @ } V_{C_1}}}$$

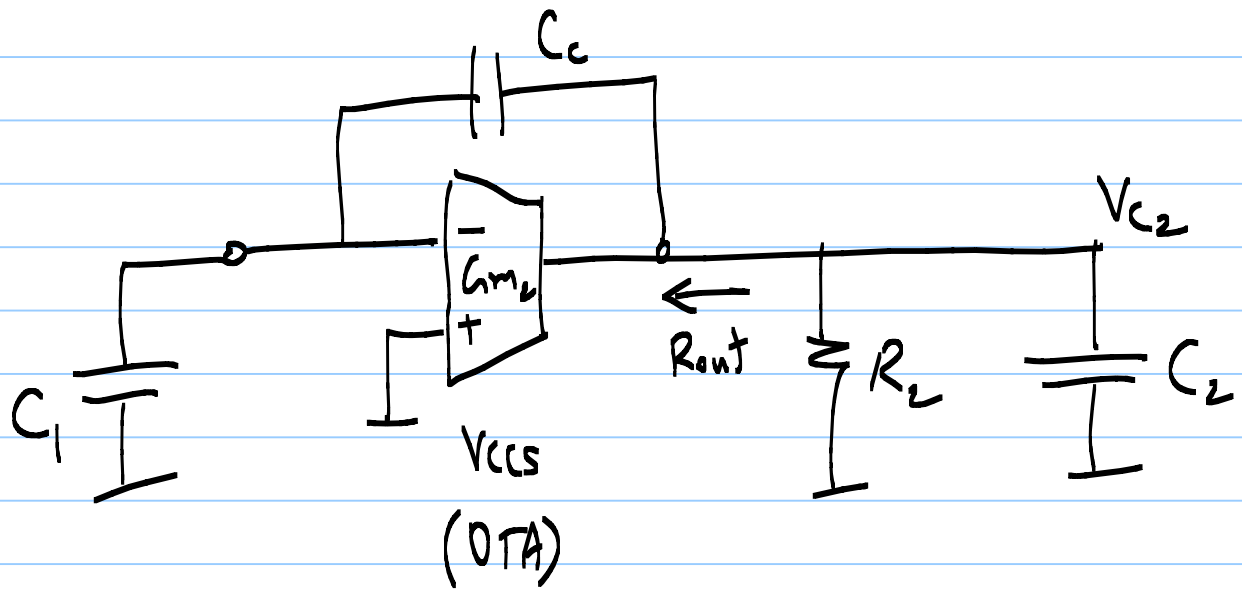
$\ll$

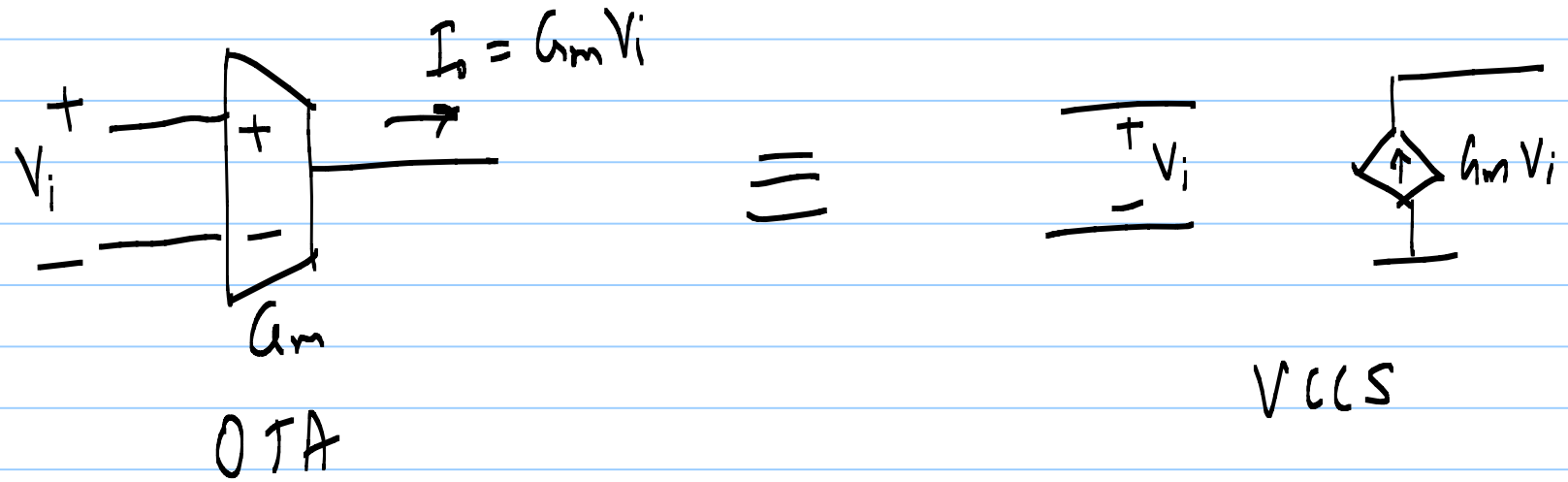
R @ V_{C_1}

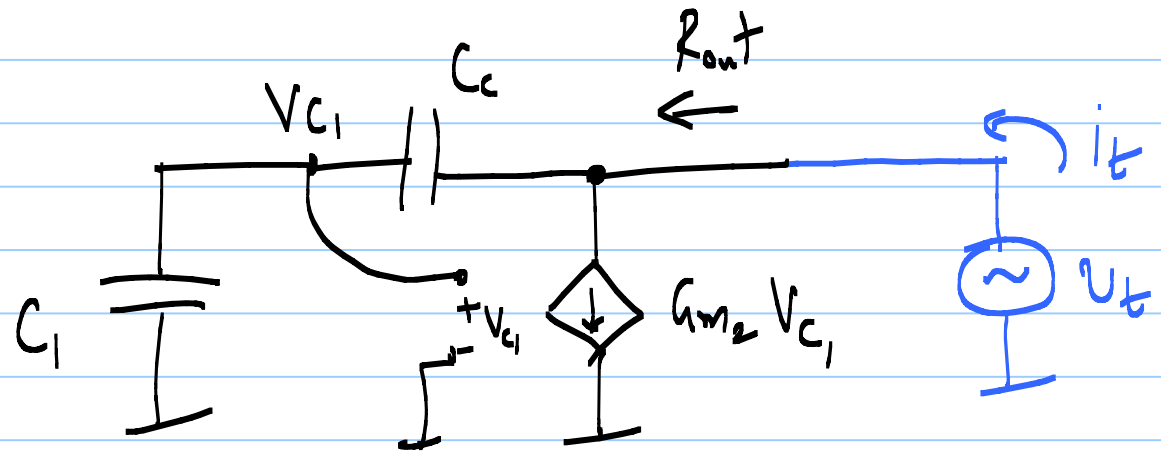
$$p_2 = \frac{-b}{a} = -\frac{R_1C_1 + R_2C_2 + R_1C_c\left\{1 + \frac{R_2}{R_1} + G_{m2}R_2\right\}}{R_1R_2\left(C_1C_c + C_2C_c + C_1C_2\right)}$$

@ V_{C_2}

$$\approx -\frac{G_{m2}R_2R_1C_c}{R_1R_2\left(C_1C_c + C_2C_c + C_1C_2\right)} \approx -\frac{G_{m2}C_c}{C_1C_2 + C_cC_1 + C_cC_2}$$





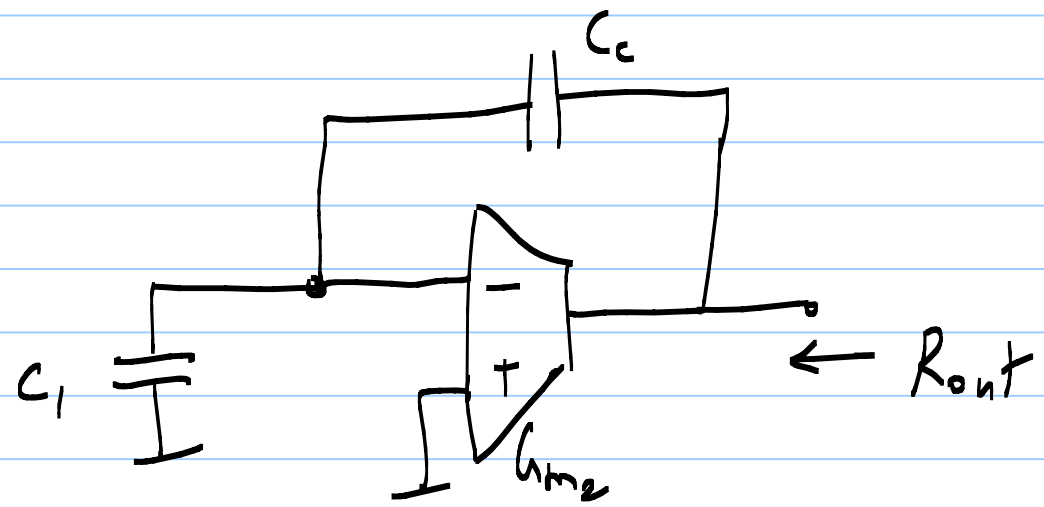


$$R_{out} = \frac{V_t}{i_t} = ?$$

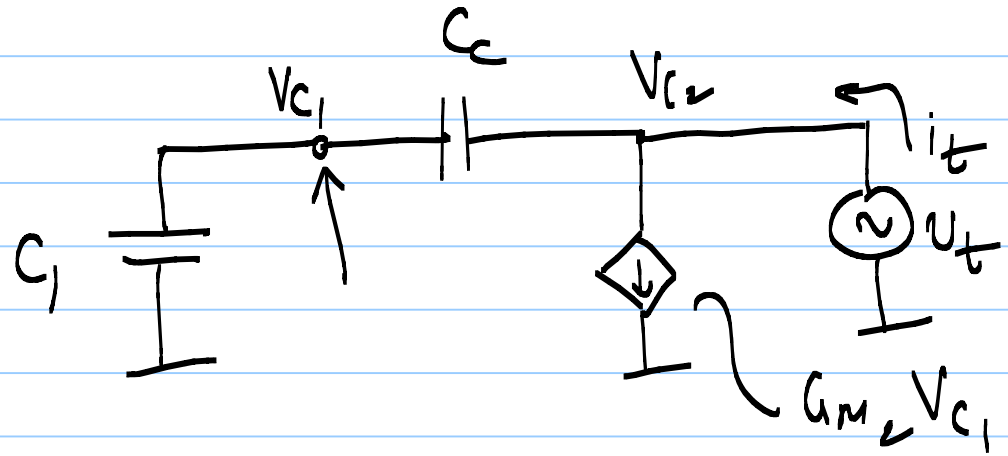
HW

03/03/2022

Lecture 23



$$R_{out} = \frac{C_1 + C_c}{C_c \cdot G_{m_2}} \times \frac{1}{1 + \frac{sC_1}{G_{m_2}}}$$



$$Y_{out} = \frac{i_t}{v_t}$$

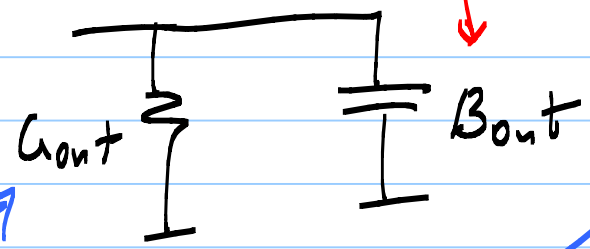
KCL @ V_{c_2} node: $(V_{c_2} - V_{c_1}) \cdot sC_c + G_{m_2} V_{c_1} = i_t$

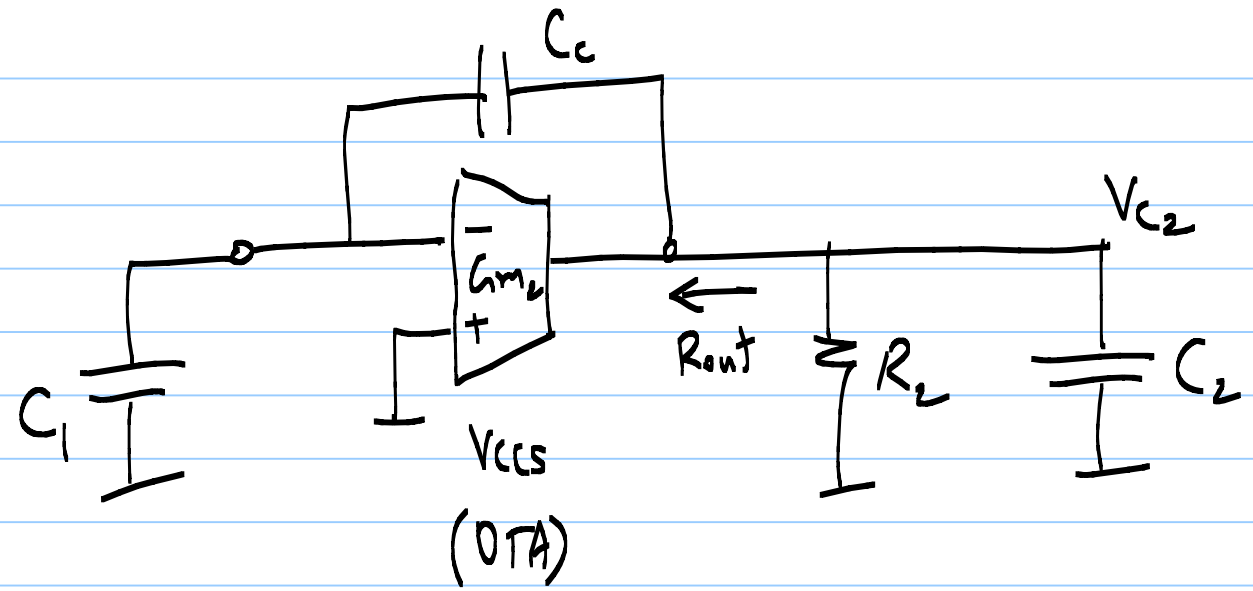
$$v_{c_1} = \frac{v_t \cdot C_c}{C_1 + C_c} \quad ; \quad v_{c_2} = v_t$$

$$i_t = sC_c \cdot v_t + (G_{m_2} - sC_c) \cdot \frac{C_c}{C_1 + C_c} \cdot v_t = i_t$$

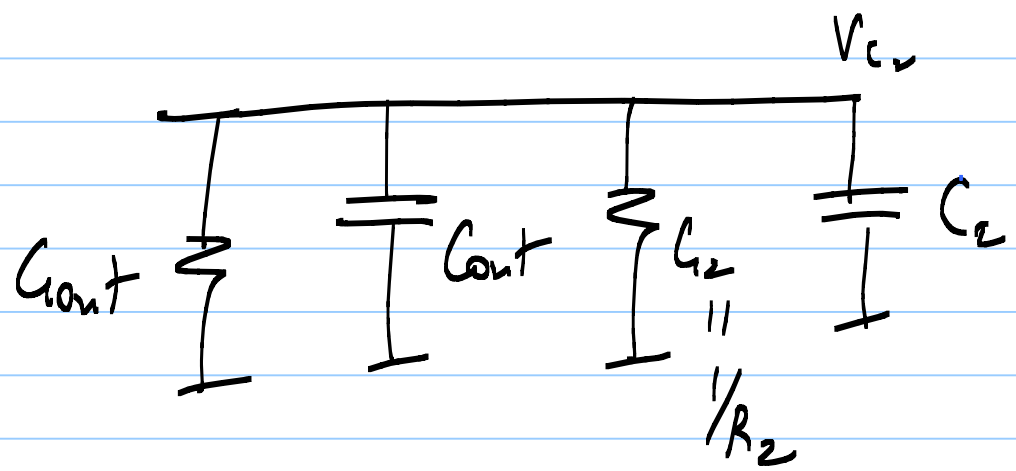
$$Y_{out} = \frac{i_t}{v_t} = sC_c - \frac{sC_c^2}{C_1 + C_c} + \frac{G_{m_2} \cdot C_c}{C_1 + C_c}$$

$Y_{out} = G_{out} + jB_{out}$





$\Downarrow$



pole p_2 is

@

$$\frac{1}{R_{eff}\, C_{eff}}$$

$$= G_{eff}/C_{eff}$$

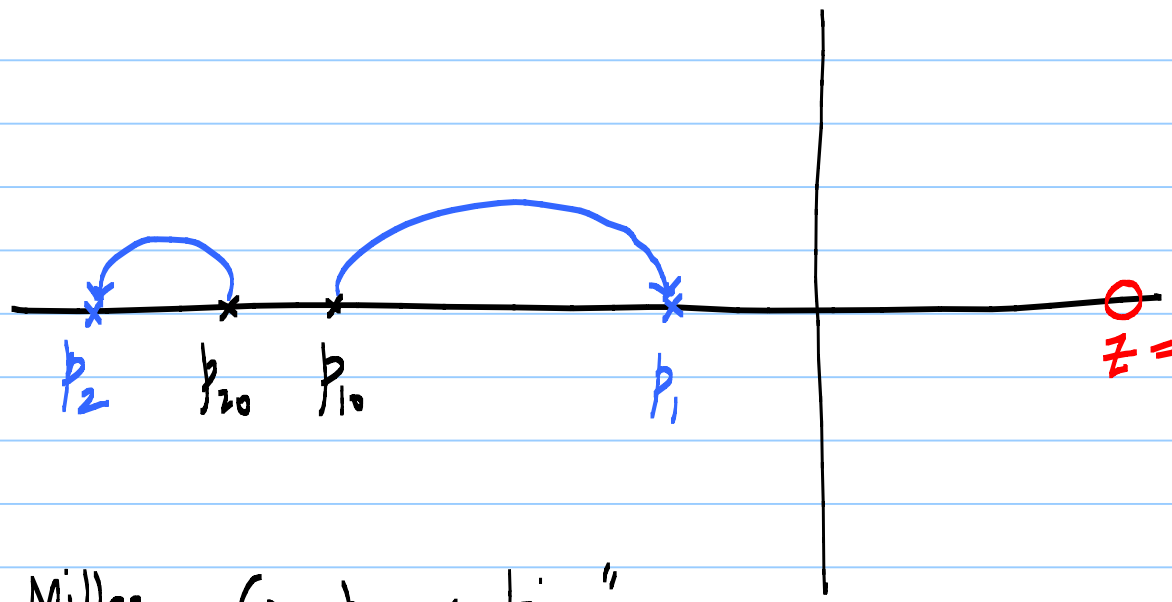
Original $\quad G_{eff_0} = \dfrac{1}{R_2} = \text{small}$

large

New $\quad G_{eff} = G_{eff_0} + \dfrac{G_{m_2} \cdot C_c}{C_1 + C_c}$

large

$$G_{eff} \gg G_{eff_0} \Rightarrow p_2 > p_{2_0}$$



$z = +\dfrac{G_{m_2}}{C_c} \Rightarrow PM \uparrow$

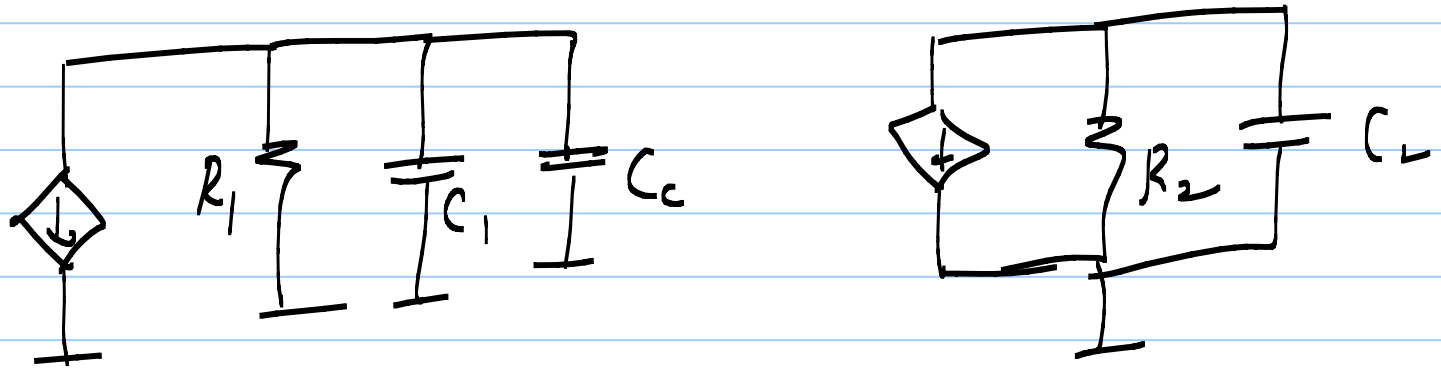
* poles have moved apart $\Rightarrow \zeta \uparrow$

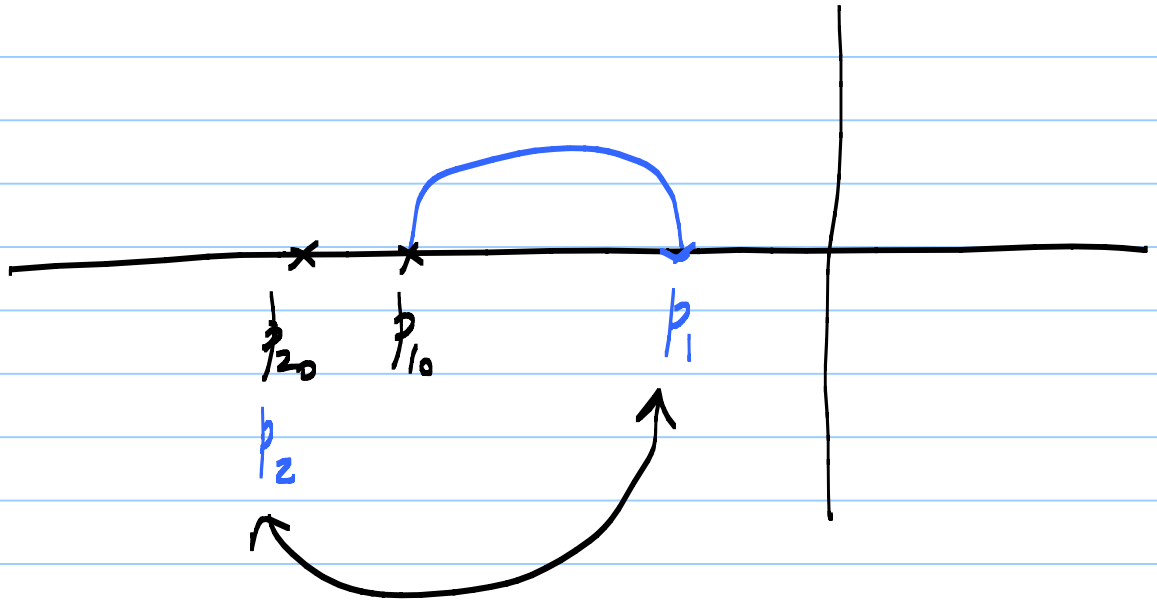
$\Rightarrow$ system becomes more stable

"Miller Compensation"

or "Pole-splitting compensation"

Dominant pole compensation:





distance between poles decides PM

ω_u is smaller than that for Miller compensation for same PM

$$z = +\frac{G_{m2}}{C_c} \quad \text{(RHP zero)}$$

@ zero z

output = 0 always

irrespective of input

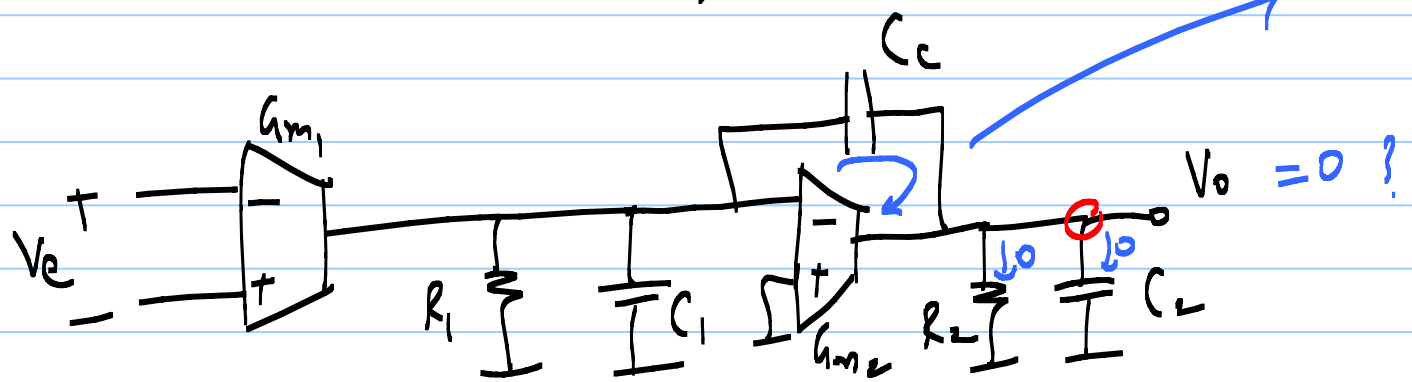
When/how does this happen?

$$\frac{V_o}{V_e}(s) = \frac{N(s)}{D(s)}$$

$$N(s) = \left(1 - \frac{sC_c}{G_{m2}}\right)$$

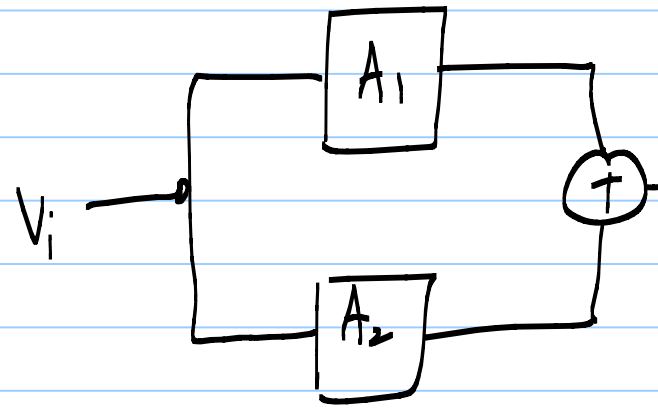
general form $\quad N(s) = \left(1 - \frac{s}{z}\right)$

2 stage op amp



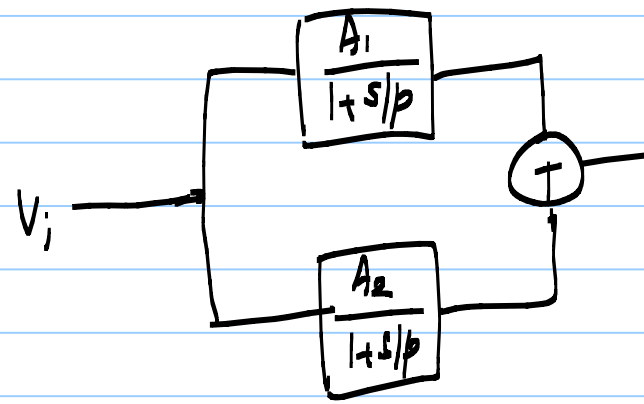
$z = \frac{+G_{m2}}{C_c}$

$V_o = 0$?



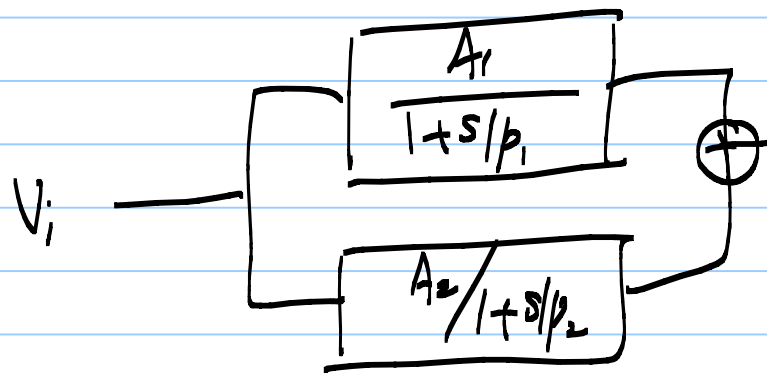
$$V_o = (A_1 + A_2) V_i$$

No zero



$$V_o = \frac{A_1 + A_2}{1 + s/p} V_i$$

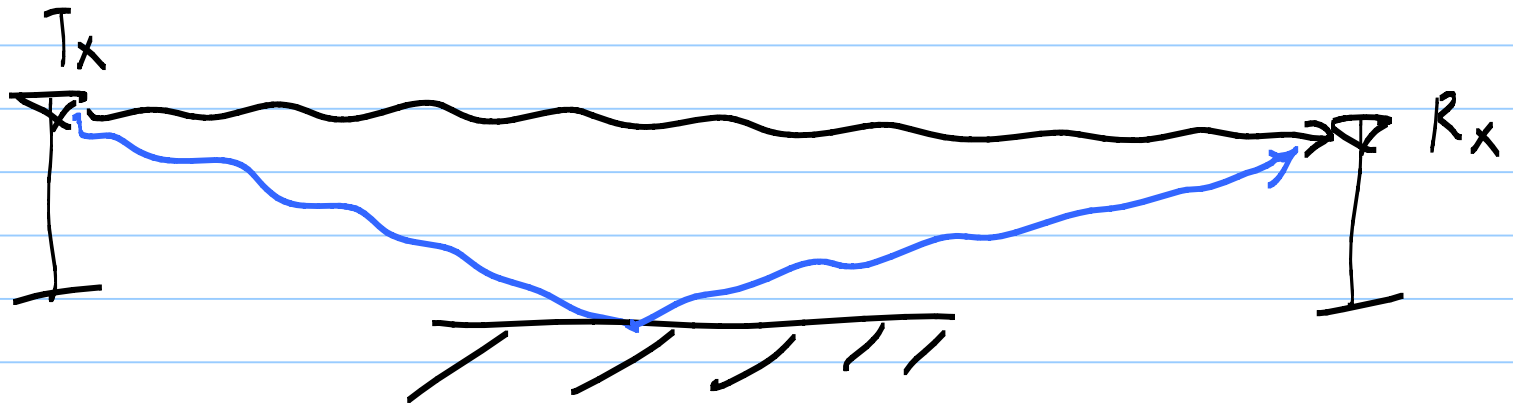
No zero



$$V_o = \left[\left(\frac{A_1}{1+s/p_1}\right) + \left(\frac{A_2}{1+s/p_2}\right)\right] V_i$$

$$V_o = \frac{A_1 + \frac{A_1 s}{p_2} + A_2 + \frac{A_2 s}{p_2}}{(1+s/p_1)(1+s/p_2)} V_i$$

zero

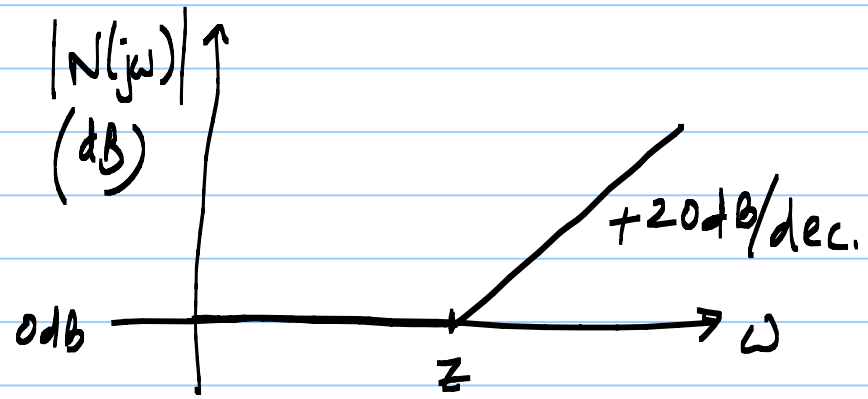


Null @ Rx is possible

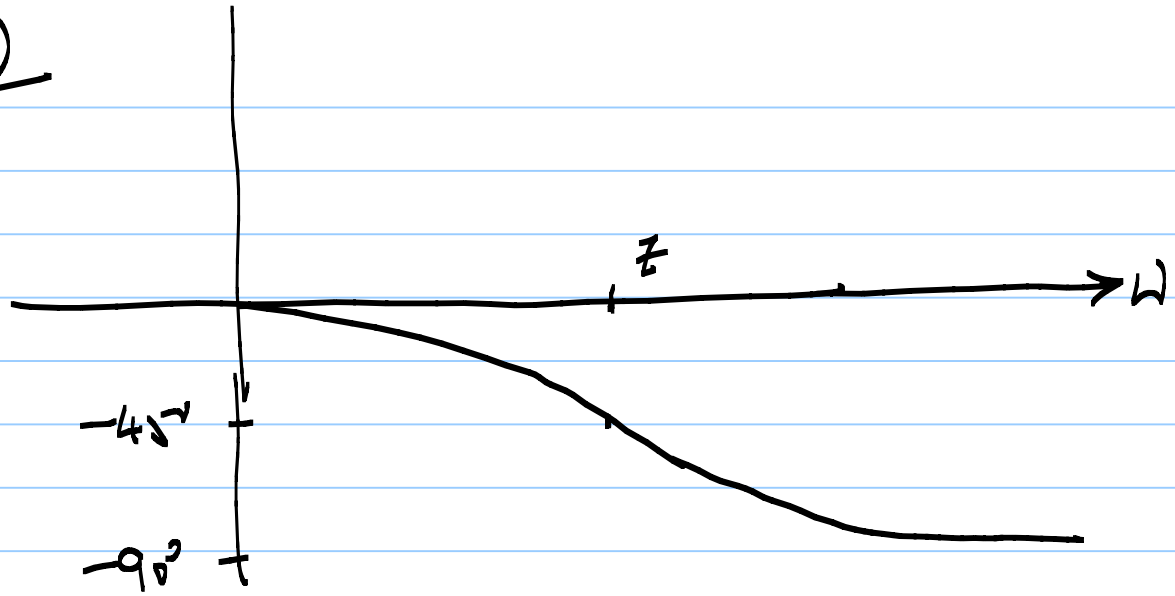
Effect of $z = +\frac{G_{m2}}{C_c}$ on stability

$$N(s) = 1 - s/z$$

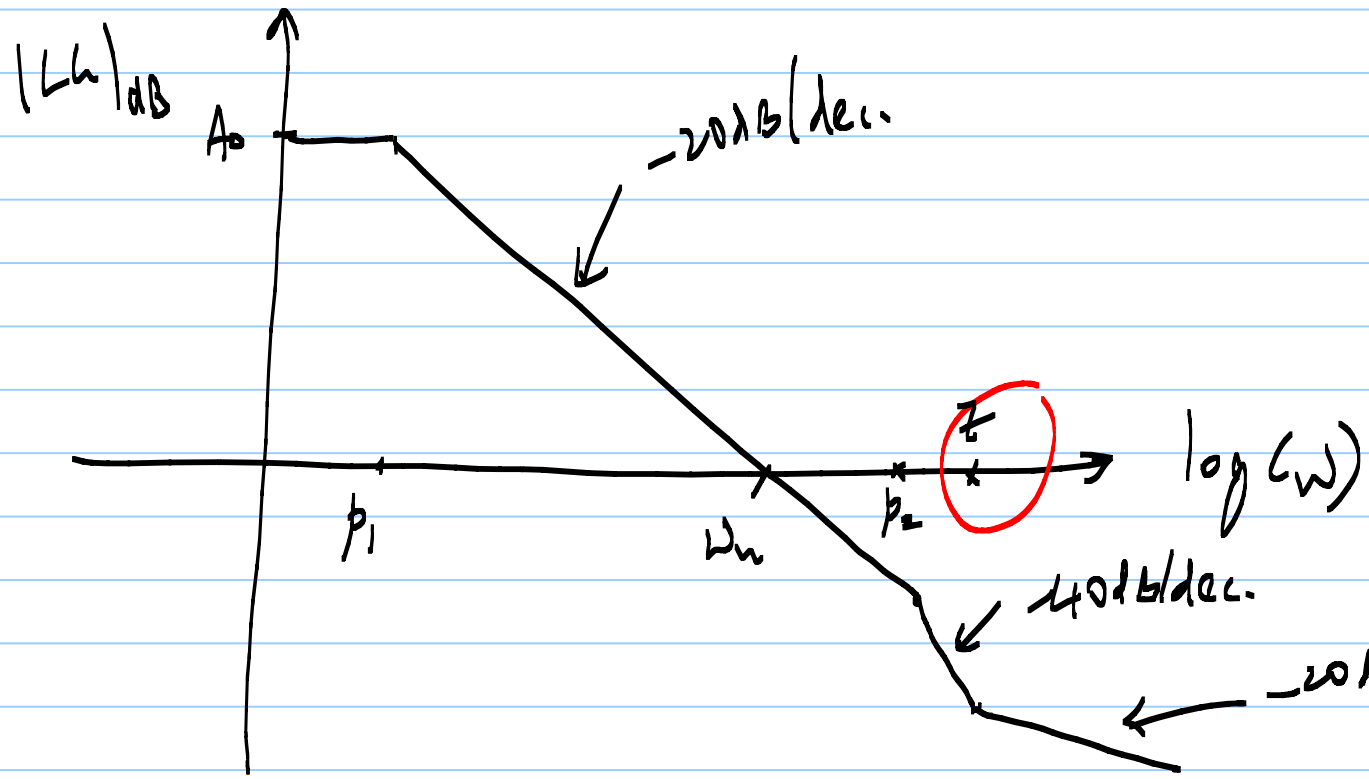
$$|N(j\omega)| = \sqrt{1 + \frac{\omega^2}{z^2}}$$



$\angle N(j\omega)$



$$\angle N(j\omega) = -\tan^{-1}\left(\frac{\omega}{z}\right)$$

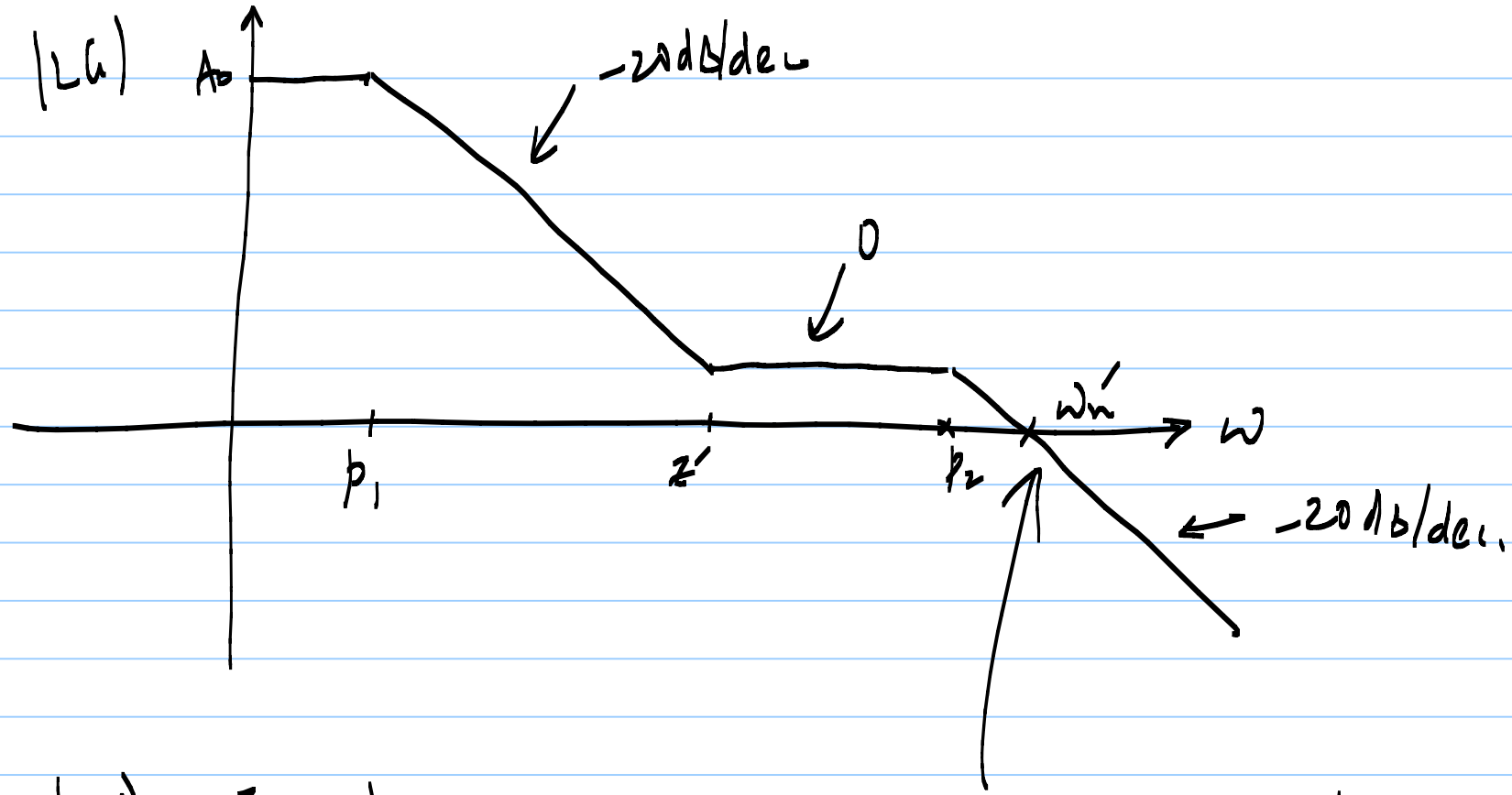


$z \gg p_2, \omega_u$

$$\omega_u = A_0 \cdot p_1$$

$$= (G_{m1} R_1)(G_{m2} R_2) \cdot \frac{-1}{C_c G_{m2} R_2 R_1}$$

$$= -\frac{G_{m1}}{C_c}$$



* push z out as far as possible

* $|z| > |\omega_n|$

Phase would have crossed $-180°$ @ ω_n

$\Rightarrow$ high chance of instability

$$|z| \gg |\omega_n|$$

$$\left|\frac{G_{m2}}{C_c}\right| \gg \left|\frac{G_{m1}}{C_c}\right| \Rightarrow \boxed{G_{m2} \gg G_{m1}}$$

$$PM \approx 180^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_n}{p_2}\right)$$

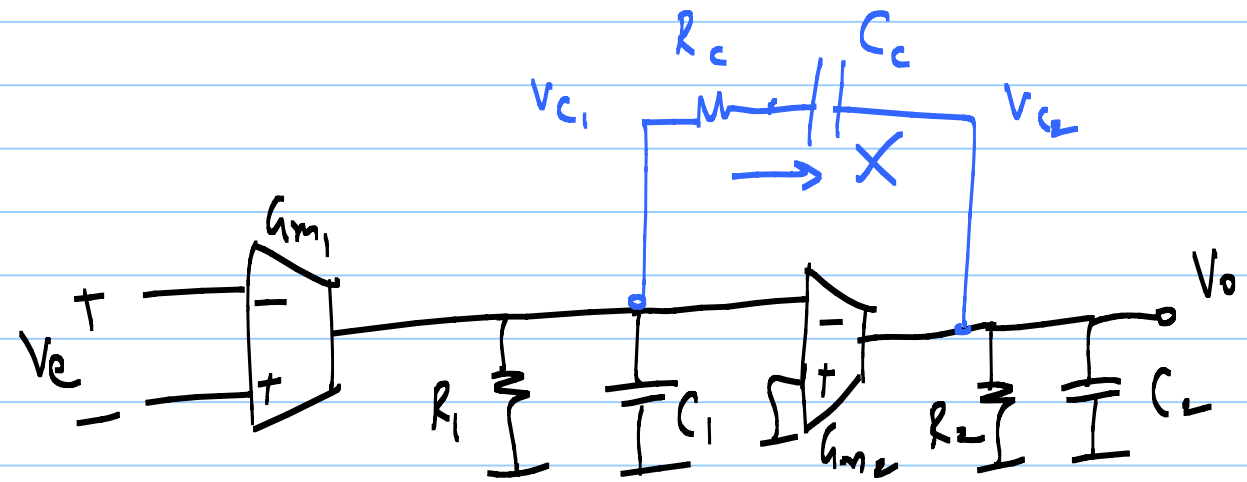
$$\approx 90^\circ - \tan^{-1}\left(\frac{\omega_n}{p_2}\right)$$

08/03/2022

Lecture 24

1 zero
3 poles

2 stage op amp



$$p_1 \approx \frac{-1}{R_1\{C_c \cdot G_{m_2} R_2\}} \quad ; \quad p_2 \approx \frac{-G_{m_2}\dfrac{C_c}{C_c+C_1}}{C_2 + \dfrac{C_c C_1}{C_c+C_1}}$$

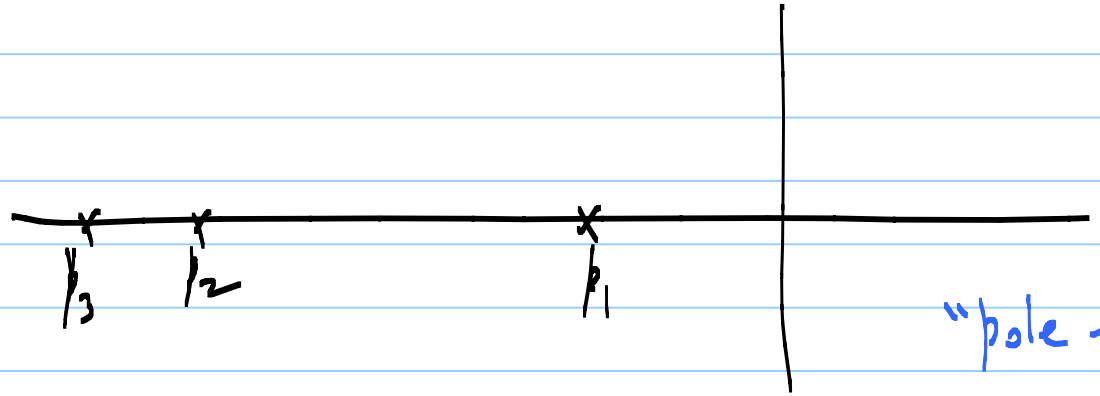
$$\approx \frac{-G_{m_2} C_c}{C_c C_2 + C_c C_1 + C_1 C_2}$$

$p_3 = HW$ (assume $p_3 > p_1, p_2$)

HW $\rightarrow$ $z = \dfrac{+G_{m_2}}{C_c(1 - G_{m_2} R_c)} = f(R_c)$

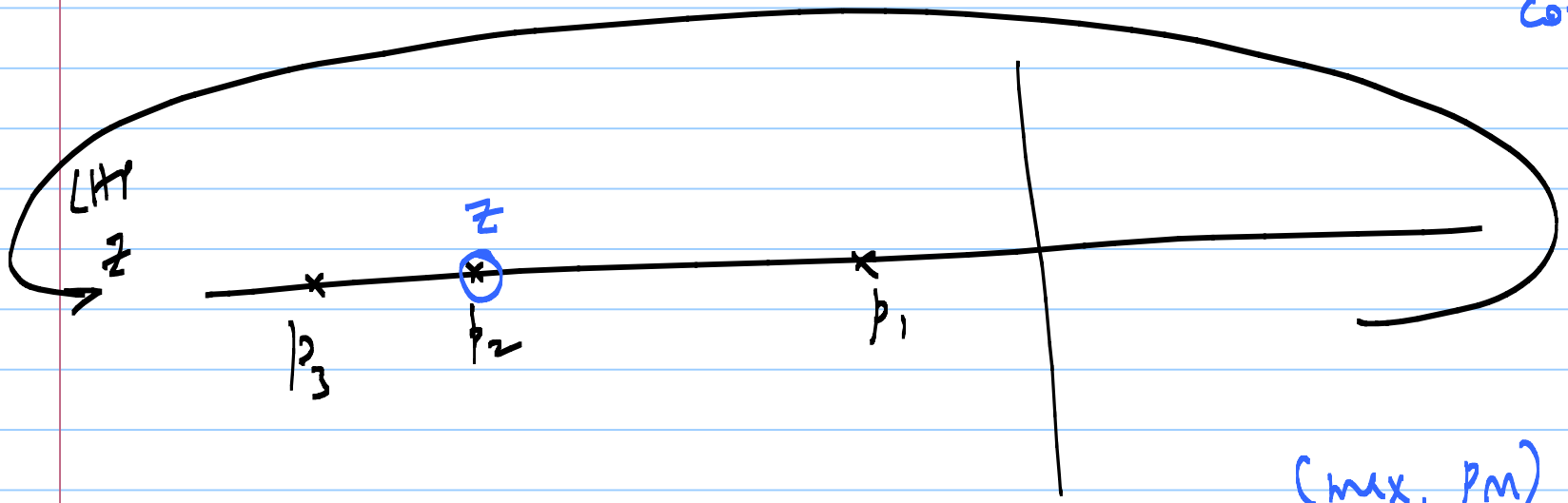
1) Set $R_c = \frac{1}{G_{m2}} \Rightarrow Z = \infty$ (no RHP zero)

2) $R_c > \frac{1}{G_{m2}} \Rightarrow$ LHP zero + 3 poles



$Z = \infty$

"pole-zero cancellation" compensation

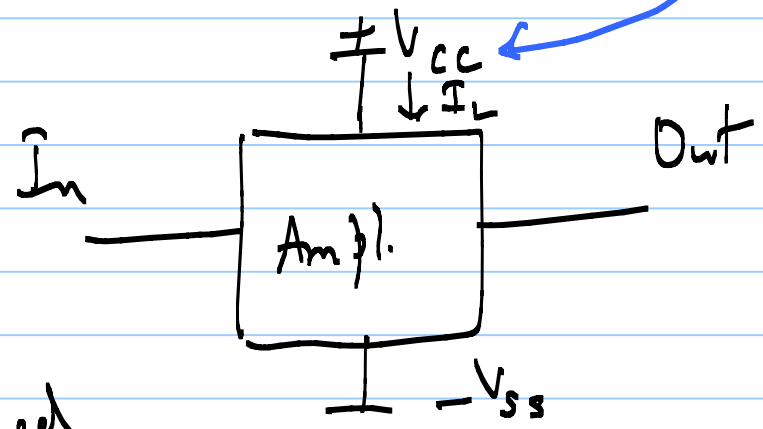


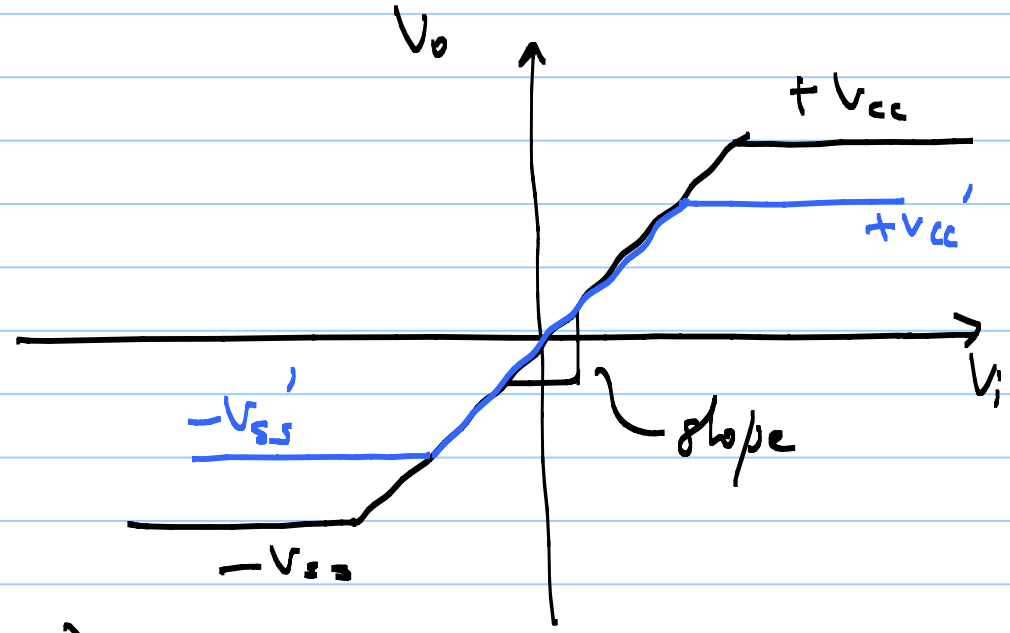
place Z on top of first non-dominant pole (max. PM)

09/03/2022

Lecture 25

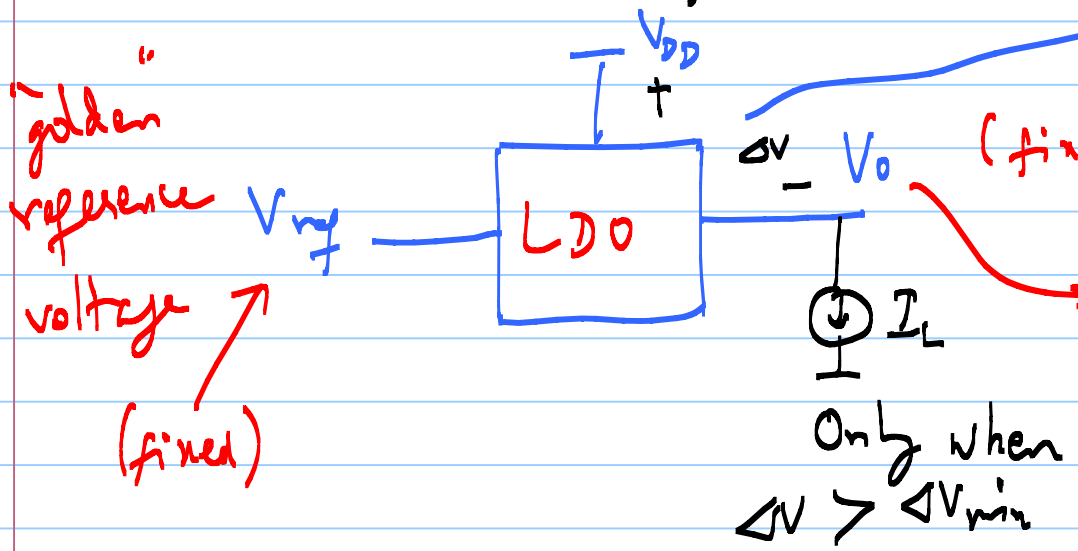
Power Management





we want well-defined Vcc & Vss

Low Dropout Regulator (LDO)

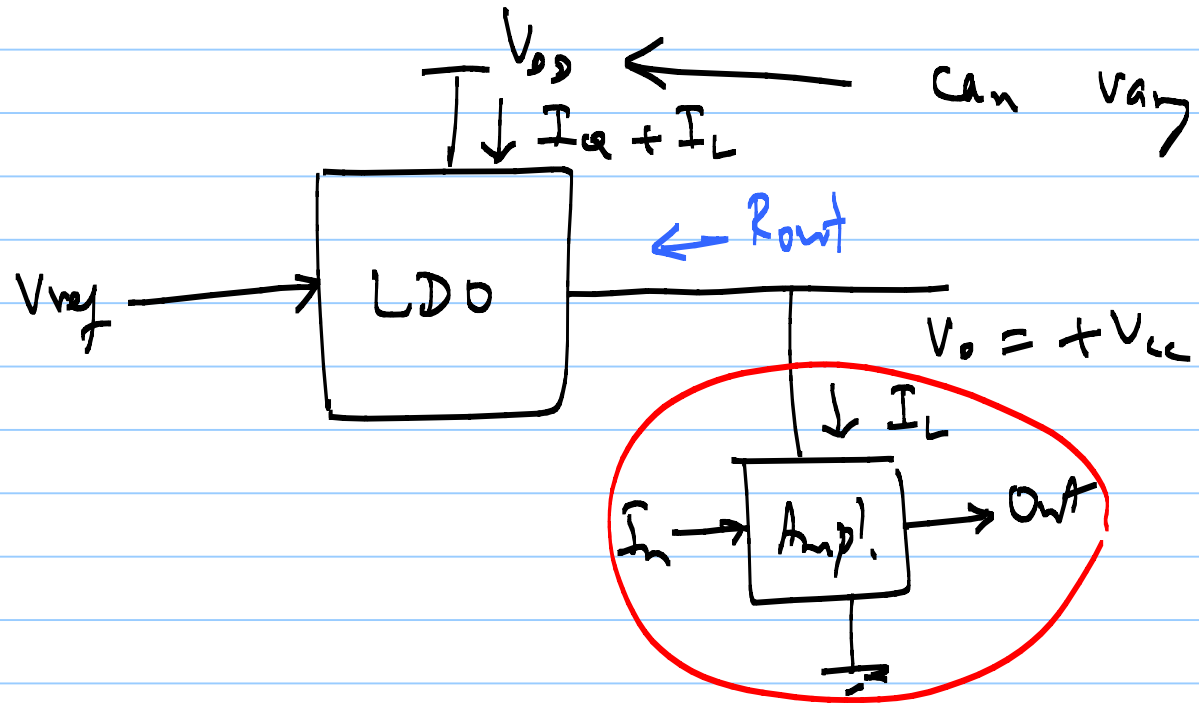


−ve feedback circuit

$V_o = f(V_{ref} \cancel{\leftarrow V_o})$

becomes +Vcc for other circuits

If $V_{DD} - V_o = \Delta V > \Delta V_{min.}$, V_o is independent of V_{DD}, and I_L



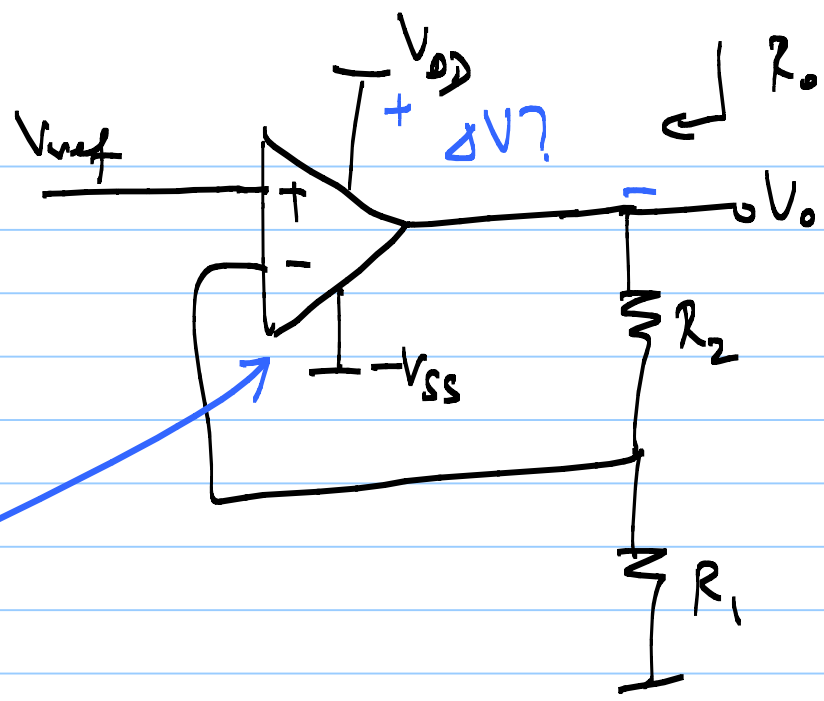
I_Q = current required for LDO to work properly

* V_o is indep. of I_L $\Rightarrow$ R_{out} is very small

$\Rightarrow$ LDO looks like a VCVS from V_{ref} to V_o

$V_o = k V_{ref}$

Reg. I



R_o is small

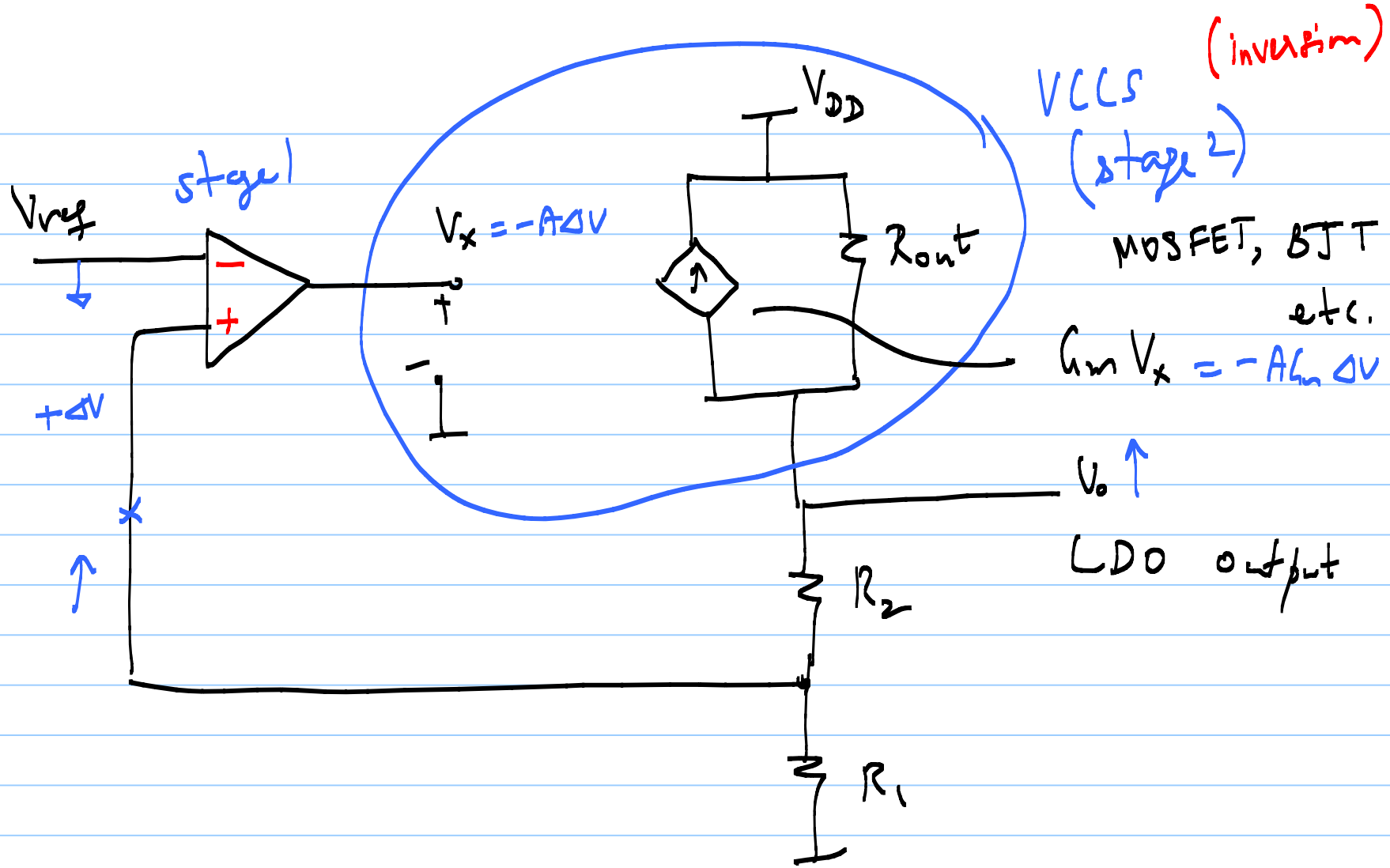
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_{ref}$$

$$V_o < V_{DD}$$

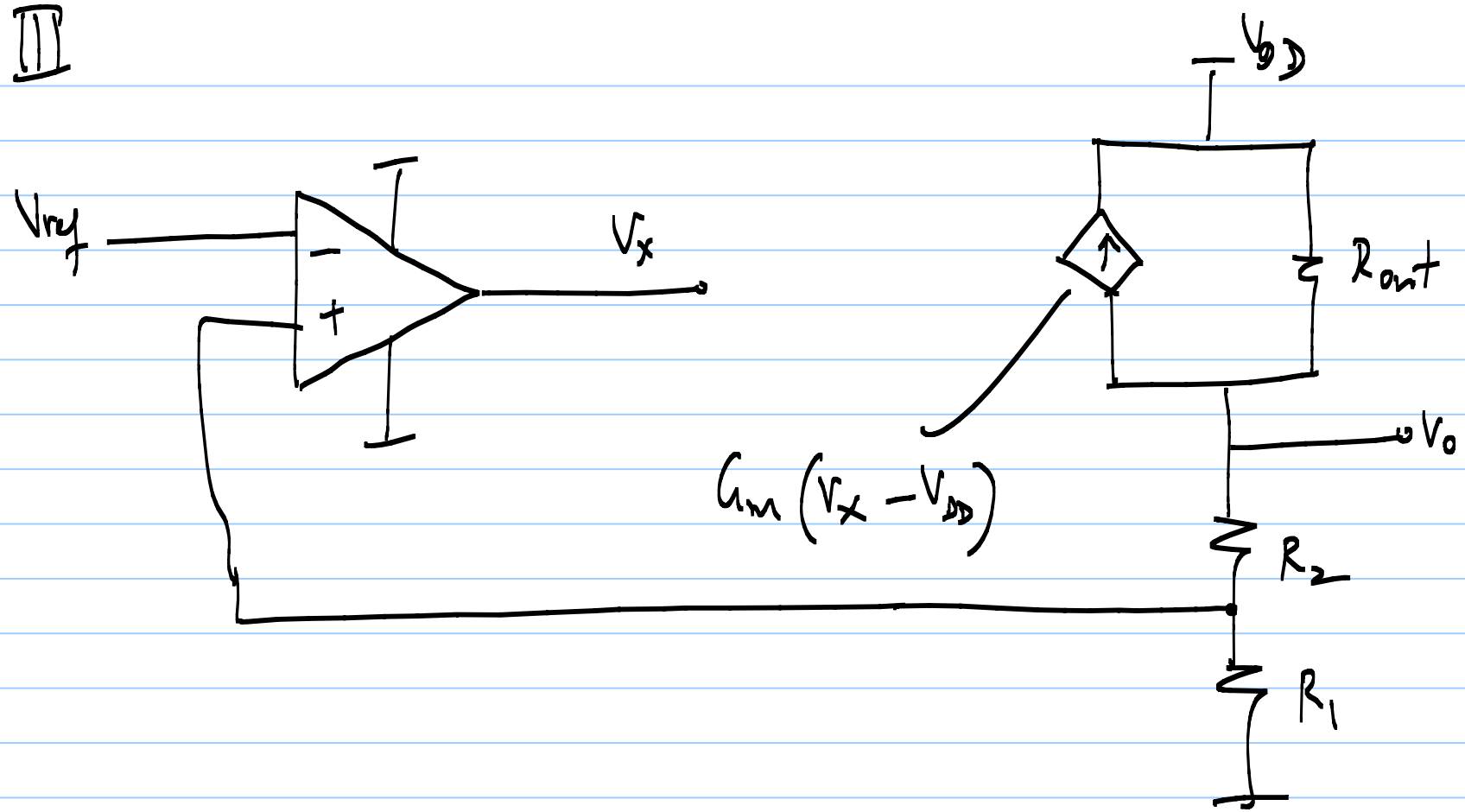
Efficiency $\eta = \dfrac{V_o \cdot I_L}{V_{DD}(I_Q + I_L)} = \dfrac{V_o}{V_o + \Delta V} \cdot \dfrac{I_L}{I_Q + I_L}$

Max η : I_Q, ΔV as small as possible

Reg. II


(inversion)
VCCS
(stage 2)
MOSFET, BJT
etc.
$G_m V_x = -AG_m \Delta V$
V_{DD}
R_{out}
$V_x = -A\Delta V$
stage 1
V_{ref}
$+\Delta V$
V_o
LDO output
R_2
R_1

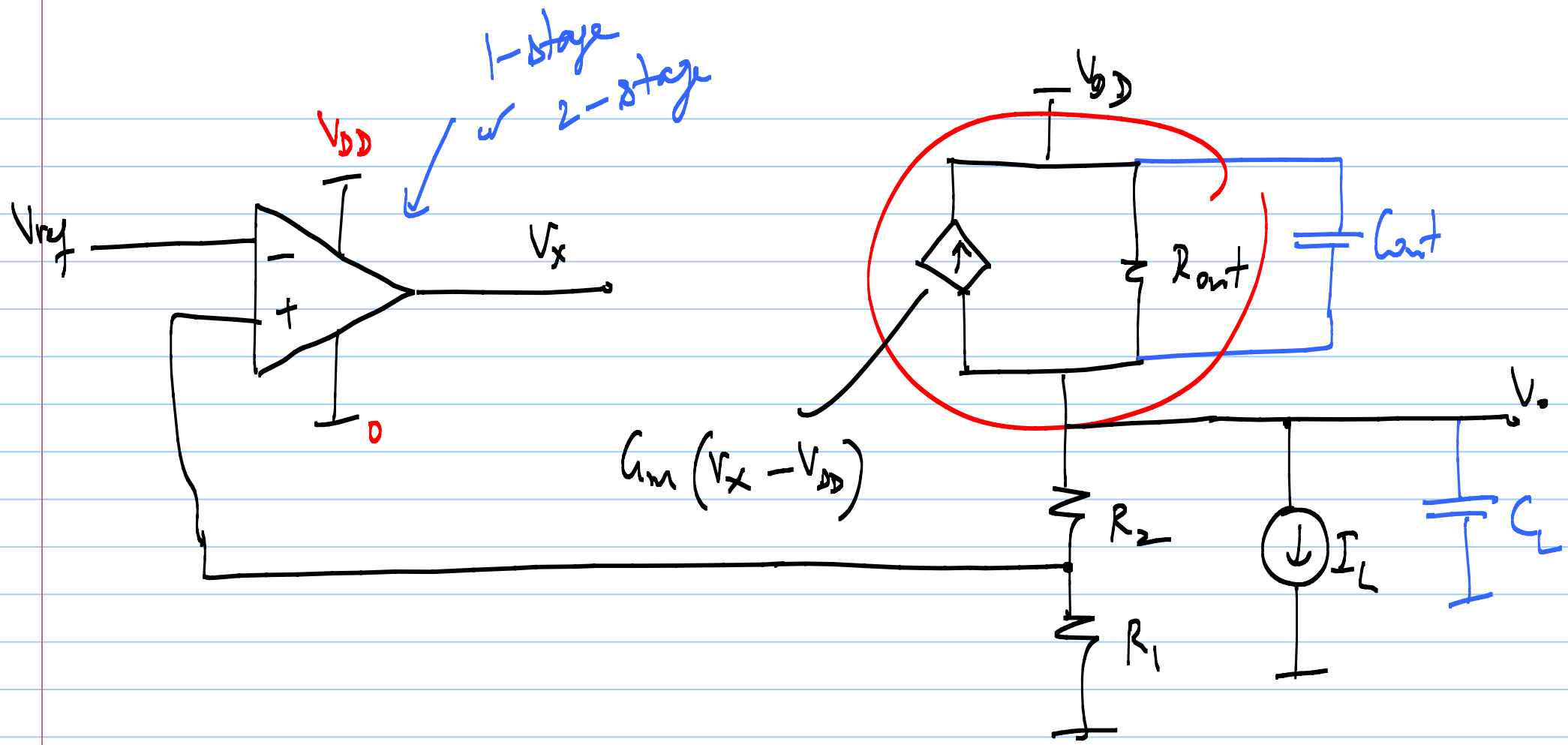
Reg. III



Stability

* $LG(s)$, plot $|LG(j\omega)|$ & $\angle LG(j\omega)$

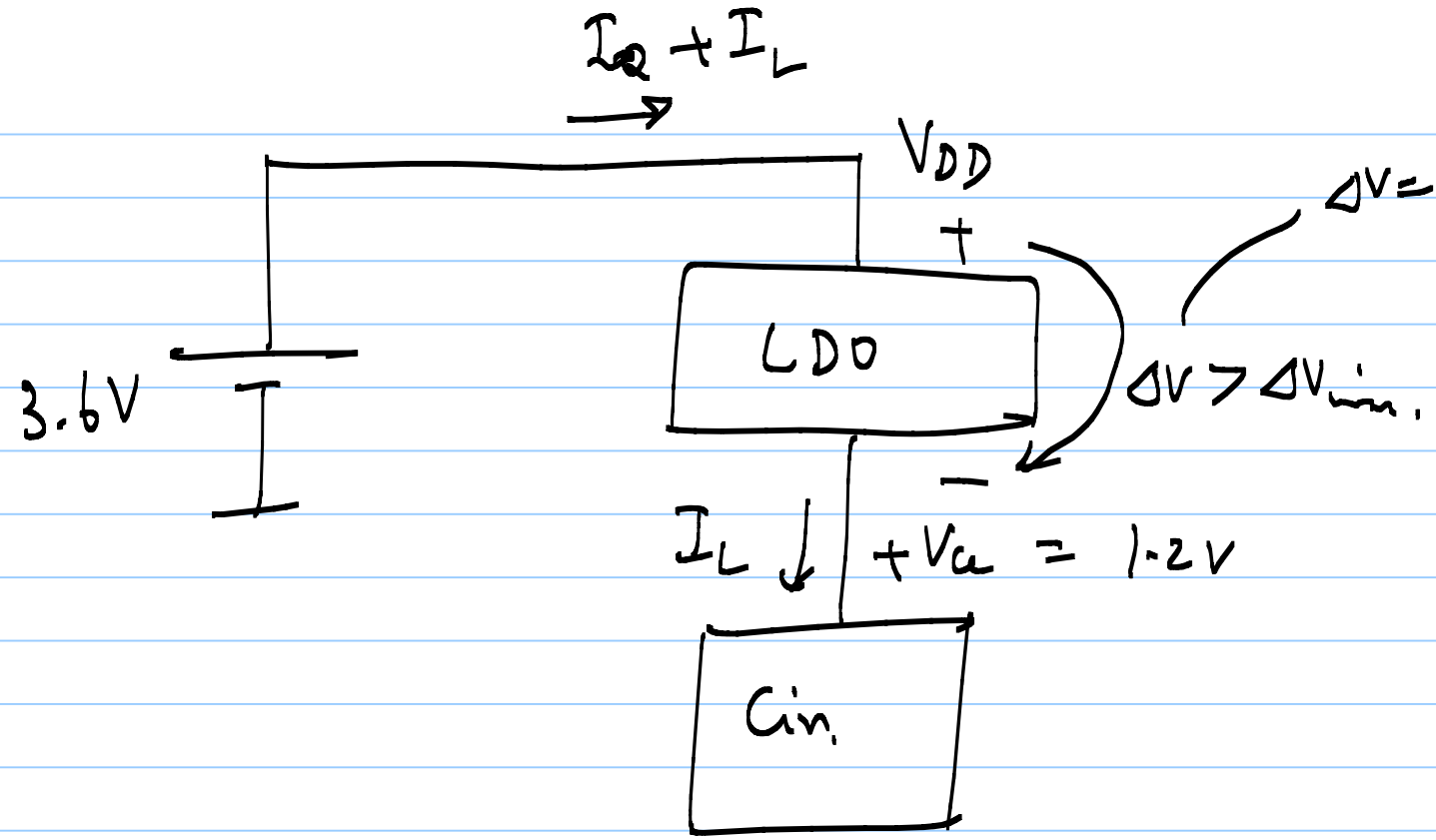
* GM & PM



1-stage opamp – 2 poles in $LG(j\omega)$

2-stage opamp – 3 poles in $LG(j\omega)$

freq. compensation



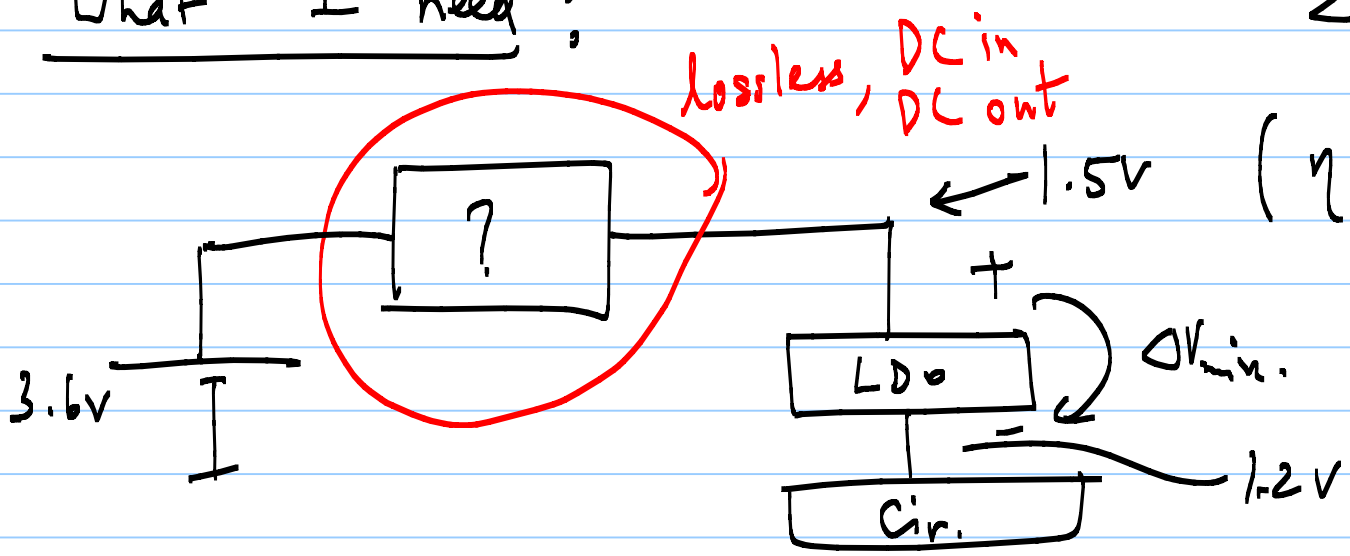
$\Delta V = 2.4V$ (large)

$\Delta V > \Delta V_{min.}$

η is very small

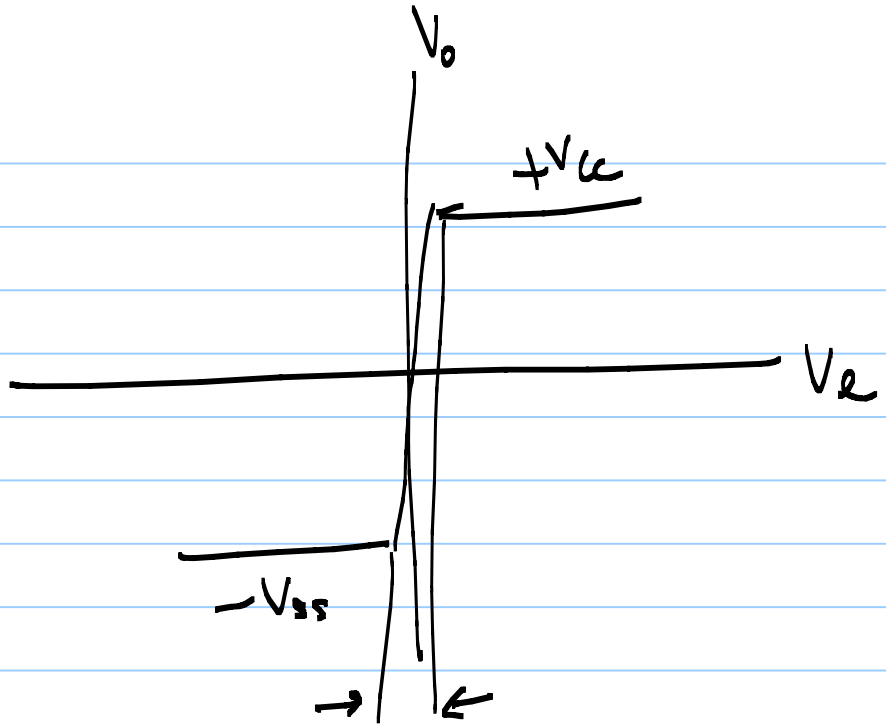
<u>What I need</u>:

$\Delta V_{min.} = 0.3V$ (say)



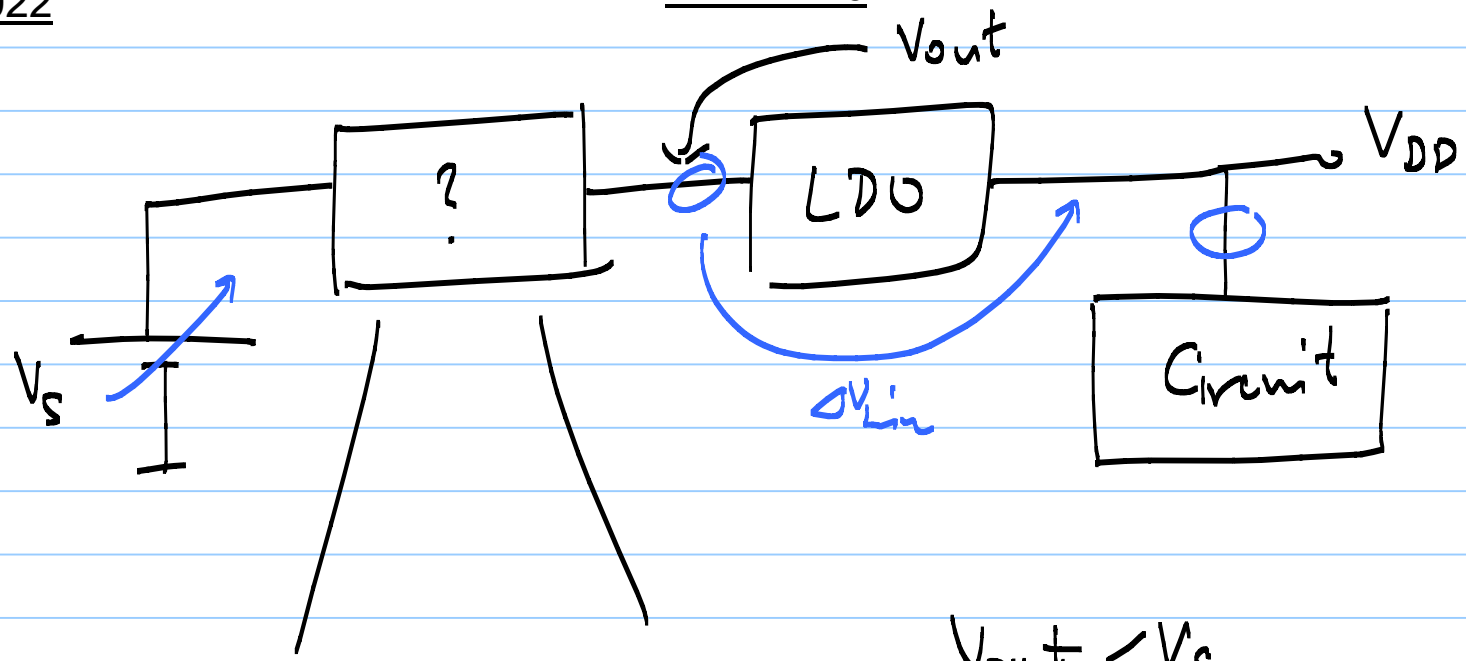
$\Delta V_{min.}$

(η = max)


V_o
$+V_{cc}$
V_e
$-V_{ss}$

10/03/2022

Lecture 26



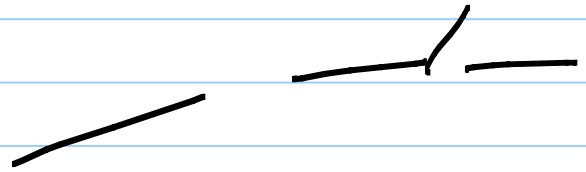
$V_{out} < V_s$

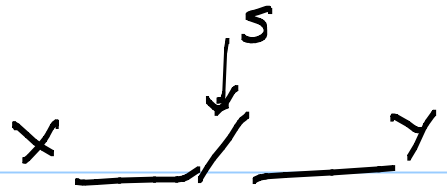
DC–DC

Converter

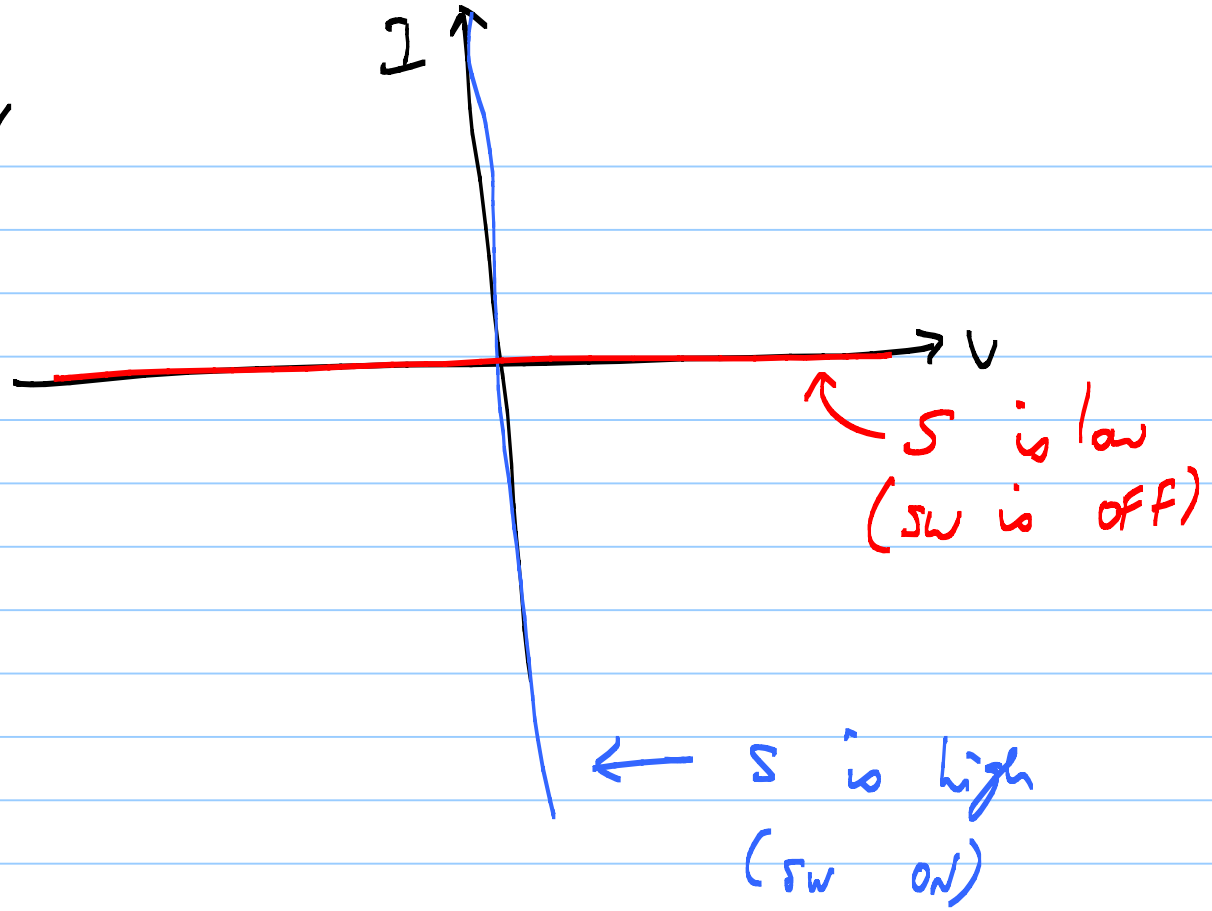
(lossless)

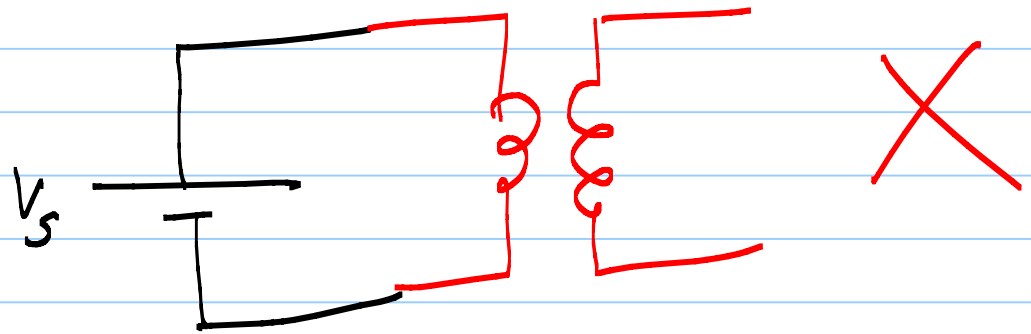
C, L, M Switch

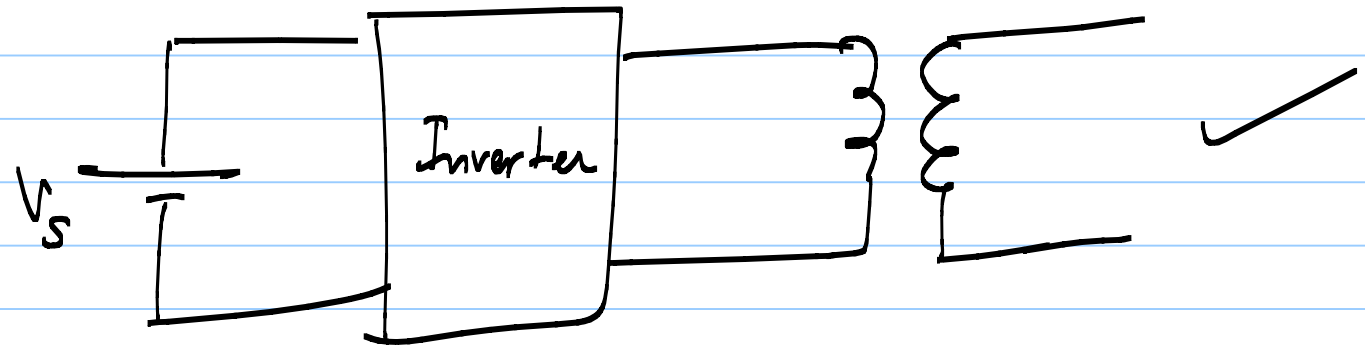


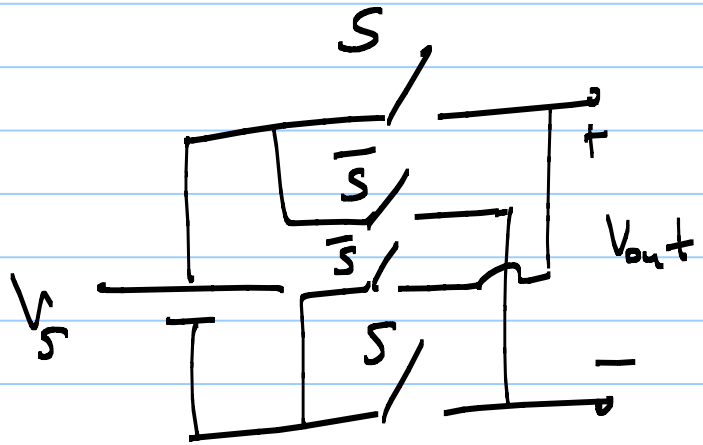


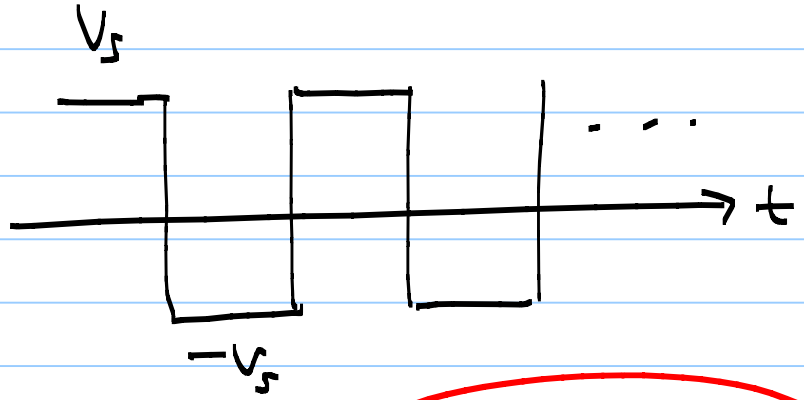
"linear"

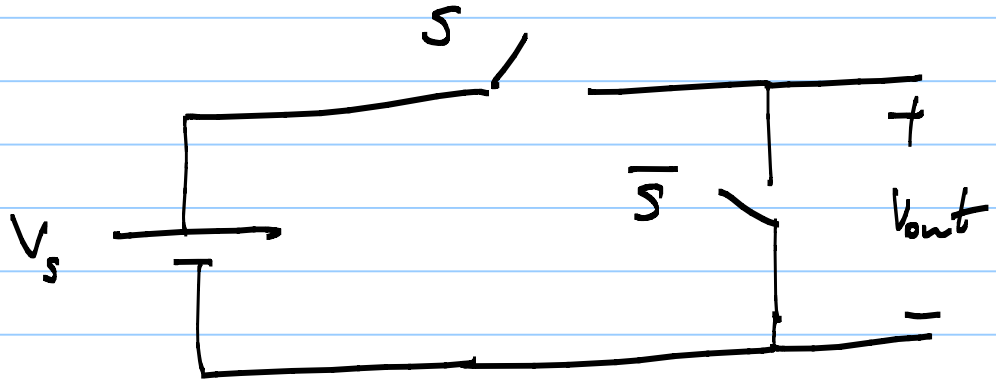


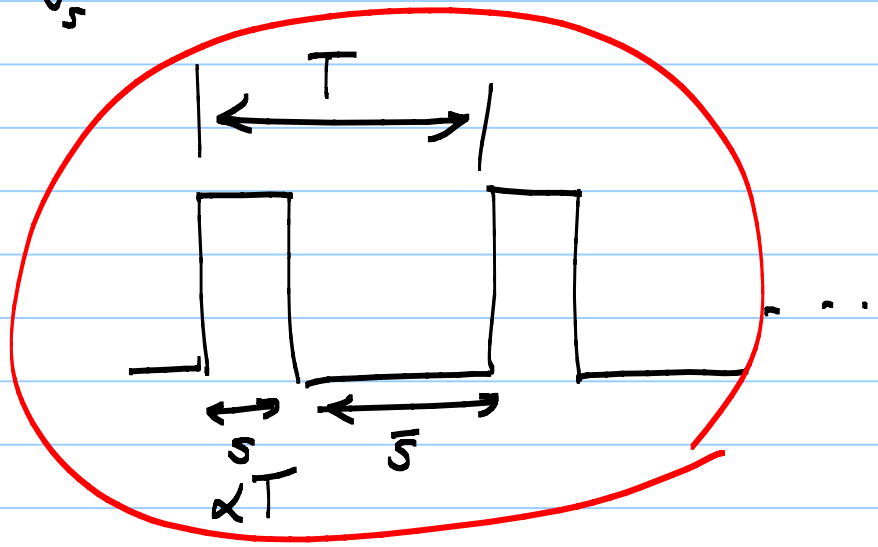



V_s
Inverter


S
$\bar{S}$
$\bar{S}$
S
V_s
+
V_{out}
−

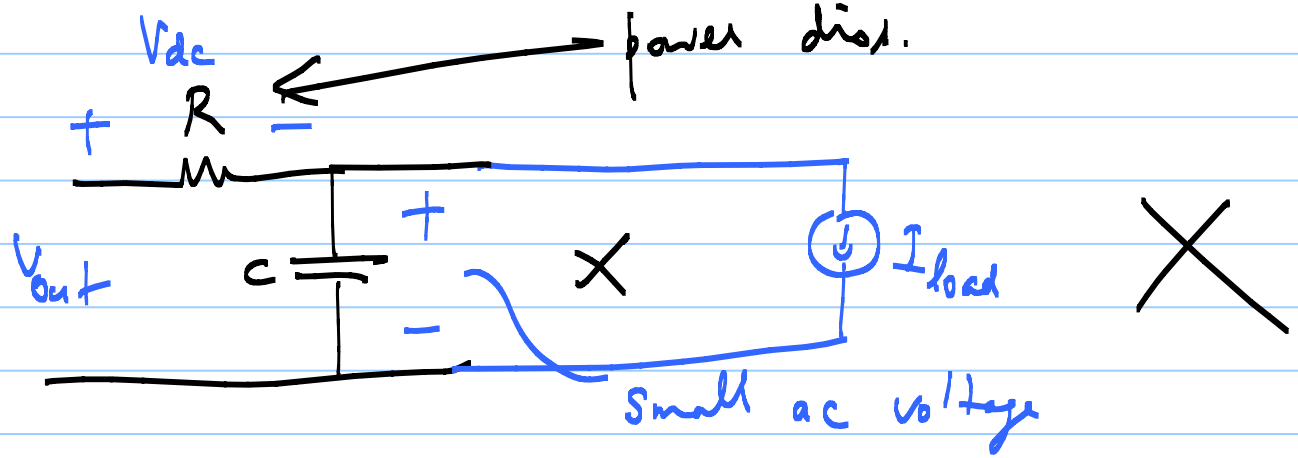

V_s
$-V_s$
t

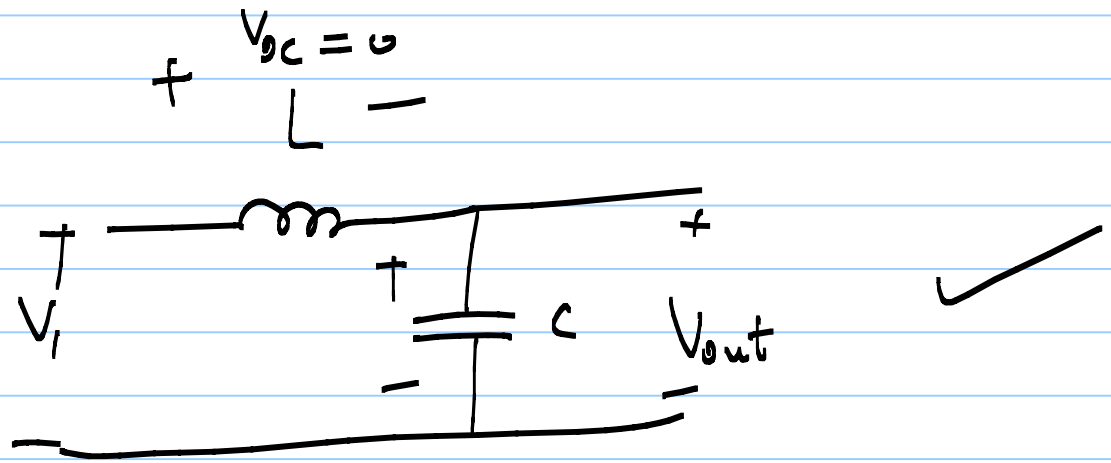

S
$\bar{S}$
V_s
+
V_{out}
−

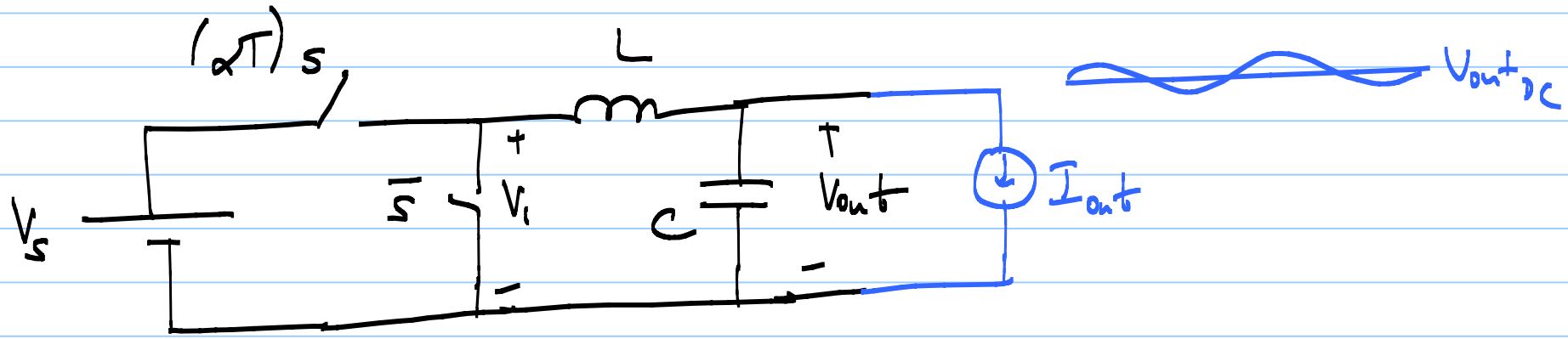

T
S
$\bar{S}$
αT

$$V_{out} = \sum_{k=0}^{\infty} A_k \sin\left(\frac{2\pi}{T} \cdot k + \phi_k\right)$$

$$\boxed{V_{out_{DC}} = \alpha V_s}$$ ← desired DC voltage



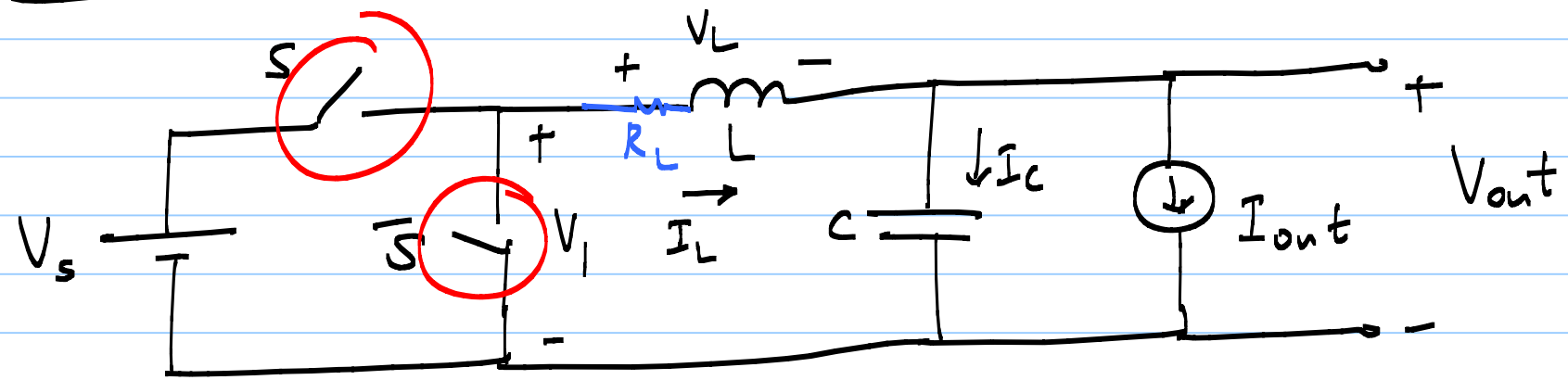


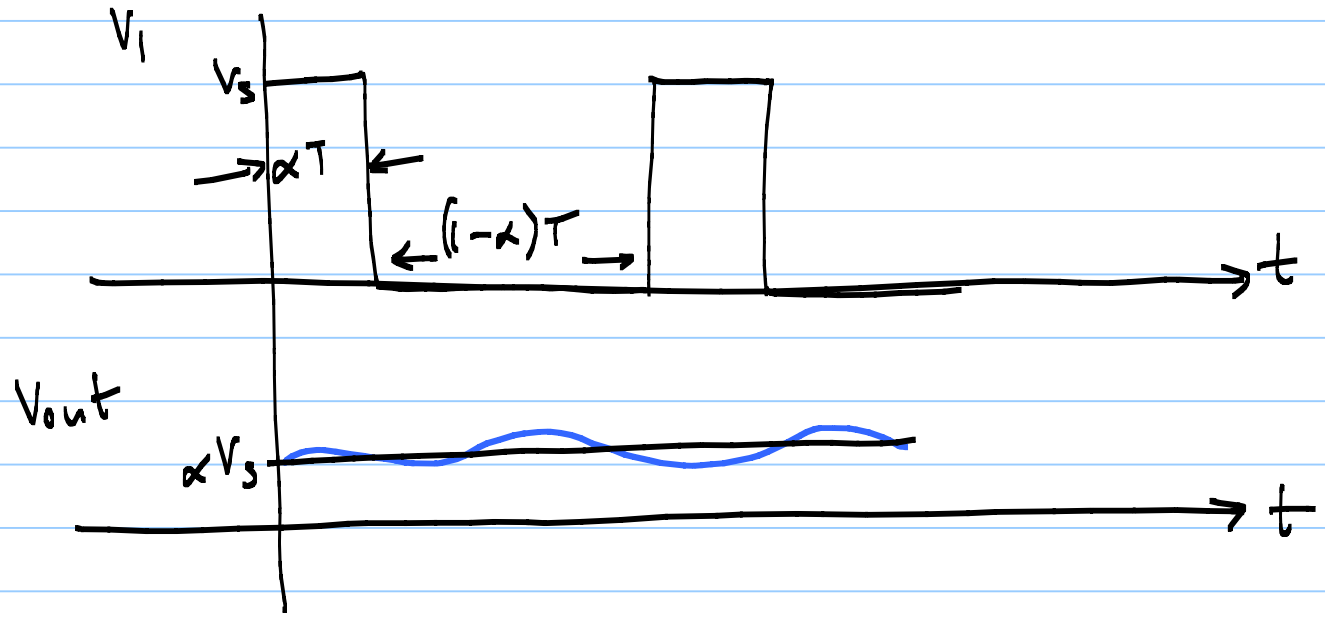

(αT) S
L
V_s
$\bar{S}$
+
V_i
−
C
+
V_{out}
−
I_{out}
$V_{out_{DC}}$

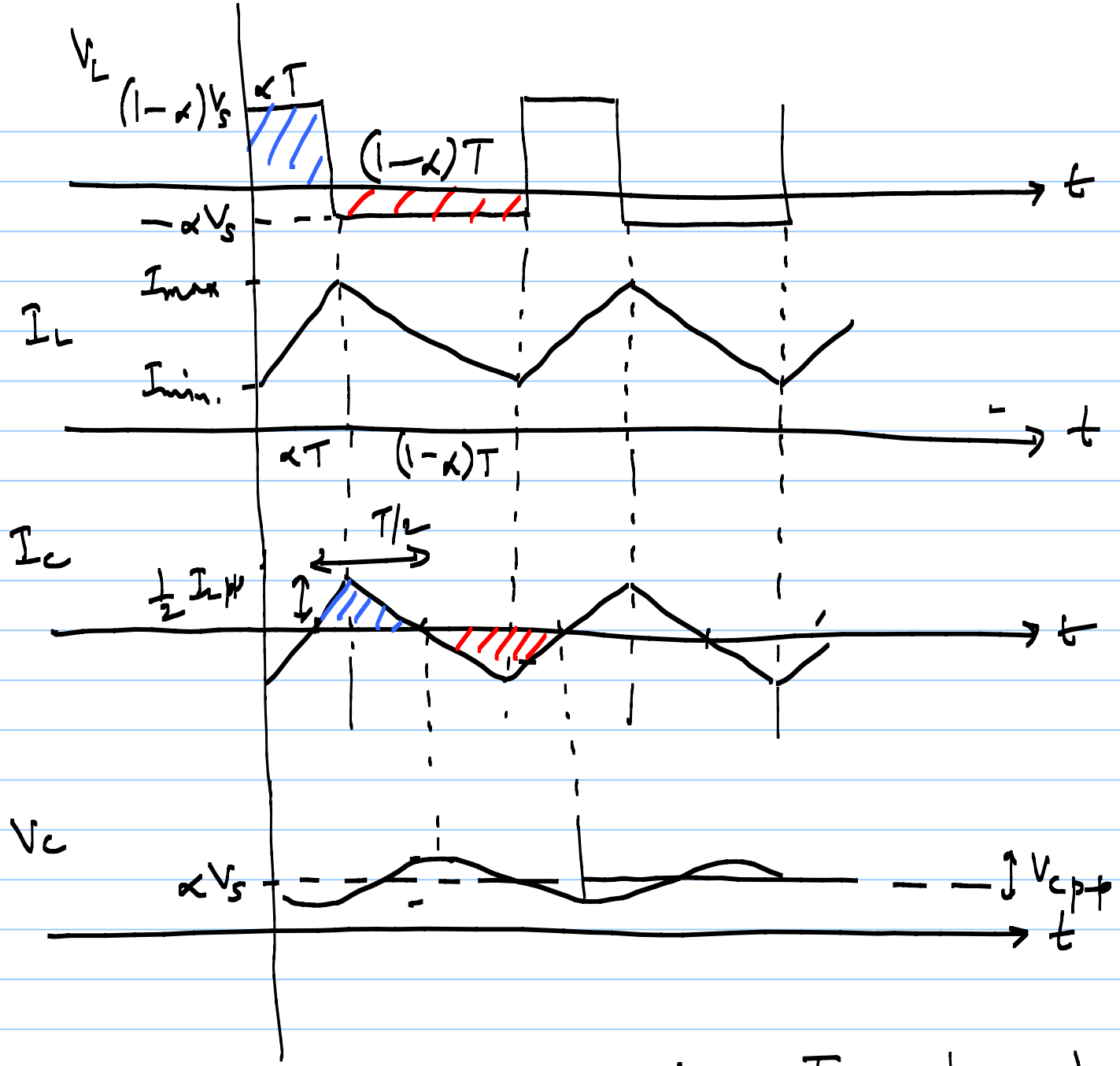
11/03/2022

Lecture 27

DC-DC Buck Converter







$$I_L = \frac{1}{L}\int V_L \, dt$$

$$I_{L\,pp} = \frac{1}{L} \times \alpha(1-\alpha) V_s T$$

$$I_{L\,avg.} = I_{out}$$

$$= \frac{I_{max} + I_{min}}{2}$$

$$I_c = I_L - I_{out}$$

$$V_c = \frac{1}{C}\int I_c \, dt$$

$$V_{c\,pp} = \frac{1}{2} \times \frac{T}{2} \times \frac{1}{2} \times \frac{1}{L} \times \alpha(1-\alpha) \cdot V_s T \times \frac{1}{C}$$

$$V_{c_{pp}} = \frac{V_s \cdot T^2 \alpha (1-\alpha)}{8LC}$$

Example

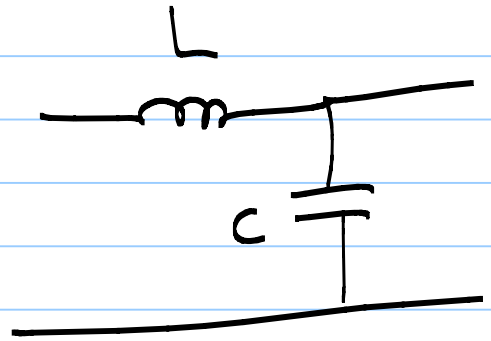
$V_s = 5V$

$V_{out} = 1V$

$V_{c_{pp}} \approx V_{out_{pp}} = 10mV$ (say)

$\alpha = 0.2$

$$\frac{T^2}{LC} = 0.1$$



$$T^2 = \frac{1}{f_{clk}^2}$$

$$f_{cutoff} = \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{1}{LC} = f_{cutoff}^2 \times 4\pi^2$$

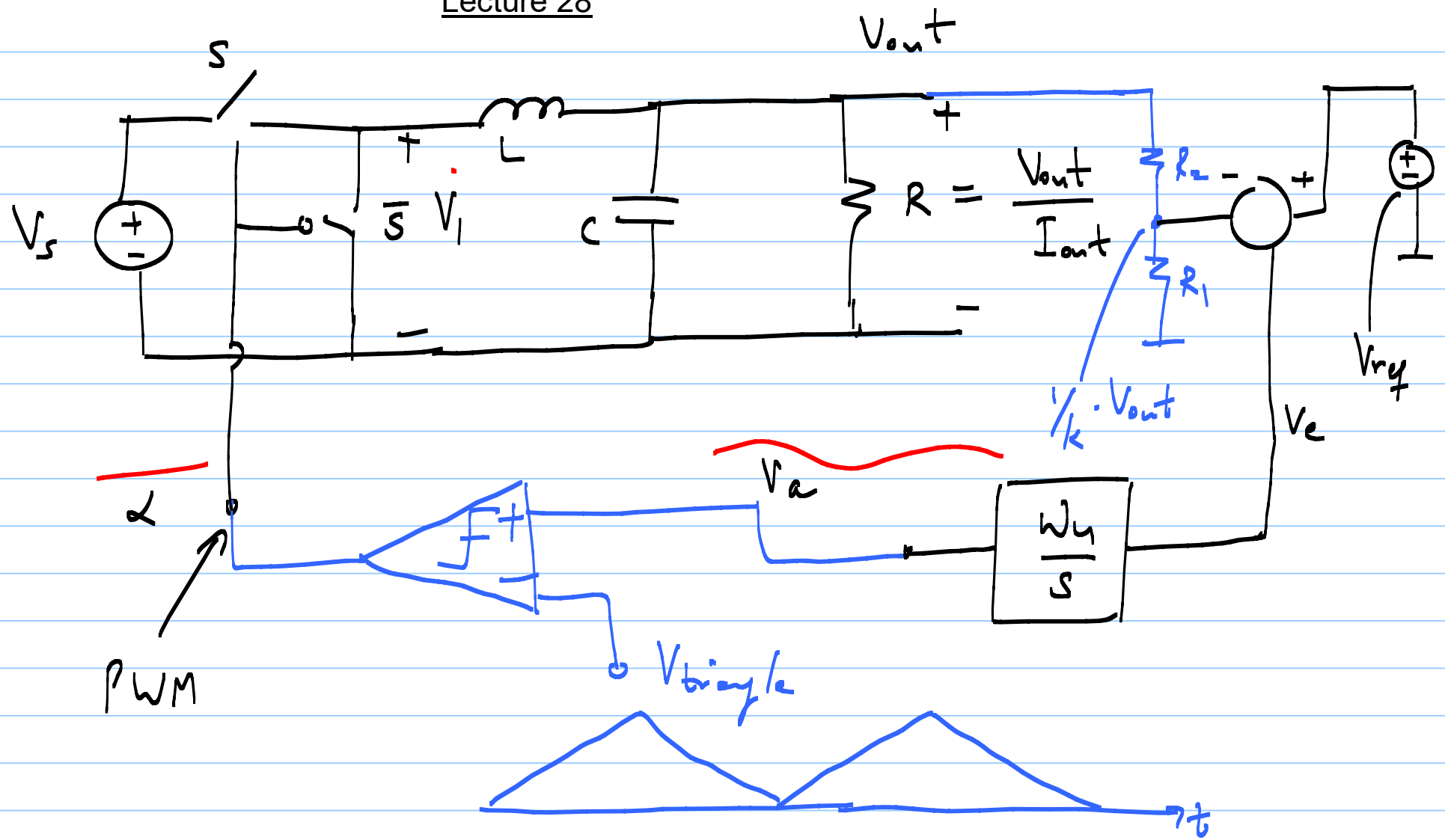
$$\frac{4\pi^2 f_{cutoff}^2}{f_{clk}^2} = 0.1$$

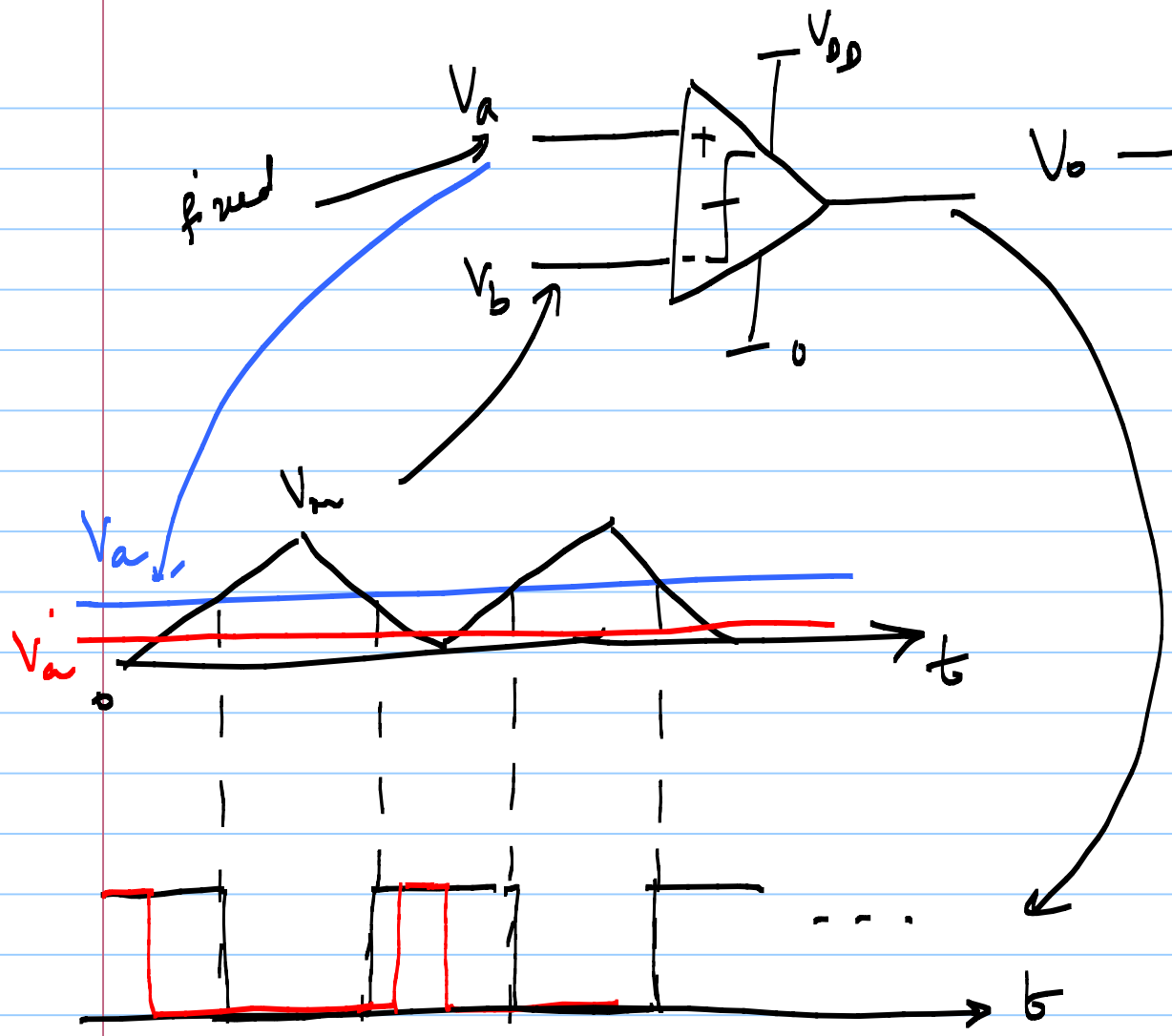
$$V_{out} = \alpha V_s$$

control α through -ve f.b.

15/03/2022

Lecture 28



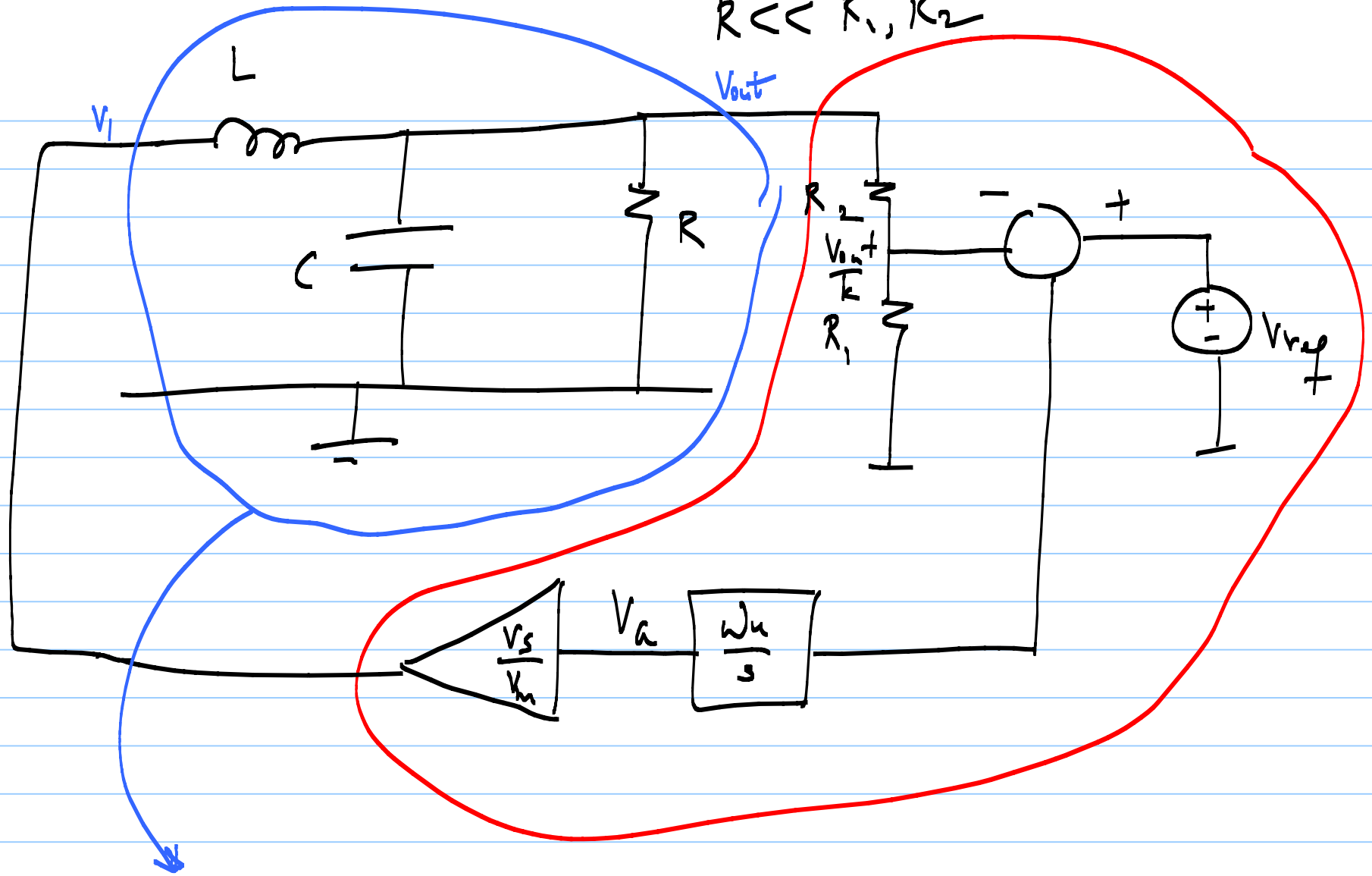


$$V_o \begin{cases} \text{"high"} \ (V_{DD}) & \text{if } V_a > V_b \\ \text{"low"} \ (0) & \text{if } V_a < V_b \end{cases}$$

$$\alpha_{eff} = \frac{V_a}{V_m}$$

LTI f.b. model

$R << R_1, R_2$

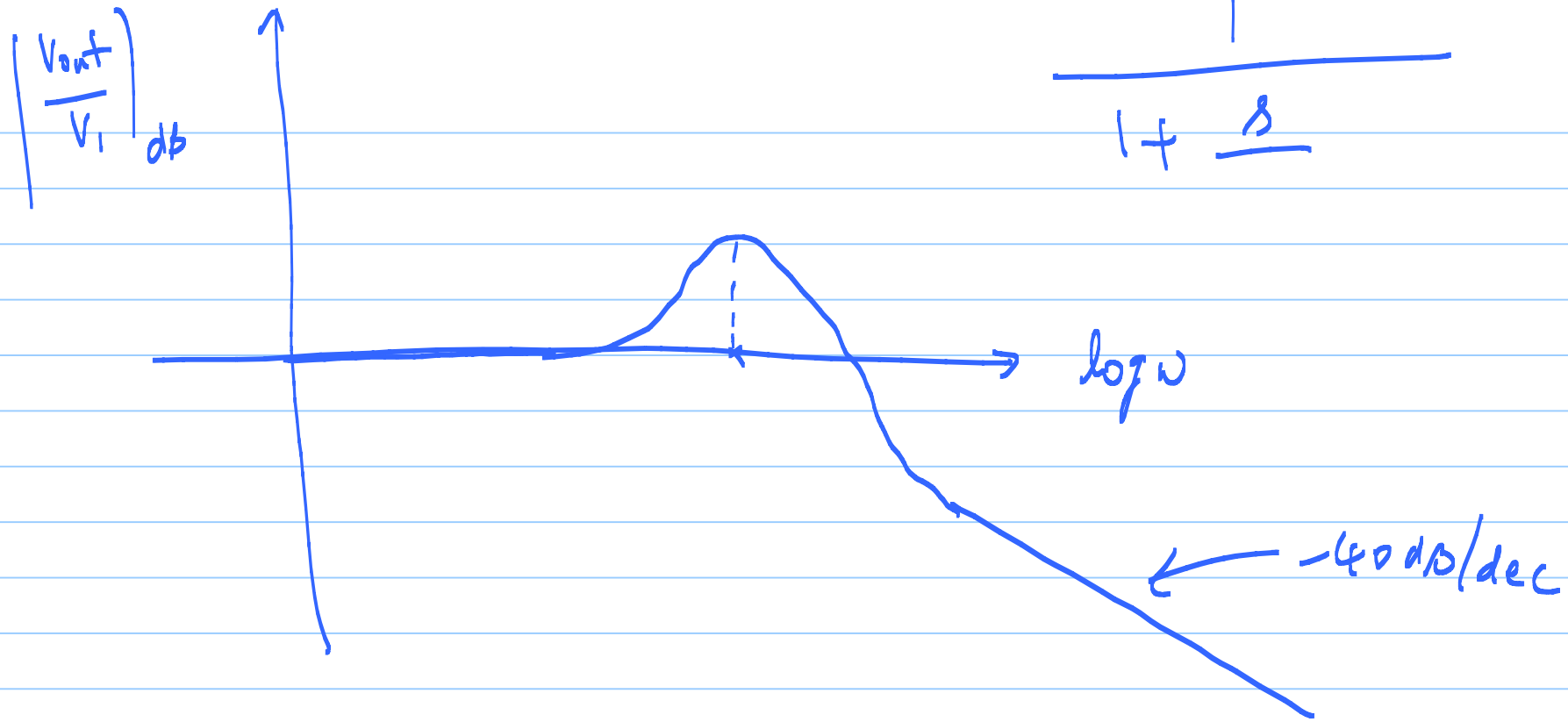


$$\frac{V_{out}}{V_1} = \frac{1}{1 + \frac{sL}{R} + s^2 LC}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$Q =$$

$$\frac{1}{1+\frac{s}{}}$$



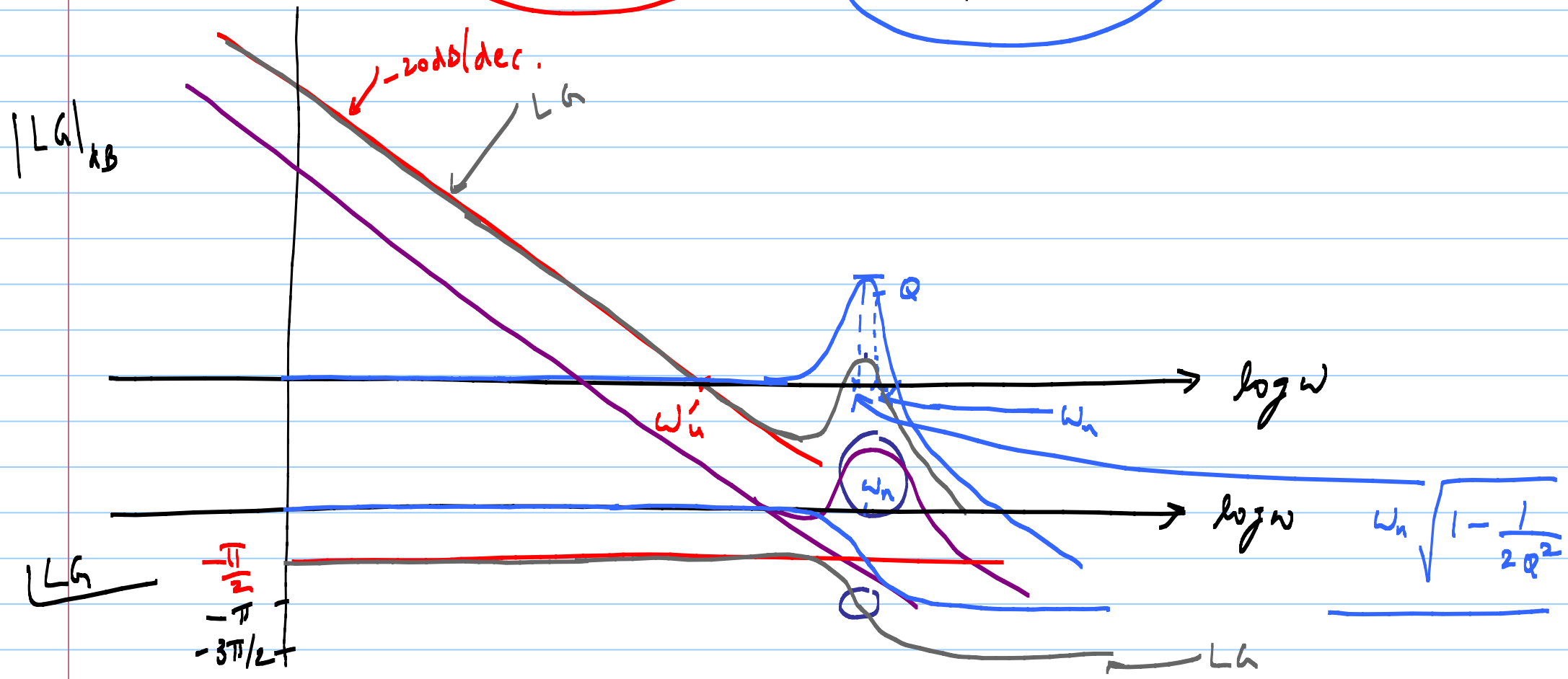
16/03/2022

Lecture 29

DC-DC converter f.b.

$$LG(s) = \frac{1}{k} \cdot \frac{\omega_u}{s} \cdot \frac{V_s}{V_m} \cdot \frac{1}{1 + \frac{sL}{R} + s^2 LC}$$

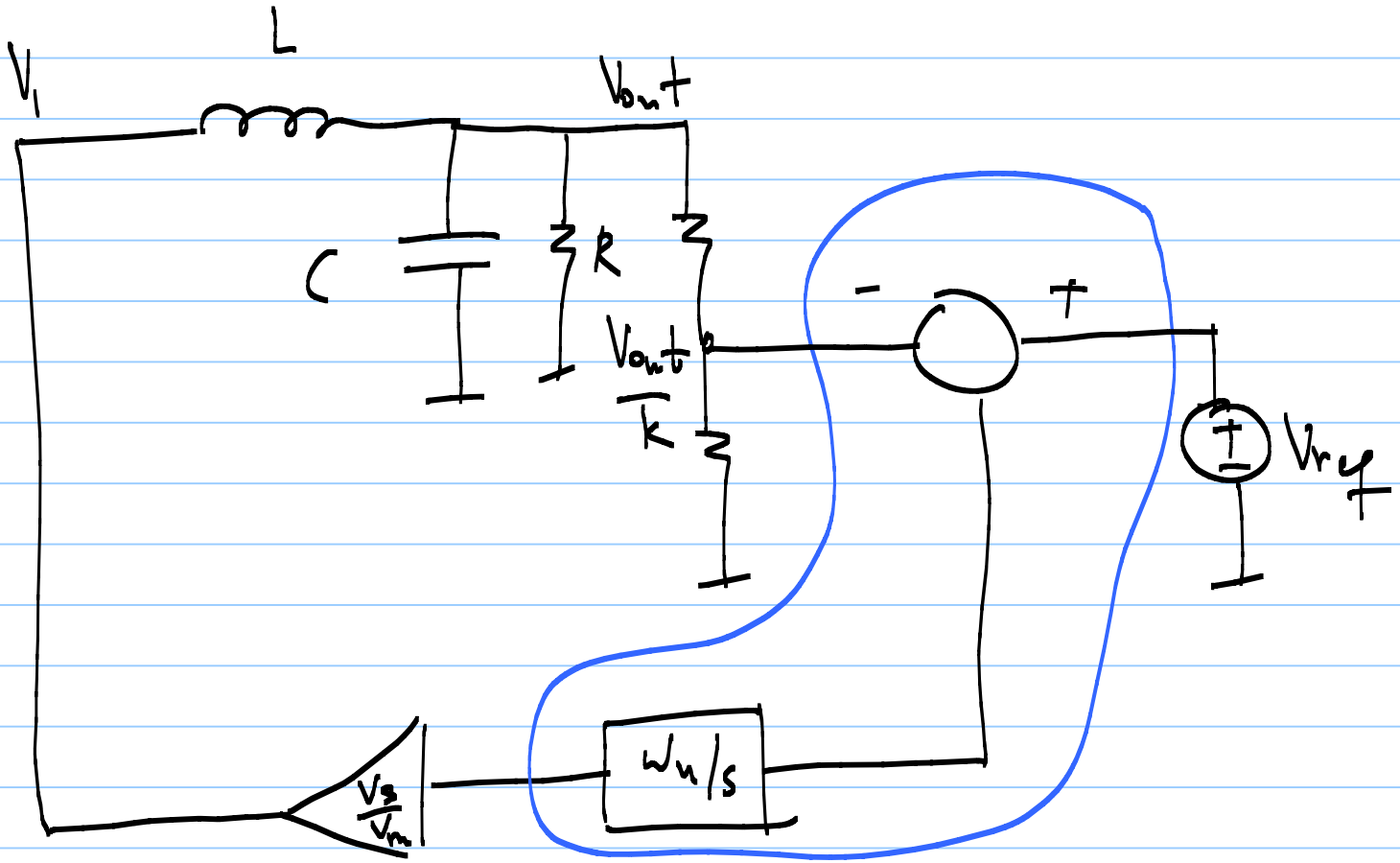
ω_u'

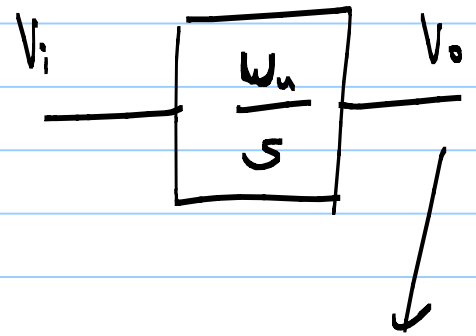


$$\omega_n = \frac{1}{\sqrt{LC}}$$

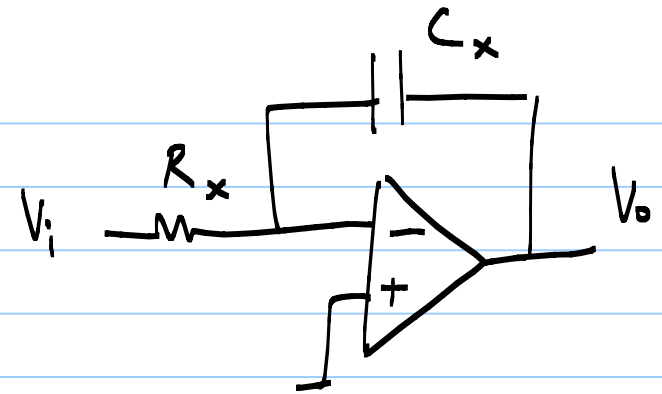
$$Q = R\sqrt{\frac{C}{L}}$$

$$\boxed{\frac{\omega_n'}{\omega_n} << \frac{1}{Q}}$$



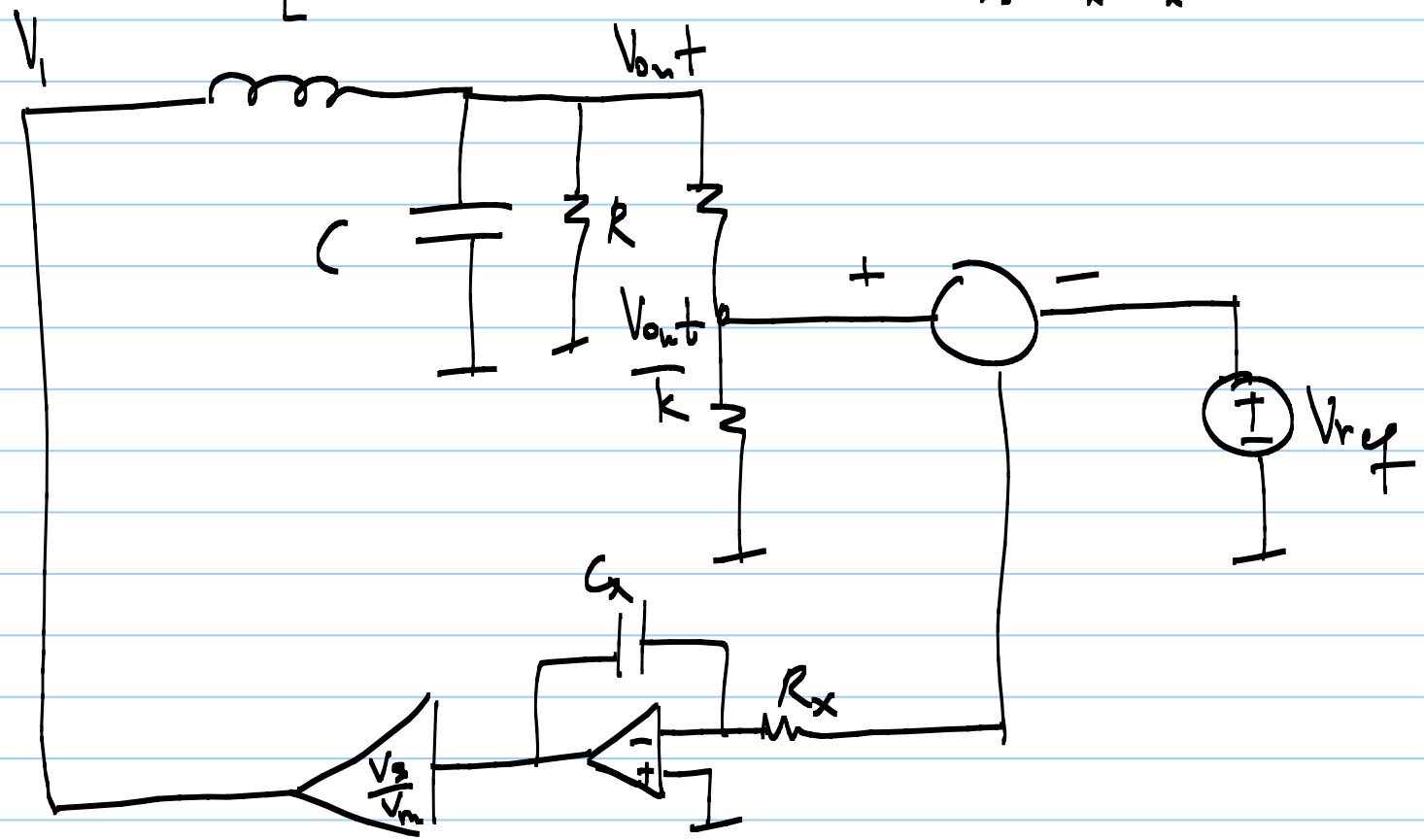


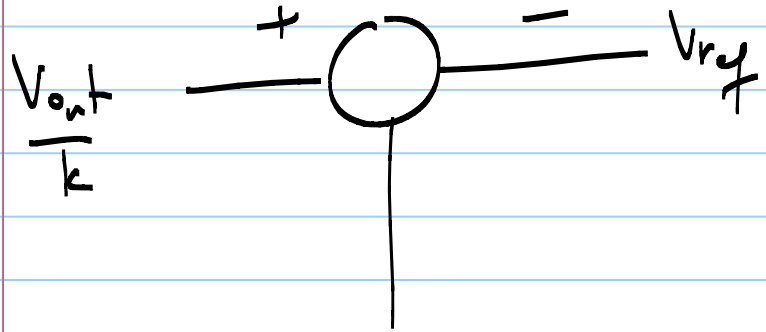
$\equiv$

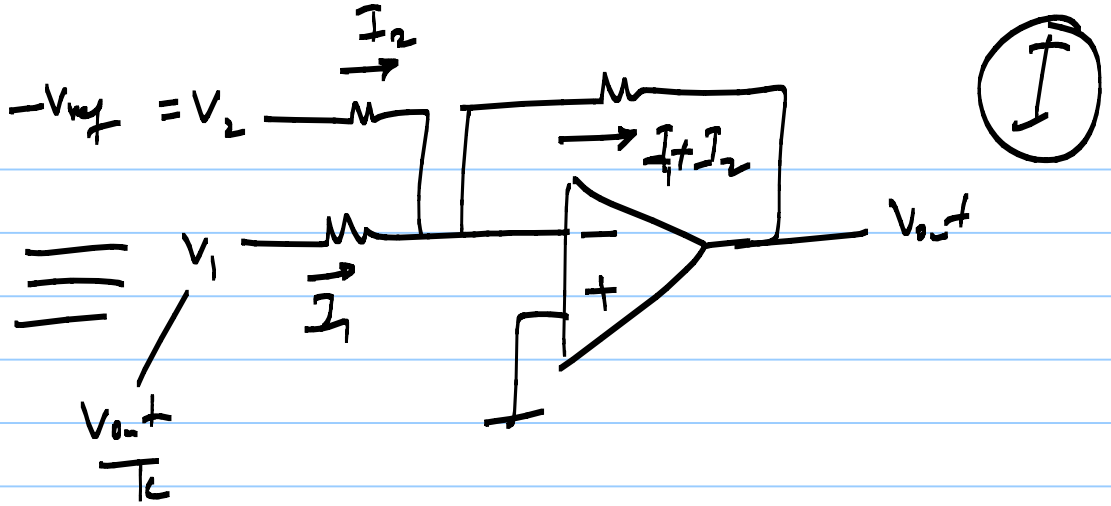


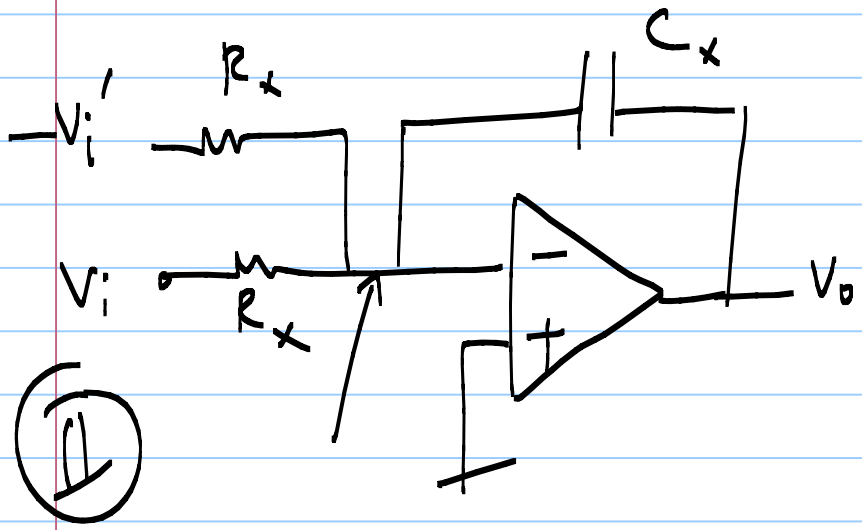
$$\frac{V_o}{V_i} = \frac{\omega_n}{s}$$

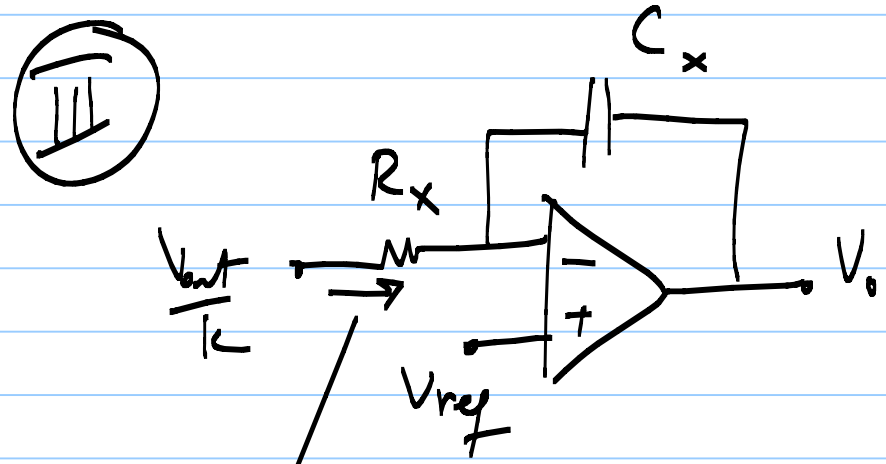
$$\frac{V_o}{V_i} = \frac{-1}{s R_x C_x}$$











$$\left(\frac{V_{out}}{k} - V_{ref}\right) \cdot \frac{1}{R_x}$$

$$V_o = \left(V_{ref} - \frac{V_{out}}{k}\right) \cdot \frac{1}{s C_x R_x} + V_{ref}$$

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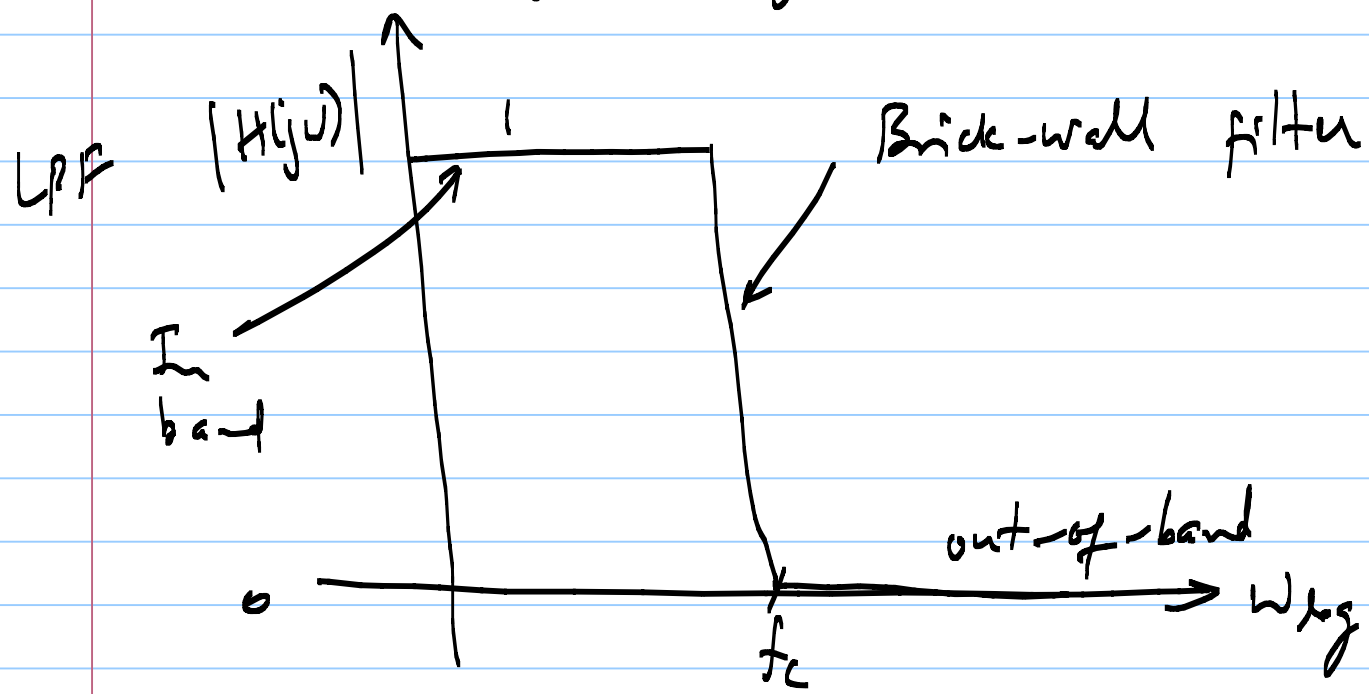
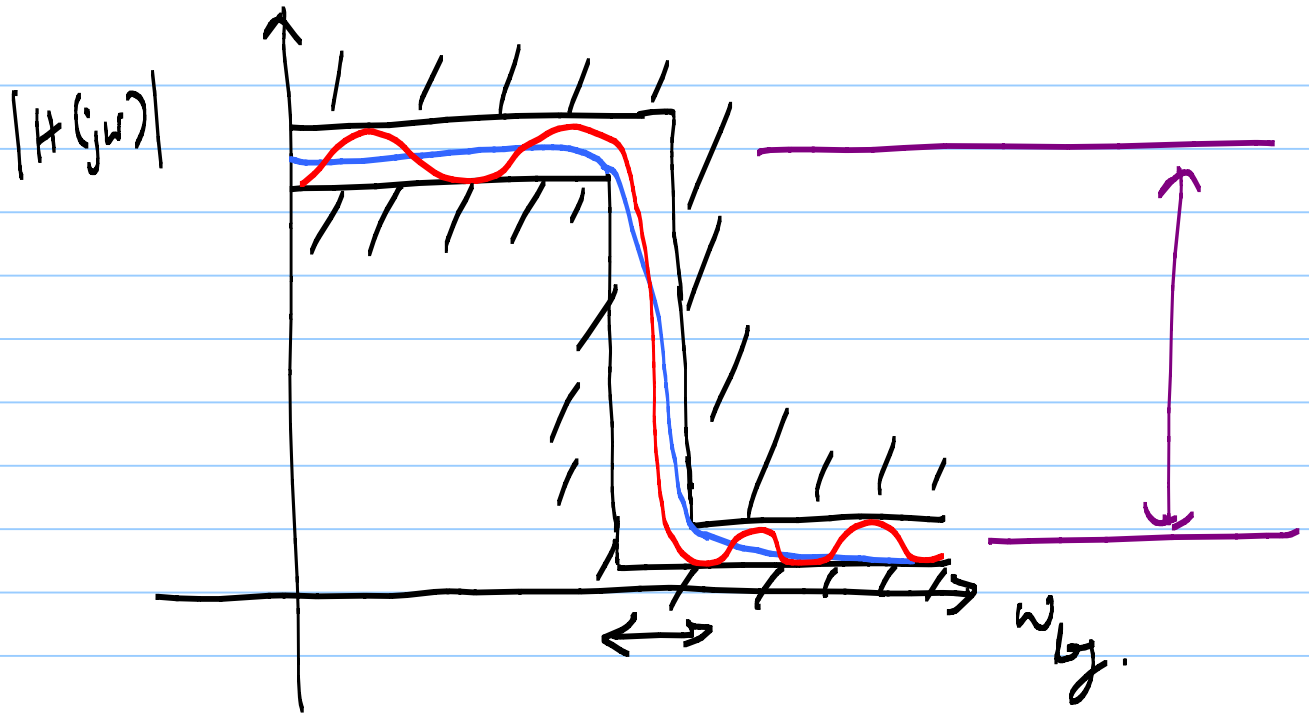
Lecture 30

Filters

allows a certain band of freq. to go through

stops another band

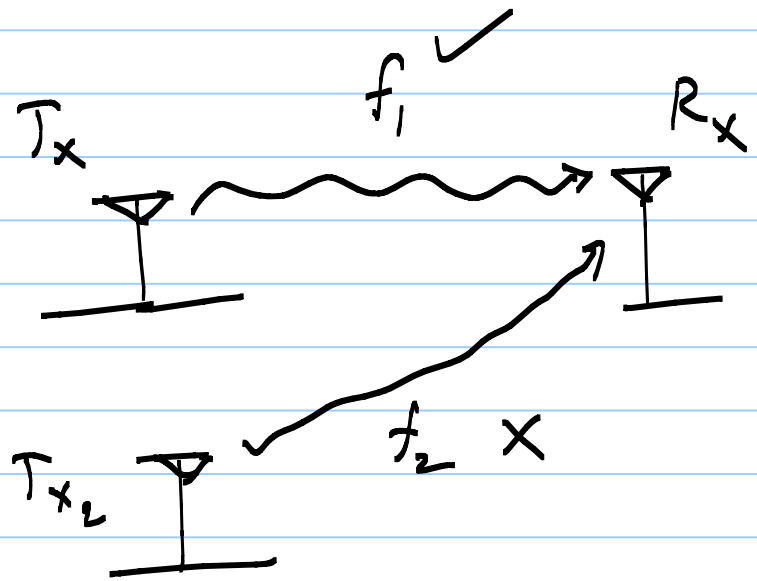




Applications

1) Radios → Remove noise

→ Remove interference

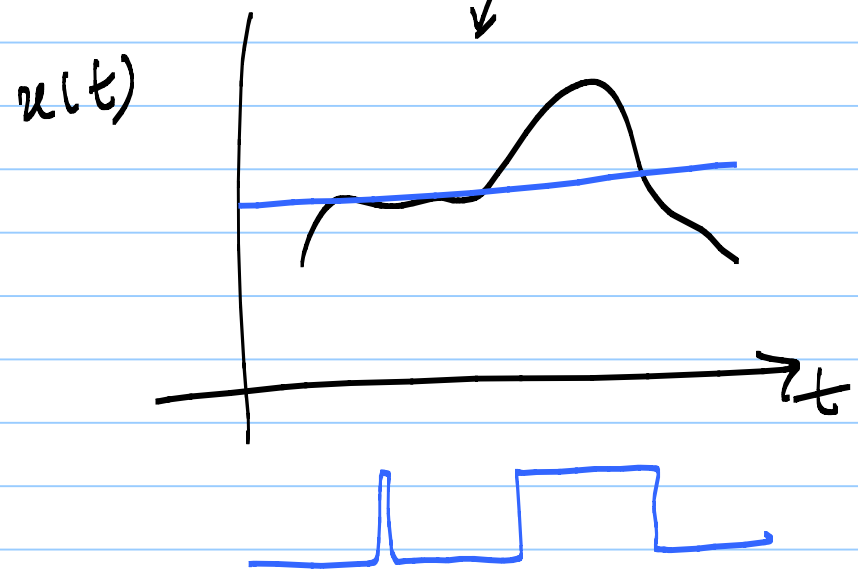


2) Audio systems

3) Data converters

DAC

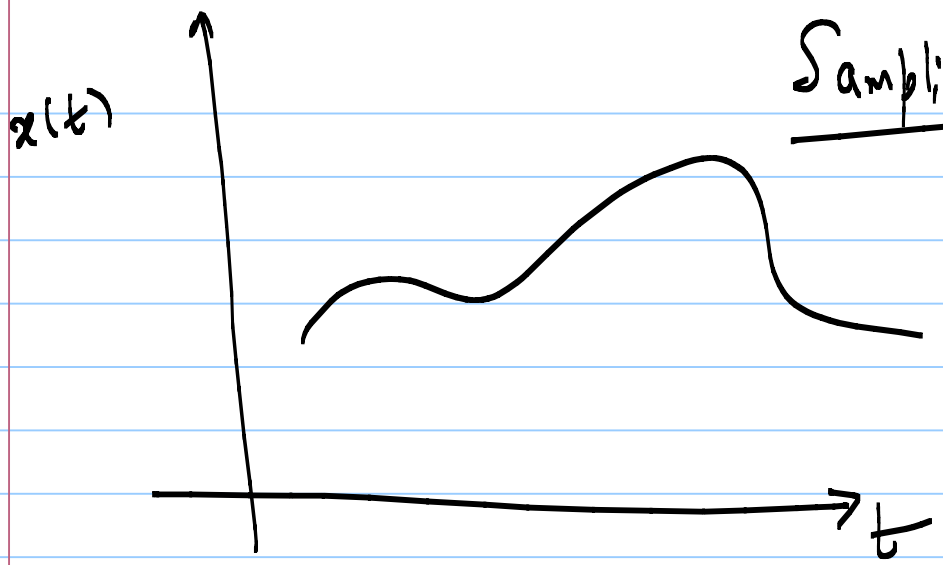
ADC

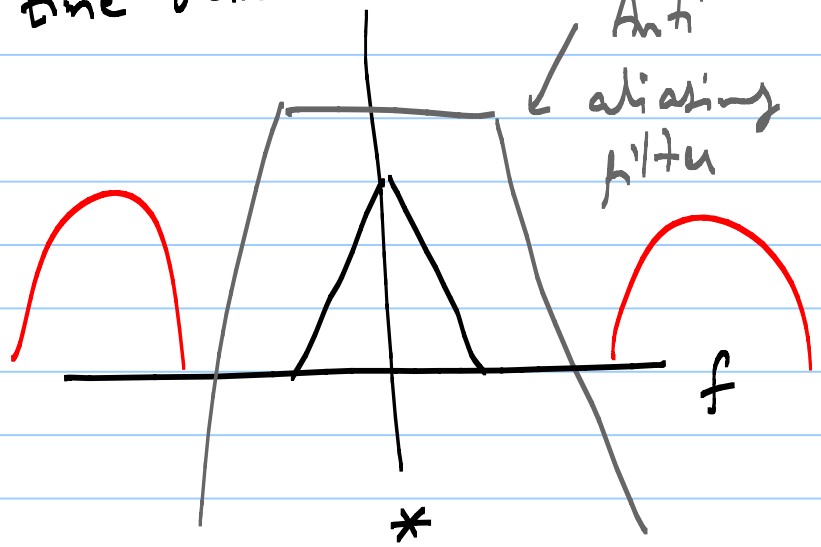


→ ...101110010 ...

Quantization → discretization in Amplitude domain

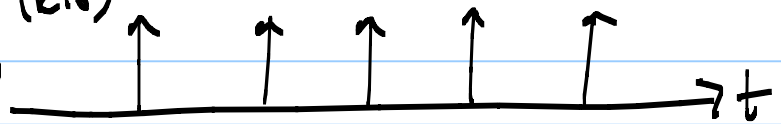
Sampling — discretisation in time domain



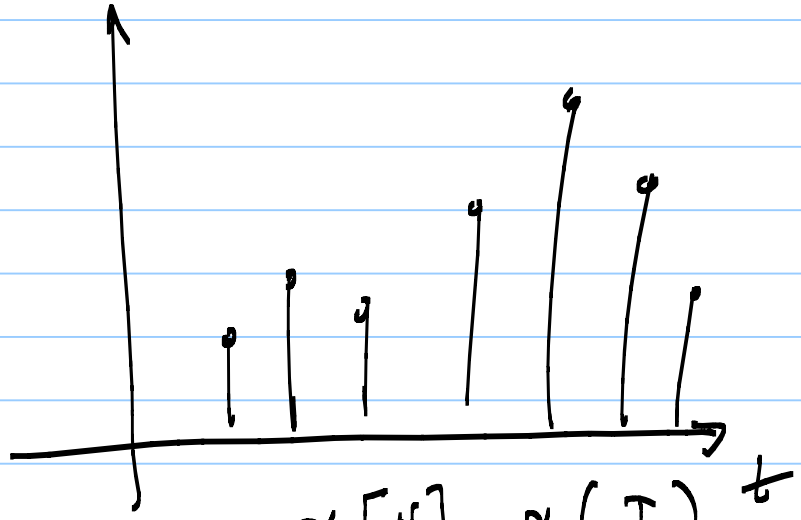


$\times$

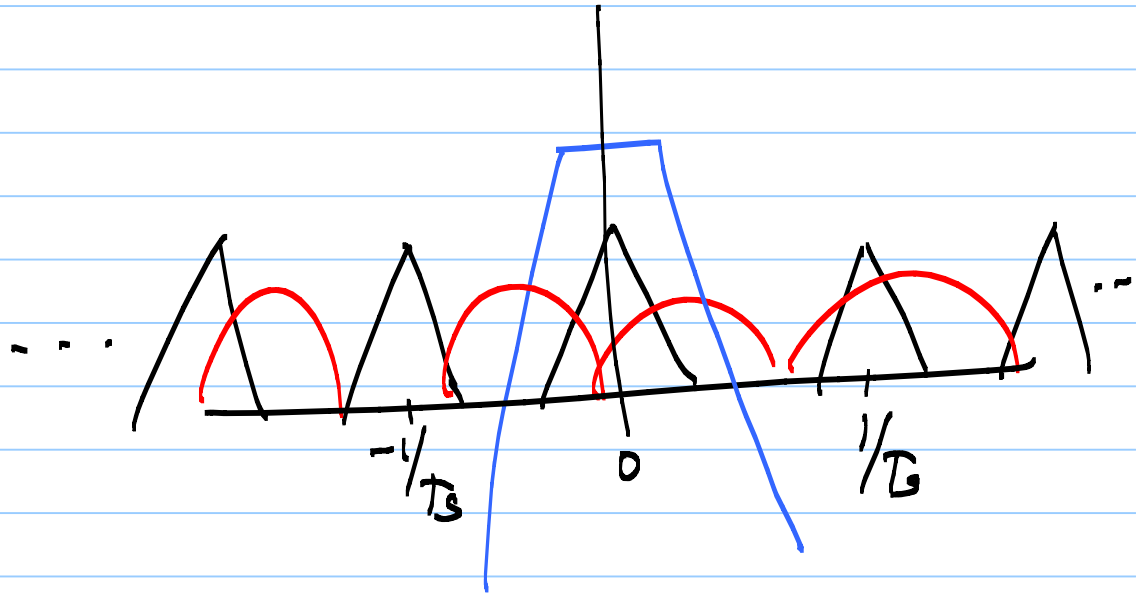
$x(t) \times \sum_{k=0}^{\infty} \delta(kT_s)$



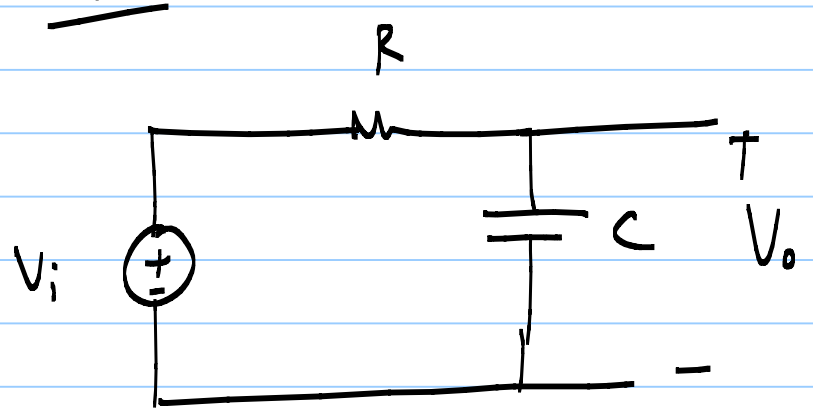
$*$



$x[n] = x(nT_s)$

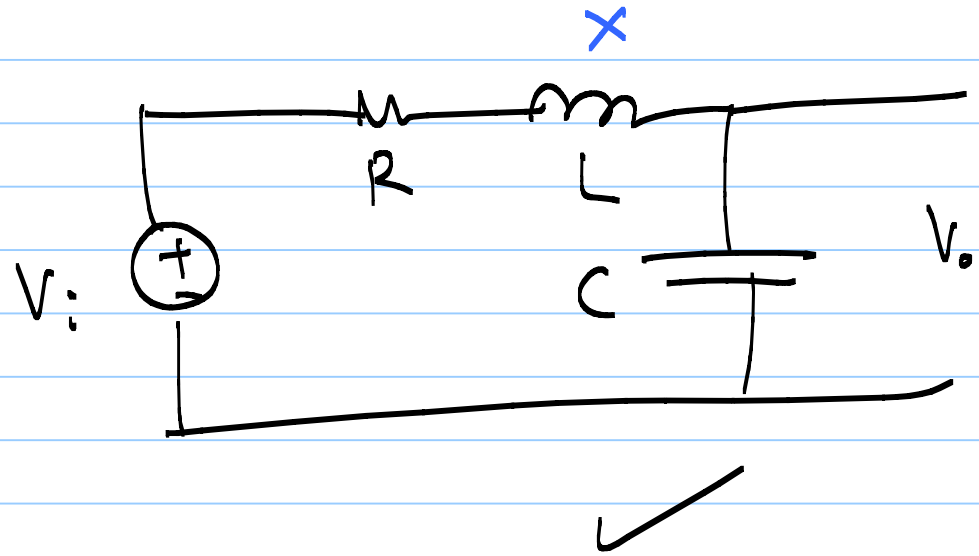


LPF



$$\frac{V_o}{V_i}(s) = \frac{1}{1 + sCR}$$

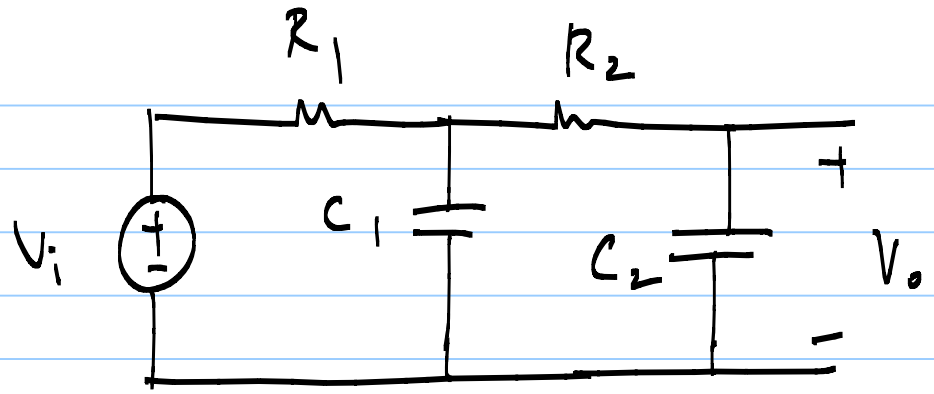
real pole



$$\frac{V_o}{V_i}(s) = \frac{1}{s^2LC + sRC + 1}$$

real poles

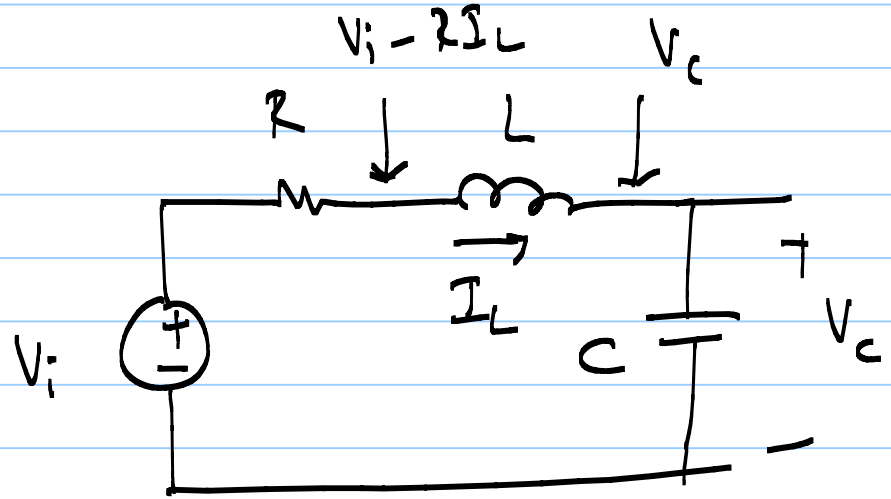
complex poles

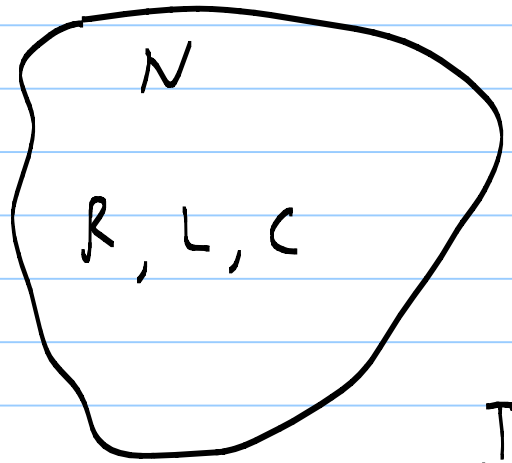


2nd order

$$\frac{1}{\left(D_2(s) \right)}$$

$Q < 1/2$





state variables → I_L's

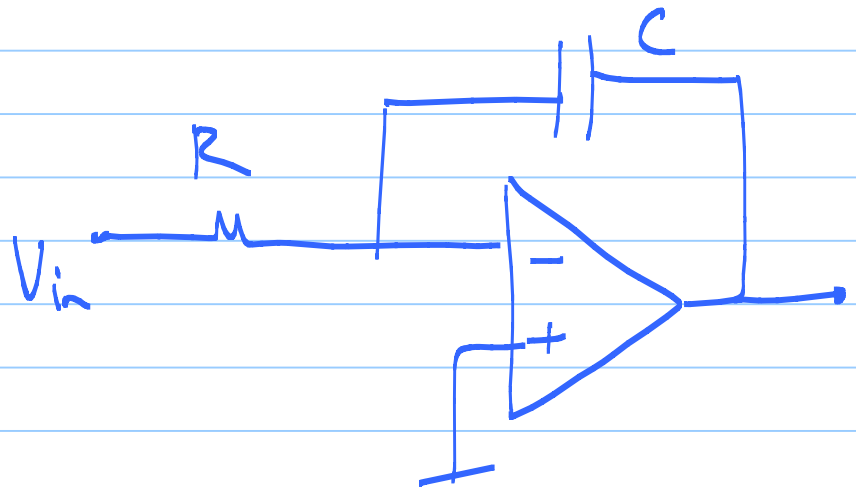
state variables → V_C's

$$V_c(t) = \frac{1}{C}\int I_L\,dt$$

$$V_c(s) = \frac{I_L}{sC} = \frac{R \cdot I_L}{sCR} \quad \text{——— (1)}$$

$$R\,I_L(s) = \frac{V_i - R I_L - V_c}{sL/R} \quad \text{——— (2)}$$

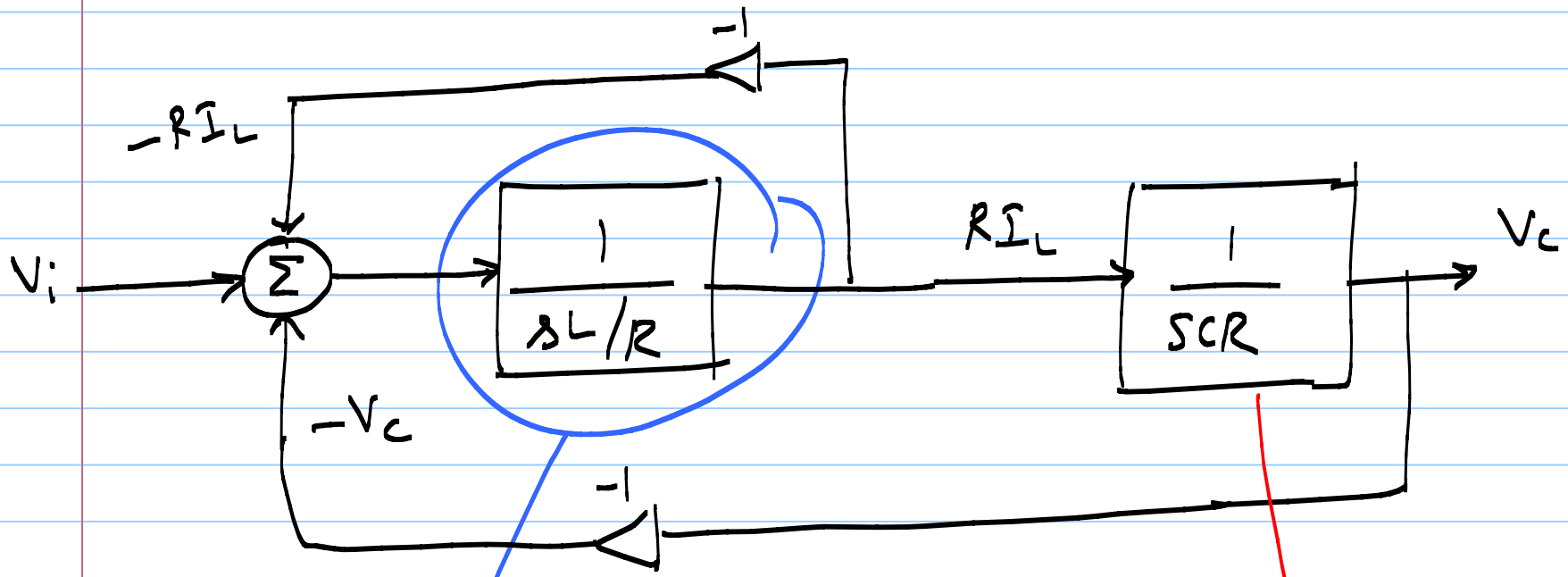
Integrators

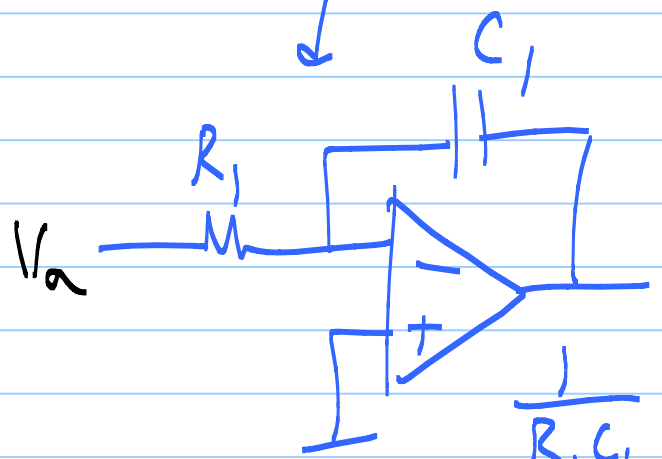


$$V_{out} = \frac{-1}{sCR} \cdot V_{in}$$

22/03/2022

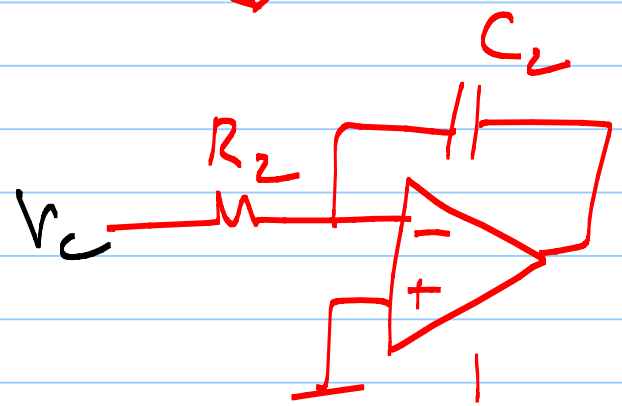
Lecture 31





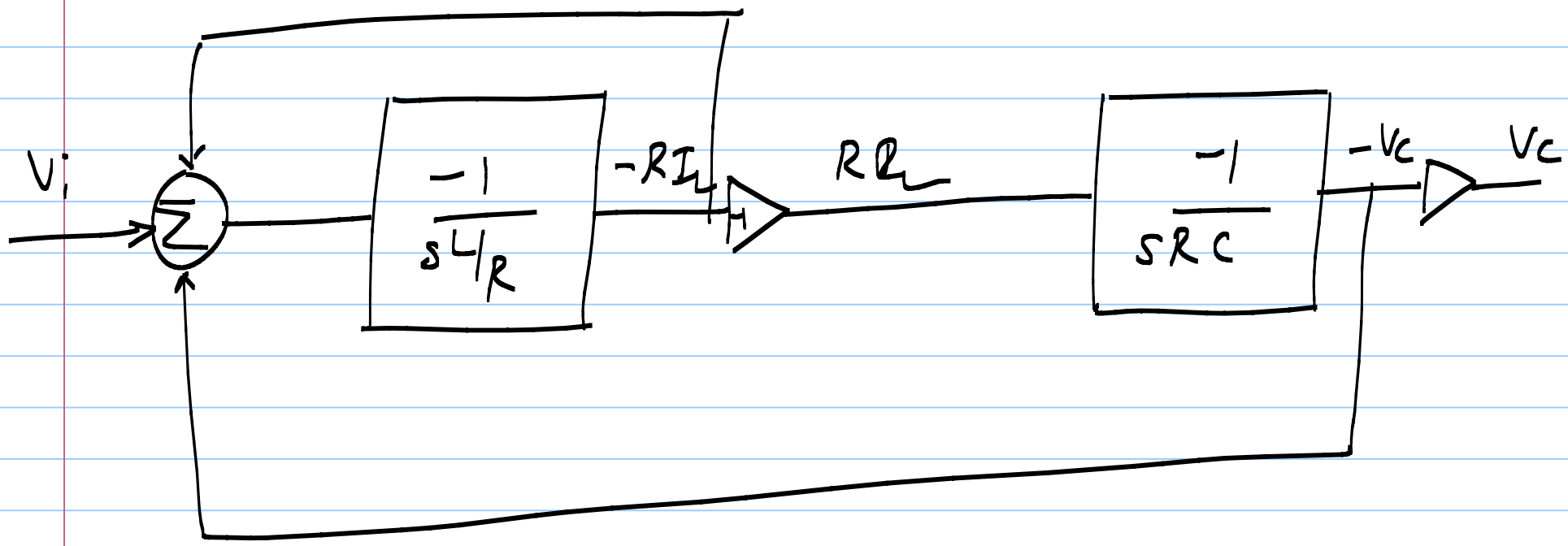
$$V_b = \frac{-V_a}{sC_1R_1}$$

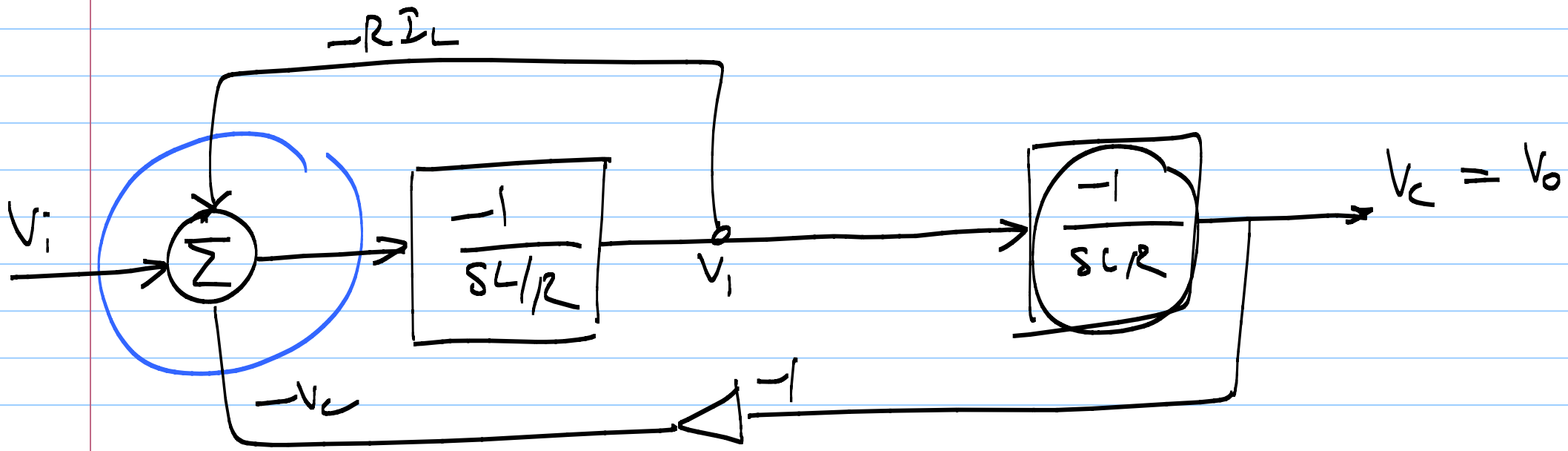
$$\frac{1}{R_1C_1} = \frac{R}{L}$$



$$V_d = \frac{-V_c}{sC_2R_2}$$

$$\frac{1}{R_2C_2} = \frac{1}{CR}$$

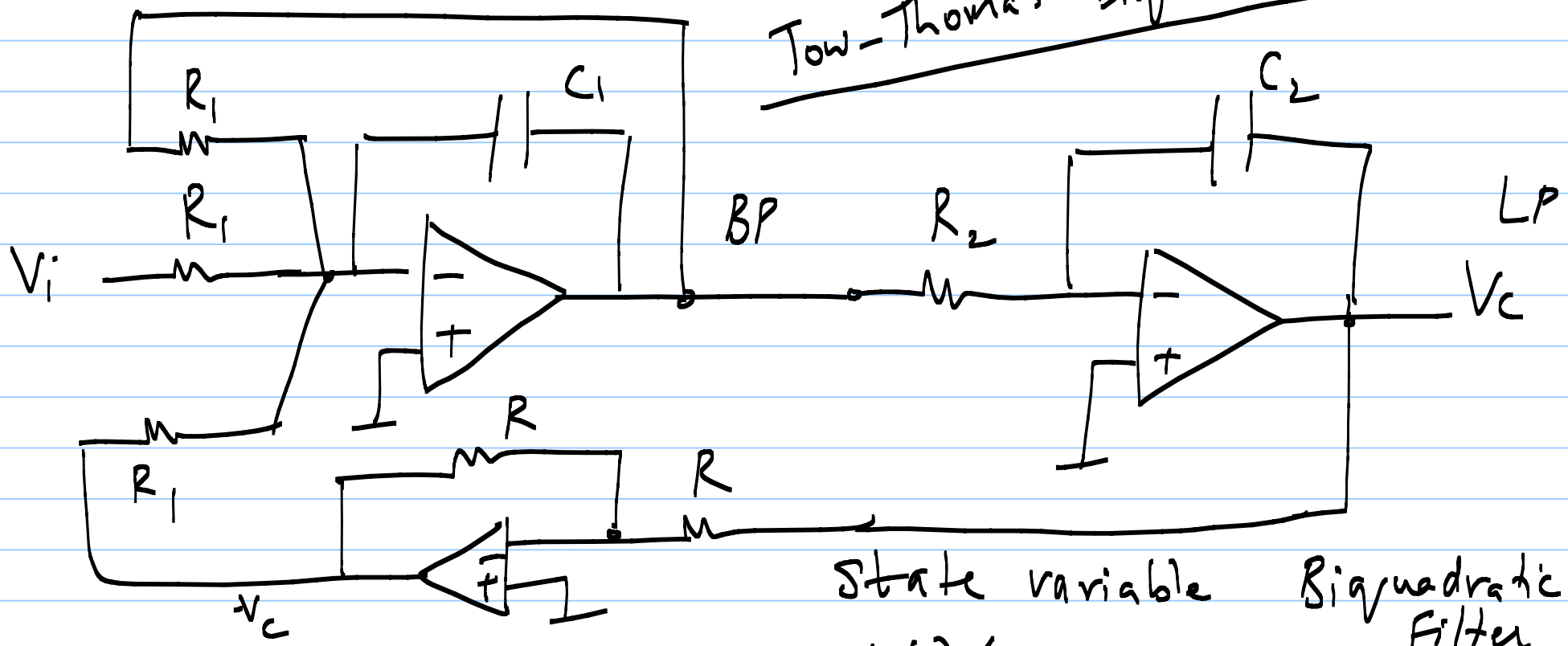

V_i
Σ
$\frac{-1}{sL/R}$
$-RI_L$
1
RI_L
$\frac{-1}{sRC}$
$-V_c$
V_c


$-RI_L$
V_i
Σ
$\frac{-1}{sL/R}$
V_1
$\frac{-1}{sL/R}$
$V_c = V_o$
$-V_c$
-1

$$\frac{V_c}{V_i} = \frac{V_o}{V_1}(s) = \frac{1}{s^2LC + sCR + 1} \quad (LPF)$$

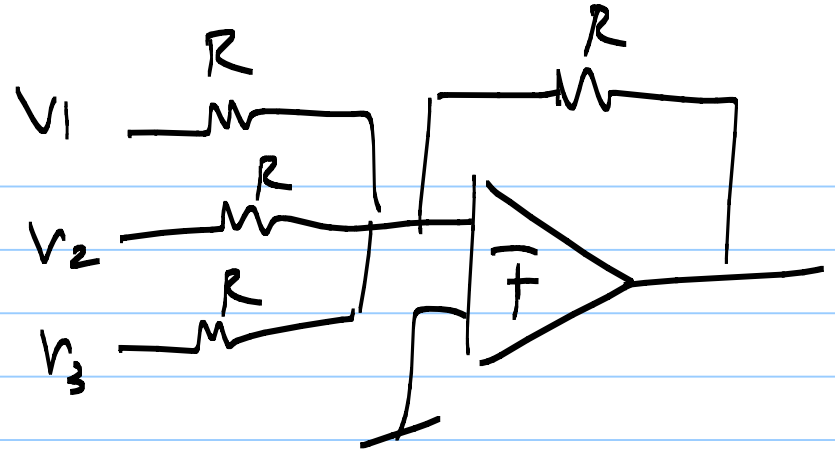
$$\frac{V_1}{V_i}(s) = \frac{-sCR}{s^2LC + sCR + 1} \quad (BPF)$$

Tow-Thomas Biquad filter



State variable Biquadratic Filter

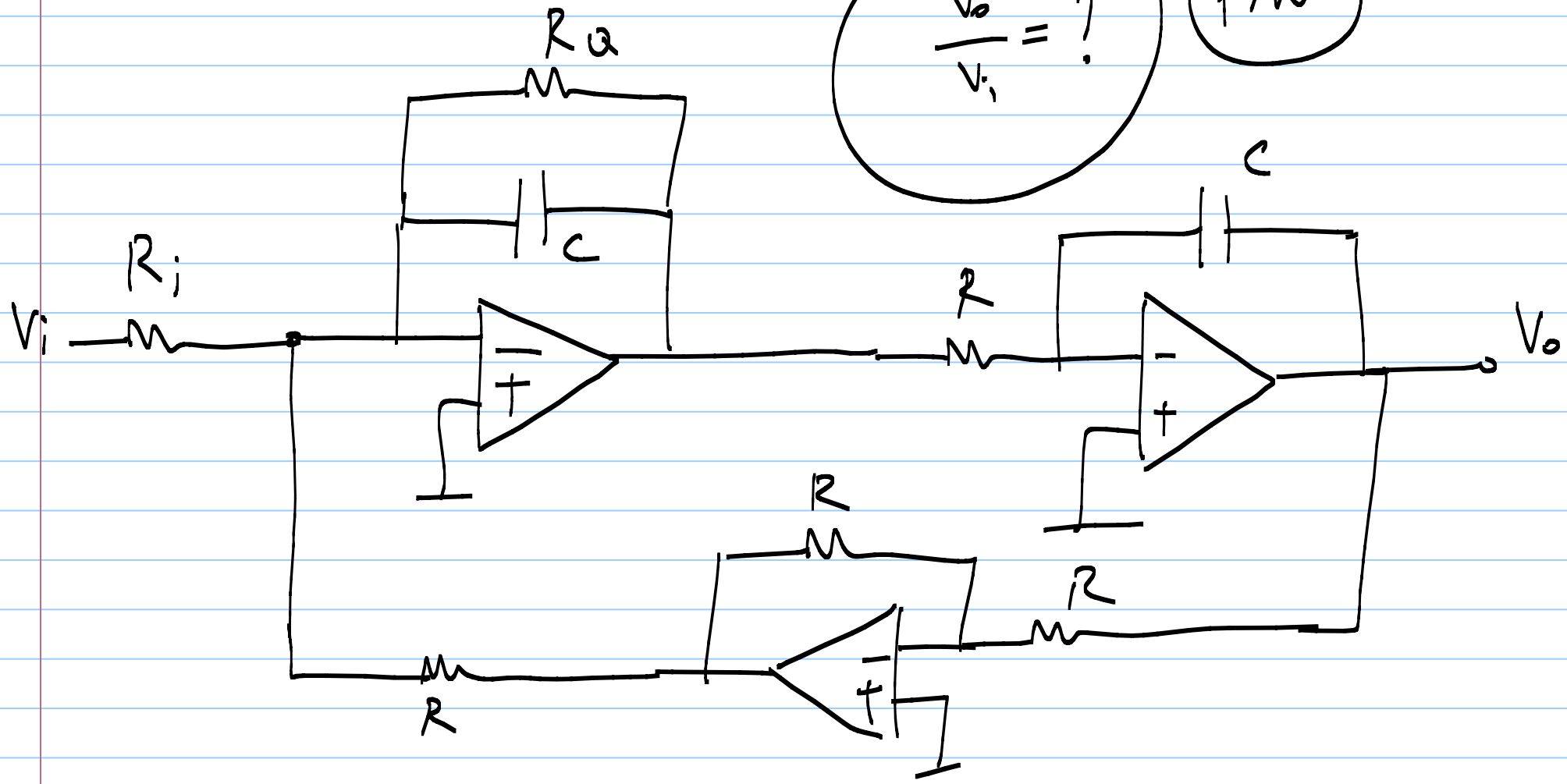
$N_2(s)/D_2(s)$



$$-(V_1 + V_2 + V_3)$$

$$\frac{V_o}{V_i} = ?$$

HW



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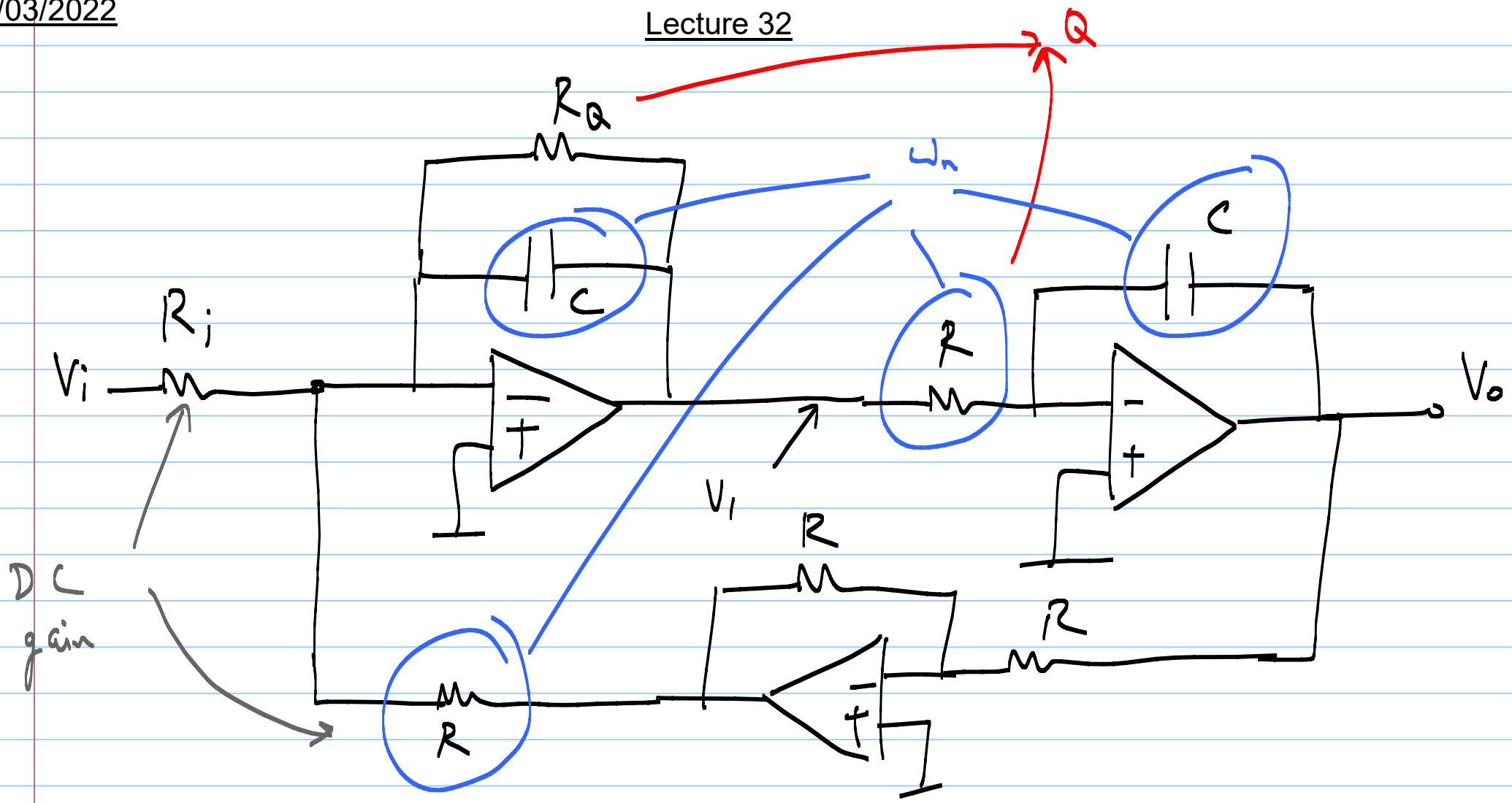
Lecture 32



$$\frac{V_o}{V_i} = \frac{1}{R_i} \cdot \frac{1}{s^2 C^2 R + \frac{sCR}{R_Q} + \frac{1}{R}}$$

$$\frac{V_o}{V_i} = \frac{R}{R_i} \cdot \frac{1}{1 + sCR \cdot \frac{R}{R_Q} + s^2 C^2 R^2}$$

$D(s)$

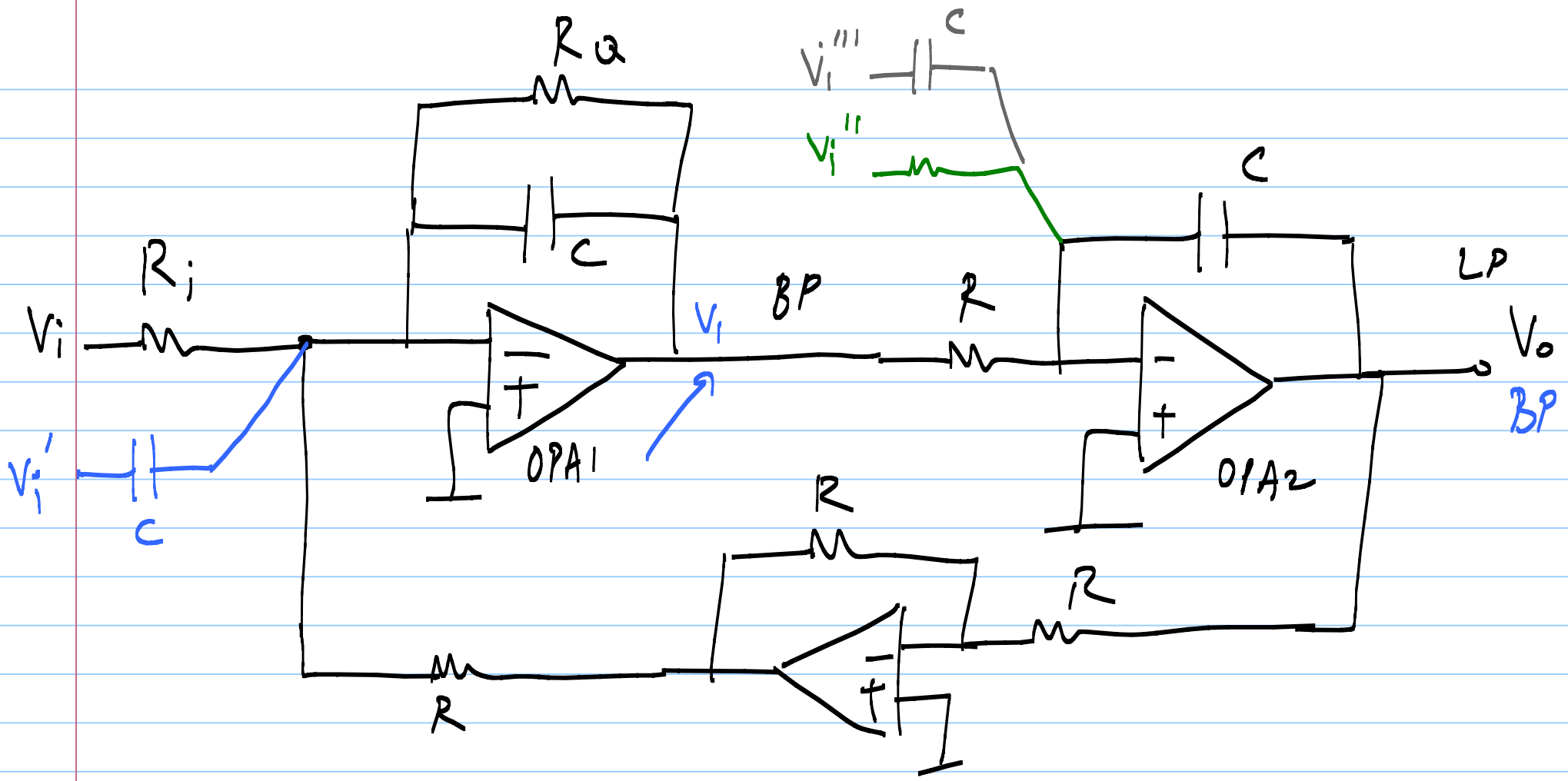
$$D(s) = 1 + \frac{s}{Q\,\omega_n} + \frac{s^2}{\omega_n^2}$$

$$\omega_n = \frac{1}{RC}$$

$$Q = \frac{R_Q}{R}$$

$$A_0 = \frac{R}{R_i}$$

$$\frac{V_1}{V_i} = \frac{-sCR}{1 + sCR \cdot \frac{R}{R_Q} + s^2 C^2 R^2} \cdot \frac{R}{R_i}$$



$$\frac{V_o}{V_i'} = + \frac{sCR}{1 + sCR \cdot \frac{R}{R_Q} + s^2C^2R^2} \quad (BP)$$

(gnd V_i)

$$\frac{V_1}{V_i'} = -\frac{s^2 C^2 R^2}{D(s)} \quad (HP)$$

<u>2nd order filters</u>

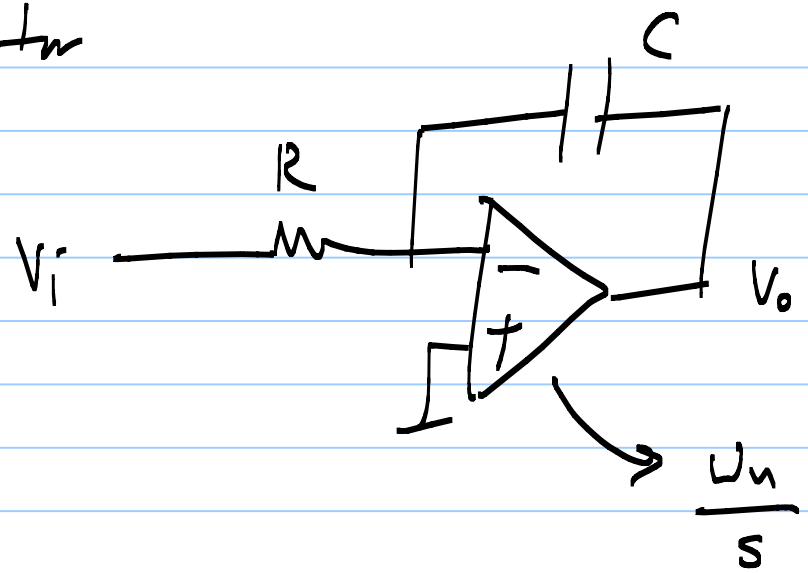
$$LP = \frac{1}{D(s)}$$

$$HP = \frac{s^2 C^2 R^2}{D(s)}$$

$$BP = \frac{sCR}{D(s)}$$

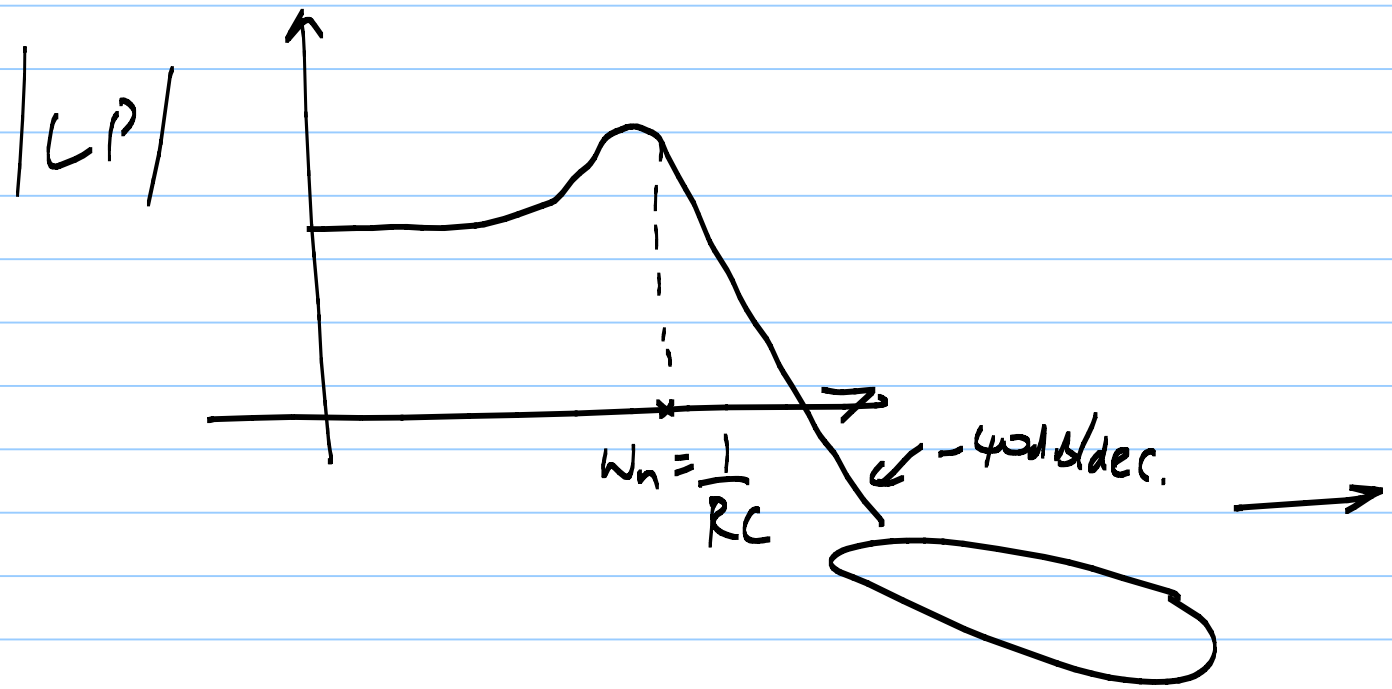
$$\text{Band stop} = \frac{1 + s^2 C^2 R^2}{D(s)}$$

Integrator

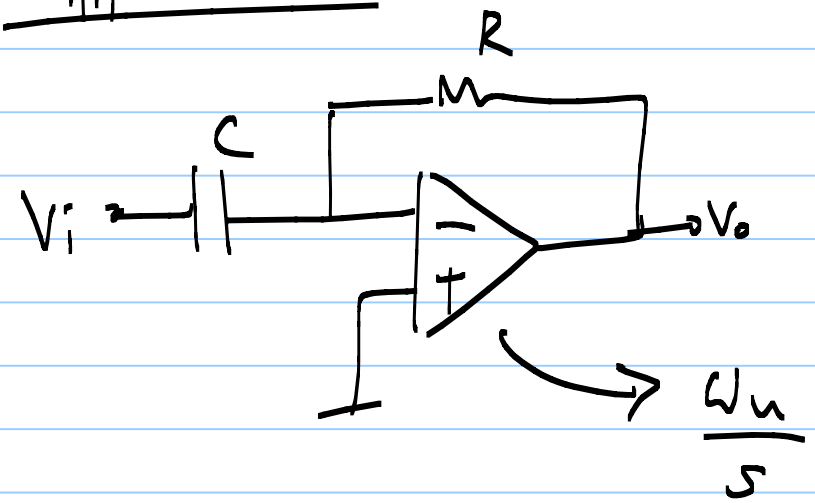


$$V_o = \frac{-V_i}{sCR}$$

$$\omega_n \gg \frac{1}{RC}$$



Differentiator



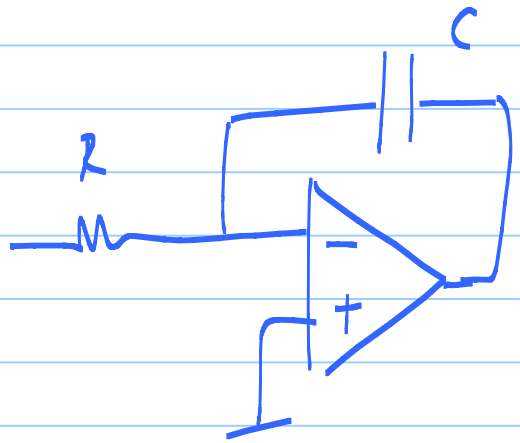
$$\frac{V_o}{V_i} = -sCR.$$

1) $\omega_u \longleftrightarrow \frac{1}{RC}$?

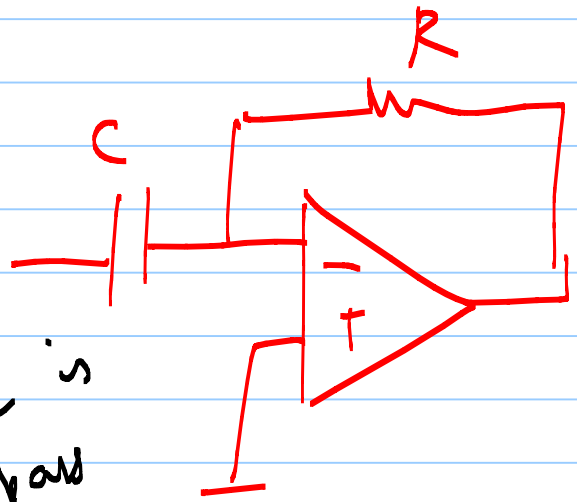
2) high freq. response?

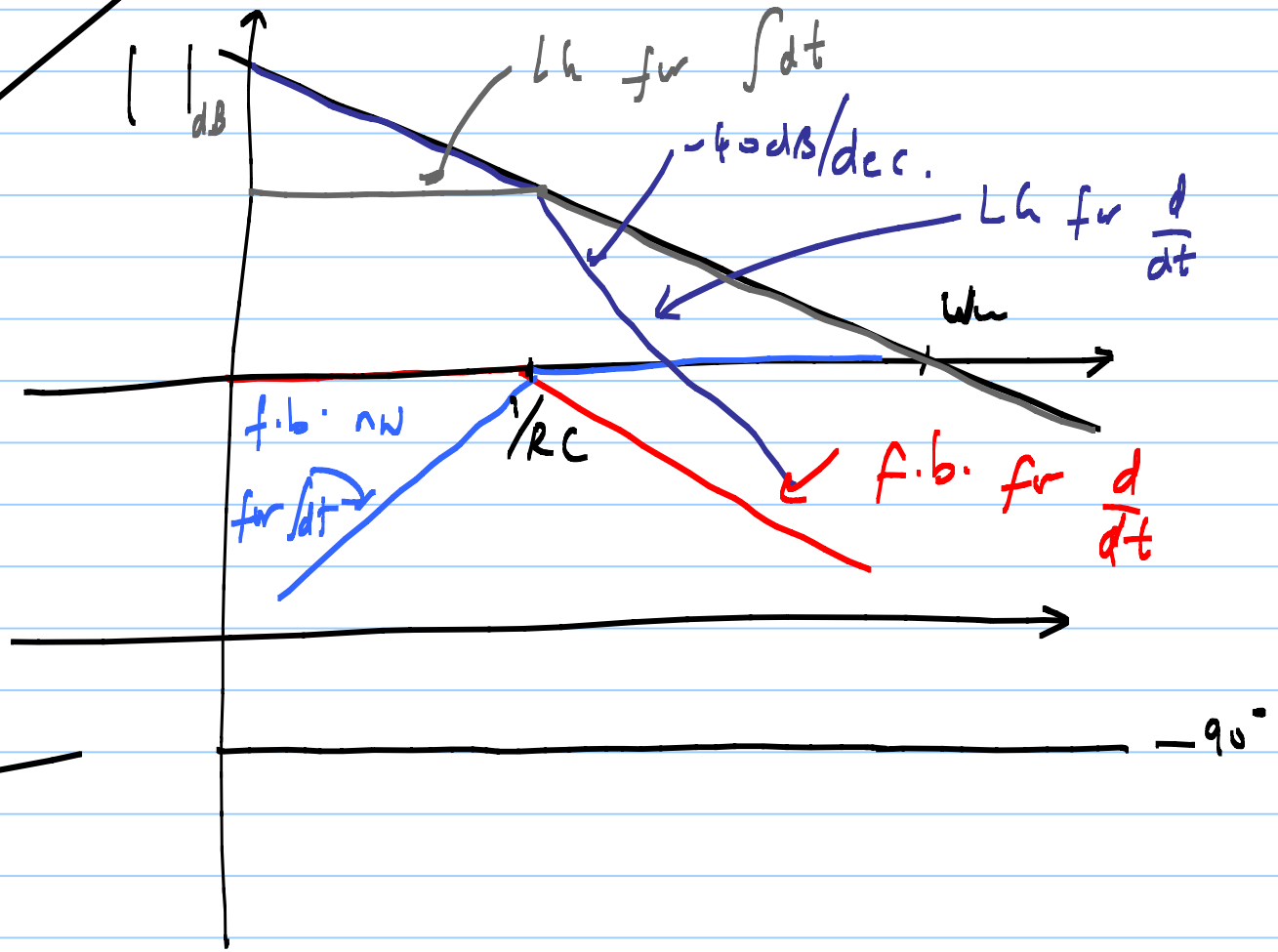
30/03/2022

Lecture 33



$$\omega_u \gg \underbrace{\frac{1}{RC}}_{\omega_n}$$

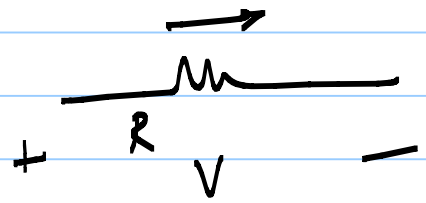


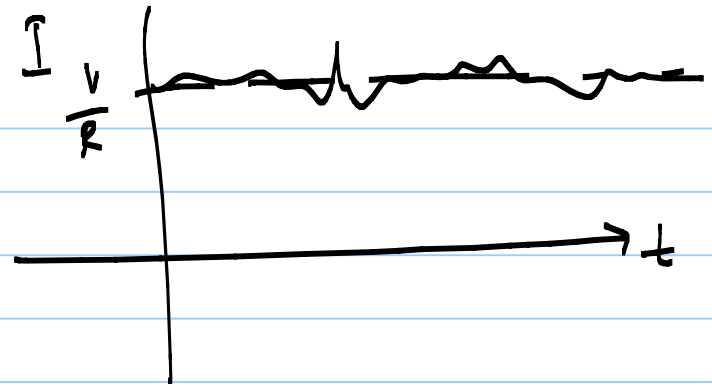


noise is high-pass

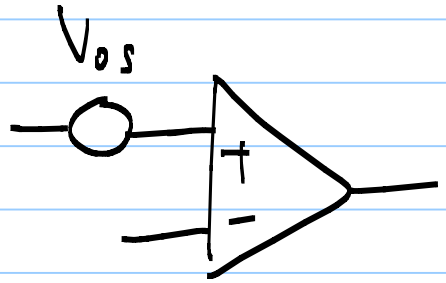
HW – plot
plots for ∫dt & d/dt & overall LG

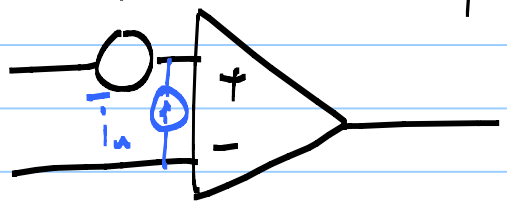
$I = V/R$



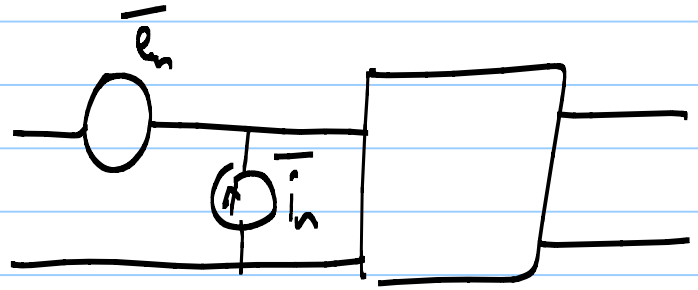


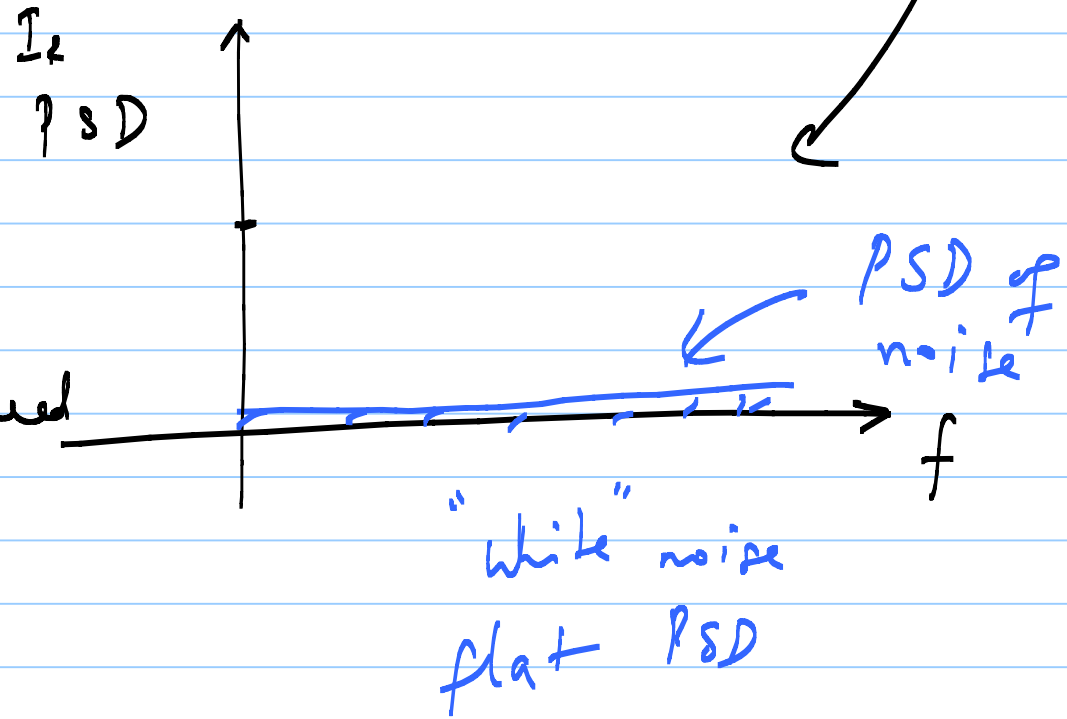
Power Spectral Density

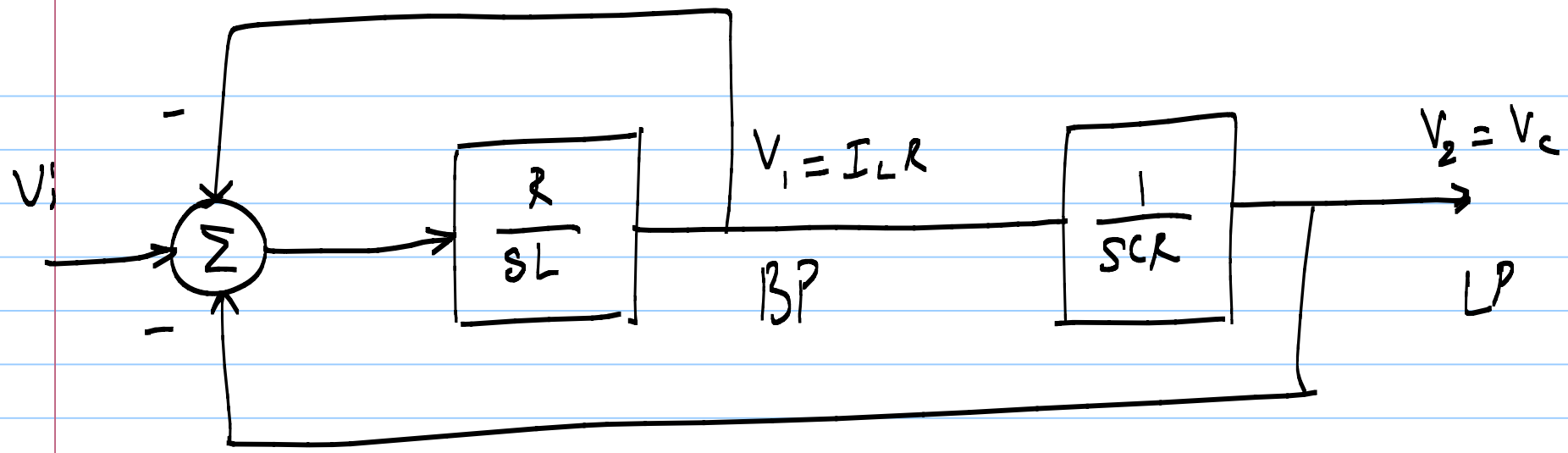




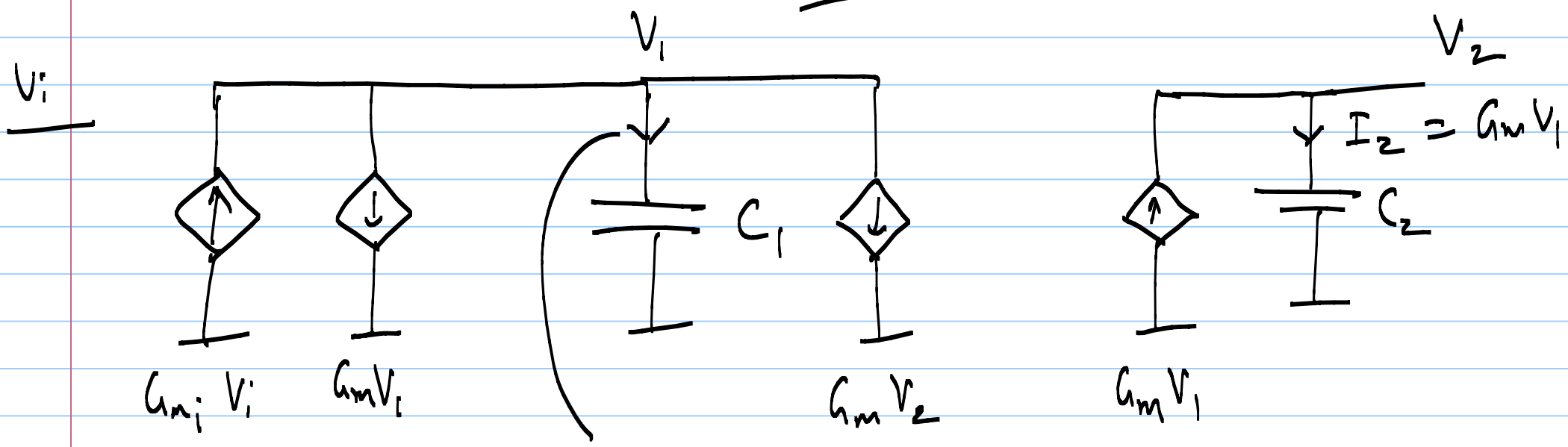
RMS input-referred noise







G_m-C filter



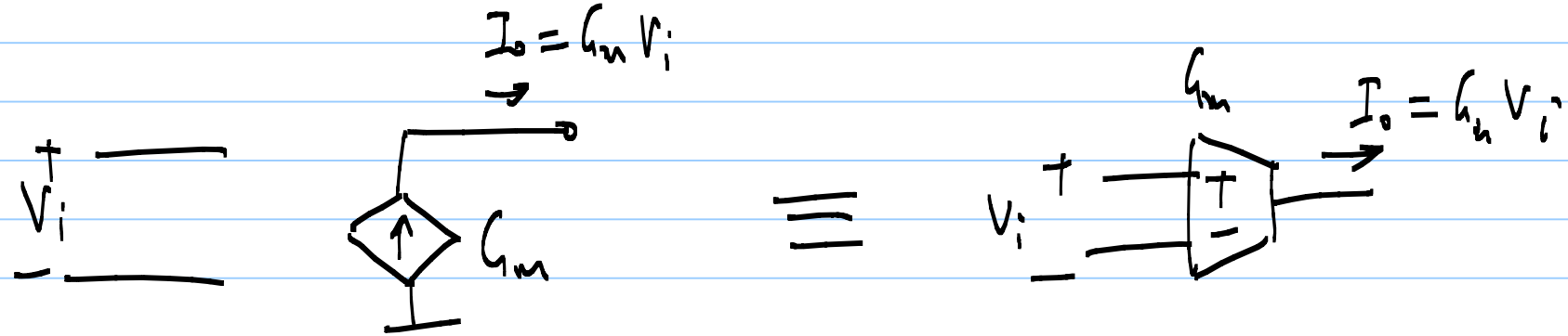
$$I_1 = G_{mi} V_i - G_m V_1 - G_m V_2$$

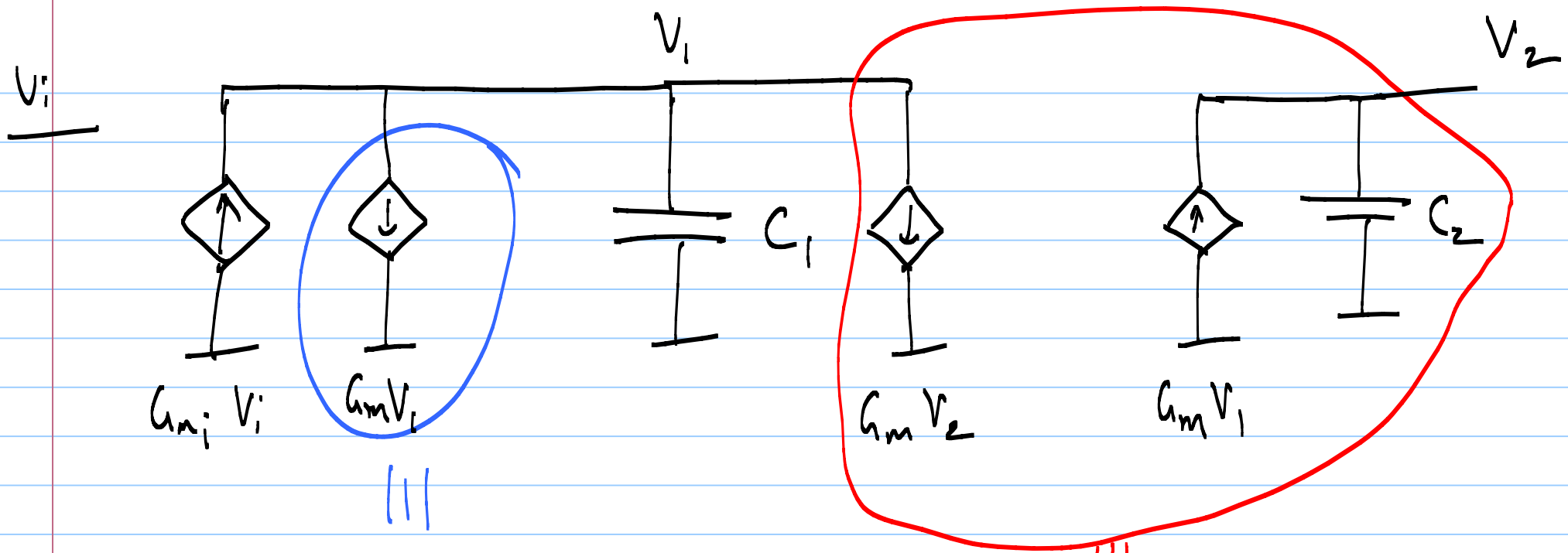
$$\frac{V_1}{V_i} = \left(\frac{V_2}{V_i}\right) \cdot \left(\frac{sC_2}{G_m}\right) \qquad (BP)$$

$$\frac{V_2}{V_i} = \frac{G_m \cdot G_{mi}}{s^2 C_1 C_2 + G_m^2 + sC_2 G_m} = \frac{G_{mi}/G_m}{1 + \frac{sC_2}{G_m} + s^2 \cdot \frac{C_1 C_2}{G_m^2}}$$

(LP)

$$\omega_n = \frac{G_m}{\sqrt{C_1 C_2}} \quad ; \quad Q = \sqrt{\frac{C_1}{C_2}}$$





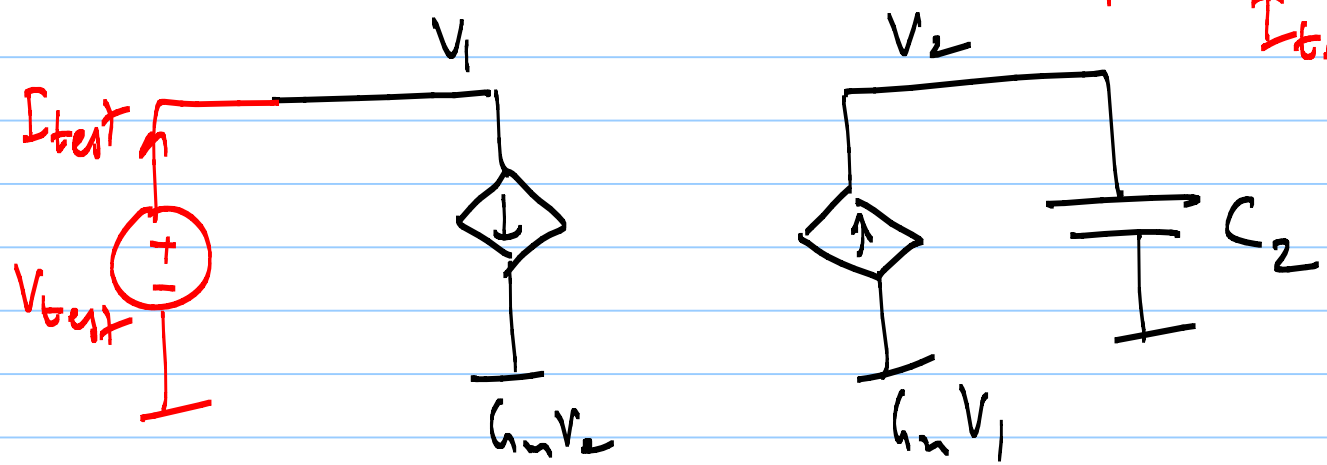
$R = \frac{1}{G_m}$

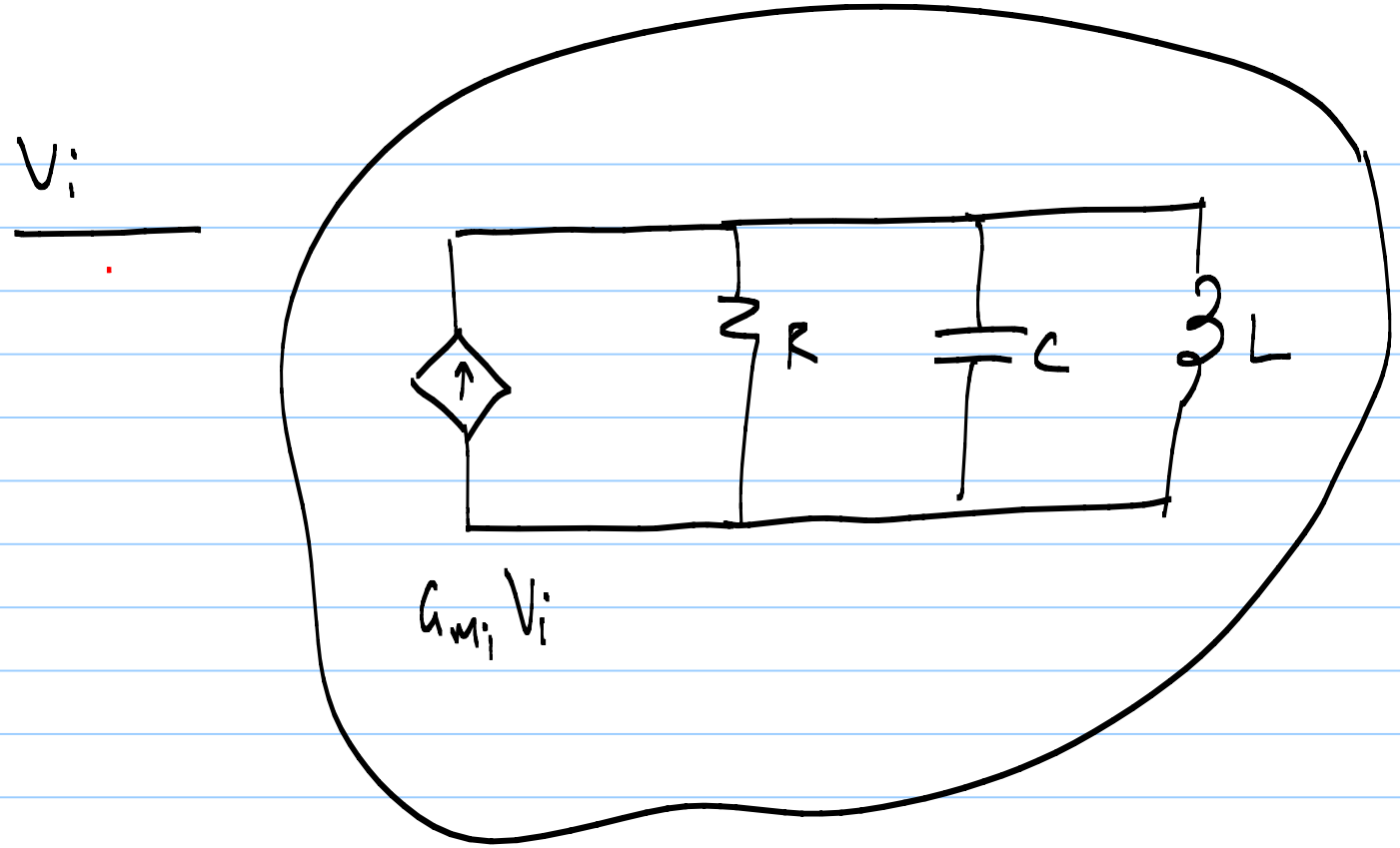
$\frac{C}{G_m^2}$

$I_{test} = G_m V_2$

$= G_m \cdot \frac{G_m V_1}{sC}$

$\frac{V_{test}}{I_{test}} = \frac{sC}{G_m^2}$

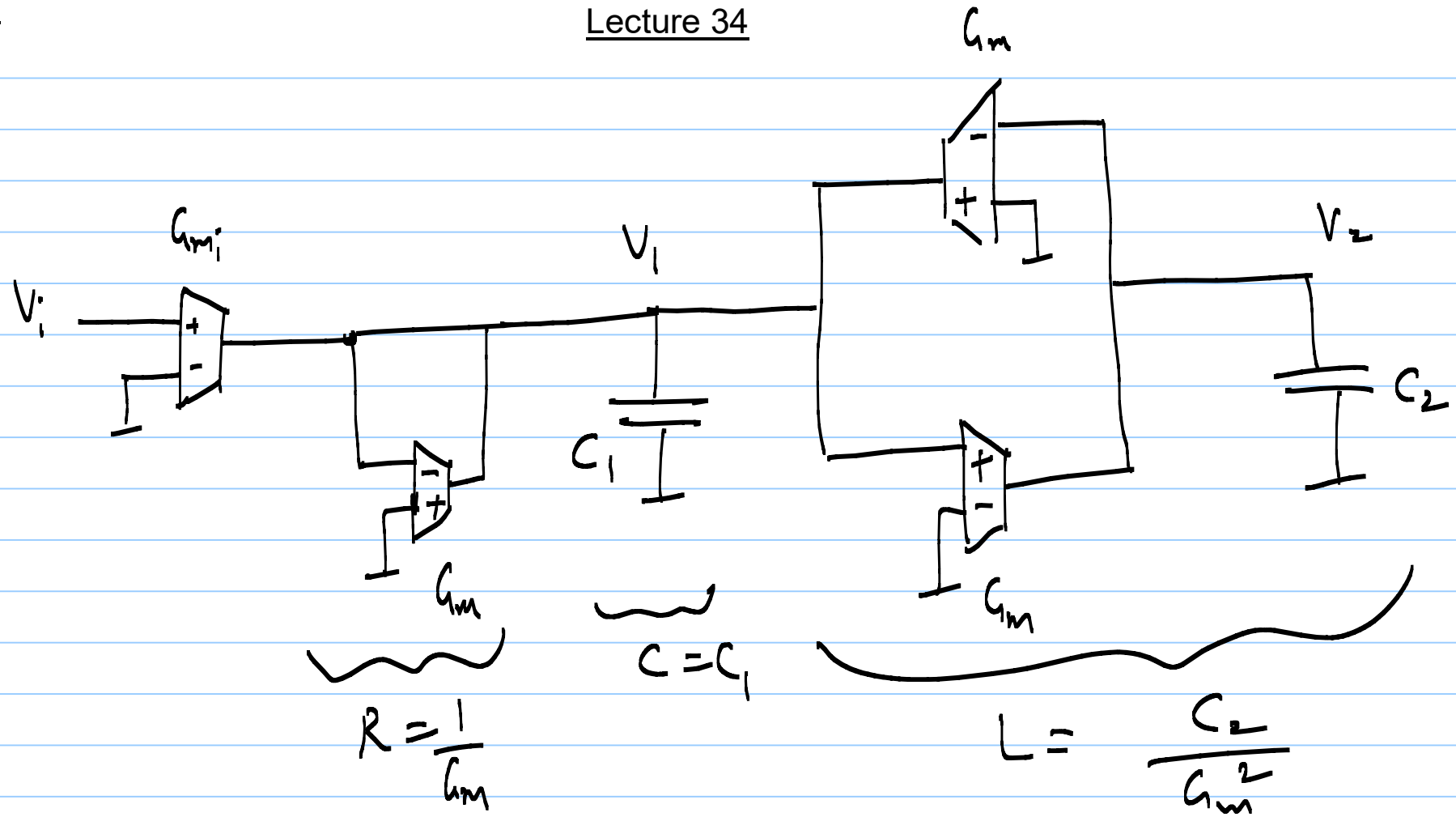


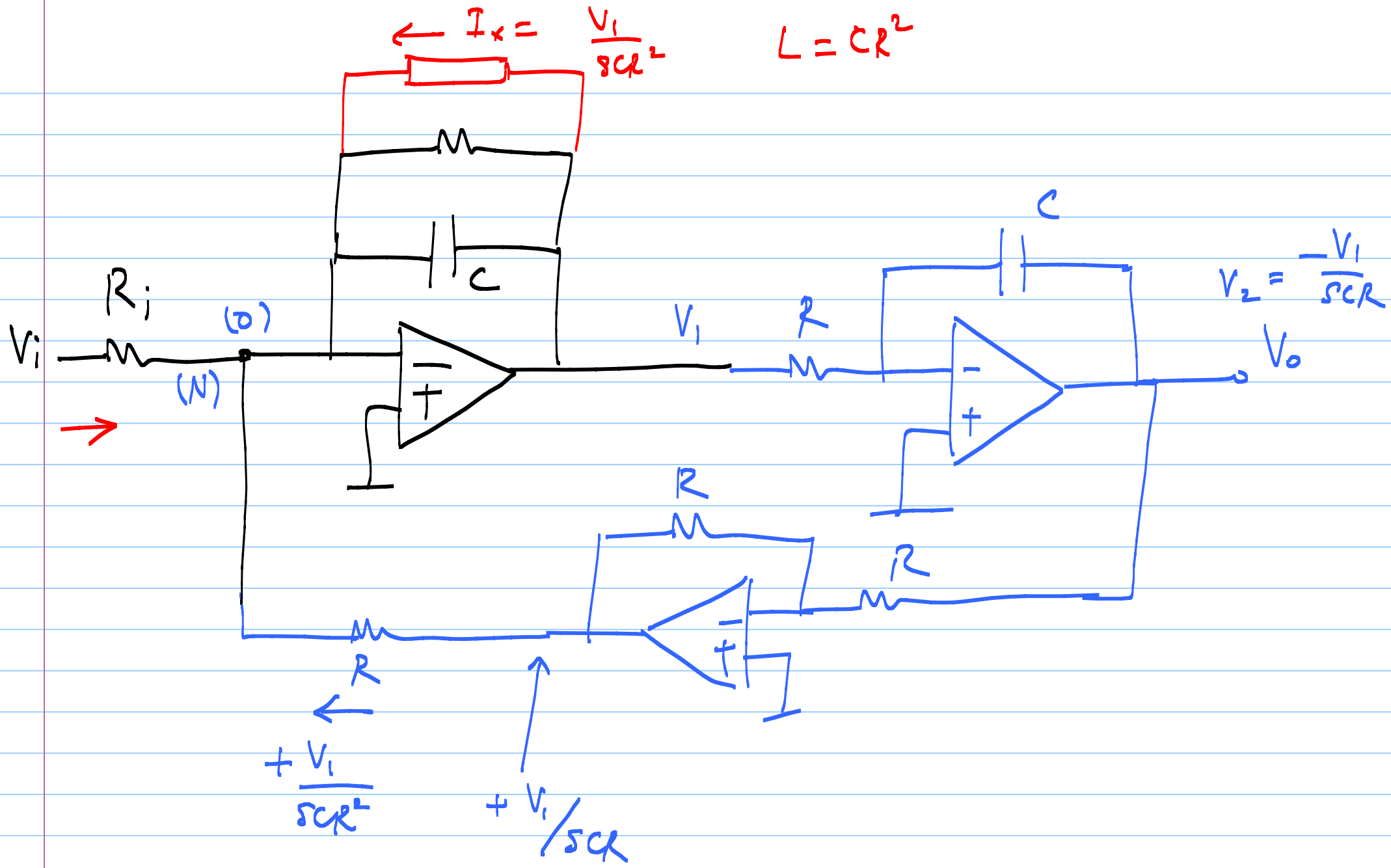

V_i
R
C
L
$G_{m_1} V_i$

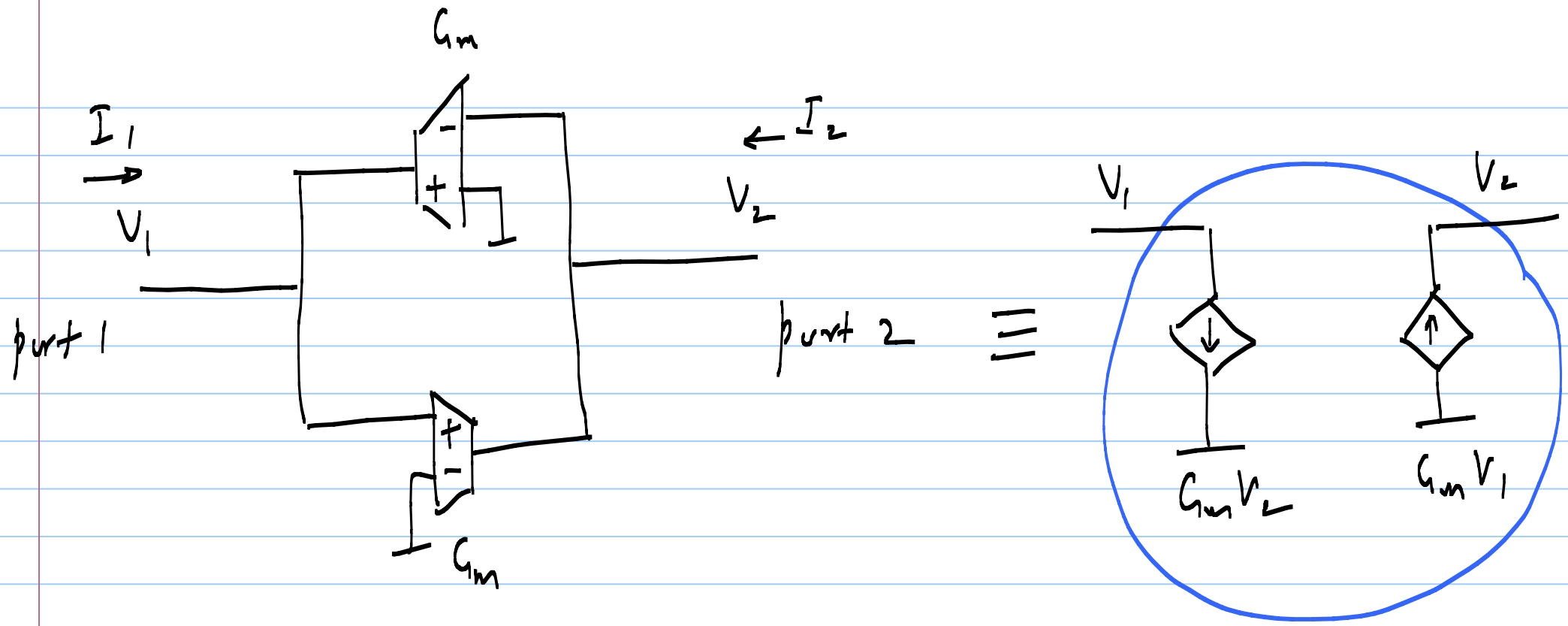
+
V_i
−

31/03/2022

Lecture 34

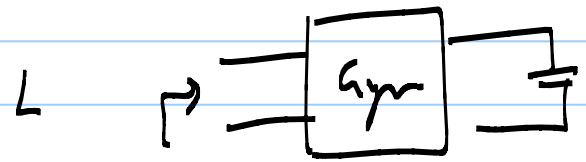


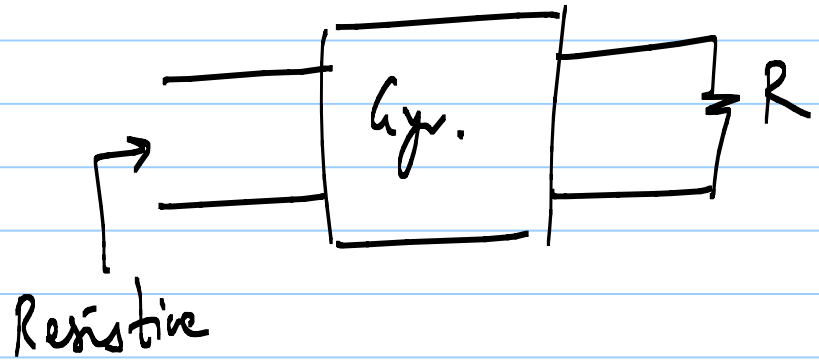

$I_x = \frac{V_1}{sCR^2}$
$L = CR^2$
C
R_i
V_i
(0)
(N)
V_1
R
C
$V_2 = \frac{-V_1}{sCR}$
V_o
R
R
R
$+\frac{V_1}{sCR^2}$
$+V_1/sCR$

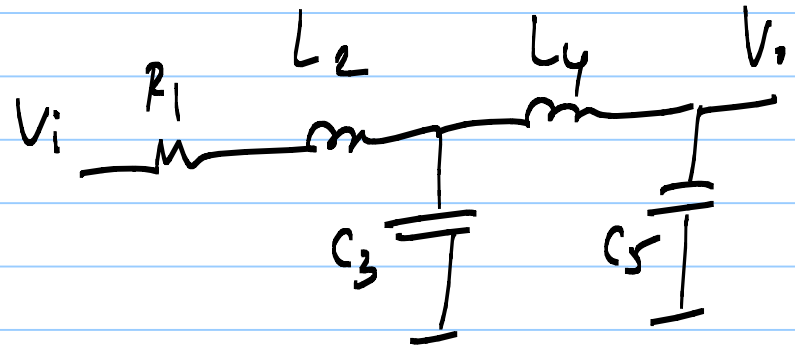


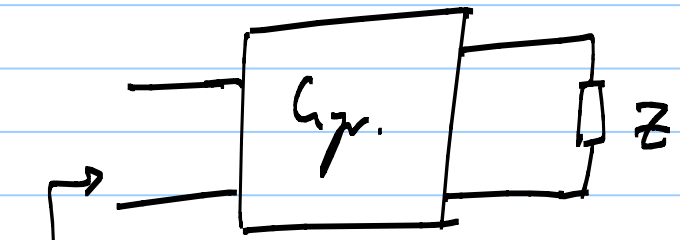
Gyrator

$$[Y] = \begin{bmatrix} 0 & G_m \\ -G_m & 0 \end{bmatrix}$$

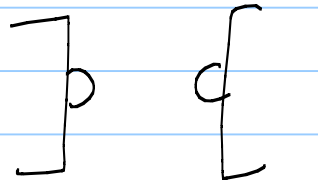




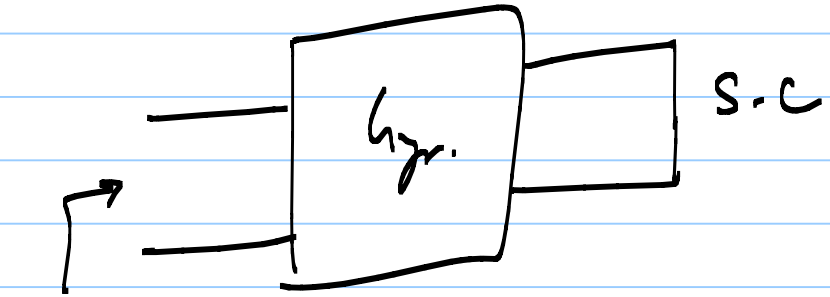




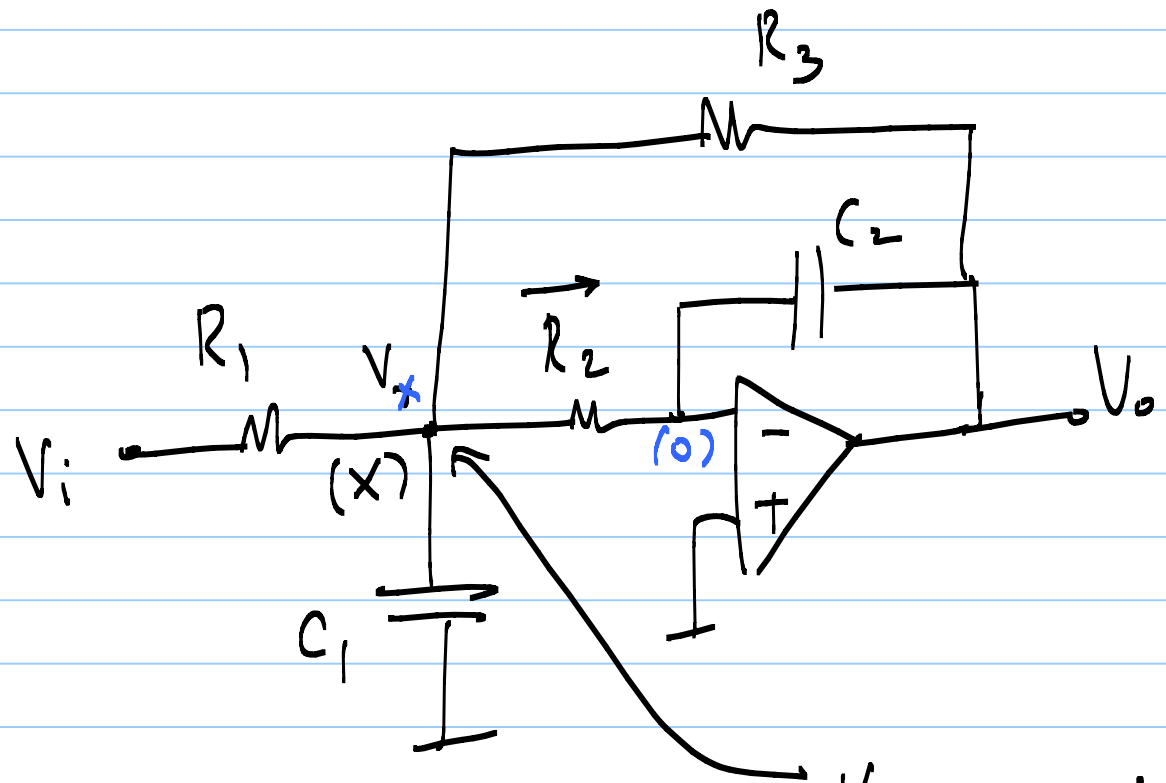
$$Z_{in} = \frac{1}{G_m^2 Z}$$



Gyrator Symbol



Rauch Filter



$$\frac{V_o}{V_i} = ?$$

$$V_x = -sC_2R_2 \cdot V_o$$

KCL @ node x

$$\left(\frac{V_x - V_i}{R_1}\right) + \left(V_x \cdot sC_1\right) + \frac{V_x}{R_2} + \frac{(V_x - V_o)}{R_3} = 0$$

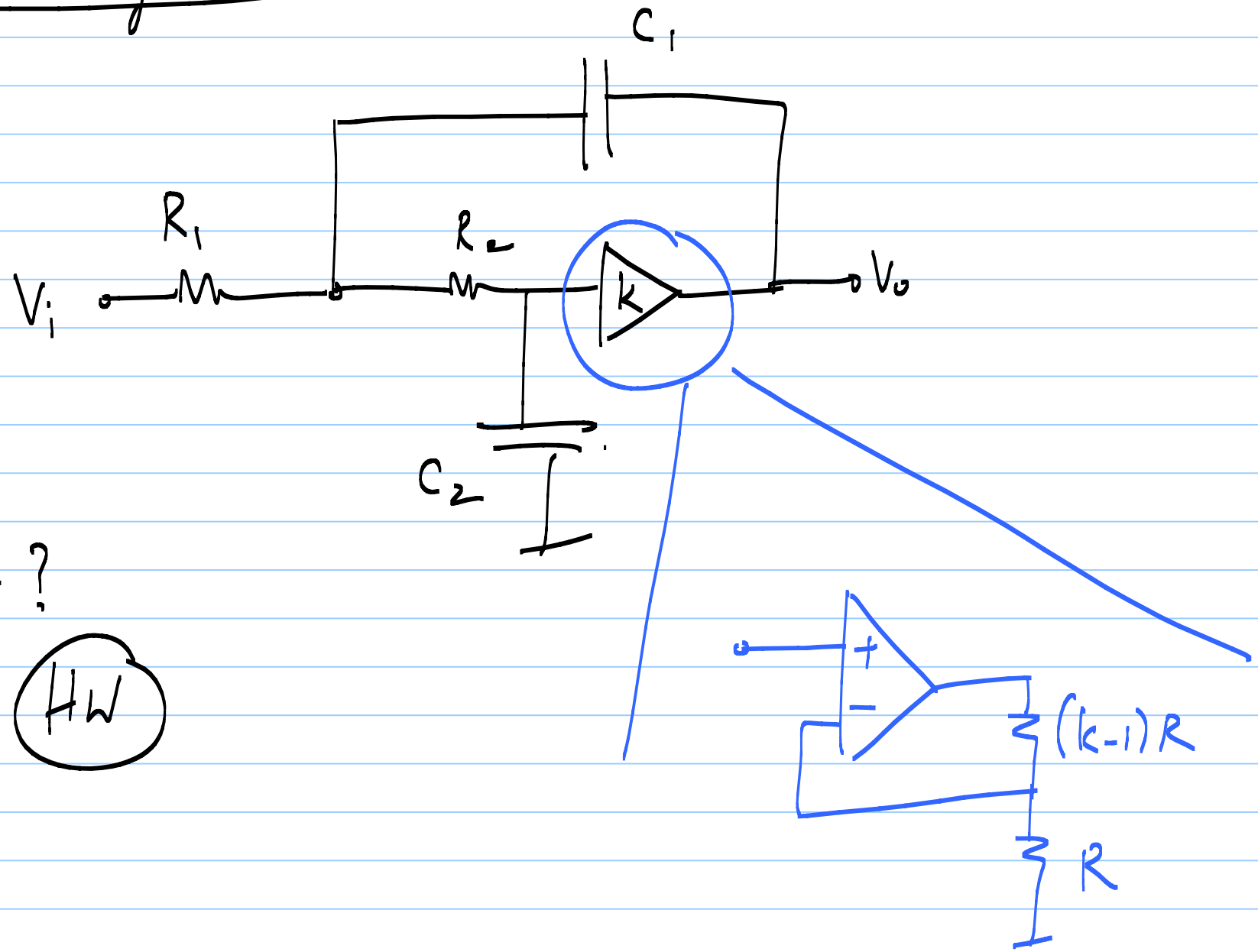
$$\frac{V_o}{V_i} = \frac{-R_3/R_1}{1 + s\cdot\left(\frac{C_2 R_2 R_3}{R_1} + C_2 R_3 + C_2 R_2\right) + s^2\cdot\left(C_1 C_2 \cdot R_2 R_3\right)}$$

DC gain

$\frac{1}{Q\cdot\omega_n}$

$1/\omega_n^2$

Sallen-Key Filter

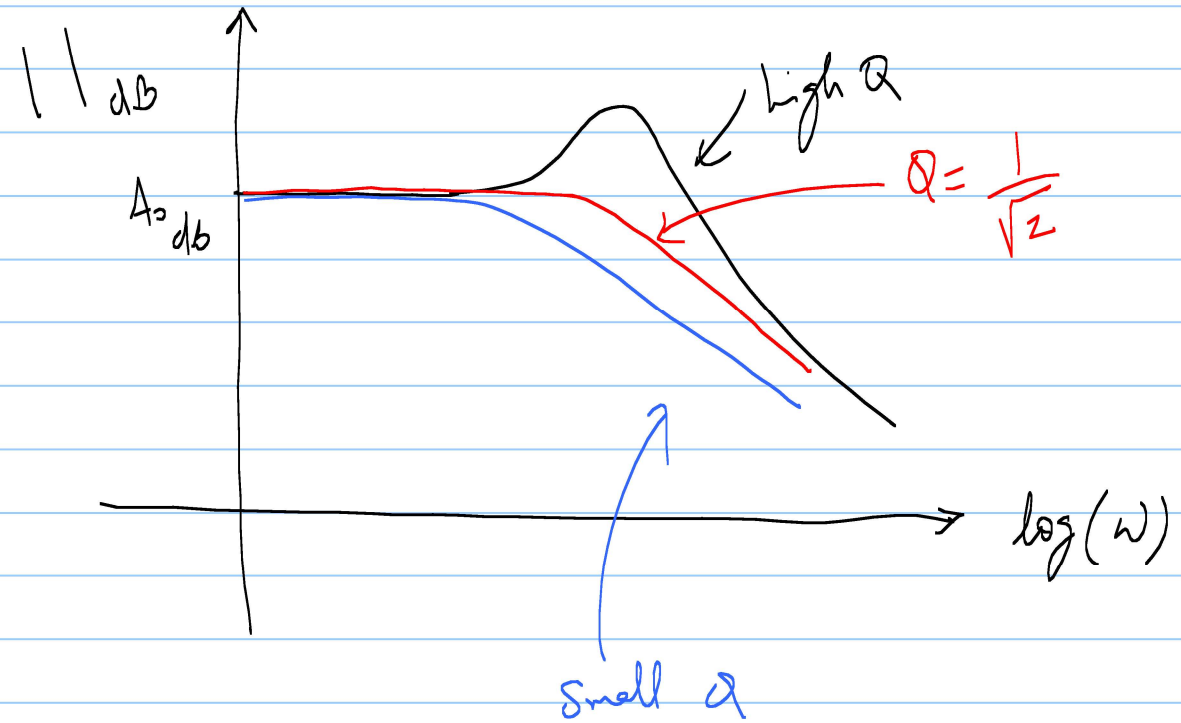


$$\frac{V_o}{V_i} = ?$$

HW

01/04/2022

Lecture 35

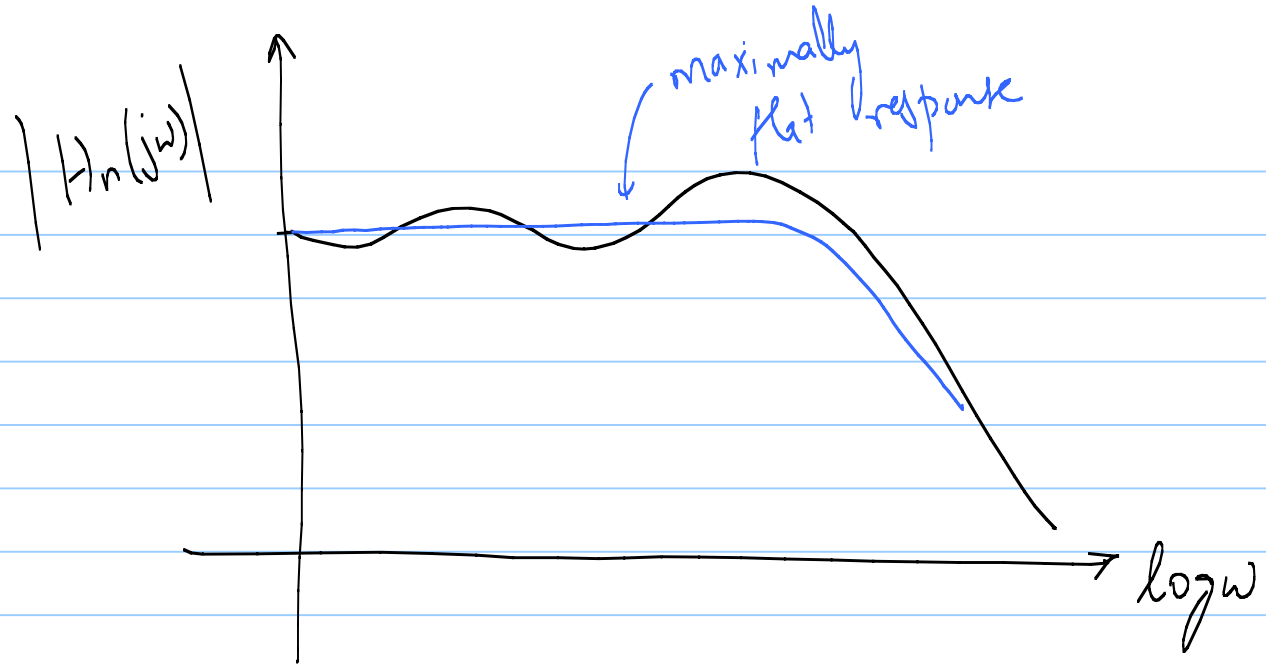


Second order filter

$$H(s) = \frac{A_0}{\frac{s^2}{\omega_n^2} + \frac{s}{Q\omega_n} + 1}$$

$Q = \frac{1}{\sqrt{2}} \Rightarrow$ Maximally flat response

$$H_n(s) = \frac{A_0}{D(s)} = \frac{A_0}{c_n \frac{s^n}{\omega_n^n} + c_{n-1} \frac{s^{n-1}}{(\ )} + \cdots + 1}$$



$$|H_n(j\omega)|^2 = \frac{A_0^2}{1 + B_2\left(\frac{\omega}{\omega_n}\right)^2 + B_4\left(\frac{\omega}{\omega_n}\right)^4 + \cdots B_{2n}\left(\frac{\omega}{\omega_n}\right)^{2n}}$$

$$|H_2(j\omega)| = \frac{A_0}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{\omega^2}{\omega_n^2 Q^2}}}$$

$$|H_2(j\omega)|^2 = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{\omega^2}{Q^2\omega_n^2}}$$

HW For maximally flat response, $B_2 = B_4 = \cdots = B_{2n-2} = 0$

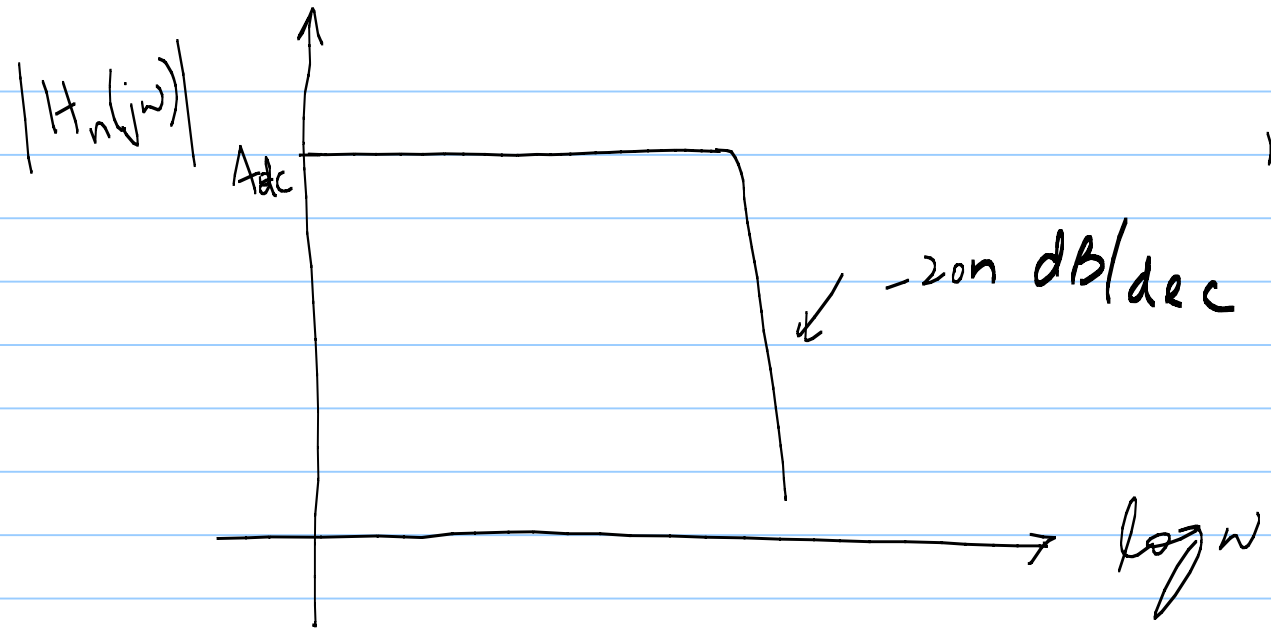
$$|H_n(j\omega)|^2 = \frac{1}{1 + B_{2n}\left(\frac{\omega}{\omega_n}\right)^{2n}}$$

(normalized $A_0 = 1$)

$$\boxed{|H_n(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2n}}}$$

Maximally flat response

-3 dB BW $= \omega_n$



n is large

"Butterworth Filter" = maximally flat response

06/04/2022

Lecture 36

Make up class

Sunday 10th April - 9:30 am - 12 noon

$$H(s) = \frac{1}{1 + \frac{s}{Q\omega_n} + \frac{s^2}{\omega_n^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{\omega}{Q\omega_n}\right)^2}}$$

$$|H(j\omega)|^2 = \frac{1}{\underbrace{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2}+\frac{\omega^2}{Q^2\omega_n^2}}$$

@ $Q = \frac{1}{\sqrt{2}}$

$$|H(j\omega)|^2 = \frac{1}{1+\frac{\omega^4}{\omega_n^4}}$$

We want poles of

$$|H_n(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_n}\right)^{2n}}$$

(2n roots)

poles of

$$\left|H(s)\right|^2_{s=j\omega} = H(s)\cdot H^*(s)$$

$$= H(s)\cdot H(-s)\Big|_{s=j\omega}$$

$$H(s)\cdot H(-s)\Big|_{s=j\omega} = \frac{1}{1+\left(\frac{\omega}{\omega_n}\right)^{2n}}$$

set $\omega = s/j$

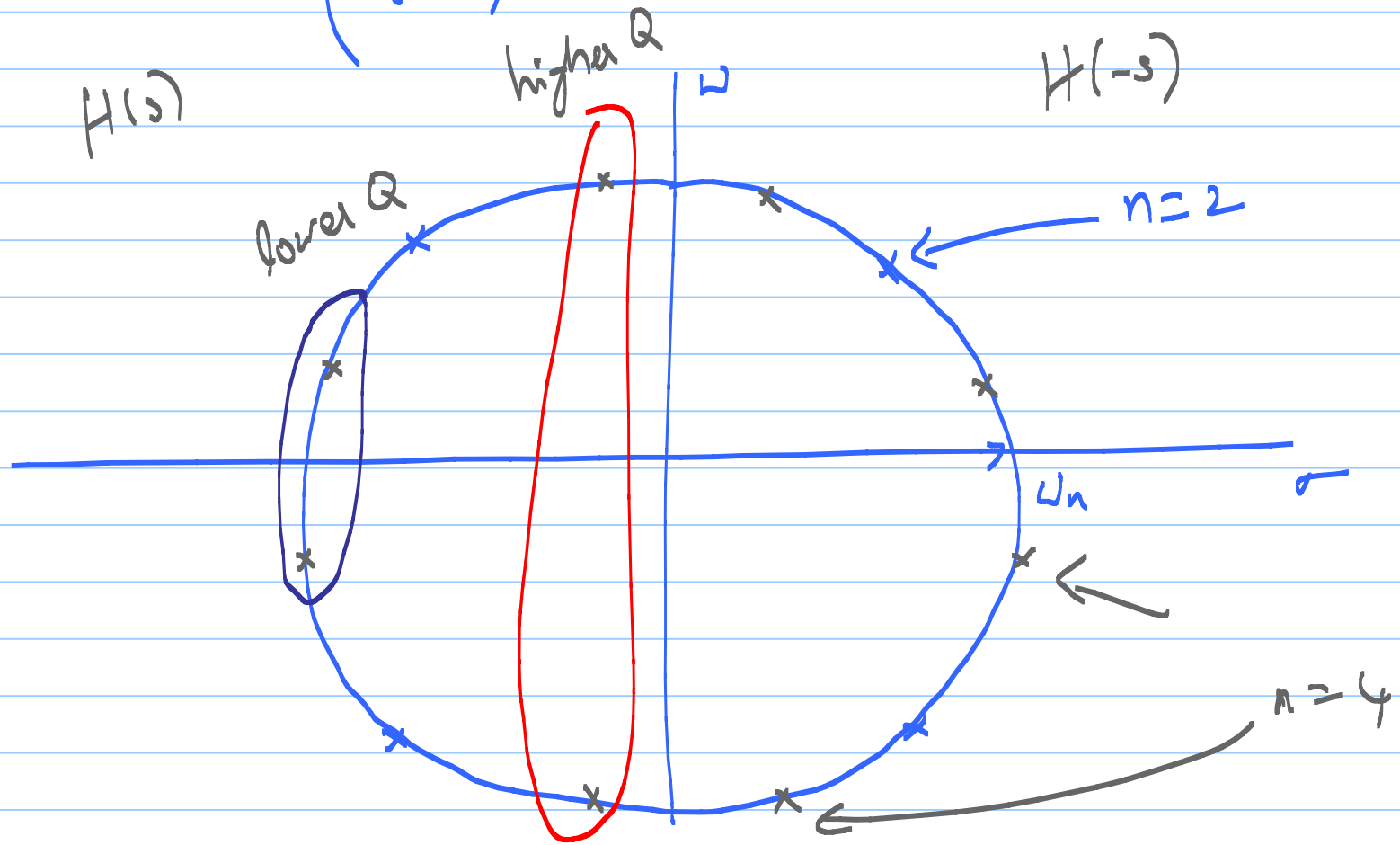
$$H(s)\cdot H(-s) = \underbrace{\frac{1}{1+\left(\frac{s}{j\omega_n}\right)^{2n}}}_{2n \text{ roots}}$$

n poles $\{p_i\}$

n poles $\{-p_i\}$

roots of $1 + \left(\frac{s}{j\omega_n}\right)^{2n} = 0$ are the poles

- n LHP poles — $H(s)$
- n RHP poles — $H(-s)$



roots are : $s = j\omega_n \cdot \exp\left[\frac{(2k+1)j\pi}{2n}\right]$

$$s = \omega_n \cdot \exp\left[j \frac{(2k+1)\pi}{2n} + \frac{j\pi}{2} \right] \Bigg|_{k=0 \text{ to } (2n-1)}$$

Higher order filter

Cascade 2nd order sections —— 2n order

1 first order, cascade end order sections —— (2n+1) order

* Put 1st order section first
* Cascade sections in increasing order of Q

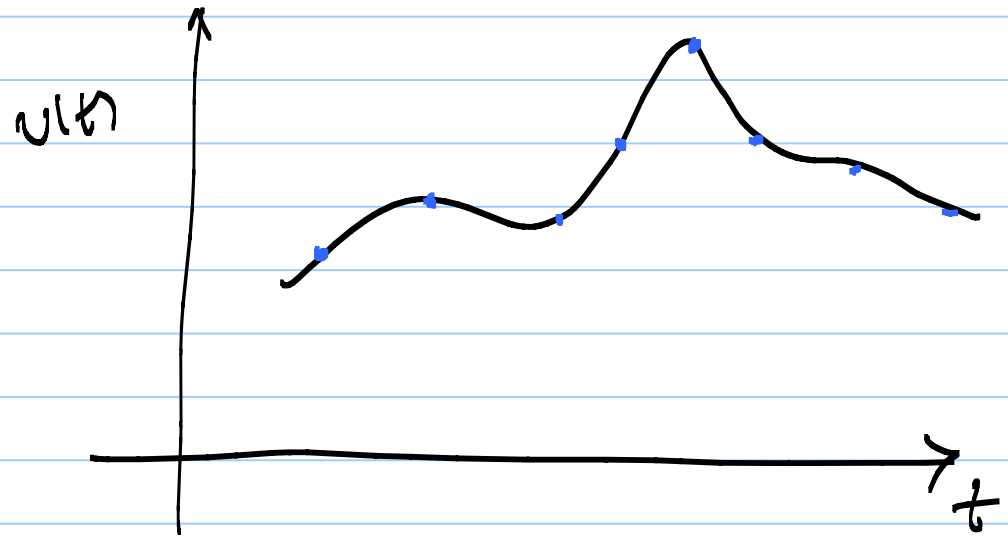
07/04/2022

Lecture 37

Data Converters

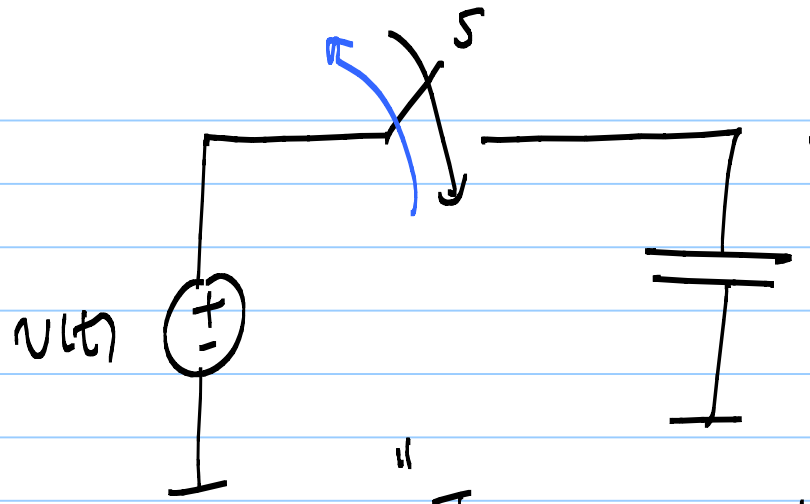
- ADC
- DAC

ADC → Sampling → Quantisation → Binary (Digital) Word



Nyquist rate = 2x max freq. content in V(f)

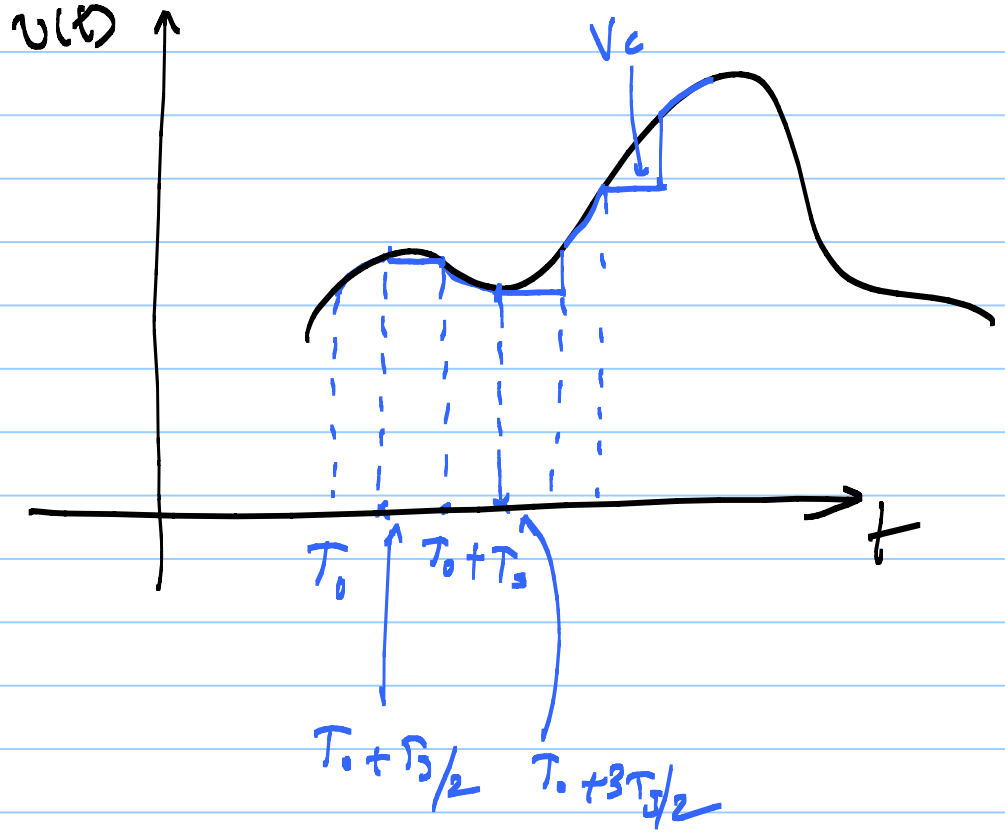
Sampling rate should be ≥ Nyquist rate

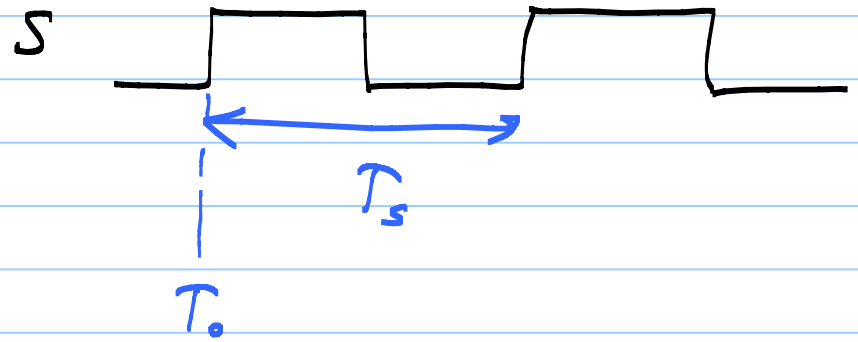


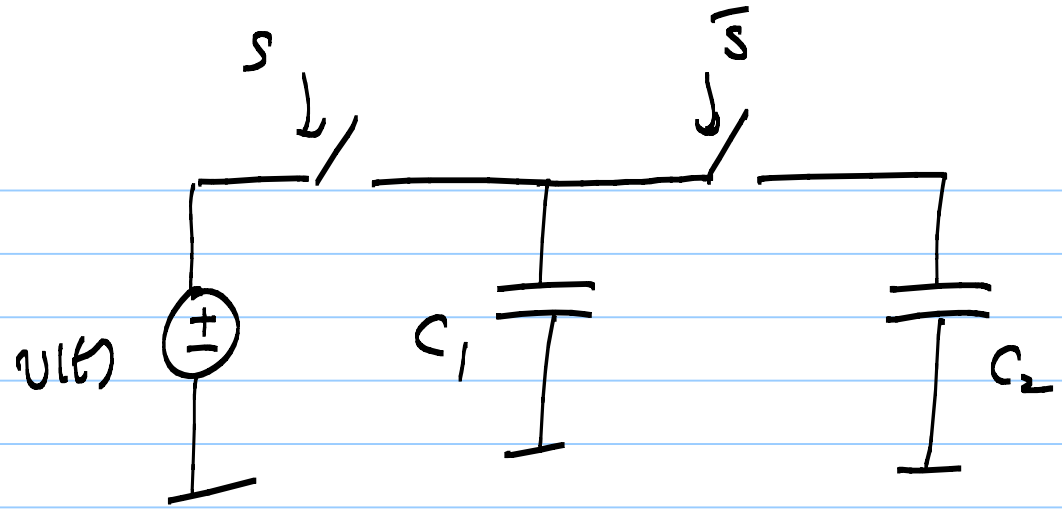
$V_c = v(t)$

V_c holds = last value of $v(t)$ just before switch opens

"Track and Hold" circuit

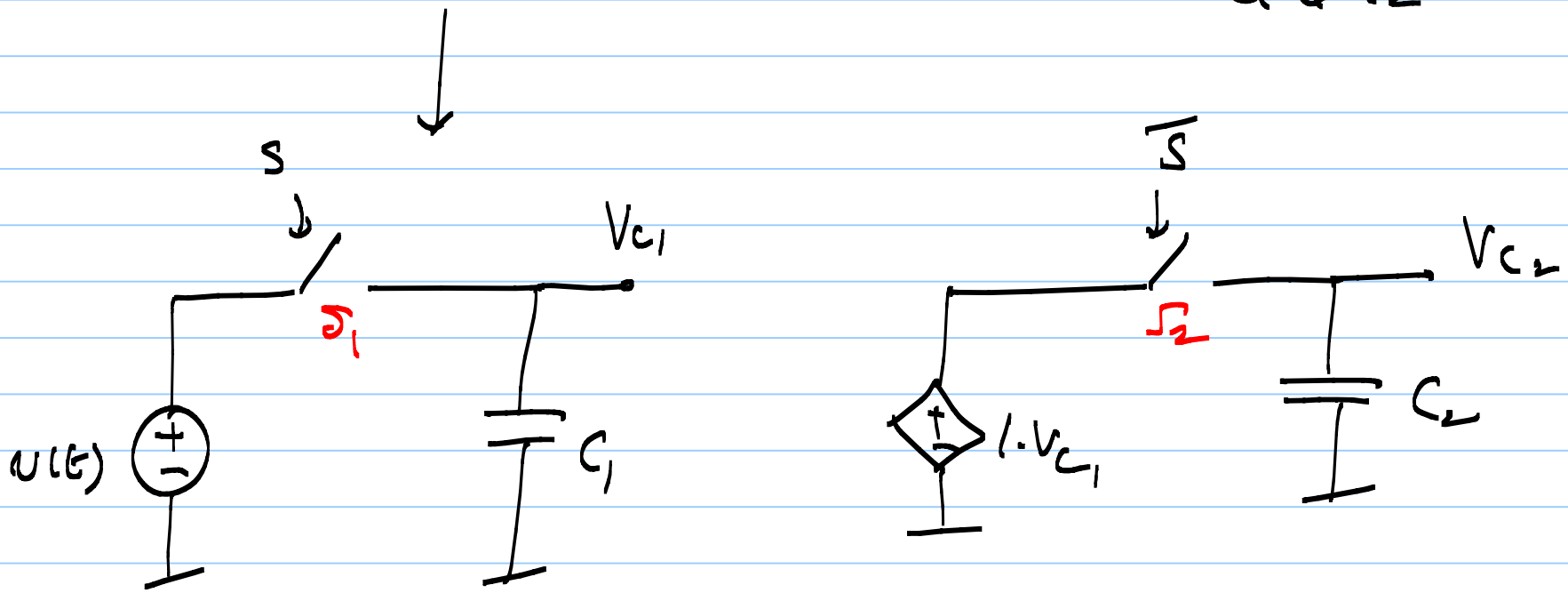




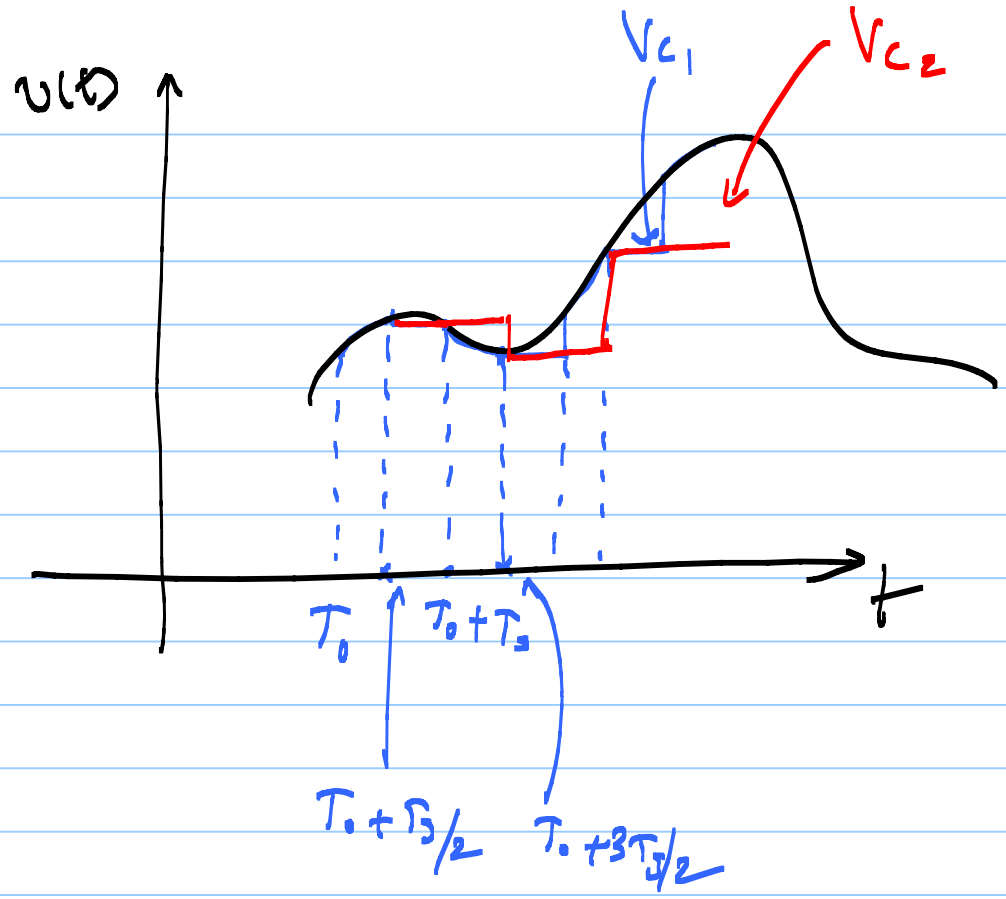


?

charge sharing between C_1 & C_2



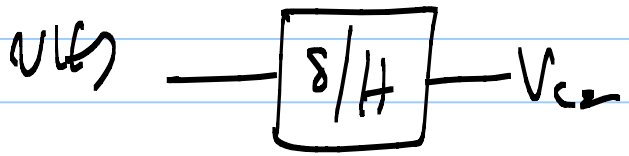
"Sample & hold"



HW:

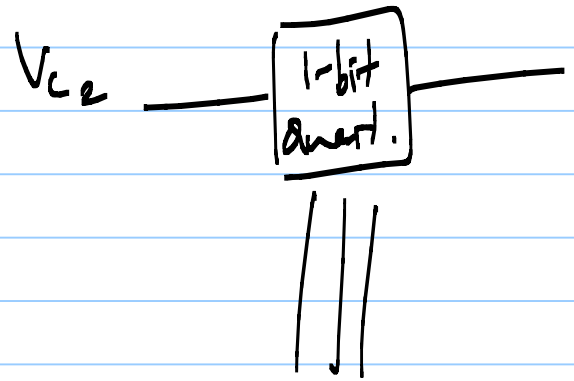
Understand in freq. domain

Quantization $\longrightarrow$ 1) Range is 0 to V_{ref}

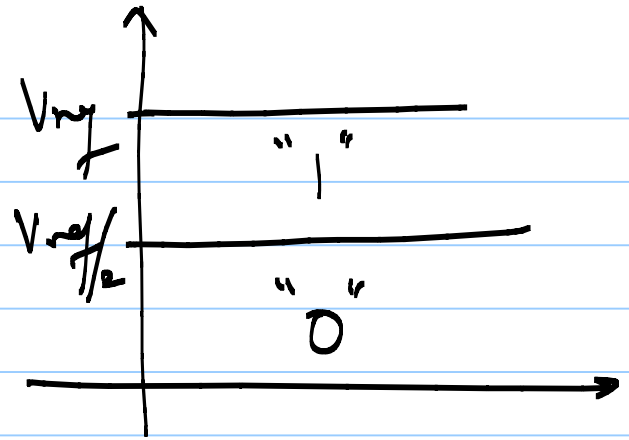


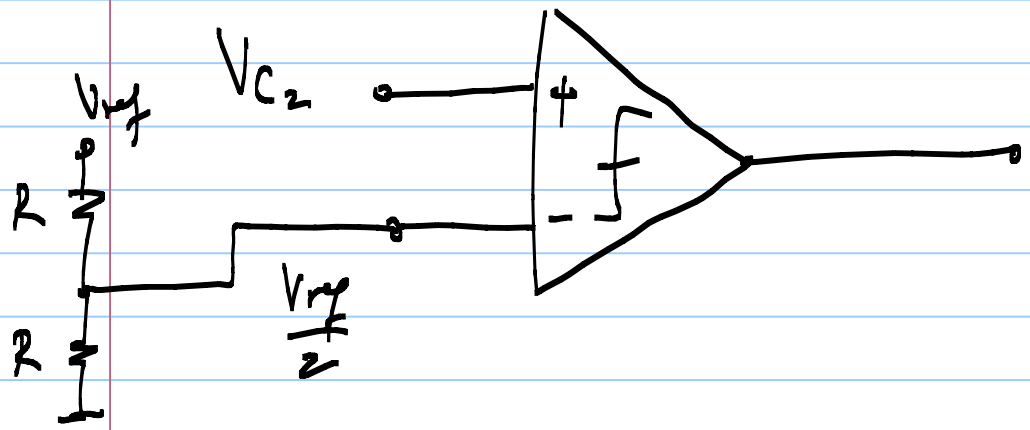
2) # of levels $\longleftrightarrow$ # of bits

1)



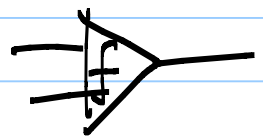
0 or 1

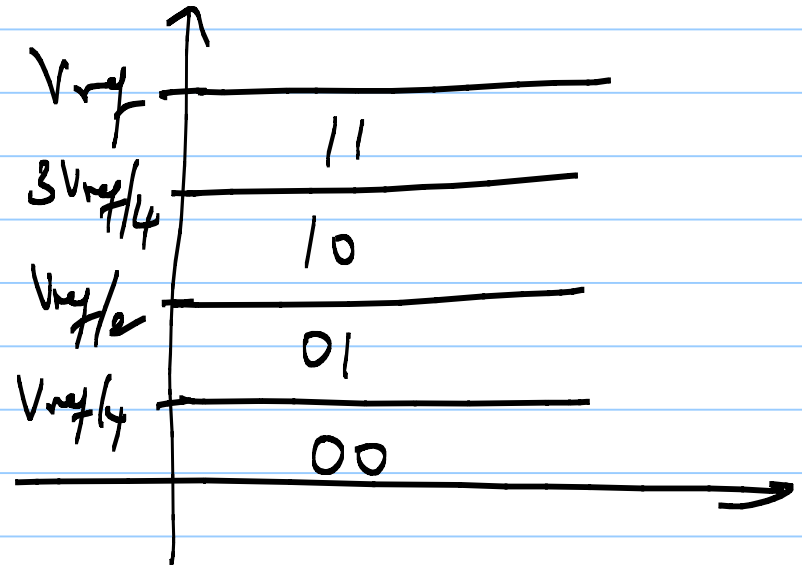


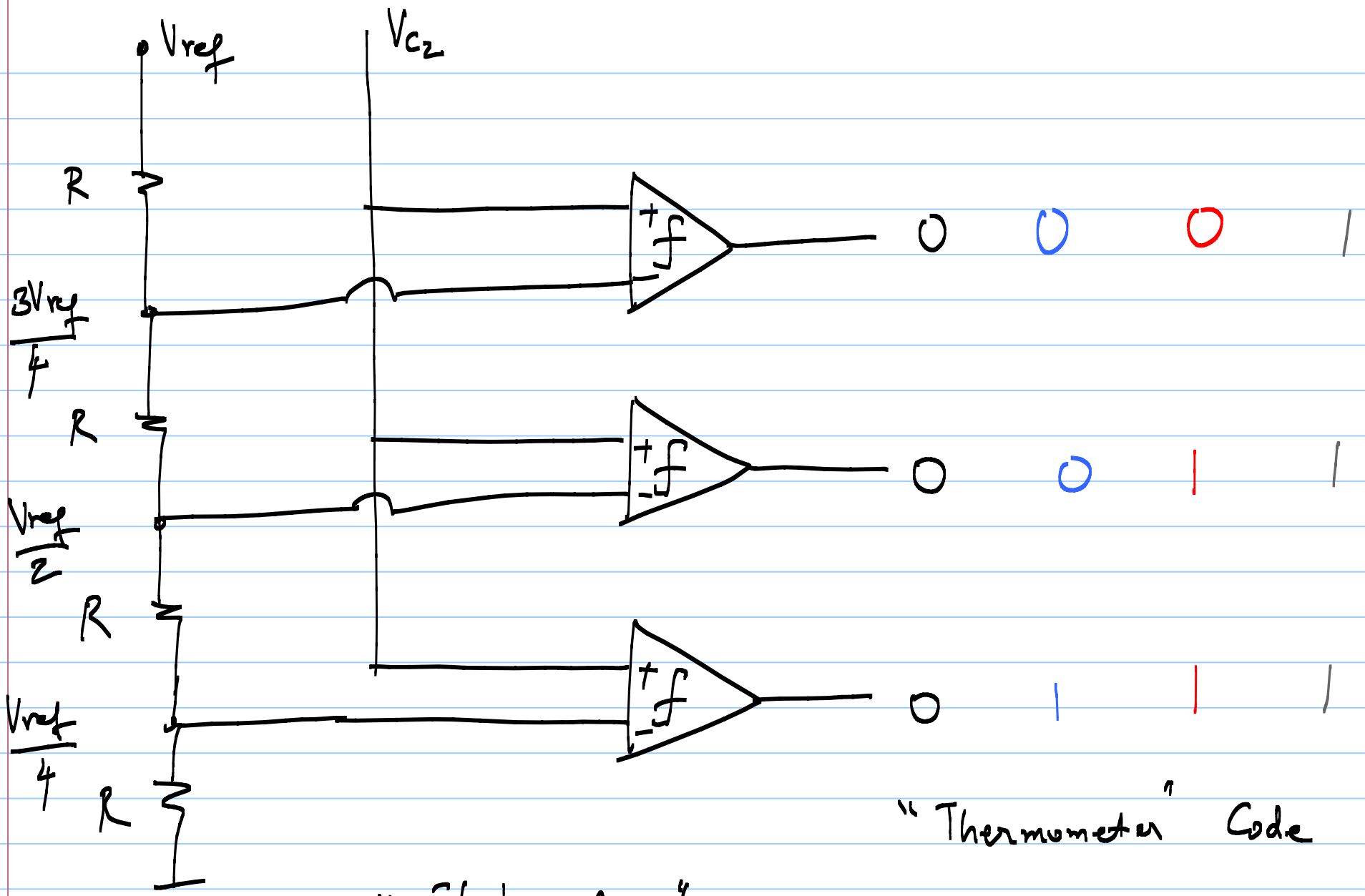


2) 2-bit quant

3

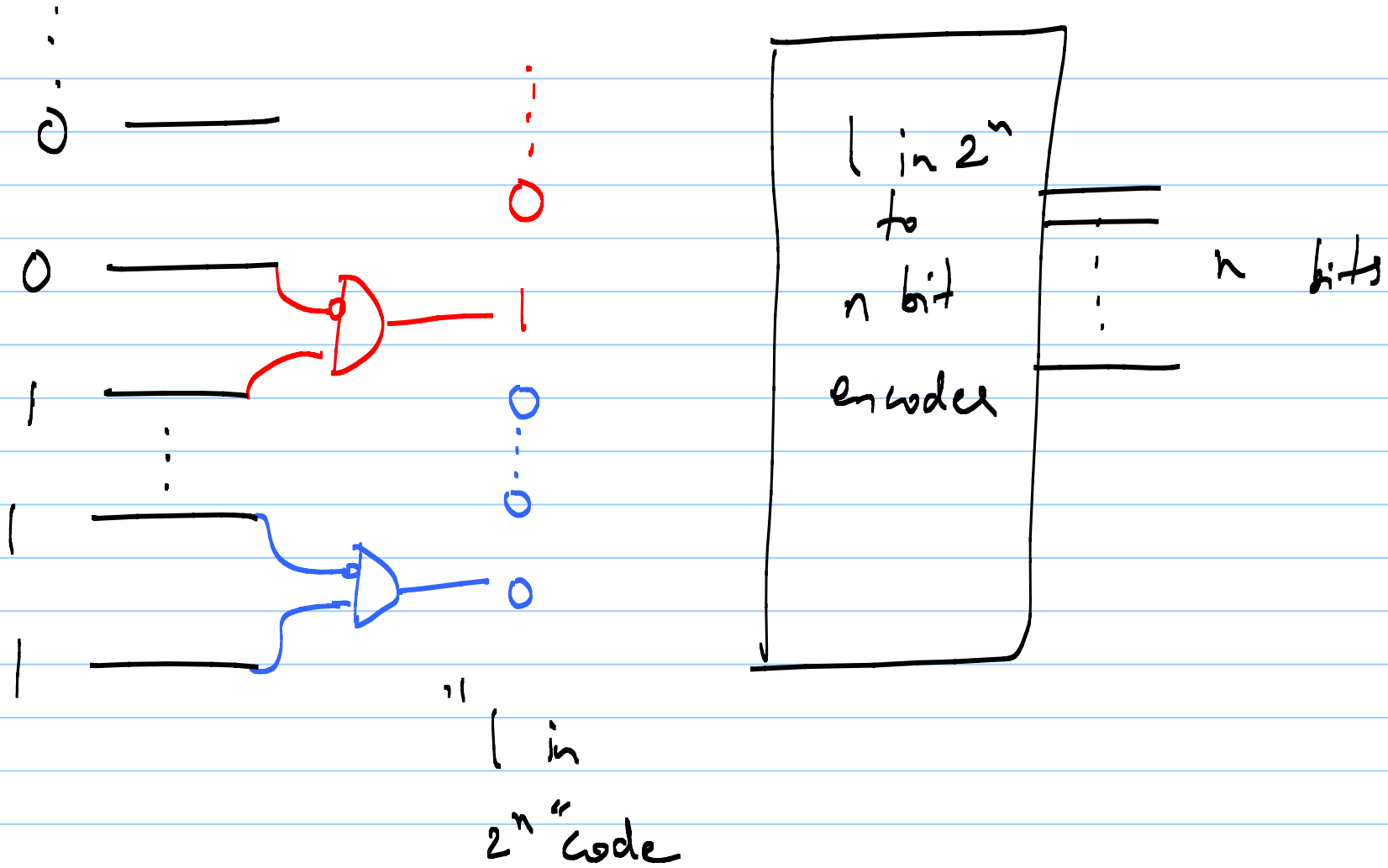


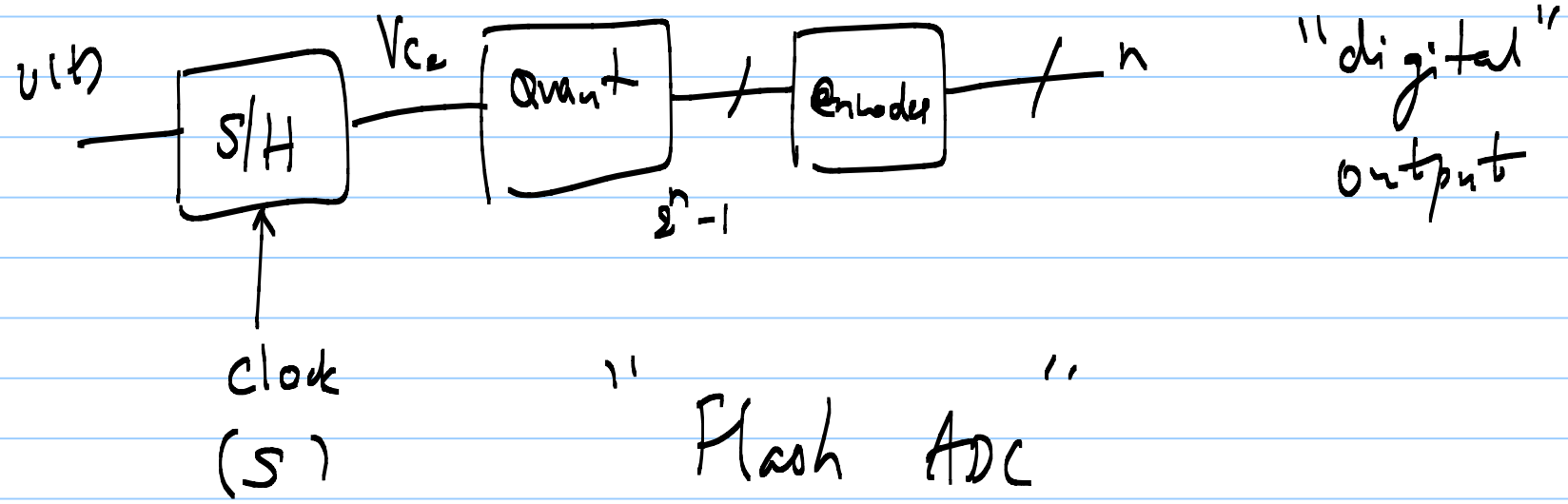




"Thermometer" Code

"Flash ADC"


0
0
1
1
1
0
1
0
0
0
"1 in 2^n" code
1 in 2^n to n bit encoder
n bits



"Flash ADC"

6 bits (low)

Fastest

High power

high IC area

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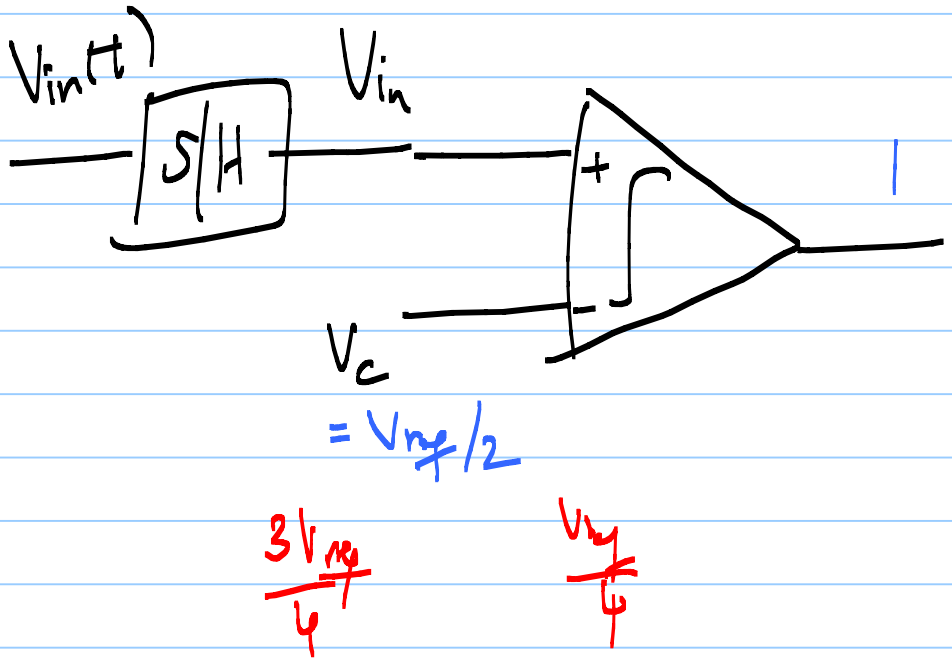
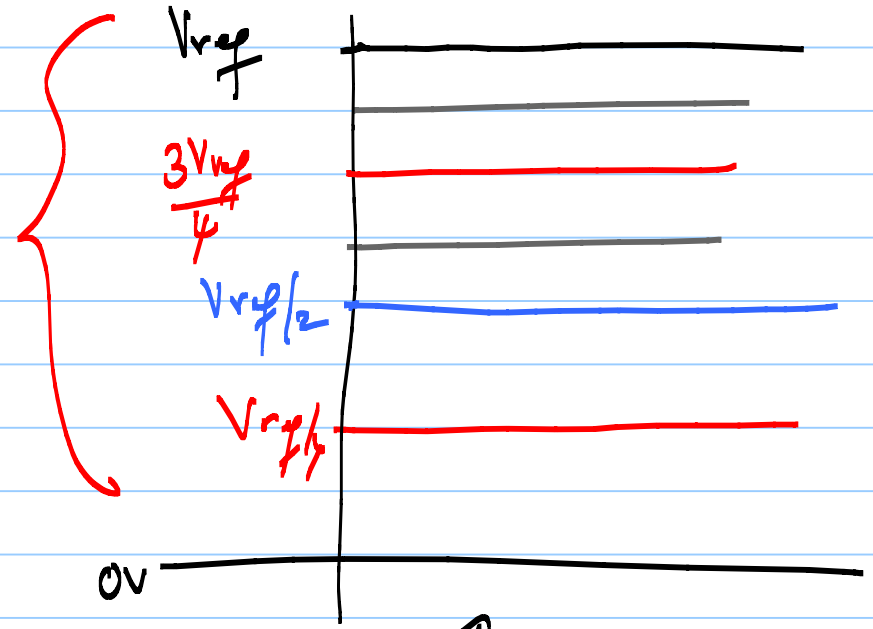
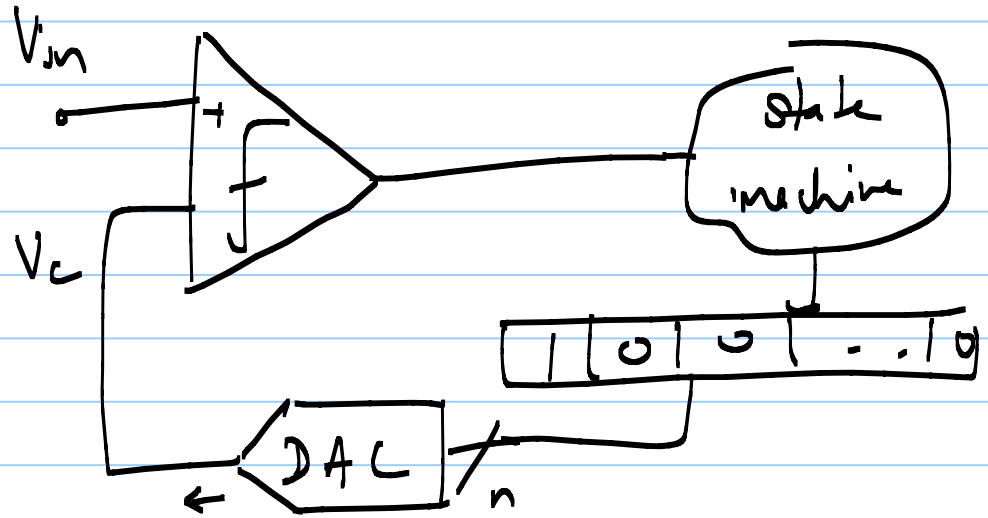
Lecture 38



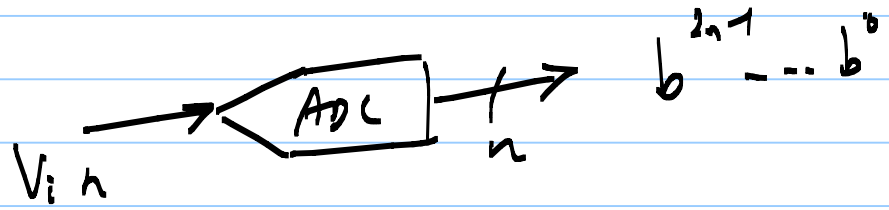


output $= b^{2n-1} b^{2n-2} \ldots b^1 b^0$

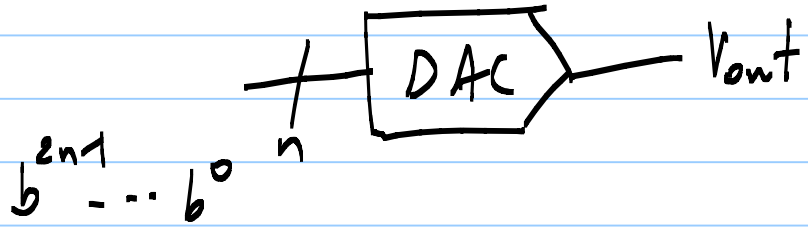
Binary Search



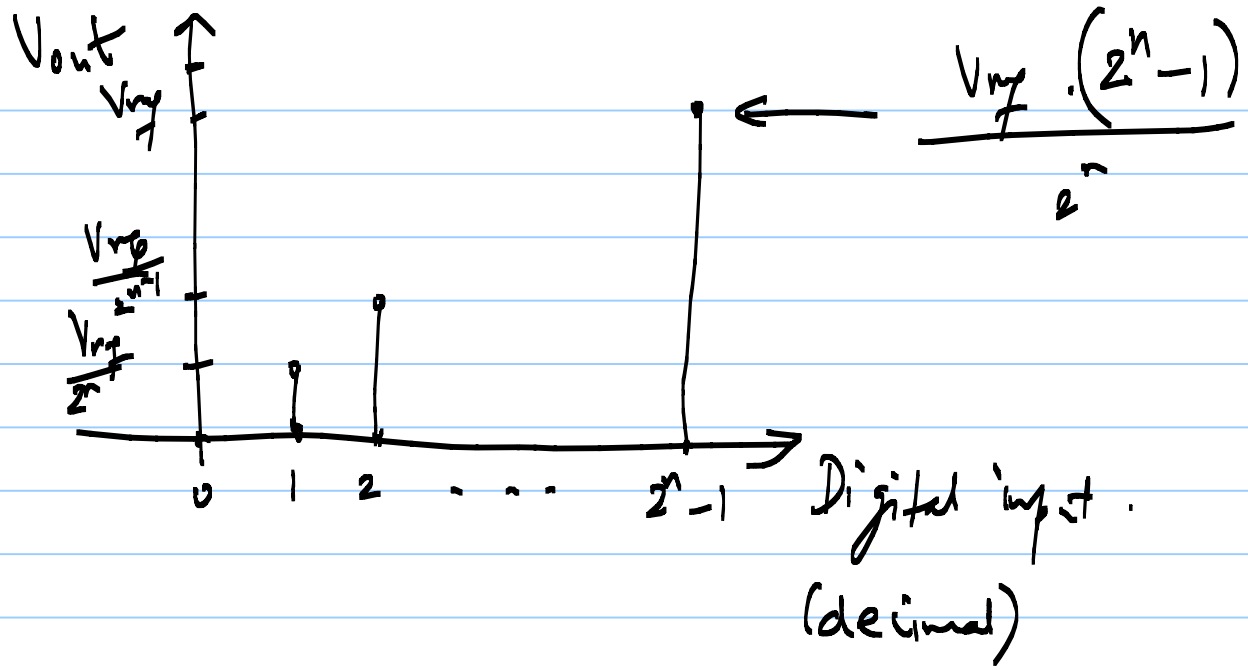
V_{in}
V_c
+
State machine
1 0 0 . . 0
DAC
n



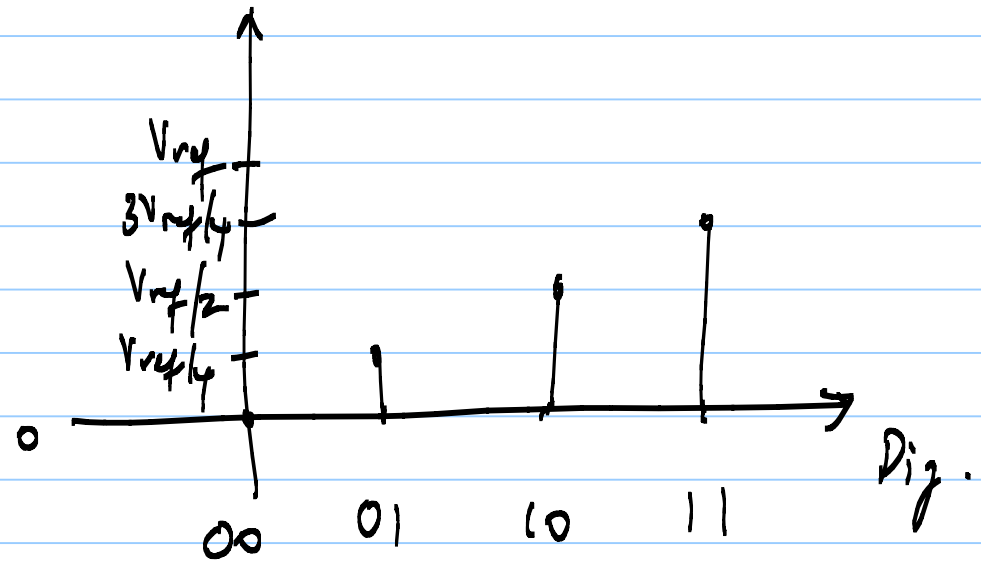
V_{in}
ADC
n
$b^{2n-1} \ldots b^0$

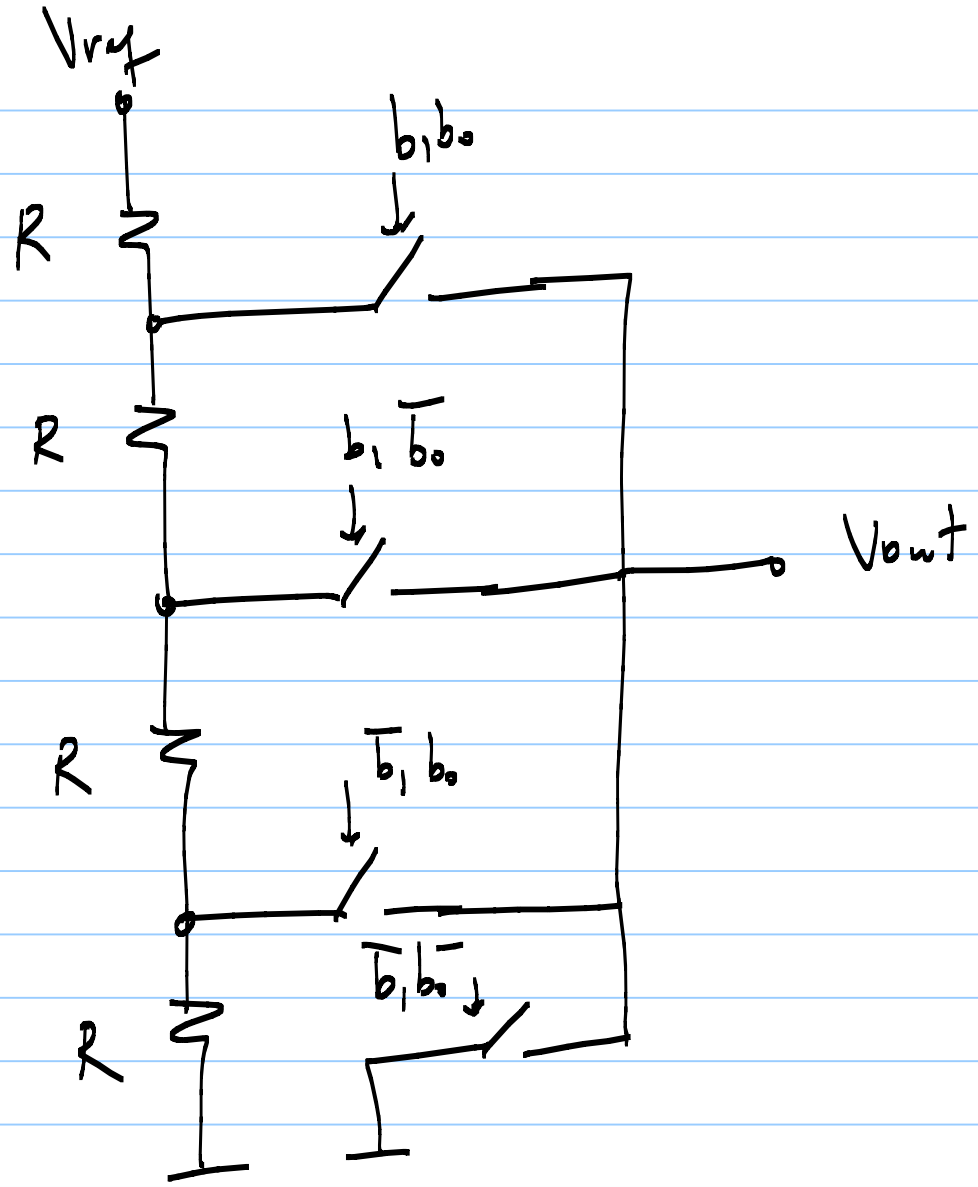


DAC
V_{out}
n
$b^{2n-1} \ldots b^0$


Vout
Vref
Vref/2^n-1
Vref/2^n
0
1
2
. . .
2^n-1
Digital input.
(decimal)
Vref.(2^n-1)/2^n

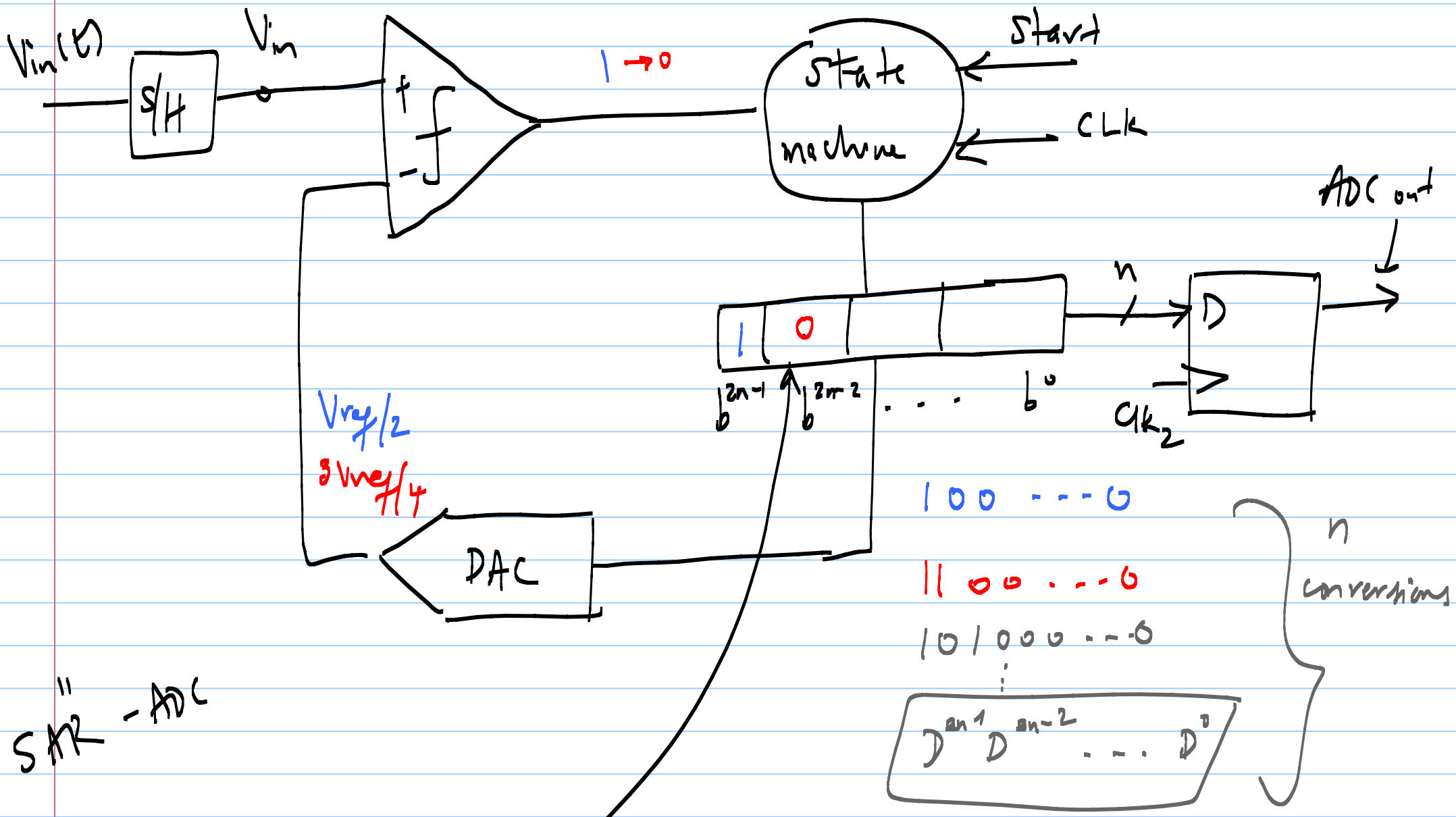
2-bit DAC


Vref
3Vref/4
Vref/2
Vref/4
0
00
01
10
11
Dig.



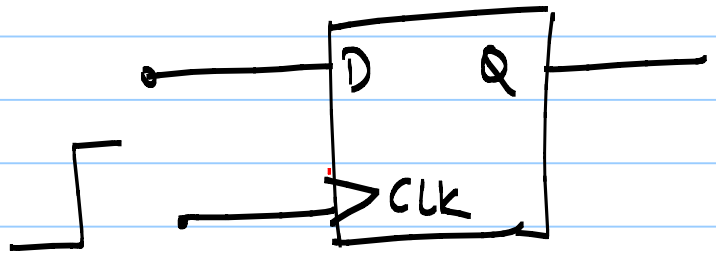
$b_1 \; b_0$

Resistor String DAC

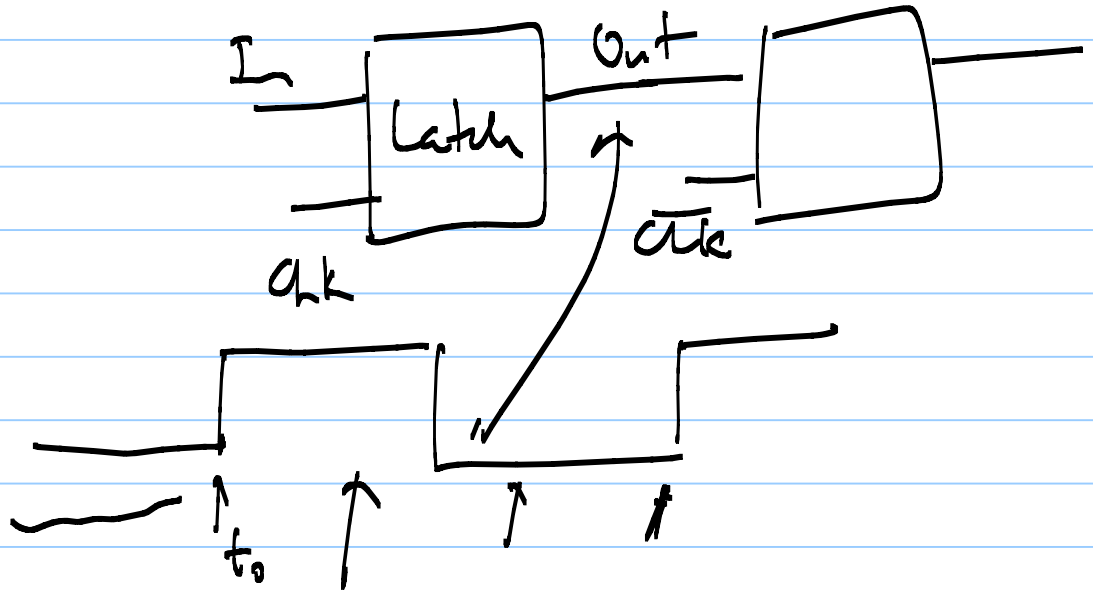


"SAR"-ADC

Successive Approximation Register



positive edge triggered



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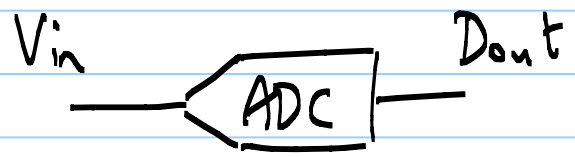
Lecture 39

ADC



# of bits = n

reference voltage = V_{ref}

Sampling frequency f_s

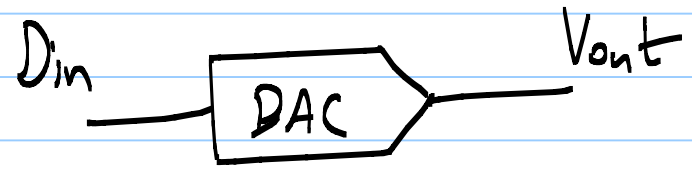
$D_{out} = k$

if

$$\frac{V_{ref}}{2^n} \cdot k < V_{in} < \frac{V_{ref}}{2^n} \cdot (k+1)$$

V_{in} Range : 0 to V_{ref}

DAC



# of bits = n

reference voltage = V_{ref}

Conversion rate f_s

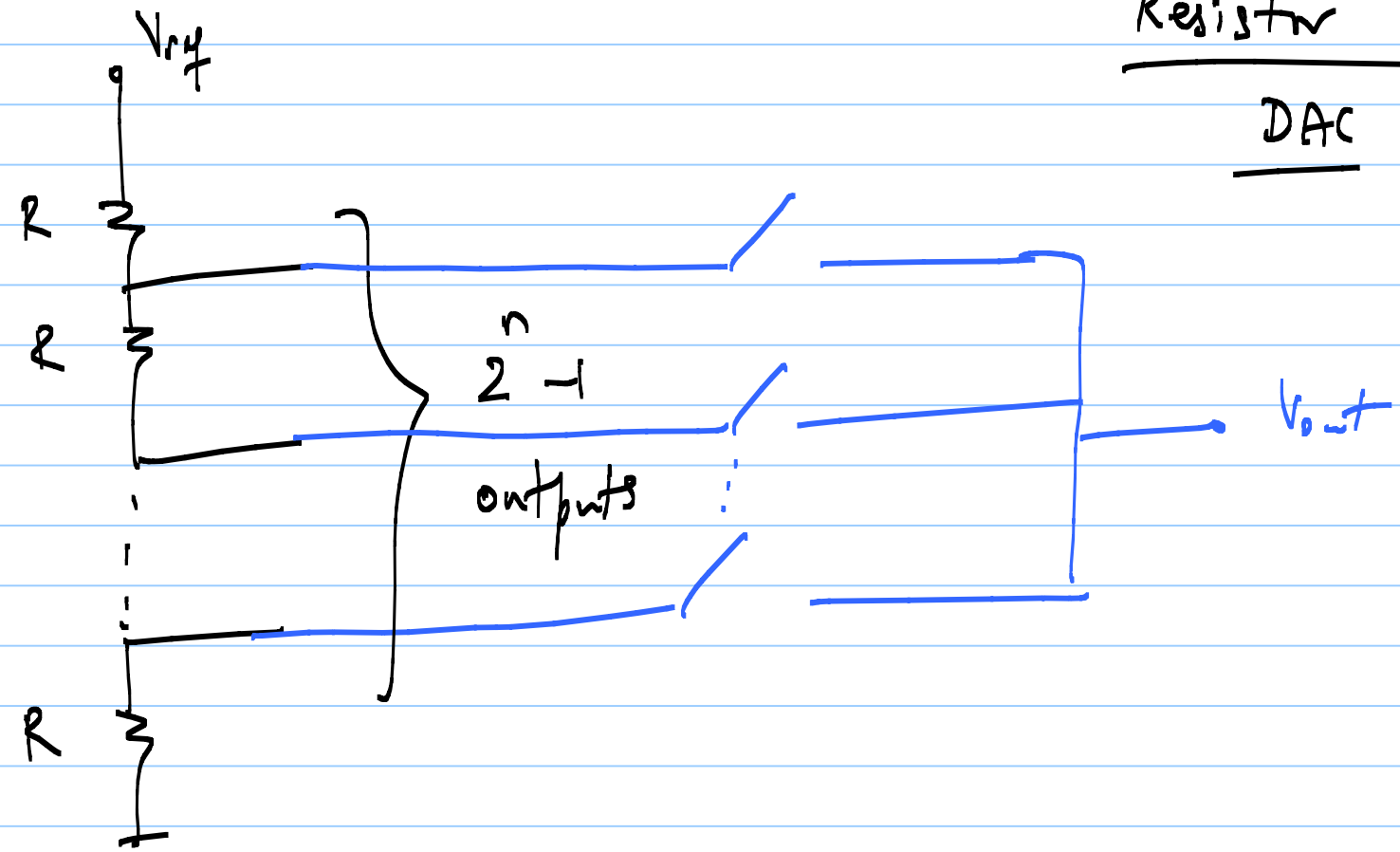
$$V_{out} = \frac{V_{ref}}{2^n} \cdot D_{in}$$

Output Range : 0 to V_{ref}

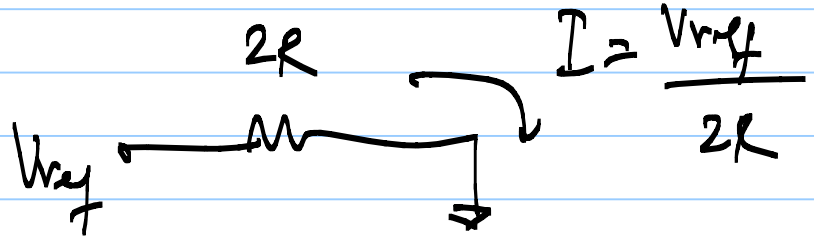
DACs

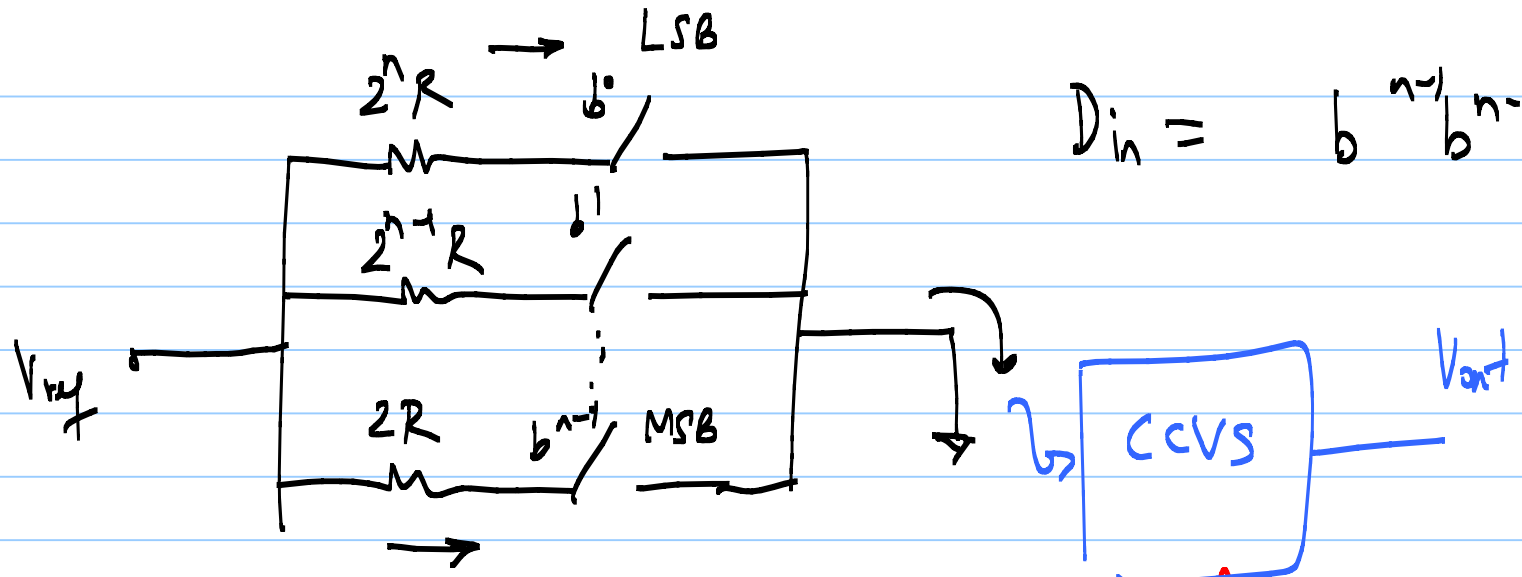
Resistor string DAC

1)

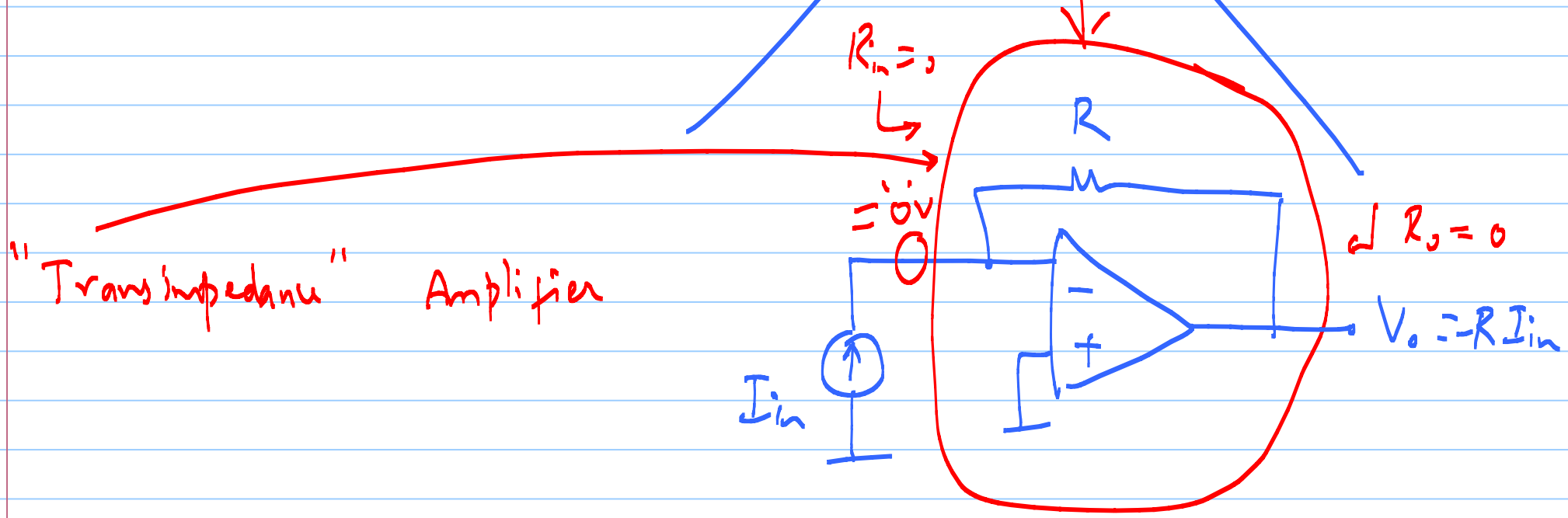


2)

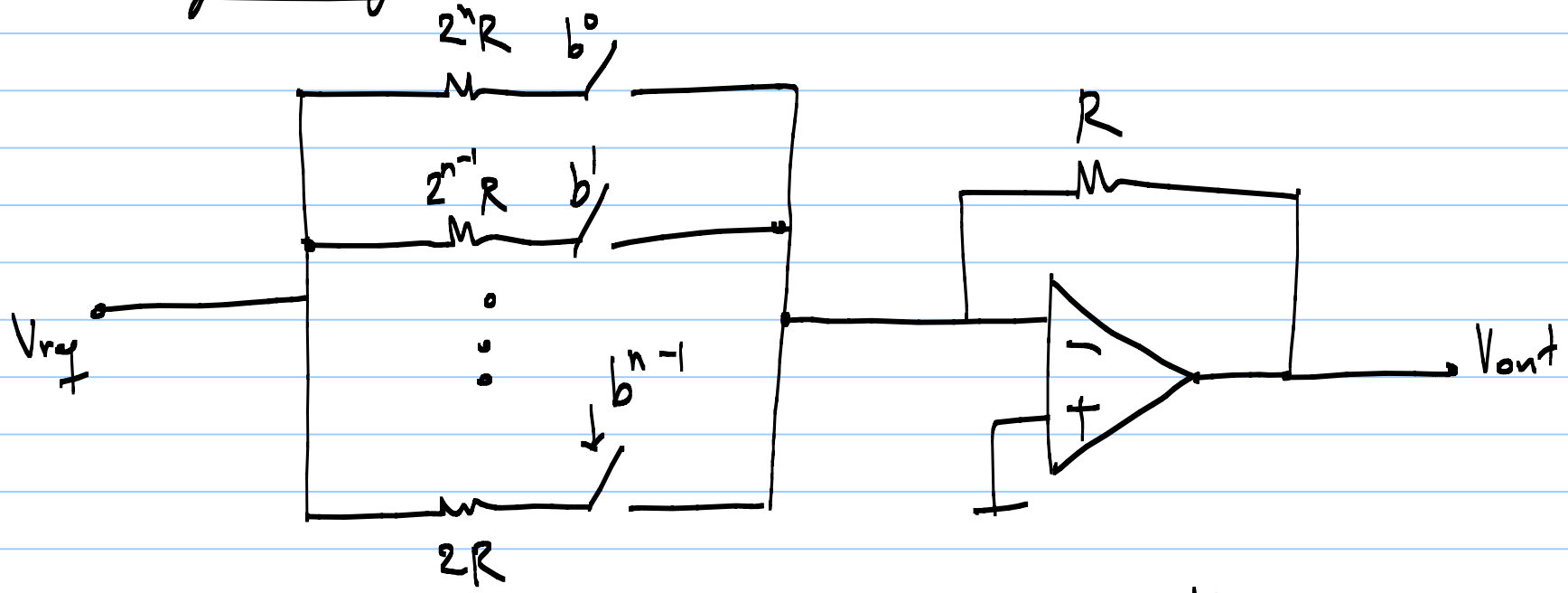




$$D_{in} = b^{n-1} b^{n-2} \ldots b^0$$



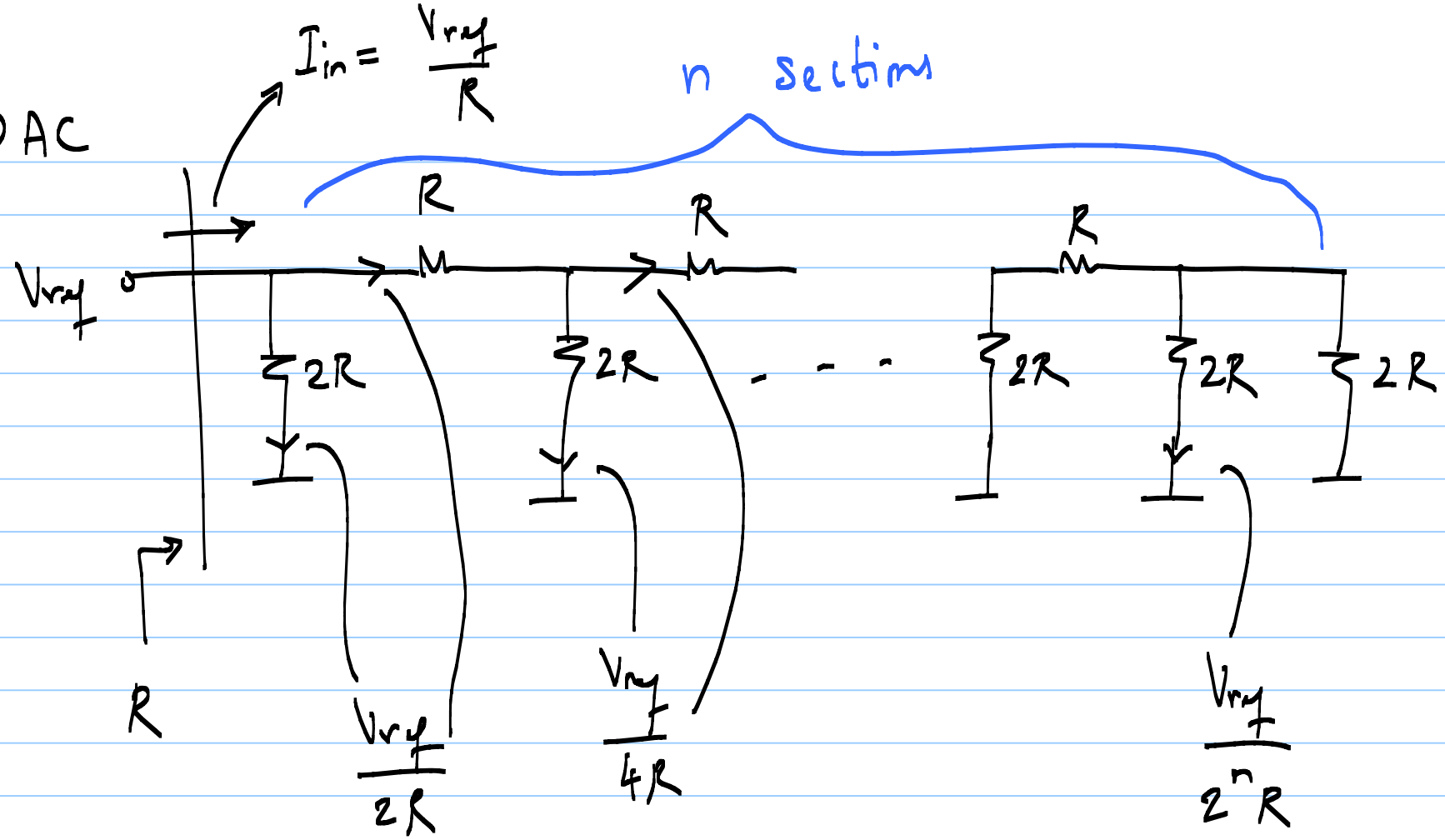
Binary Weighted DAC

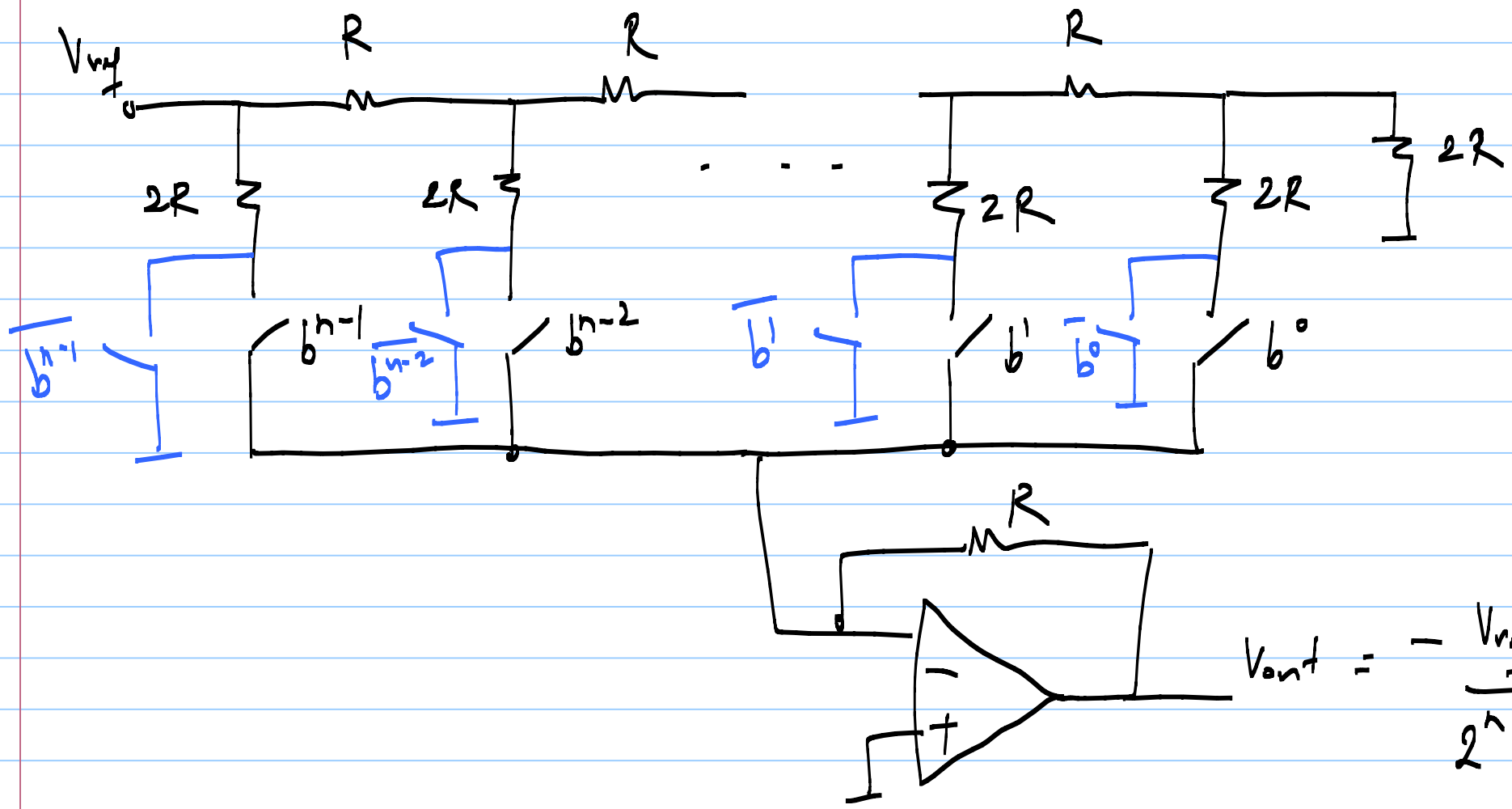


$$D_{in} = b^{n-1} b^{n-2} \ldots b^1 b^0$$

$$V_{out} = -\frac{V_{ref}}{2^n} \cdot D_{in}$$

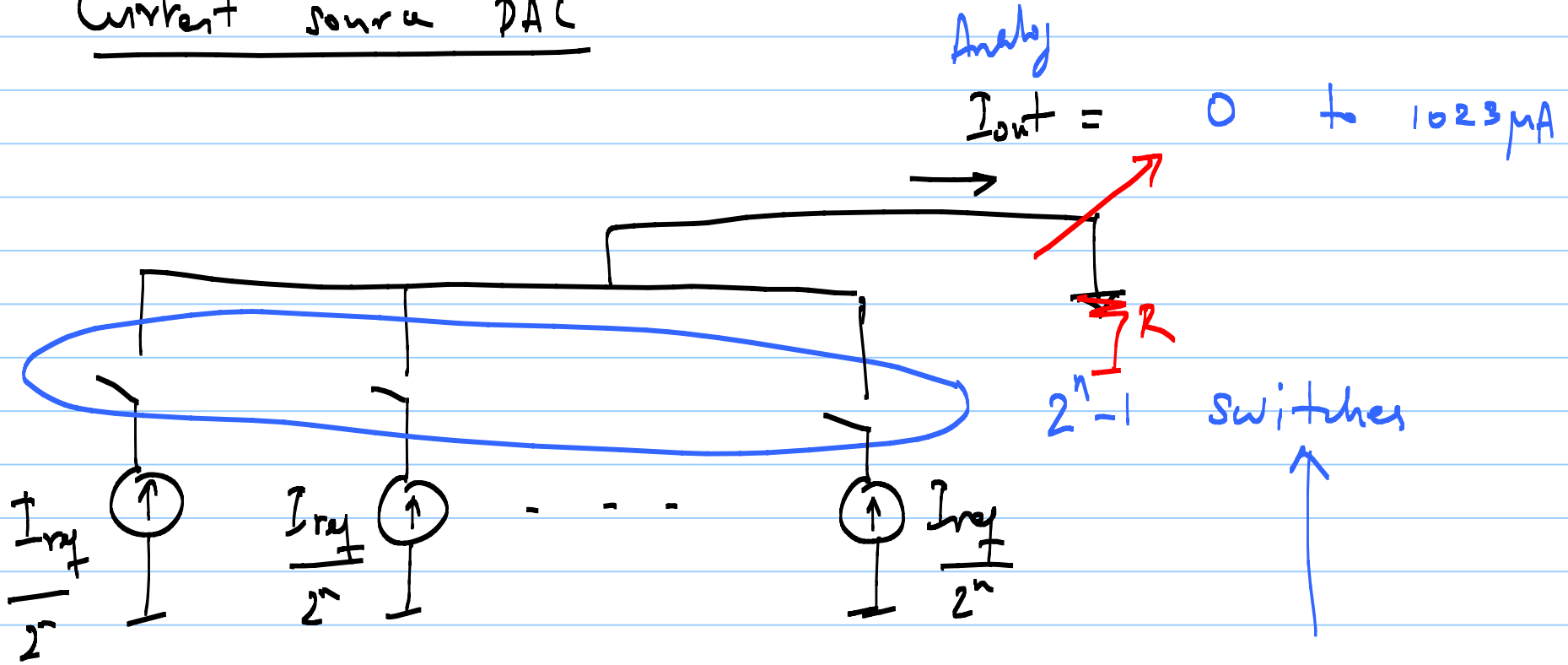
3) DAC


$I_{in} = \frac{V_{ref}}{R}$
n sections
R
R
R
V_{ref}
2R
2R
2R
2R
2R
R
$\frac{V_{ref}}{2R}$
$\frac{V_{ref}}{4R}$
$\frac{V_{ref}}{2^n R}$



$$V_{out} = -\frac{V_{ref}}{2^n} \cdot D_{in}$$

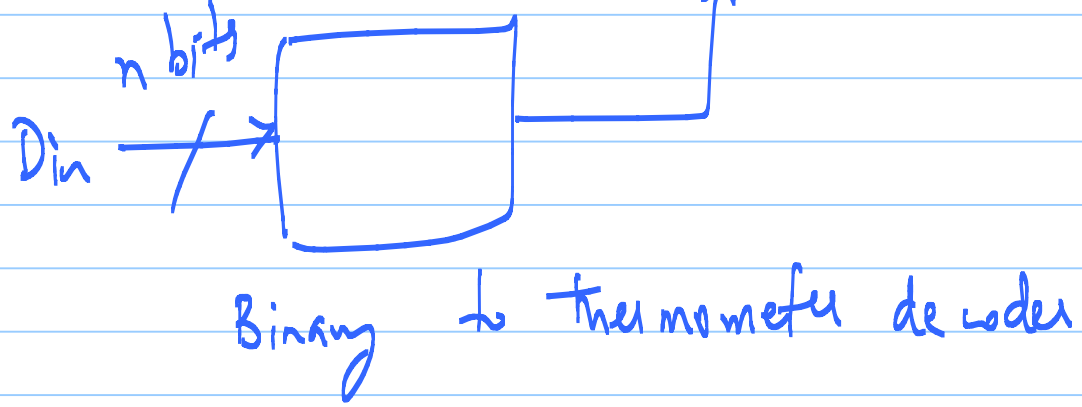
4) <u>Current source DAC</u>



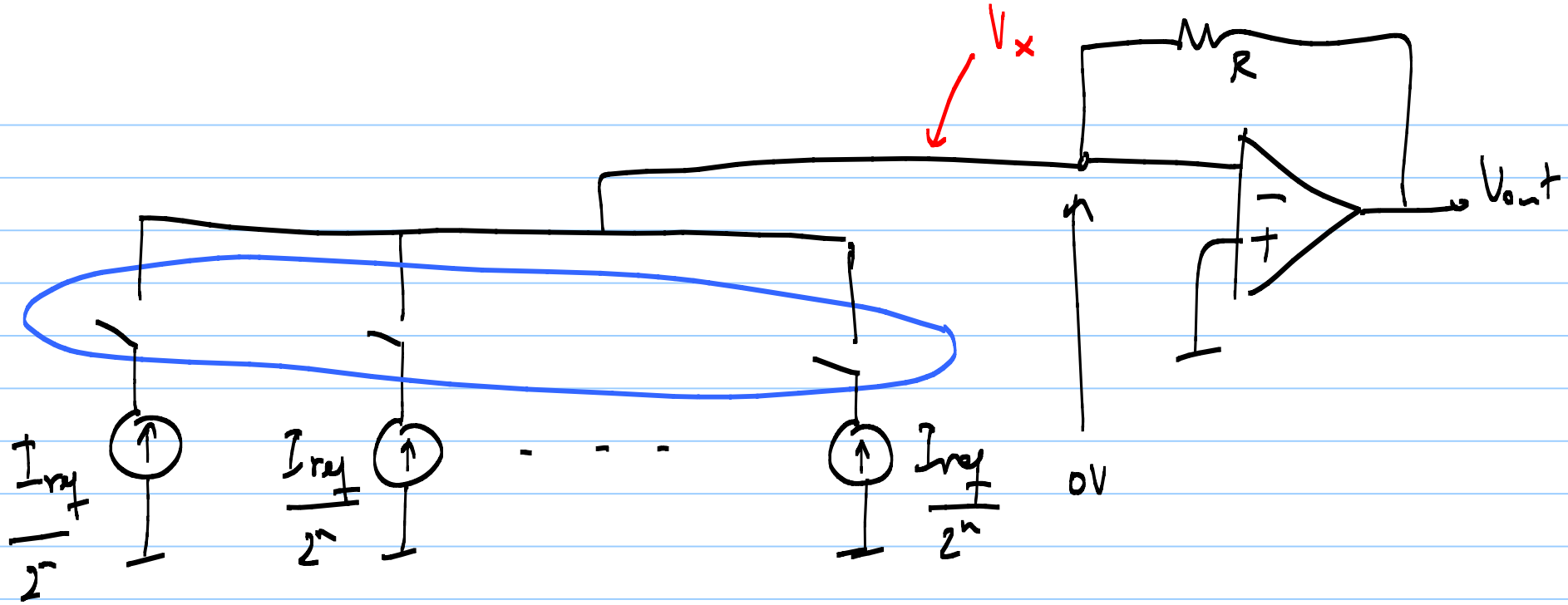
e.g 1μA

10 bit DAC

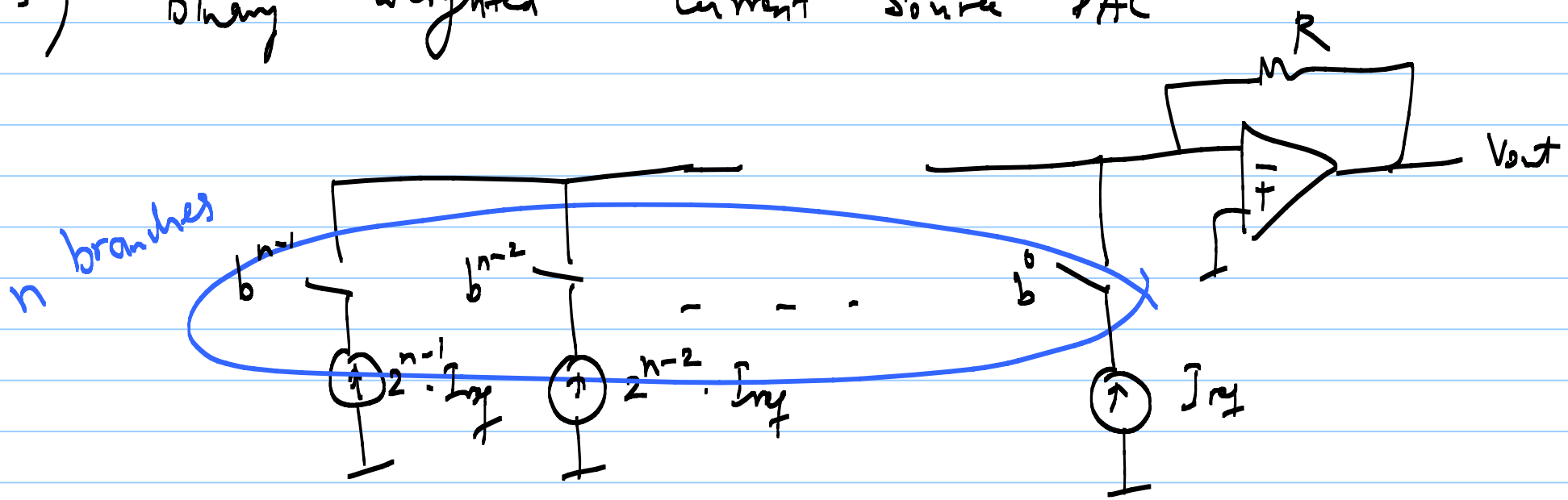
thermometer code



Binary to thermometer decoder



5) Binary Weighted Current source DAC



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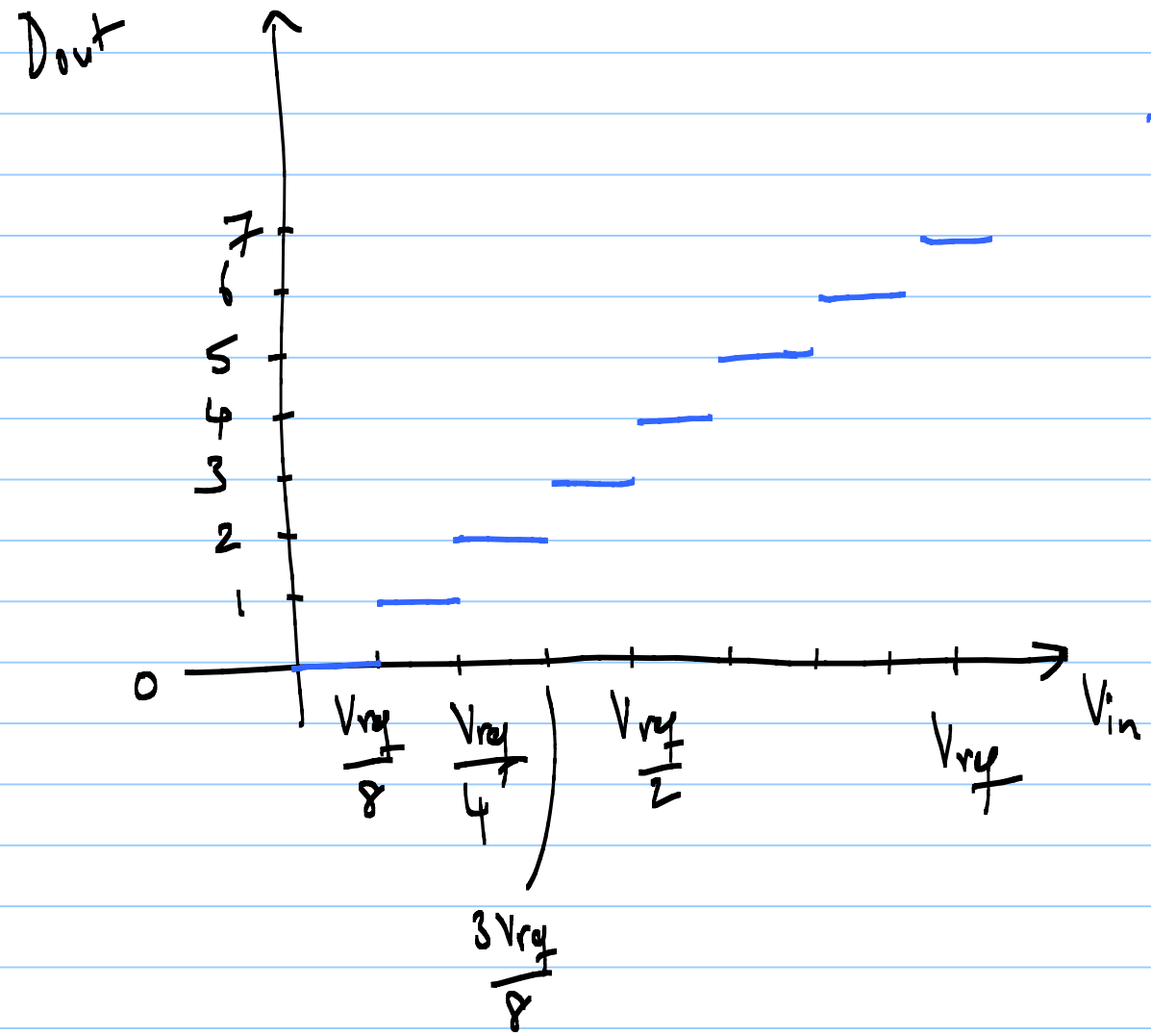


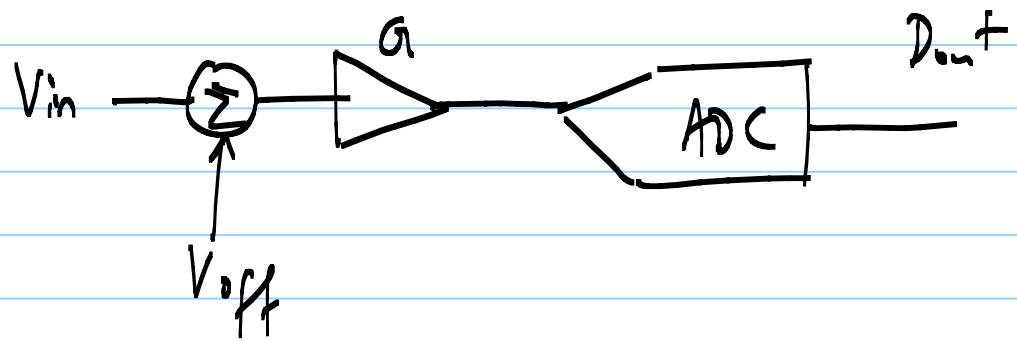
transition points

Ideally $V_{ADC}[k] = \underbrace{\frac{V_{ref}}{2^n}}_{1\ LSB} \cdot k$

3-bit ADC

1) End points : 0 & V_{ref}

$$\left(\frac{V_{ref}}{2^n} + V_{off}\right)G = V_{ADC}[1]$$



$$\left(\frac{V_{ref}}{2^n}\cdot(2^n-1) + V_{off}\right)G = V_{ADC}[2^n-1]$$

2) <u>Intermediate points</u> → can vary

$$\text{INL error} = \frac{V_k - \frac{V_{ref}}{2^n}\cdot k}{\left(V_{ref}/2^n\right)}$$

$2 \leq k \leq 2^n - 2$

integral non-linearity

ideal transition

actual transition

$$INL < \frac{1}{2}\,LSB$$

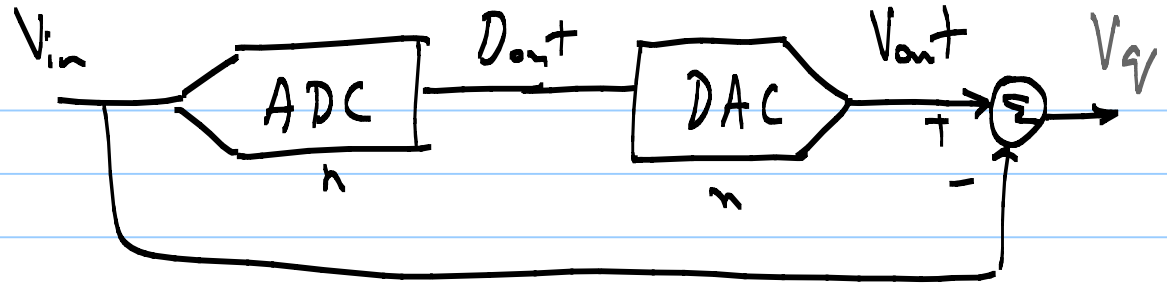
DNL error $= \dfrac{\left(V_k - V_{k-1}\right) - \dfrac{V_{ref}}{2^n}}{\left(\dfrac{V_{ref}}{2^n}\right)}$

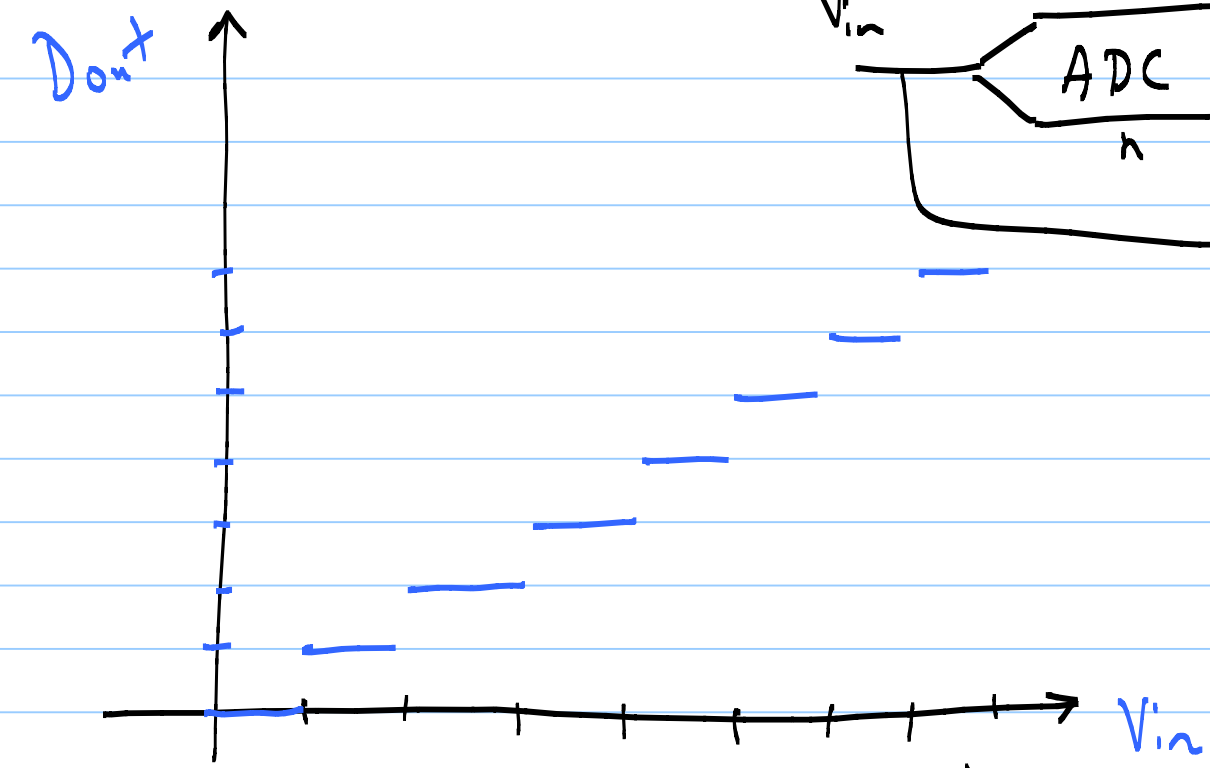
Differential non-linearity

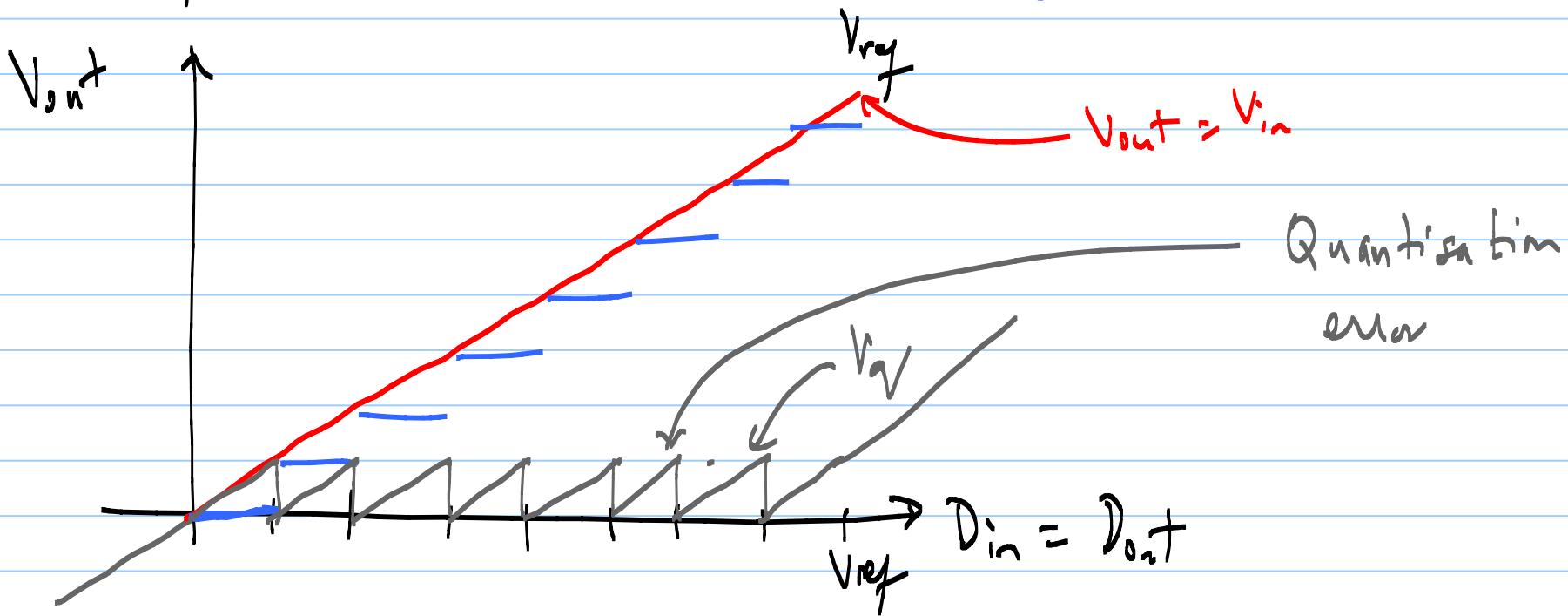
$$DNL < \frac{1}{2} LSB$$

ADC $\rightarrow$ Errors in transition voltage levels

DAC $\rightarrow$ Errors in output voltage levels


V_{in}
ADC
n
D_{out}
DAC
n
V_{out}
+
−
Σ
V_q


D_{out}
V_{in}

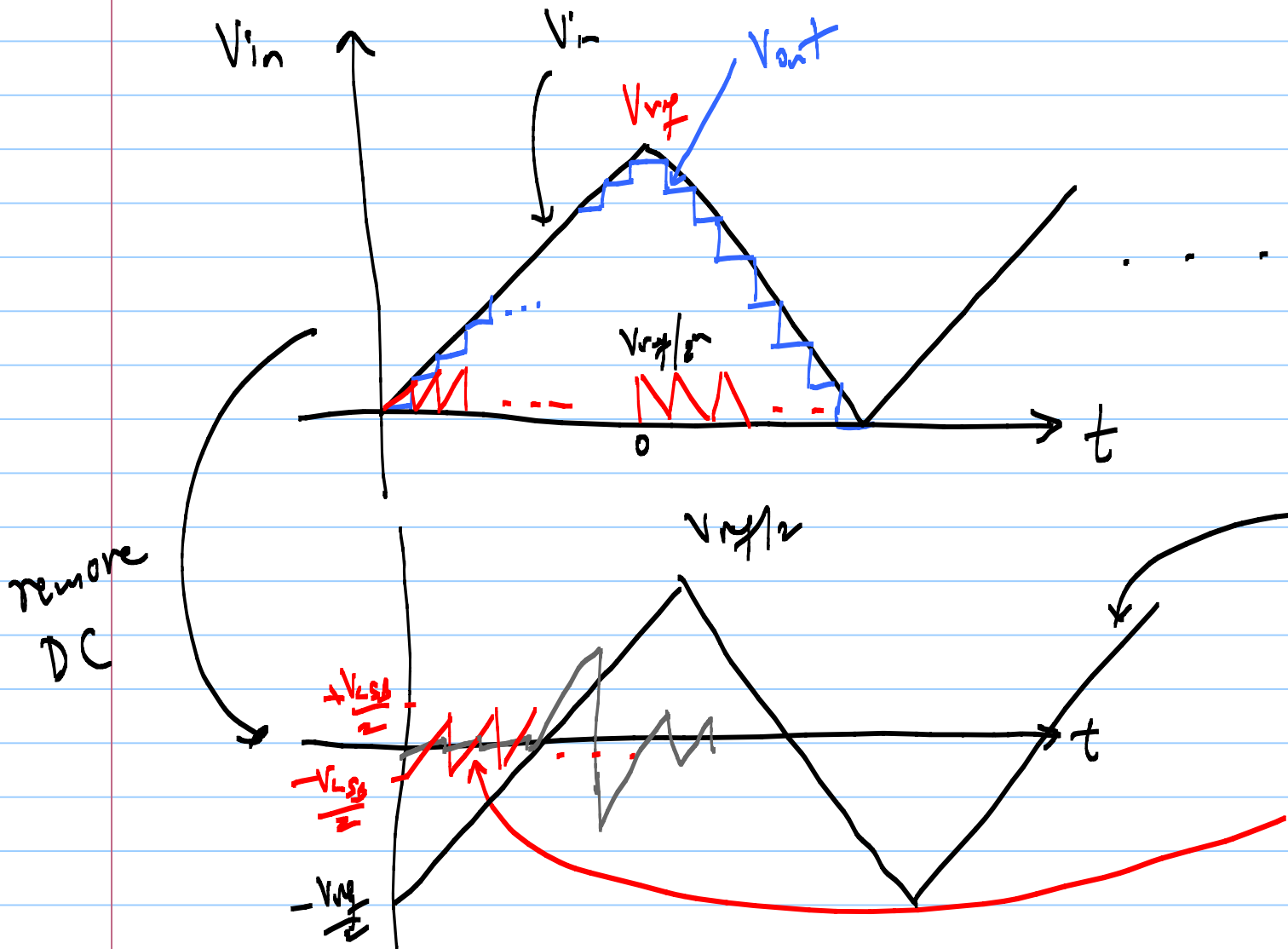

V_{out}
V_{ref}
$V_{out} = V_{in}$
Quantisation error
V_q
$D_{in} = D_{out}$
V_{ref}

$$SNR = 20 \log_{10}\left(\frac{S}{N}\right)$$

signal power

noise power

Signal-to-noise ratio



$$\text{rms value} = \frac{V_{ref}}{2\sqrt{3}} = S$$

$$\text{rms value} = \frac{V_{LSB}}{2\sqrt{3}} = N$$

Triangular input :

$$SNR = 20\log_{10}\left(\frac{S}{N}\right)$$

$$= 20\log_{10}\left(\frac{V_{ref}/2\sqrt{3}}{\frac{V_{ref}/2^n}{2\sqrt{3}}}\right)$$

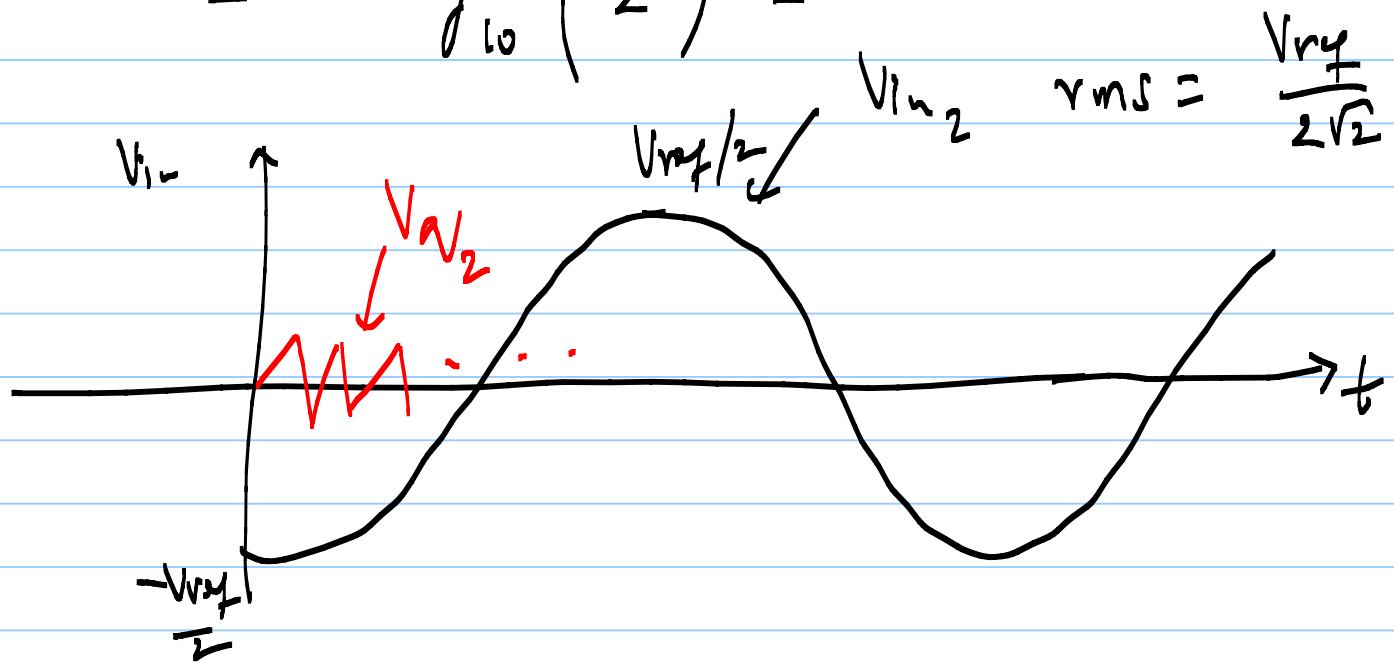
$$= 20\log_{10}\left(2^n\right) = 6.02\,n \text{ dB}$$

Sinusoidal input

It can be shown that:

V_{q_2} power $= V_{q_1}$ power

N is same as before



$$V_{in_2}\ rms = \frac{V_{ref}}{2\sqrt{2}}$$

$$S = \frac{V_{ref}}{2\sqrt{2}} \quad ; \quad N = \frac{V_{ref}/2^n}{2\sqrt{3}}$$

$$SNR = 20\log_{10}\left(2^n \cdot \sqrt{\frac{3}{2}}\right) = 6.02n + 1.76 \text{ dB}$$

e.g. 10 bit ADC should have SNR $\approx$ 62 dB

If actual SNR is lower $\Rightarrow$ ENOB

effective # of bits

is lower than actual expected # of bits

e.g. actual SNR = 55 dB

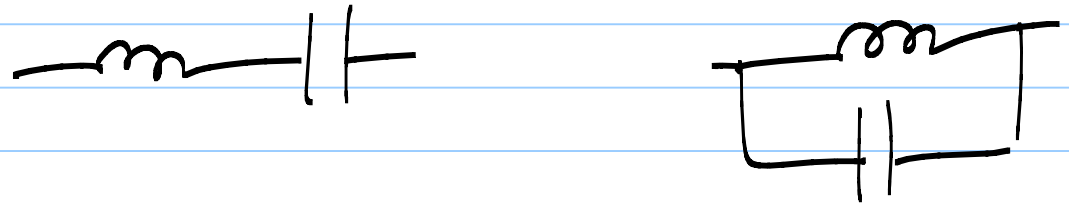
ENOB = 8.8

13/04/2022

Lecture 41

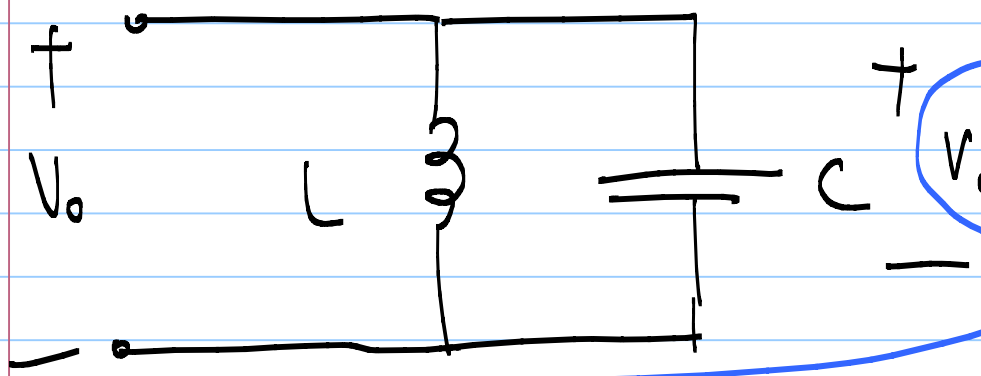
Sinusoidal Oscillators

* LC



* pair of complex conjugate poles on $j\omega$ axis

Ideal, lossless L & C



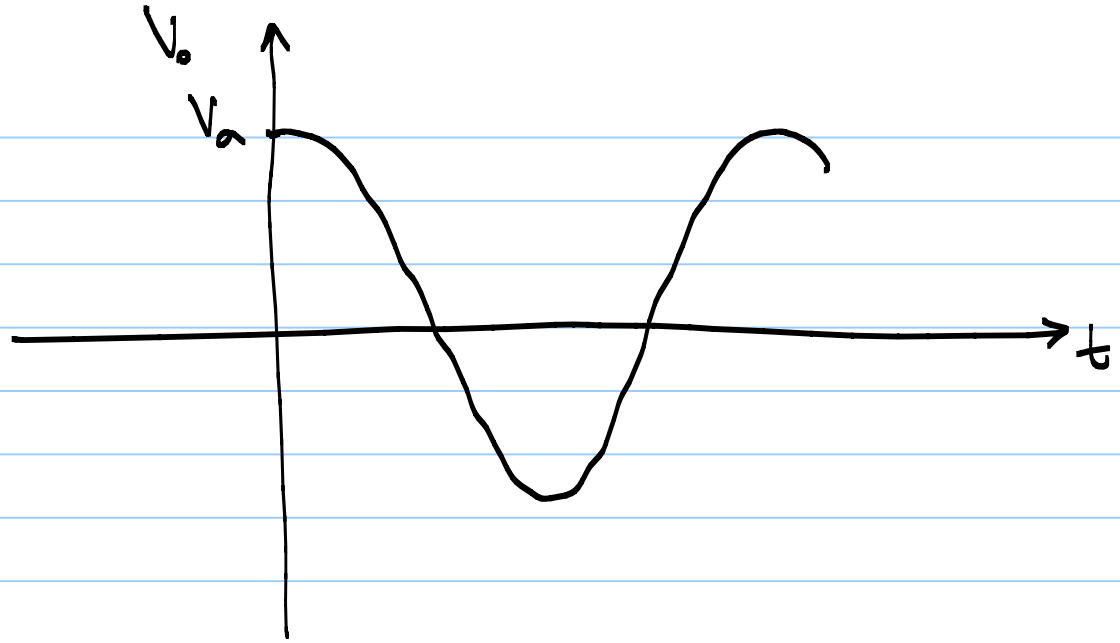
$V_c(t=0^-) = V_a$

$I_L(t=0^-) = 0$

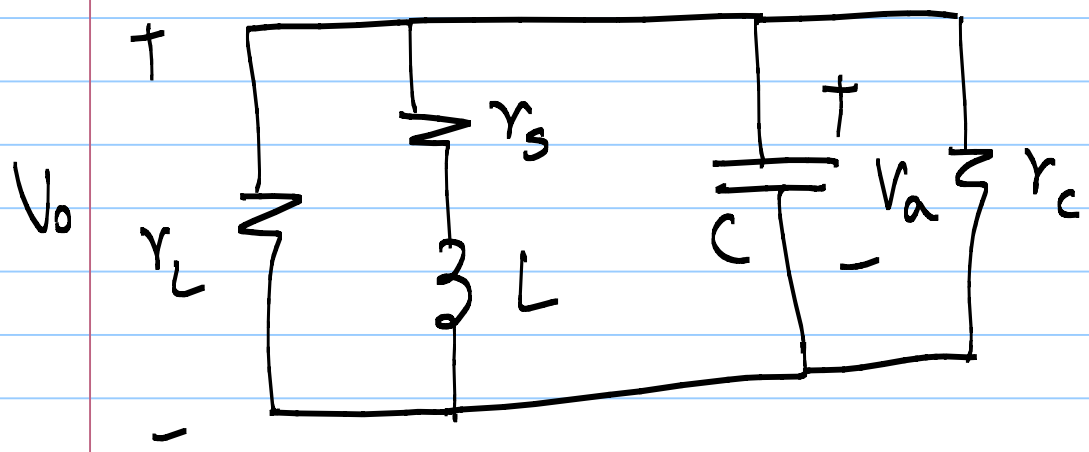
$V_o(t) = ?$

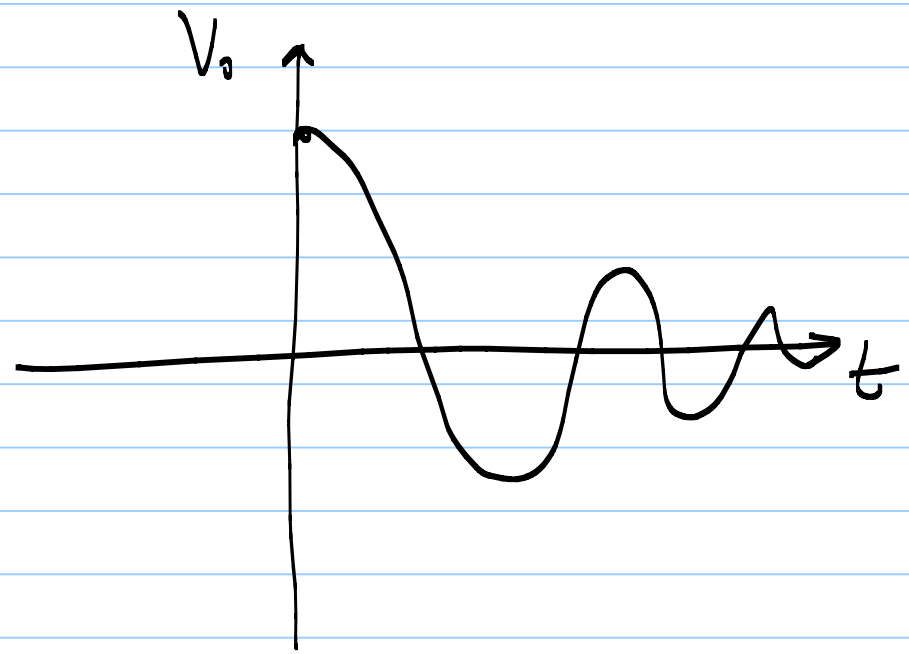
general form: $V_o(t) = V_p \cos\left(\frac{t}{\sqrt{LC}} + \varphi\right)$

here: $V_p = V_a$; $\varphi = 0$



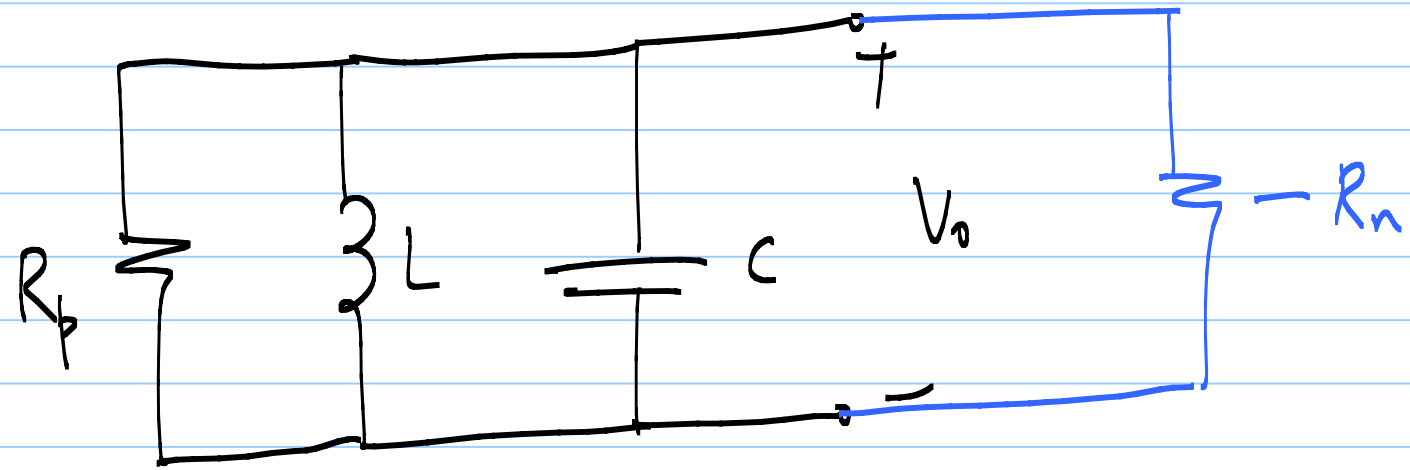
Real system

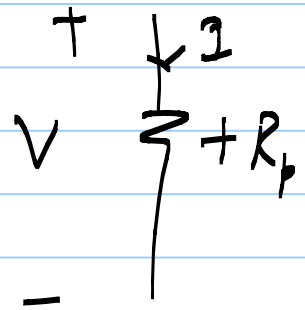


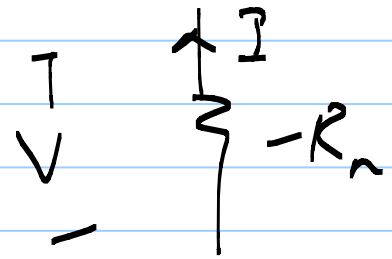


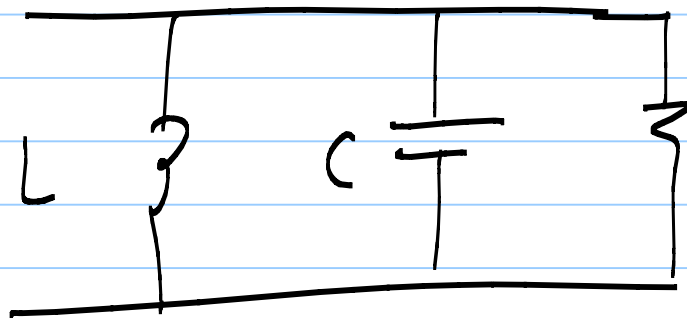
LHP poles

Representative LC system





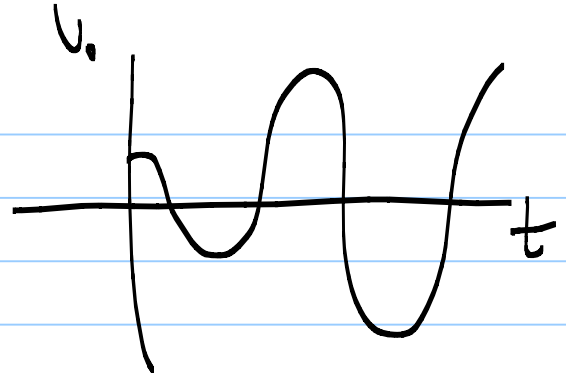




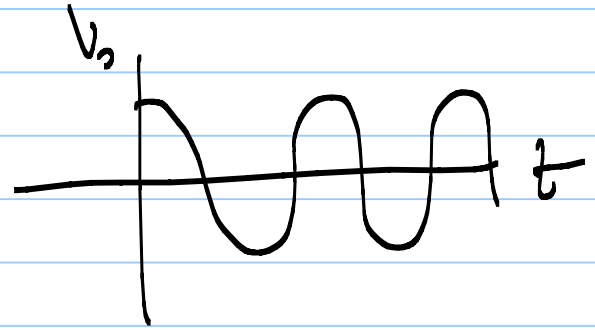
$$R_{eq} = \frac{-R_p \cdot R_n}{R_p - R_n} = \frac{R_p R_n}{R_n - R_p}$$

$$= \infty \text{ if } R_n = R_p$$

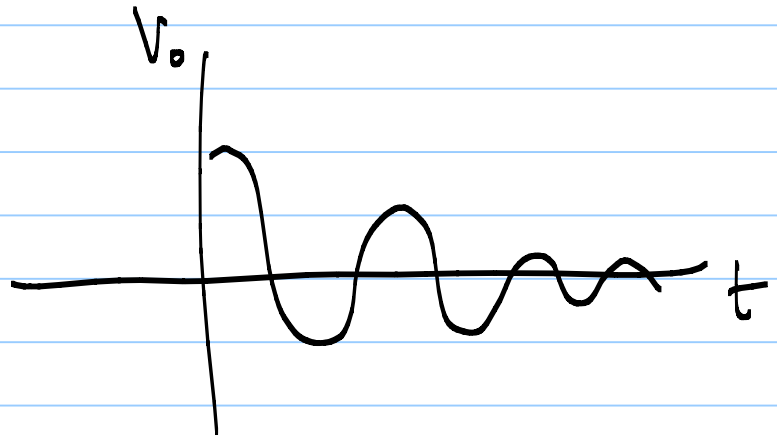
$|-R_n| < |R_p| \Rightarrow$ RHP poles

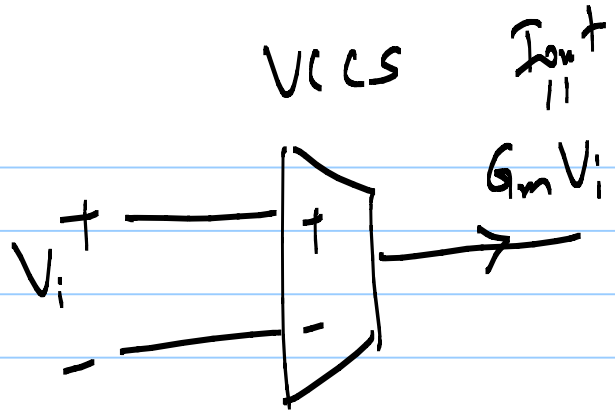


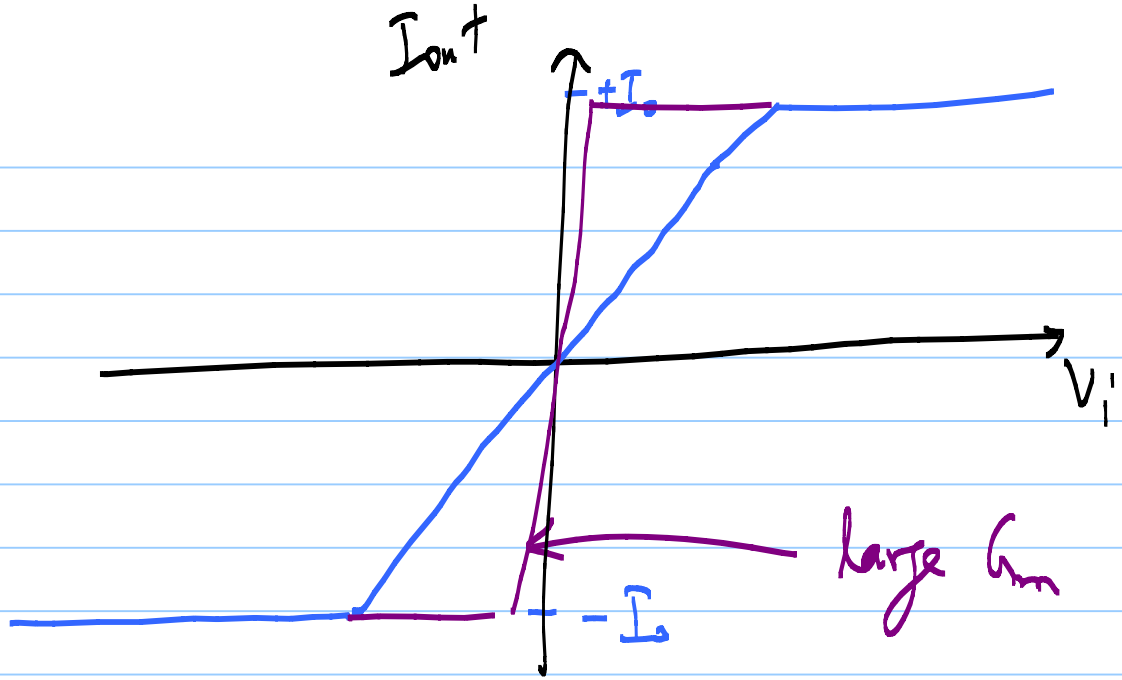
$|-R_n| = |R_p| \Rightarrow$ poles on $j\omega$ axis

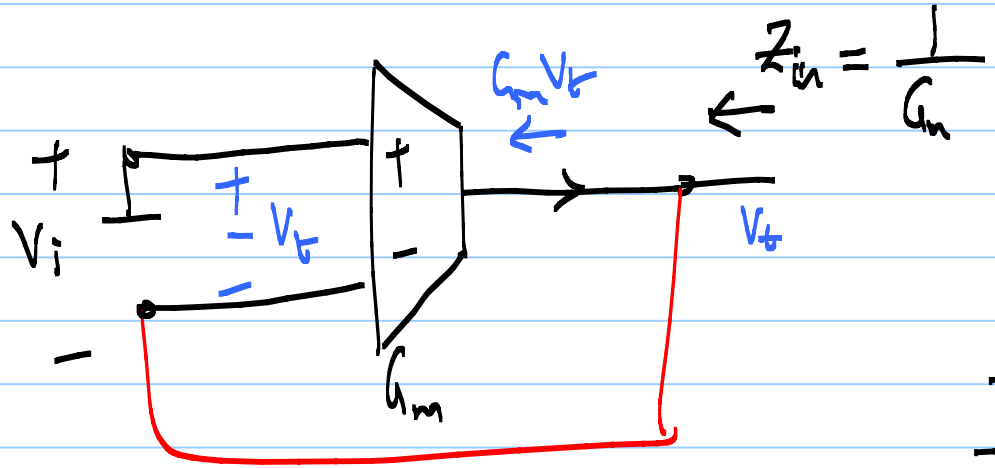


$|-R_n| > |R_p| \Rightarrow$ LHP poles

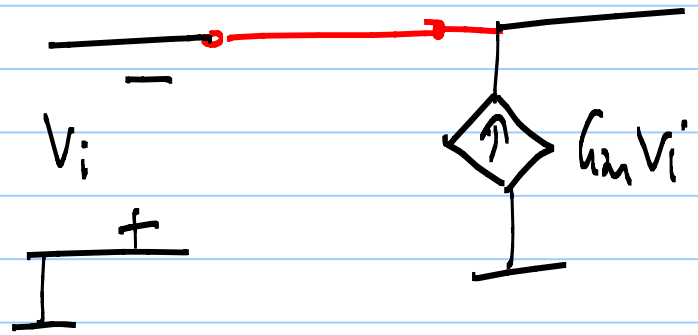


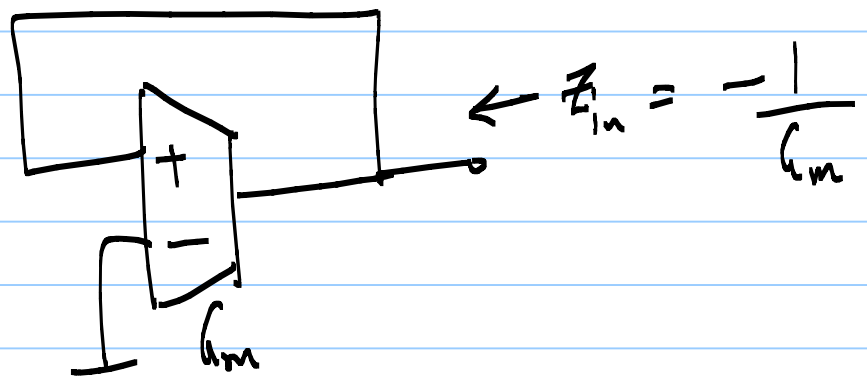




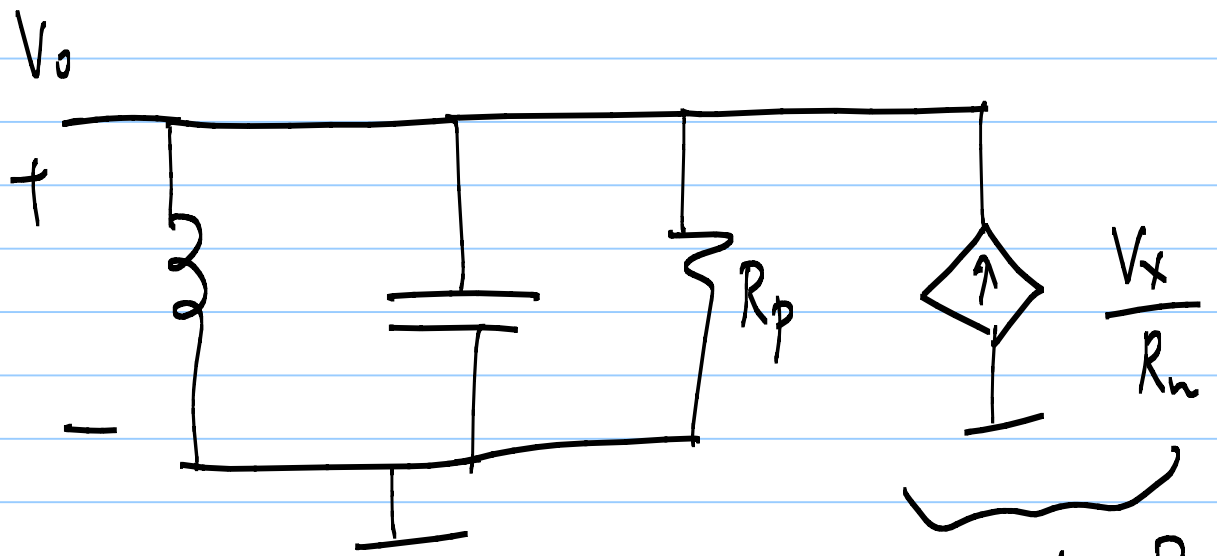


$$Z_{in} = \frac{1}{G_m}$$





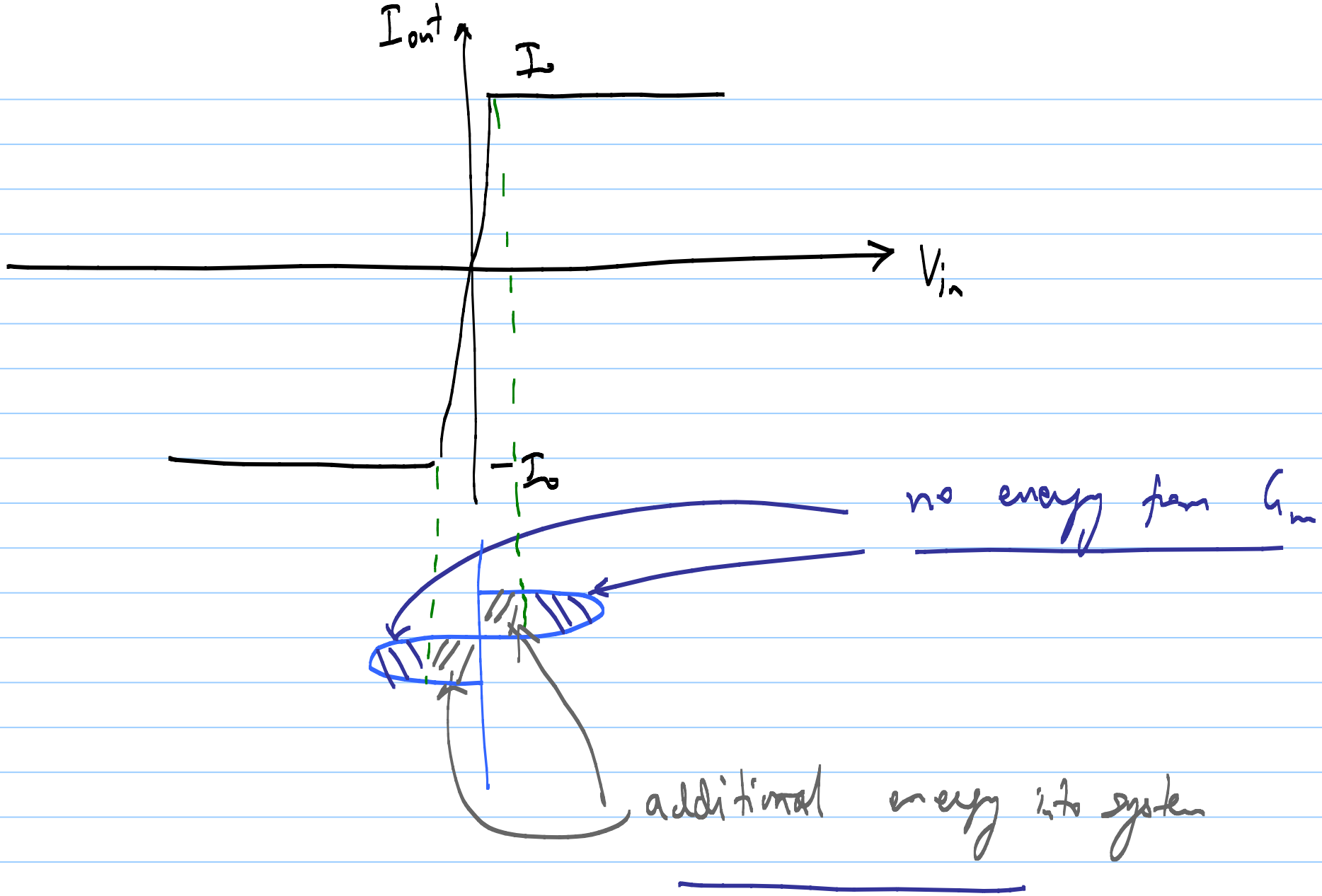
$\leftarrow Z_{in} = -\dfrac{1}{G_m}$ (small signal)



$\dfrac{V_x}{R_n} = \dfrac{V_o}{R_n}$

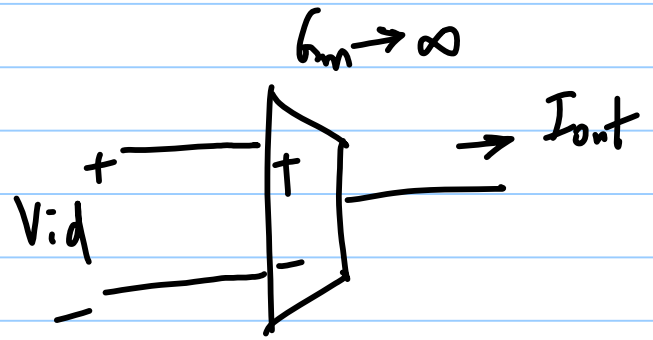
$\underbrace{\qquad\qquad}_{-ve\ R_n}$

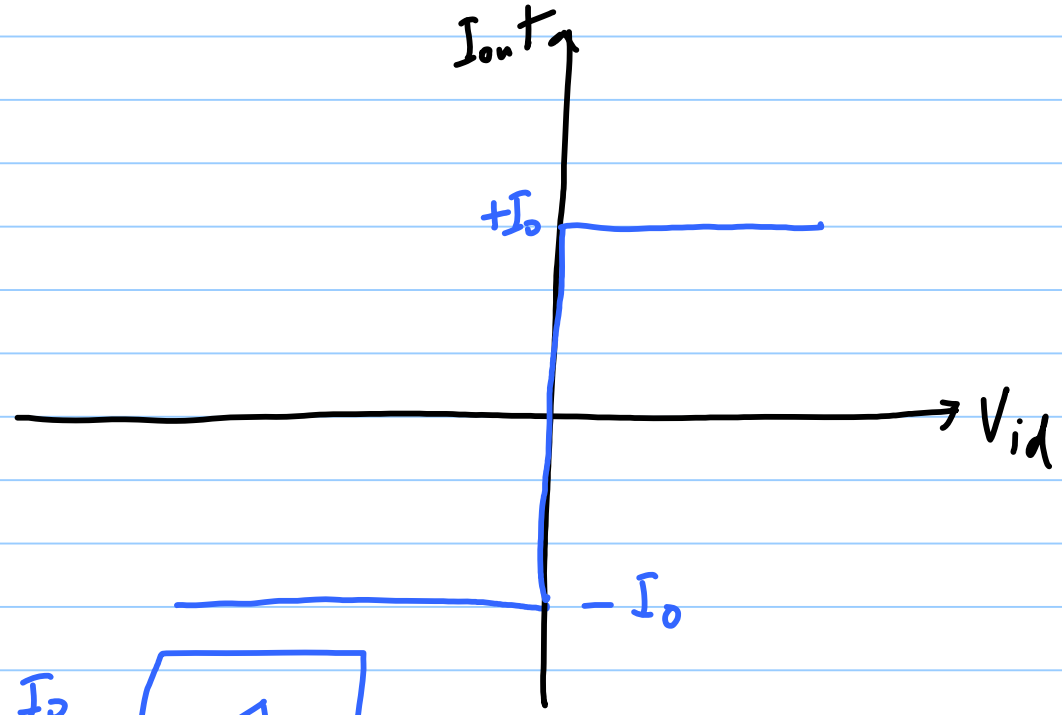
Set $|-R_n| < |R_p|$

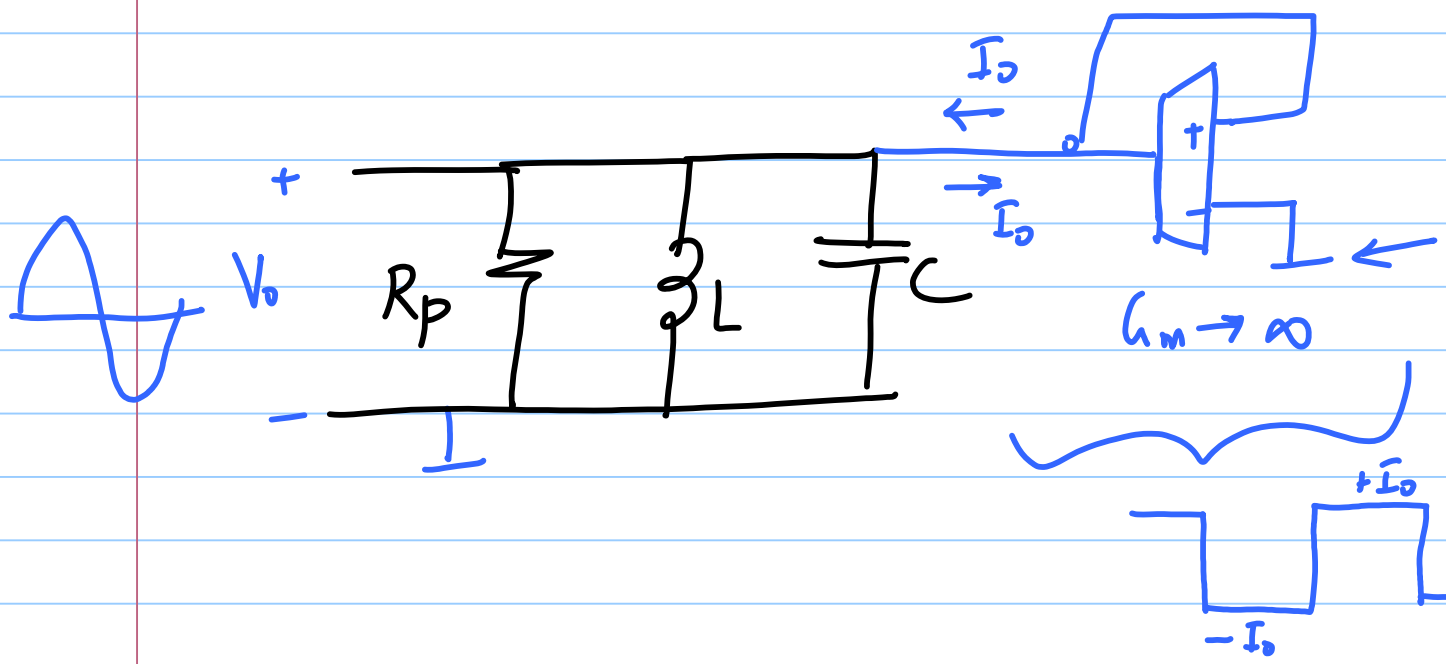

I_{out}
I_0
V_{in}
$-I_0$
no energy from G_m
additional energy into system

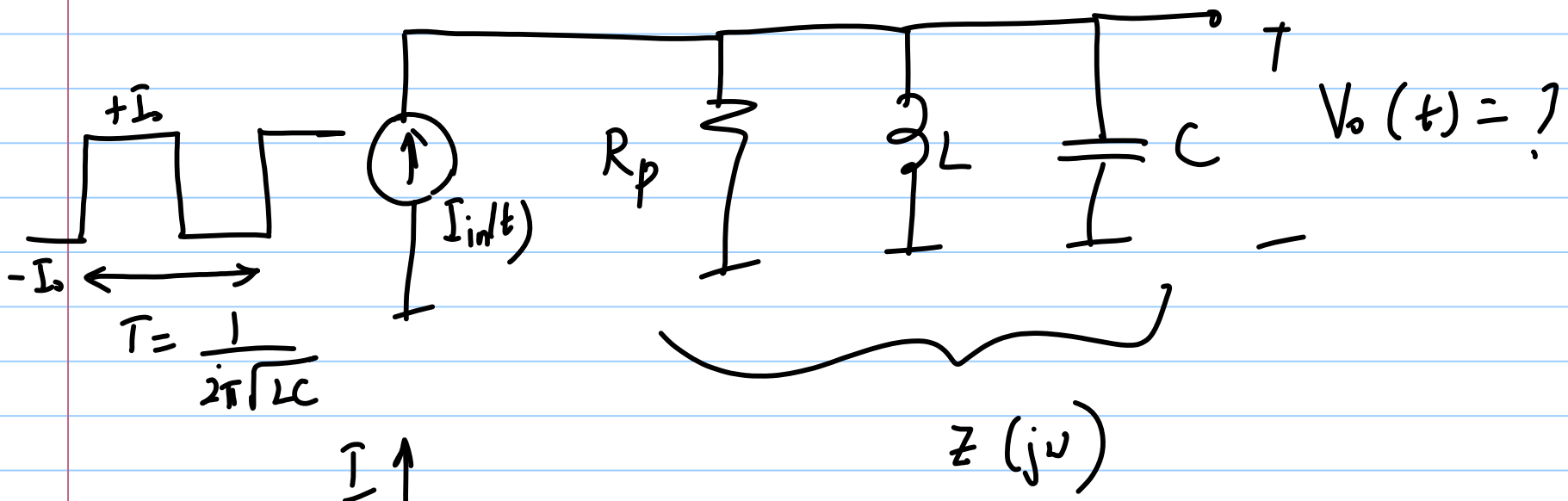
19/04/2022

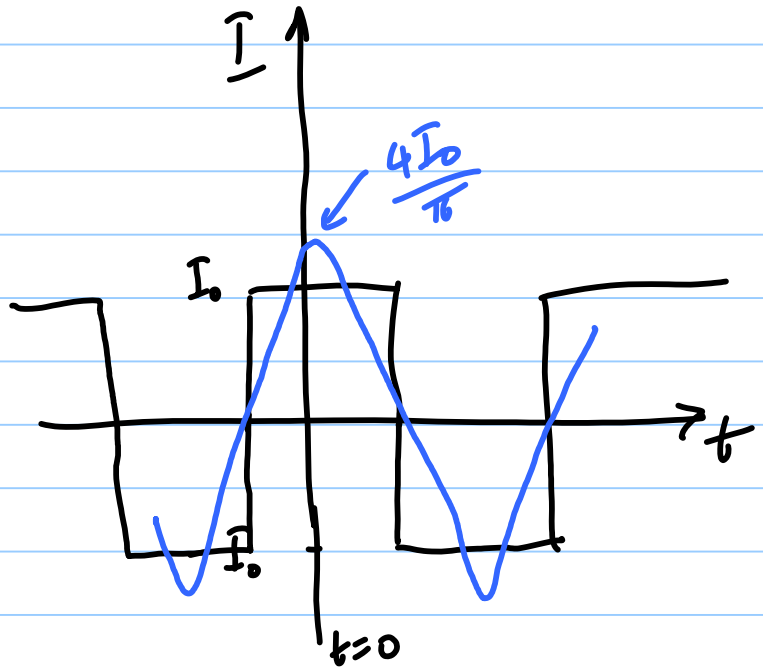
Lecture 42







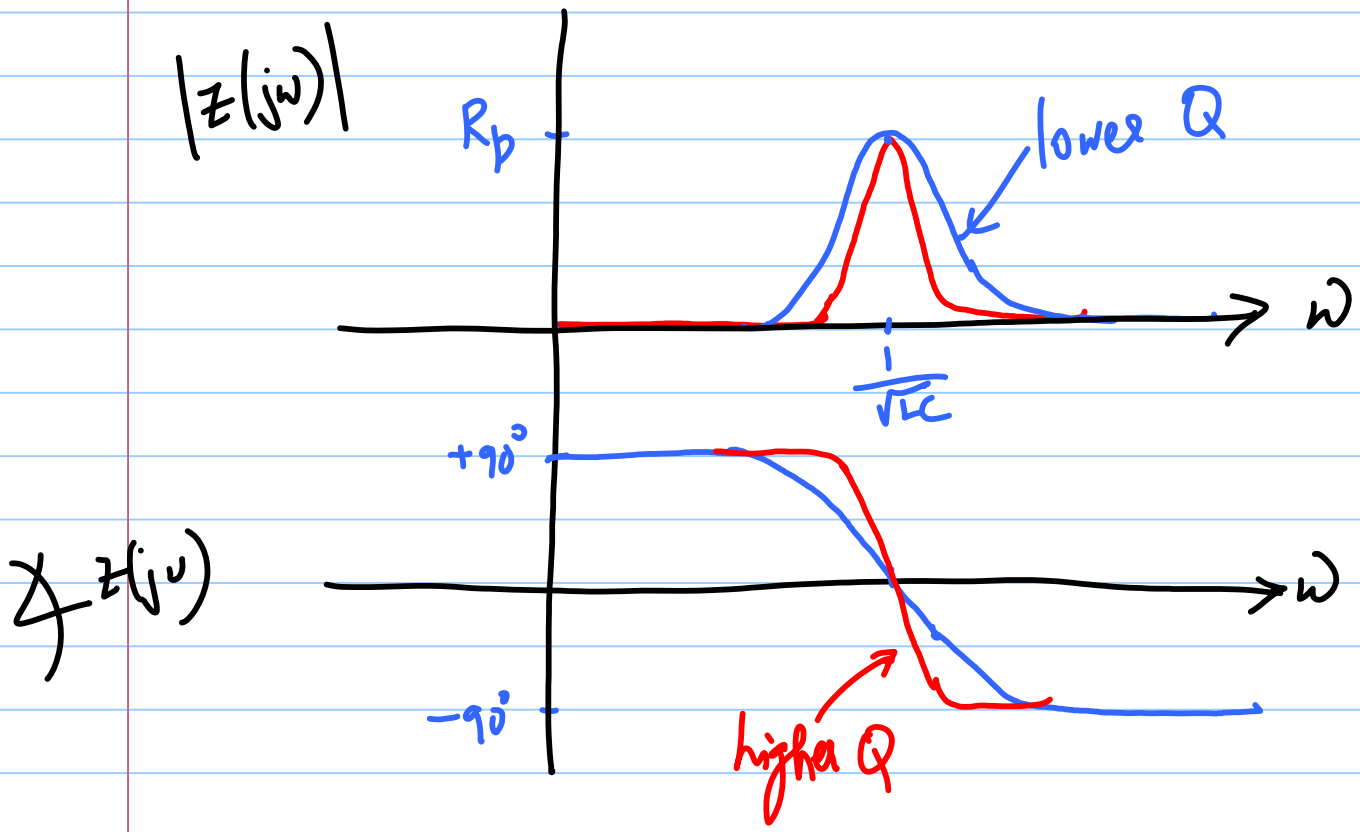


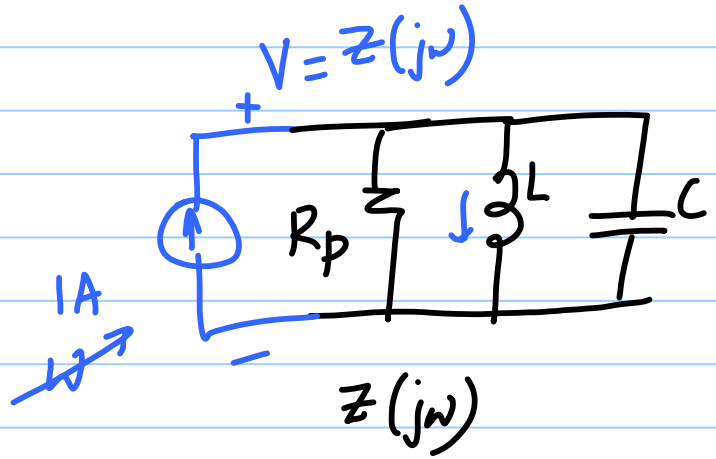


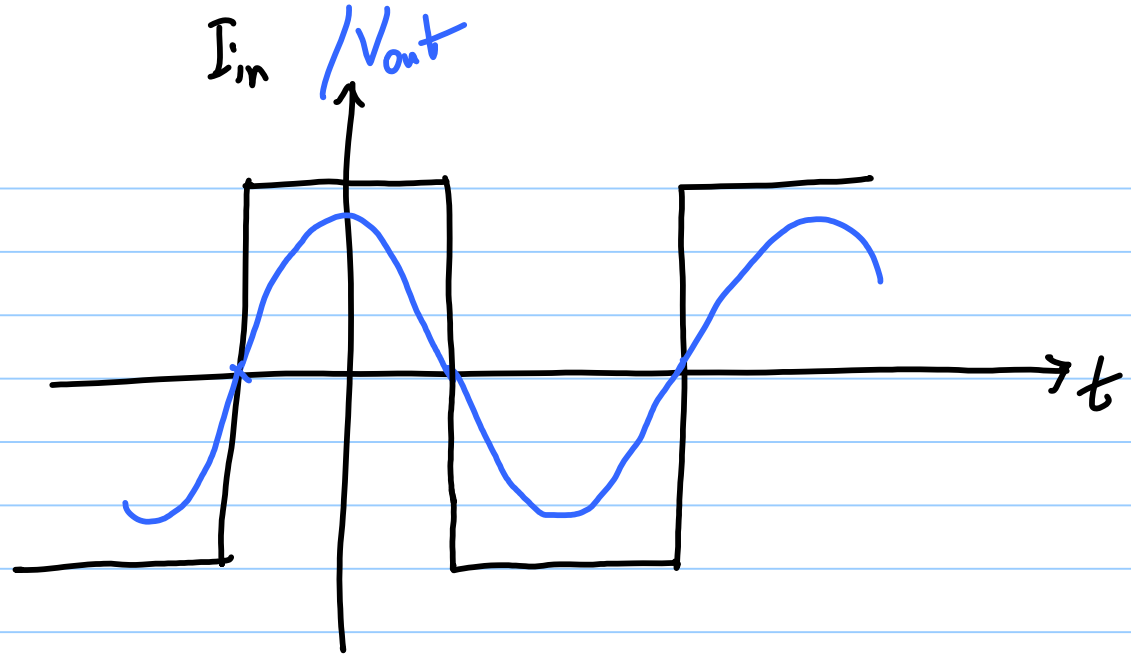
$$I_{in}(t) = \sum_{k=1}^{\infty} \frac{4 I_0}{(2k-1)\pi} \cos\left((2k-1)\frac{t}{\sqrt{LC}} + \phi_k\right)$$

* Only add terms
* higher terms go down in amplitude

$$V_o(t) = \sum_{k=1}^{\infty} \left| Z\left(j\frac{(2k-1)}{\sqrt{LC}}\right) \right| \frac{4I_o}{(2k-1)\pi} \cdot \cos\left[\frac{(2k-1)t}{\sqrt{LC}} + \phi_k + \angle Z\left(\frac{j(2k-1)}{\sqrt{LC}}\right)\right]$$







$$V_o = V_a \cos \omega_0 t$$

$$V_o(t) = \left[V_a + v_n(t)\right] \cdot \cos(\omega_0 t + \varphi_n)$$

Amplitude noise

phase noise

20/04/2022

Lecture 43

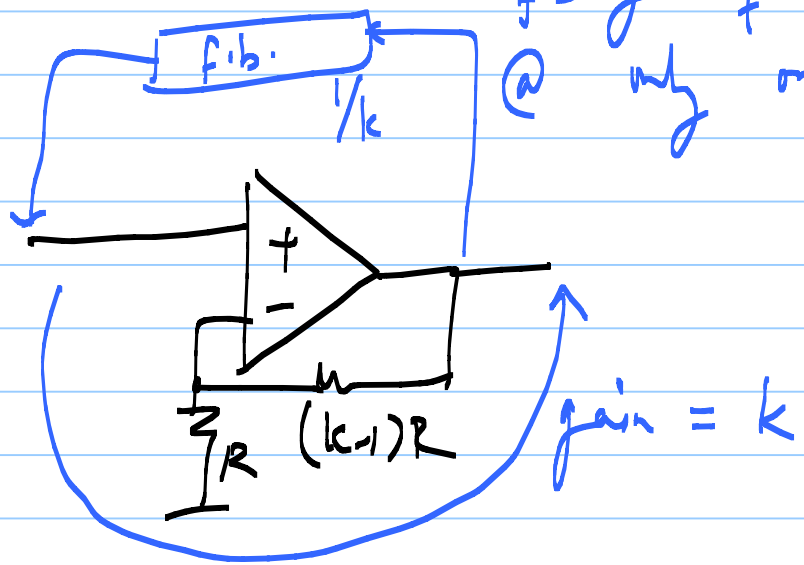
LC Oscillators

Ring Osc

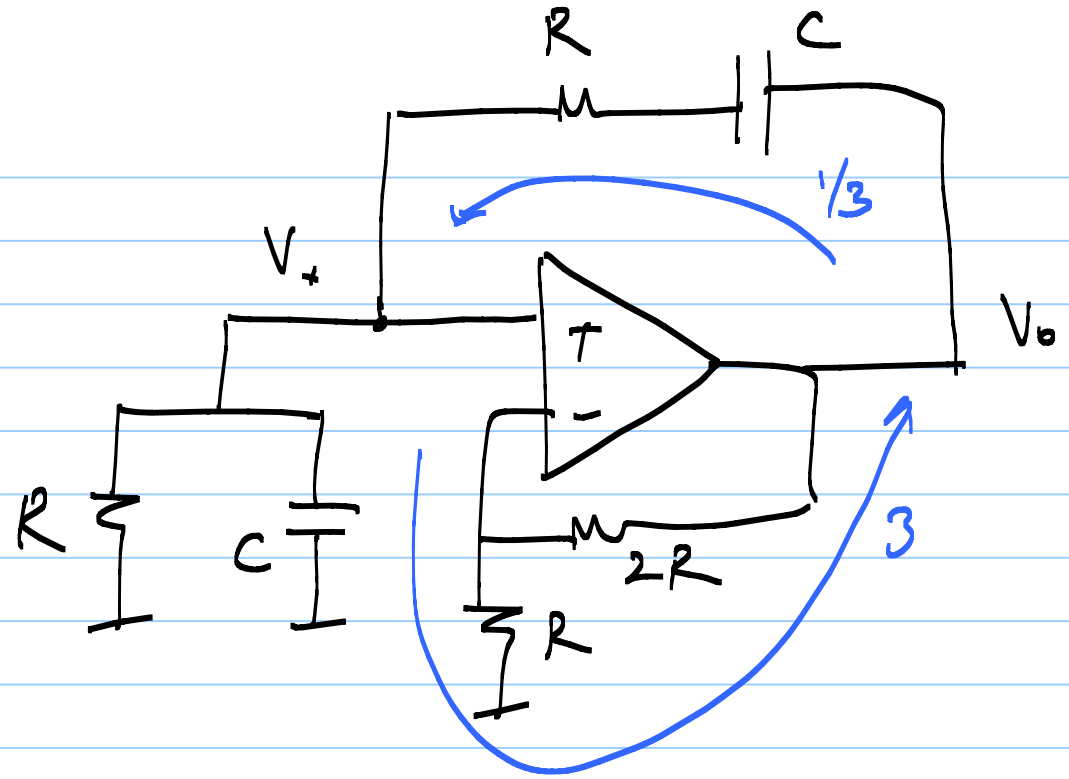
Relaxation Osc (Schmitt trigger)

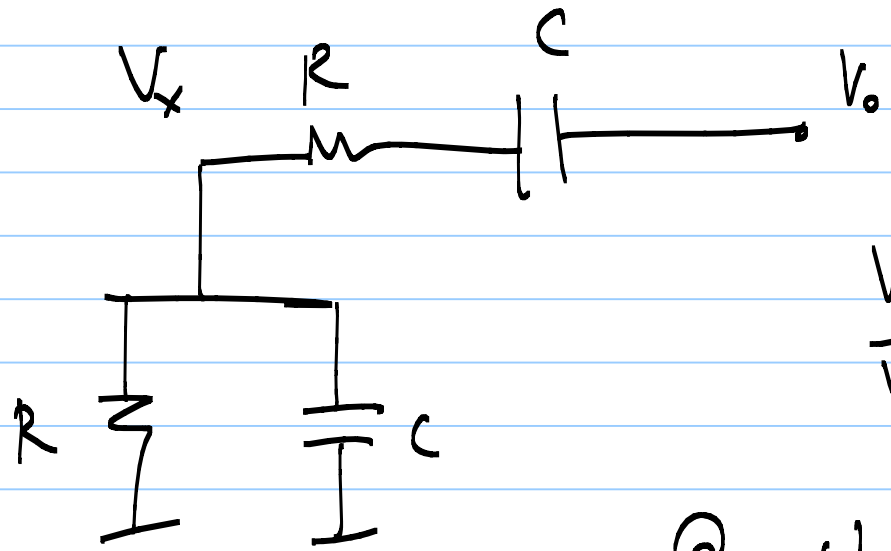
increasing spectral purity ($\downarrow \phi_n$)

Wien Bridge Osc.



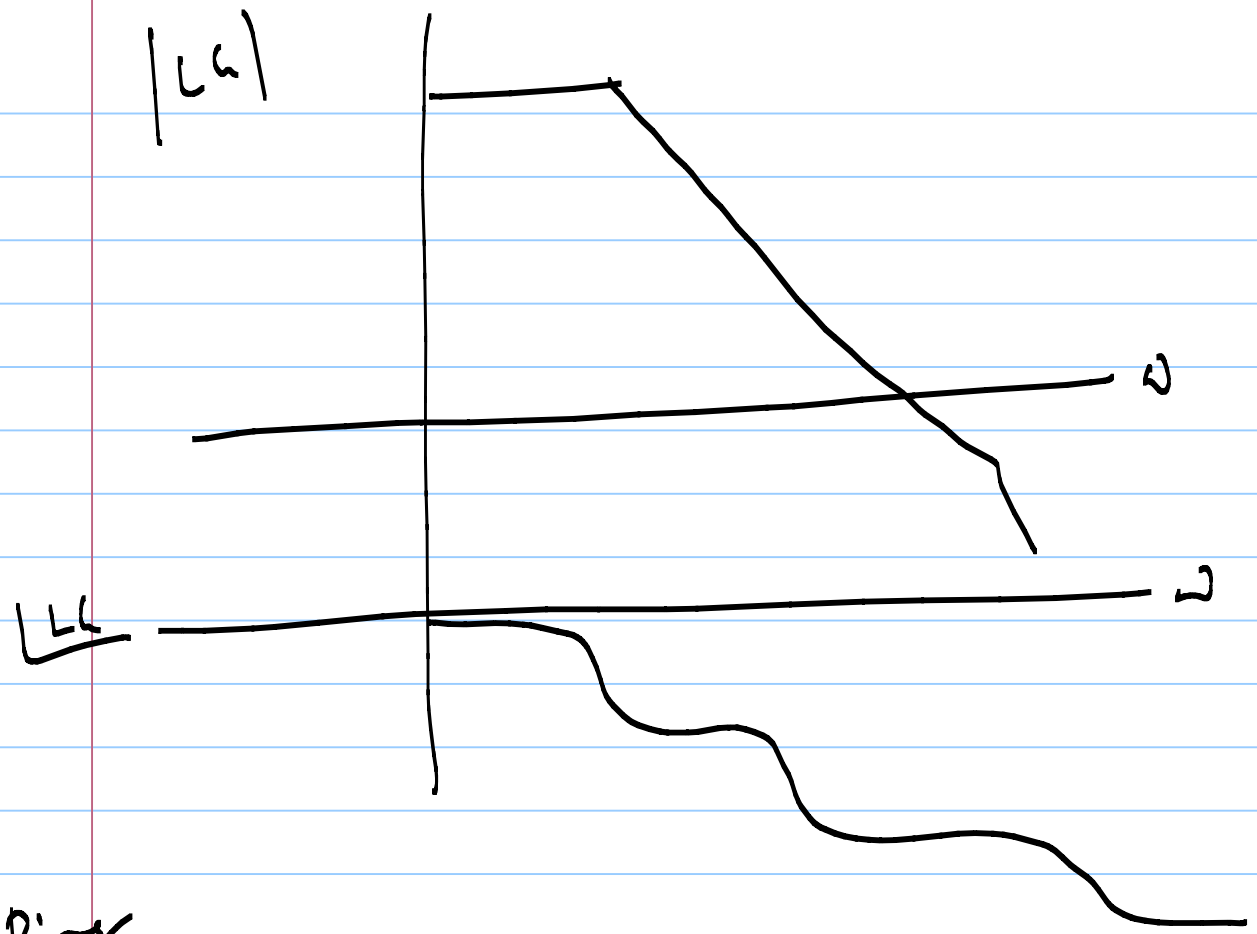
fib. gain of 1/k @ only one freq. ω_0





$$\frac{V_x}{V_o}(s) = \frac{sCR}{s^2C^2R^2 + 3sCR + 1}$$

$$@ \quad \omega_0 = \frac{1}{RC} \quad \Rightarrow \quad \frac{V_x}{V_o} = \frac{1}{3}$$



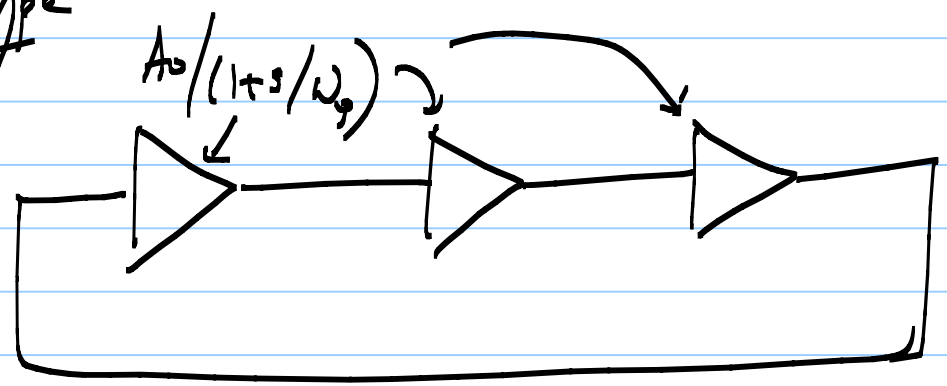
$$CLG(s) = \frac{LG(s)}{1+LG(s)} \cdot \frac{1}{f}$$

$$PM = 180^\circ + \angle LG(j\omega)\Big|_{|LG|=1}$$

$$\left.\begin{array}{l} |LG| = 1 \\ \angle LG = -180^\circ \end{array}\right\}$$ CLG poles on $j\omega$ axis

CL system can have output without input

Ring Osc. Prototype



At least a 3rd order system

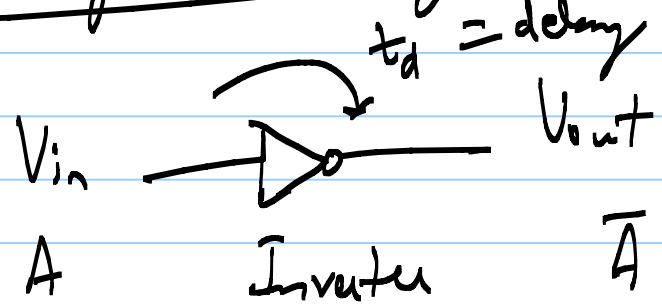
$$LG(s) = \frac{A_0^3}{\left(1 + s/\omega_p\right)^3}$$

$$|LG| = 1 \quad ; \quad \angle LG = -180^\circ$$

$$A_0 = 2$$

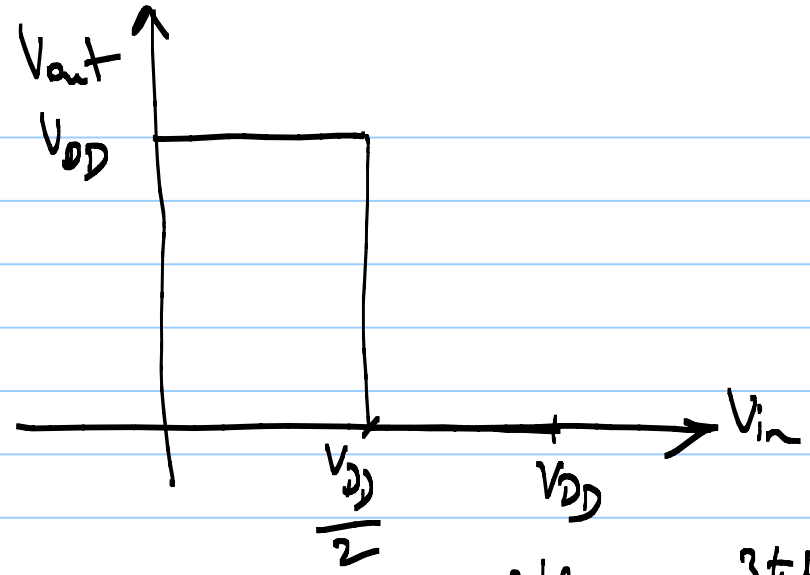
$$\omega_0 = \sqrt{3}\,\omega_p$$

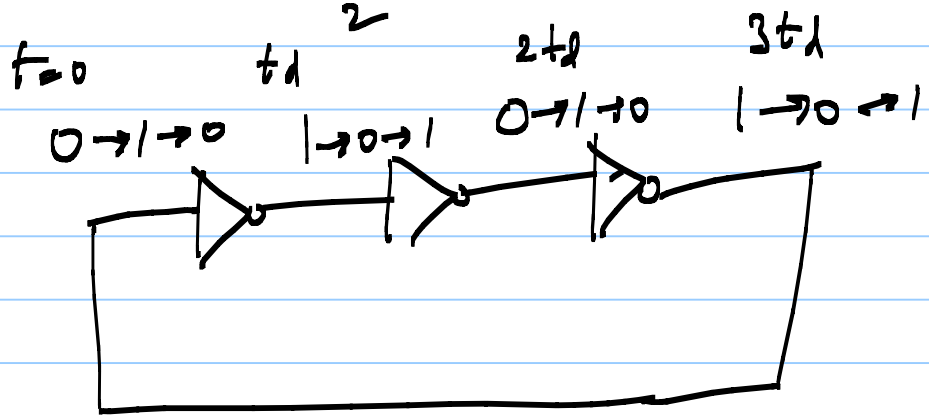
Digital Ring Osc.

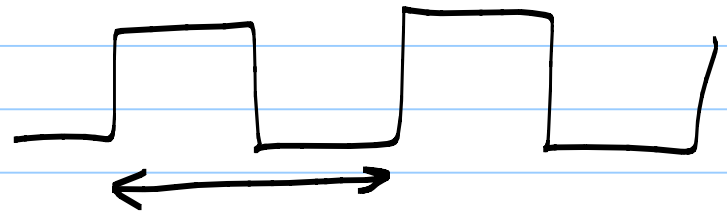


$1 = V_{DD}$

$0 = GND$ '0V'

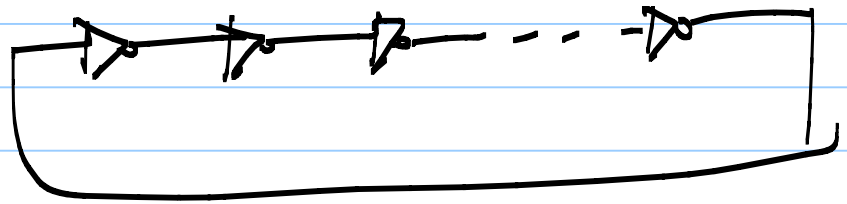






$T = 6t_d$

$(2n+1)$ inv.



$$T = \frac{1}{2(2n+1)t_d}$$

21/04/2022

Lecture 44

1) Dominant pole compensation

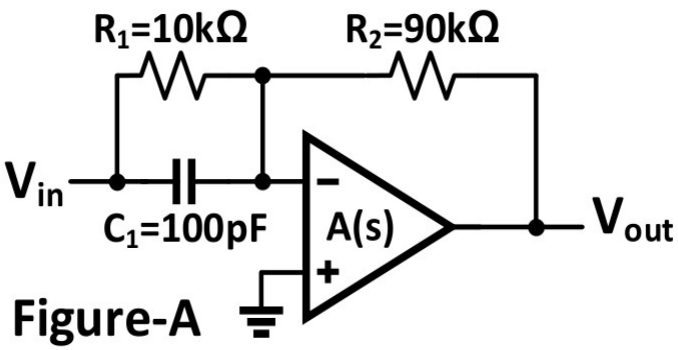


Figure-A

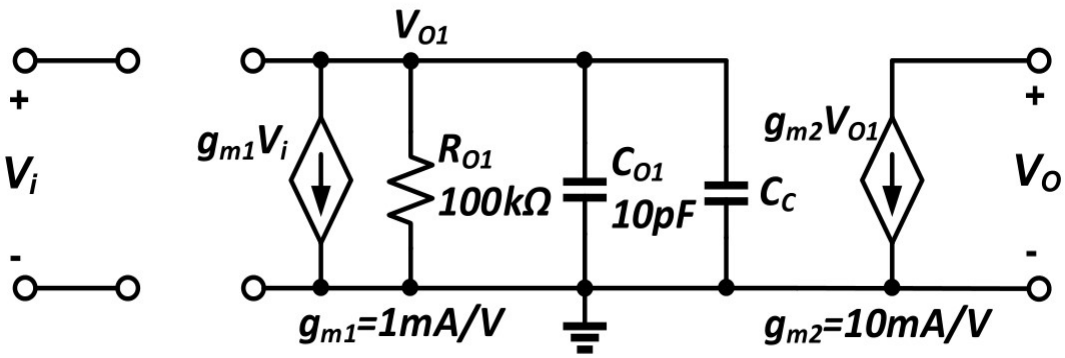


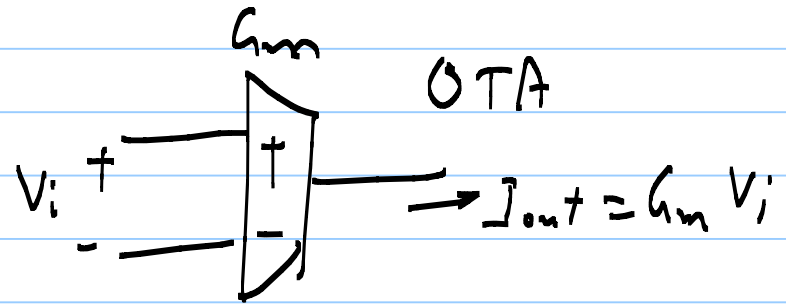
Figure-B

Closed loop amplifier shown in Figure-A needs to be designed for stable operation. Considering the op-amp (OA) model shown in Figure-B

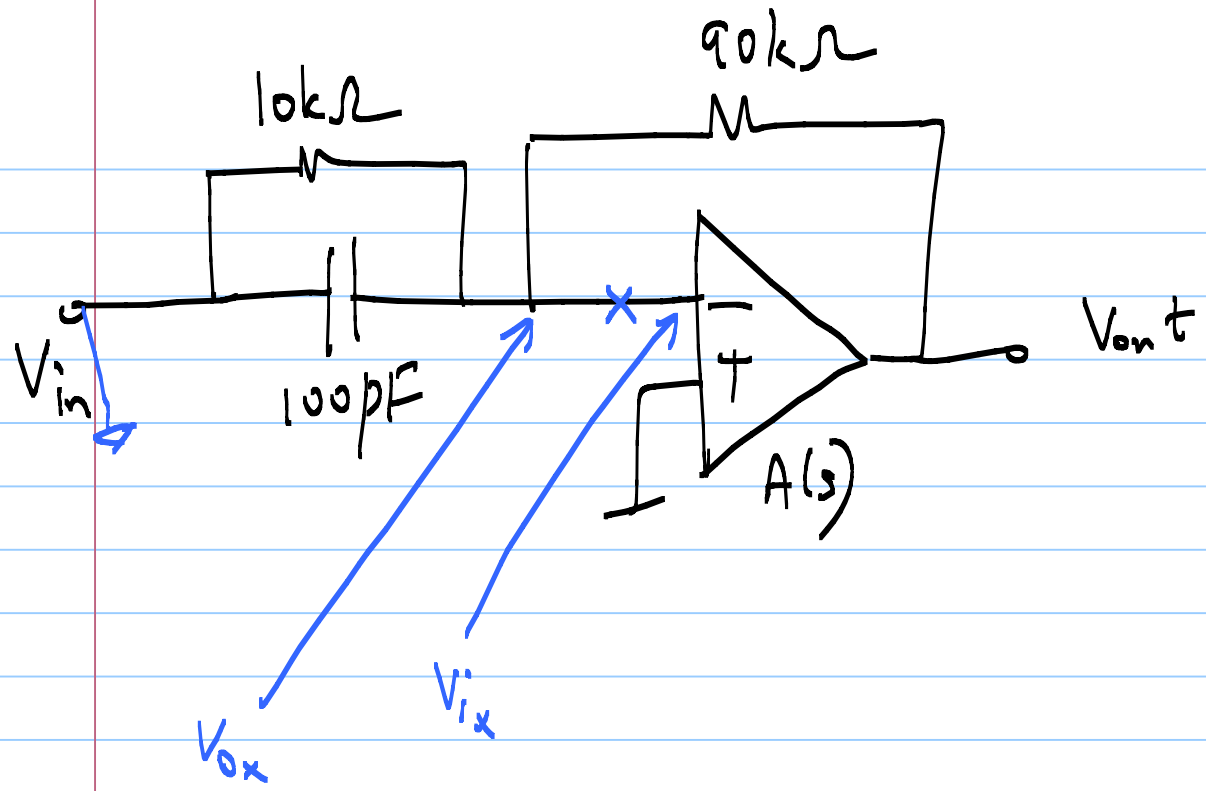
a) Find the loop gain transfer function

b) Calculate the value of capacitor C_C to achieve the phase margin of 60 degrees

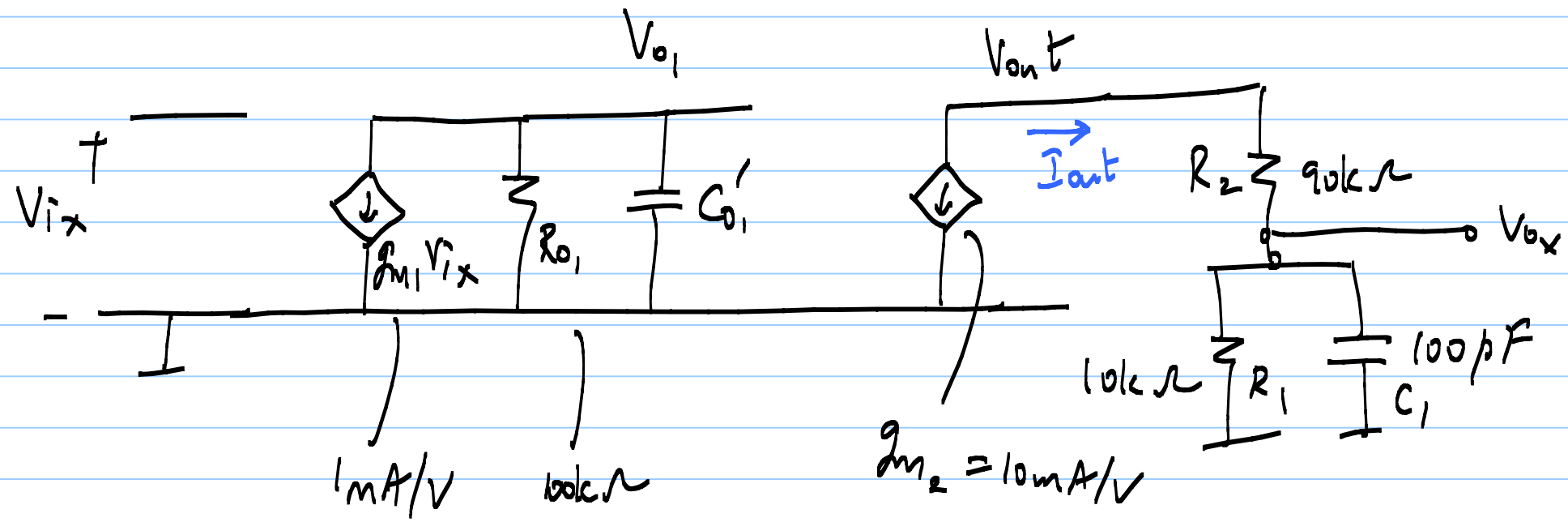
c) Draw the bode magnitude and phase plot of the loop gain transfer function after the compensation. Clearly mention values of gain, phase and location of poles, zeroes and unity gain frequency.



opamp $A(s) = \dfrac{g_{m1} R_{o1} \cdot g_{m2}}{1 + s\,(C_{o1} + C_c)\,R_{o1}}$ $\left(s \sim A/V\right) = \dfrac{I_{out}}{V_i}(s)$



$$L_G(s) = \frac{V_{ox}}{V_{ix}}(s)$$



$$V_{ox} = I_{out} \cdot \frac{R_1}{1 + sC_1R_1}$$

$$= V_{ix} \cdot A(s) \cdot \frac{R_1}{1 + sC_1R_1}$$

$$LG(s) = \frac{V_{ox}}{V_{ix}}(s) = \frac{g_{m1} R_{o1} \cdot g_{m2} \cdot R_1}{(1 + sC_{o1}'R_{o1})(1 + sC_1R_1)}$$

<u>without C_c</u>:

$$LG_0(s) = \frac{g_{m1} R_{o1} \; g_{m2} R_1}{\underbrace{(1 + sC_{o1}R_{o1})}_{p_1}\underbrace{(1 + sC_1R_1)}_{p_2}}$$

$$LG_o(s) = \frac{10^4}{(1+10^{-6}s)(1+10^{-6}s)}$$

Add C_c to move in p_1
need $60°$ PM

Dominant pole compensation

$$\angle LG = -120°$$

$$\left(\frac{\omega_n}{p_2}\right) = \frac{1}{\sqrt{3}} \Rightarrow \omega_n = \frac{10^6}{\sqrt{3}} = 577.35 \text{ k rad/s}$$

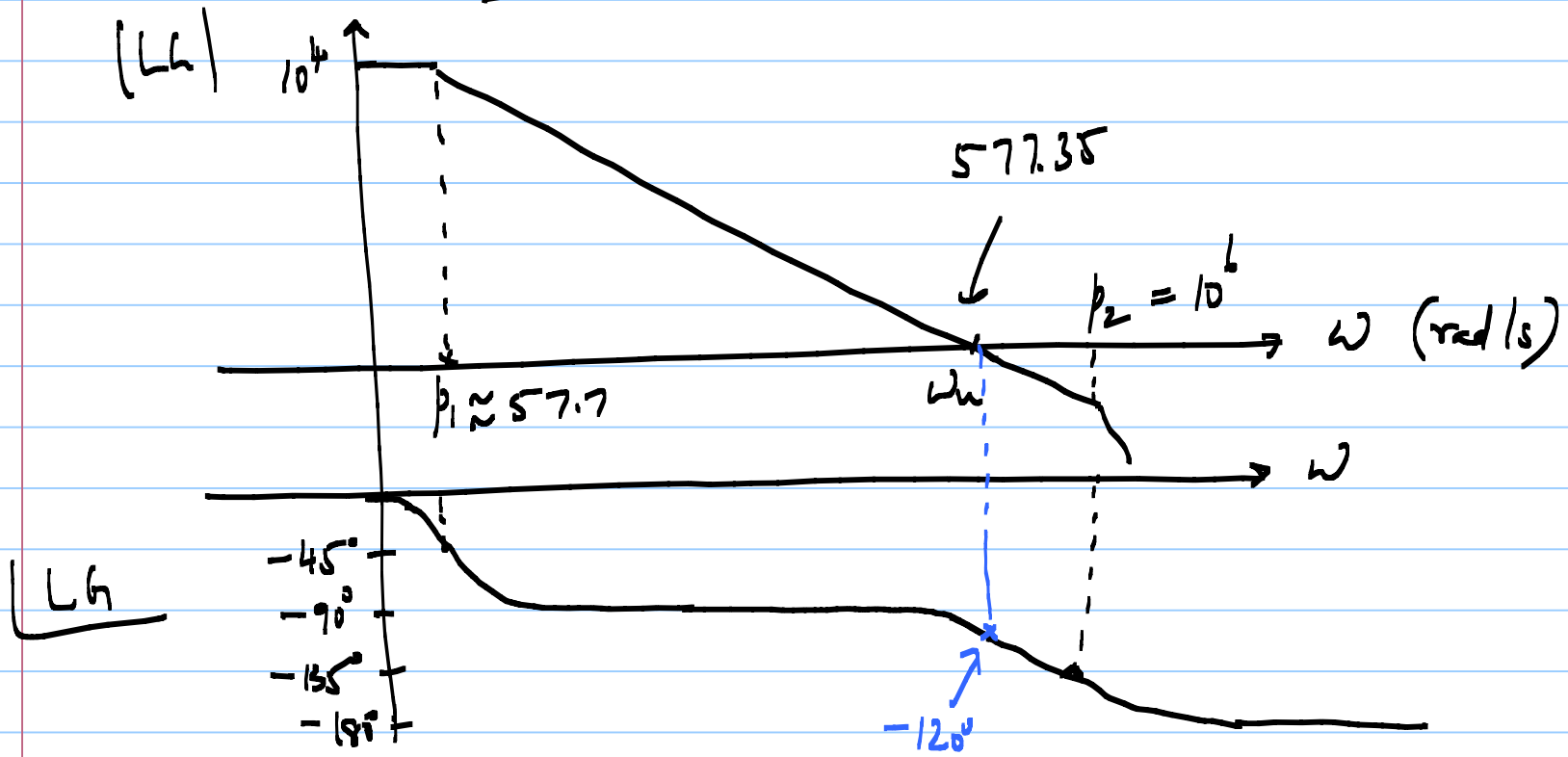
$$p_1 = \frac{\omega_n}{LG(0)} = \frac{577.35 \times 10^3}{10^4} = 57.735 \text{ rad/s}$$

$$p_1 (Hz) = 9.2 \text{ Hz}$$

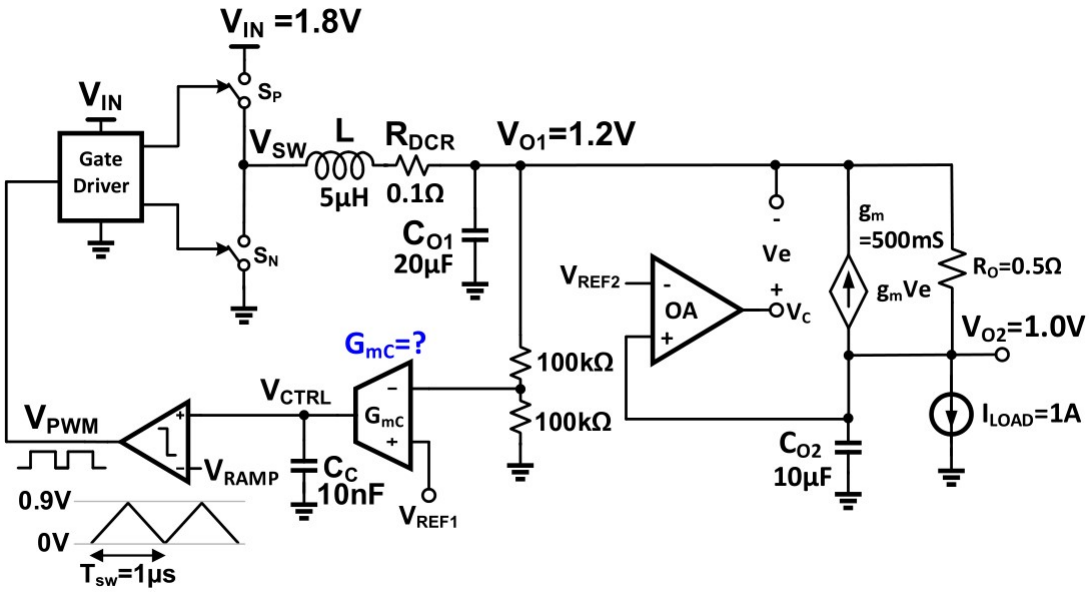
$$p_1 = \frac{1}{R_{o1}(C_{o1}+C_c)}$$

$$C_{o1}+C_c = \frac{1}{57.735 \times 100\,k\Omega} = 173.2\ nF$$

$$\underline{C_c = 173.19\ nF}$$



2) Buck Converter



Above voltage regulator system consisting of a switching buck converter followed by a linear regulator is designed for following specifications: V_{IN}=1.8V, output of the switching buck converter, V_{O1}=1.2V, output of the linear regulator, VO2=1.0V, load current of the linear regulator, I_{LOAD}=1A, ramp switching period=1μS, ramp amplitude=0.9V, inductor L=5μH, series resistance R_{DCR}=0.1Ω, capacitors C_{O1}=20μF, C_{O2}=10μF and integrator capacitor, C_C=10nF. Assuming ideal switches S_P and S_N where S_P a turned ON when V_{PWM} is high and S_N is turned ON when V_{PWM} is low.

a) Determine the steady state value of control voltage, V_{CTRL}, PWM duty cycle and linear regulator control voltage, V_C.
b) Find the loop gain unity gain frequency and value of integrator transconductance, G_{mc} to achieve the minimum gain margin of -20dB for I_{LOAD}=0 to 1A.
c) Find peak-to-peak inductor ripple current ΔI_L and output ripple voltage ΔV_{O1}.
d) Determine the overall efficiency of the entire voltage regulator system.

$$V_{R_{DCR}} = 1A \times 0.1\Omega$$
$$= 0.1V$$

$$V_{SW} = 1.3V$$

duty cycle $\alpha = \frac{1.3}{1.8} = 0.72$

$$\alpha = \frac{V_{ctrl}}{V_m}$$

$$V_{ctrl} = \frac{1.3}{1.8} \times 0.9 = 0.65\,V$$

Current through R_o (LDO) $= \frac{V_{o1} - V_{o2}}{R_o} = \frac{0.2}{0.5} = 0.4A$

Current through $g_m V_e$ VCCS (LDO) $= 0.6A$

$g_m V_e = -0.6A \Rightarrow V_e = -1.2V$

$V_c - V_{o1} = V_e \Rightarrow V_c = 0V$