Properties

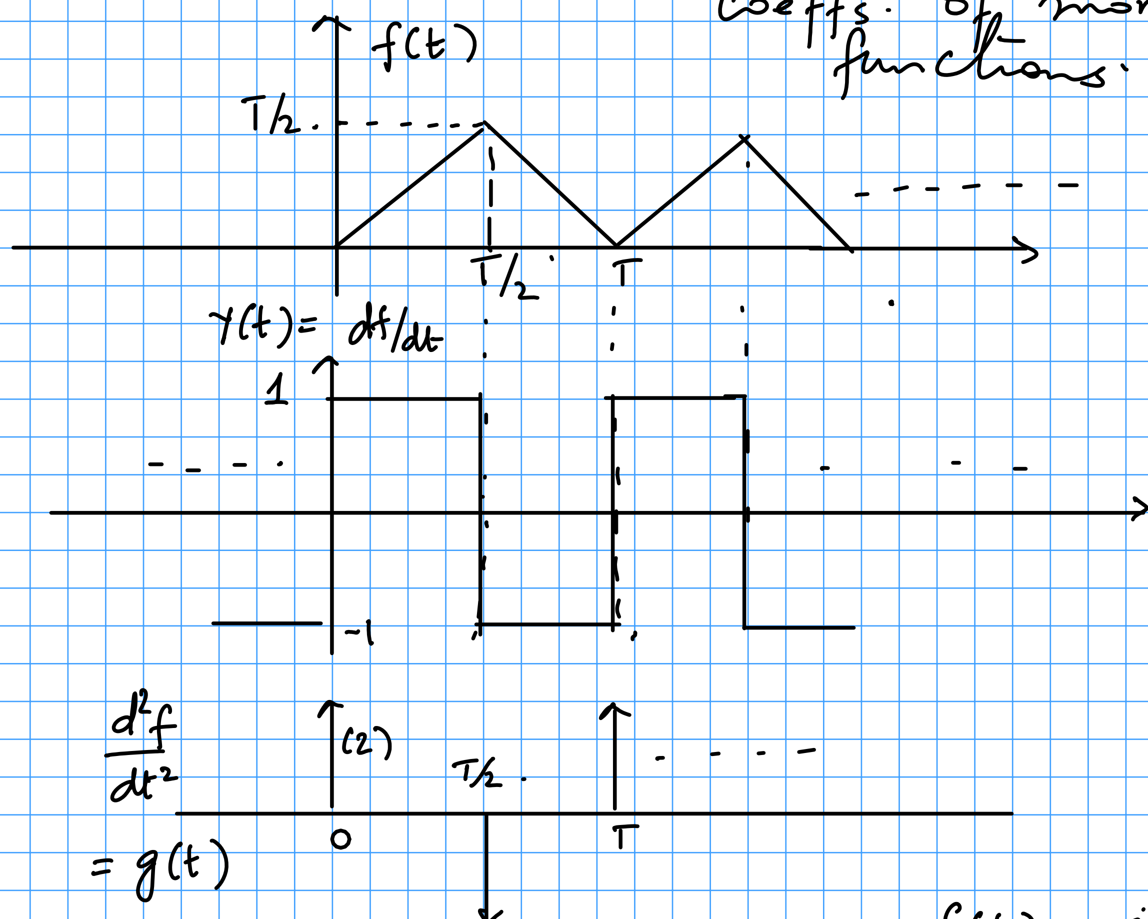
$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

1. Linearity – Coeffs. combine linearly.
2. Time shift – Phase shift.
3. Time reversal – a_{-k}.
4. Time scaling – fundamental frequency changes.
5. Real signals – $a_{-k} = a_k^*$
6. Real and even, real and odd – a_k real, a_k imag.
7. Multiplication – Discrete time convolution
8. Derivative – $(jk\omega_0) a_k$.

Integral – $\dfrac{a_k}{jk\omega_0}$; {x(t) does not have a DC component.}

$$\int_{-\infty}^{t} x(\tau)\, d\tau .$$

Use it to get Coeffs. of more complex functions.



$$f(t) = c_0 + \sum_k c_k e^{jk\omega_0 t}$$

$\frac{df}{dt}$ will not have a DC component

Let $g(t) = \sum_k c_k e^{jk\omega_0 t}$.

$$\Rightarrow c_k = \frac{1}{T}\int_0^T g(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{2}{T}\int_0^T \left(\delta(t) - \delta(t - T/2)\right) e^{-jk\omega_0 t}\, dt$$

$\underline{c_0 = 0}$

$$= \frac{2}{T}\left[1 - e^{-jk\pi}\right]$$

odd harmonics

Fourier coeffs. of $y(t) = \frac{\left(1 - e^{-jk\pi}\right) 2}{jk\omega_0 T}$.

$$= 2\, e^{-jk\pi/2} \frac{\left(2j \sin k\pi/2\right)}{jk \cdot 2\pi}$$

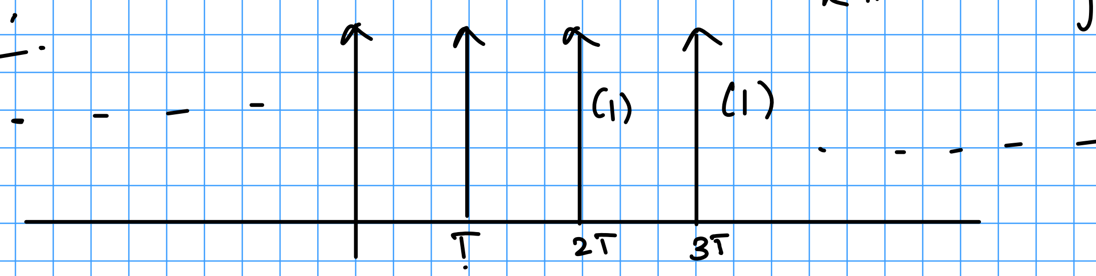
$$= 2\, \frac{\sin k\pi/2}{k\pi} \cdot e^{-jk\pi/2}$$

To find coeffs of $f(t)$; divide the coeffs. of $y(t)$ by $j\omega_0 k$..

$\therefore$ If $f(t) = \sum_k a_k e^{jk\omega_0 t}$

$$a_k = 2\, \frac{\sin k\pi/2}{k\pi} \cdot \frac{e^{-jk\pi/2}}{j\omega_0 k}.$$

eg:



$c_k = \frac{1}{T}$.

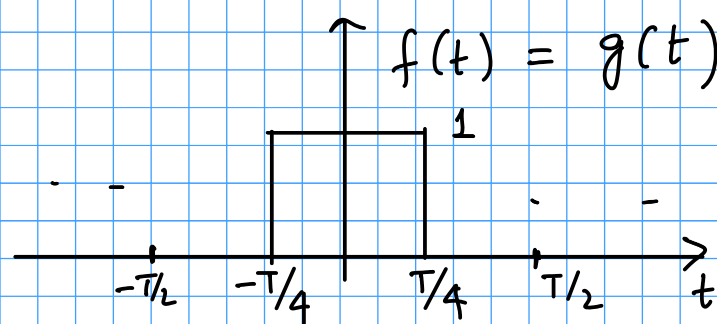
Differentiate and reduce the functions to impulses. Easy to find Fourier coeffs. Divide by $j\omega_0 k$ for each differentiation to get the Fourier series coeff. of the original function.

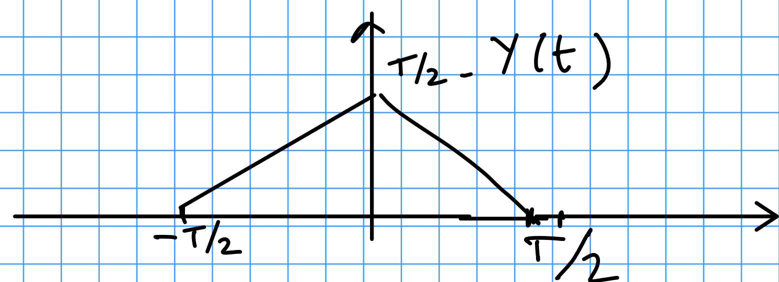
Periodic convolution.

$f(t)$, $g(t)$ ⟷ Take only one period of the signal and convolve.

$$y(t) = \int_0^T f(t-\tau)\, g(\tau)\, d\tau$$

$f(t)$, $g(t)$ have the same period.





Same as regular convolution but integration done only for one period.

Fourier series coefficients for $y(t)$

if $f(t) = \sum_k a_k e^{jk\omega_0 t}$

$g(t) = \sum_k b_k e^{jk\omega_0 t}$

$$y(t) = \int^T \left(\sum_l \sum_k a_l b_k e^{jl\omega_0 (t-\tau)} e^{jk\omega_0 \tau} \right) d\tau$$

$$= \sum_{l} \sum_{k} a_k b_k e^{jl\omega_0 t} \underbrace{\int_0^T e^{j(k-l)\omega_0 \tau} d\tau}_{\substack{=0 \quad k \neq l \\ =T \quad k = l}}$$

$$Y(t) = \sum_{k} (a_k b_k T) e^{jk\omega_0 t}$$

$\therefore$ Coeff. of the k^{th} harmoni is

$\underline{T a_k b_k}$.

If $Y(t) = f(t) g(t)$; F. coeff. of $Y(t)$ is

$\sum_{m} a_m b_{k-m}$.

<u>Parsevals relationship</u>.

Periodic signals are finite power signals.

$$\lim_{N \to \infty} \frac{1}{2NT} \int_{-NT}^{NT} |x(t)|^2 dt = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt.$$

$$x(t) = \sum_{k} a_k e^{jk\omega_0 t}.$$

$$\int_0^T |x(t)|^2 dt = \int_0^T (x(t)) x^*(t) \, dt$$

$$= \sum_{k} \sum_{l} a_k a_l^* \int_0^T e^{jk\omega_0 (k-l) t} dt$$

$$= T \sum_{k} |a_k|^2.$$

$$\therefore \text{Signal power} = \frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{k} |a_k|^2$$