# Fourier series.

Review.

* Any periodic $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ ; $T = \frac{2\pi}{\omega_0}$.

$$c_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t}\, dt$$

* Orthogonal: $\frac{1}{T}\int_0^T e^{j(k-\ell)\omega_0 t}\, dt = 0 \quad k \neq \ell$

$$= 1, \quad k = \ell.$$

* LTI system : $\sum_k c_k e^{jk\omega_0 t} \longrightarrow \sum_k c_k H(jk\omega_0) e^{jk\omega_0 t}$

where $H(jk\omega_0) = \int_{-\infty}^{\infty} h(\tau) e^{-jk\omega_0 t} dt$.

<u>$c_k$ : complex</u>

eg. 1. $x(t) = \sin \pi t = \dfrac{e^{j\pi t} - e^{-j\pi t}}{2j}$

$$c_1 = 1/2j \quad c_{-1} = -1/2j$$

$$c_k = 0, \quad k \neq 1, -1.$$

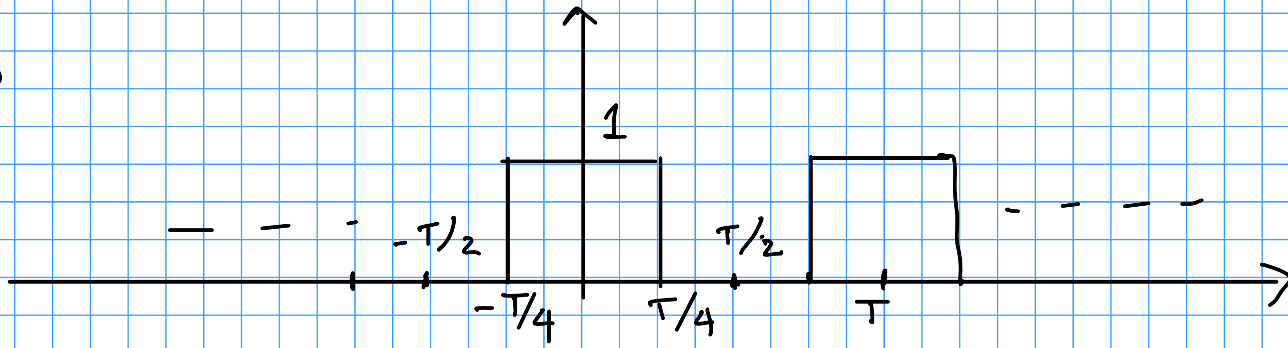
2. $x(t) = 2 + \sin \pi t + \cos(2\pi t + \pi/4)$

$$= 2 + \frac{e^{j\pi t} - e^{-j\pi t}}{2j} + \frac{e^{j2\pi t} \cdot e^{j\pi/4} + e^{-j2\pi t} \cdot e^{-j\pi/4}}{2}.$$

$$c_0 = 2, \quad c_1 = \frac{1}{2j}, \quad c_{-1} = \frac{-1}{2j}$$

$$c_2 = \frac{e^{j\pi/4}}{2}, \quad c_{-2} = \frac{e^{-j\pi/4}}{2}.$$

3




$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

average over one period
`DC' component.

$$= \frac{1}{2}.$$

$$\frac{1}{T}\int_{-T/4}^{T/4} 1 \cdot dt = \frac{T}{2T} = \frac{1}{2}.$$

$$C_k = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jk\omega_0 t}\,dt.$$

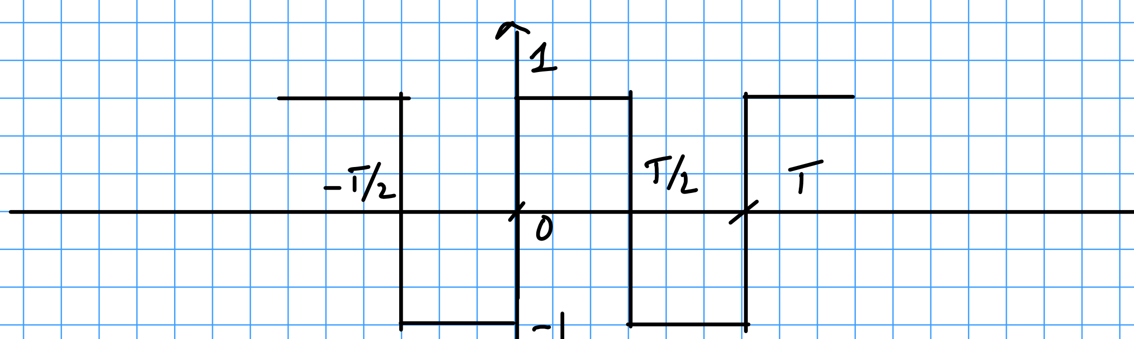
$$= \frac{1}{T}\left[\frac{e^{jk\omega_0 T/4} - e^{-jk\omega_0 T/4}}{jk\omega_0}\right]$$

$$\omega_0 T = 2\pi$$

$$C_0 = 1/2$$

$$= \frac{\sin k\pi/2}{k\pi}$$

4.




$$C_0 = 0$$

$$C_k = 2\sin^2 k\pi/2 \,/\, jk\pi = |C_k|\, e^{j\theta_k}.$$

## Properties:

(a) $x_1(t) = \sum a_k e^{jk\omega_0 t}$ & $x_2(t) = \sum_k b_k e^{jk\omega_0 t}$

$$c_1 x_1(t) + c_2 x_2(t)$$
$$= \sum_k (c_1 a_k + c_2 b_k) e^{jk\omega_0 t}$$

coeff. of the $k^{th}$ harmonic is
$c_1 a_k + c_2 b_k$.

Linear.

Note: $x_1(t)$ and $x_2(t)$ have the same fundamental period.

(2) $x(t) = \sum_k a_k e^{jk\omega_0 t}$

$$x(t-t_0) = \sum_k a_k e^{jk\omega_0 (t-t_0)}$$
$$= \sum_k \left(a_k e^{-jk\omega_0 t_0}\right) e^{jk\omega_0 t}.$$

magnitude remains the same, phase (angle) changes.

(c) $x(t) = \sum_k a_k e^{jk\omega_0 t}$

$x(-t)$ what is the coeff. of the $k^{th}$ harmonic in $x(-t)$

$$x(-t) = \sum_k a_k e^{-jk\omega_0 t} \qquad l = -k.$$

$$= \sum_l a_{-l} e^{jl\omega_0 t}.$$

Coeff. of the $k^{th}$ harmonic is $a_{-k}$.

(d) $x^*(t)$ ?

$$x^*(t) = \sum_k a_k^* e^{-jk\omega_0 t}.$$

$$= \sum_k \underline{\underline{a_{-k}^*}} e^{jk\omega_0 t}$$

(e) $x(ct) = \sum_k a_k e^{jk\omega_0(ct)}$ $\quad c \neq -1$

Same Fourier coefficients but the fundamental frequency $= c\omega_0$.

(f) $x(t)$ real, what can you say about its coeffs?

$$x(t) = x^*(t)$$

$a\hat{i} + b\hat{j} = 0$
$a = b = 0$.

$$\sum_k (a_k - a_{-k}^*) e^{jk\omega_0 t} = 0.$$

$$\Rightarrow a_k = a_{-k}^*$$

$$x(t) = \cdots a_{-k} e^{-jk\omega_0 t} + \cdots + a_k e^{jk\omega_0 t}$$

$$= \cdots a_k^* e^{-jk\omega_0 t} + \cdots + a_k e^{jk\omega_0 t}$$

$$2\mathrm{Re}\left(a_k e^{jk\omega_0 t}\right)$$

(g) $x(t)$ real and even.

$$x(t) = x^*(t)$$

$$x(-t) = x(t) \Rightarrow a_k = a_{-k}.$$

$$a_k = a_{-k}^* = a_{-k}.$$

Coeffs are real and even.

$(a_{-k} = a_k)$

f $x(t)$ real and odd.

$$a_k = a_{-k}^* = -a_{-k}.$$

purely imaginary.