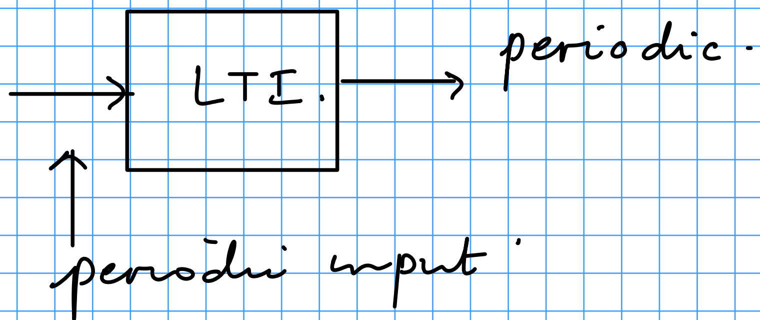
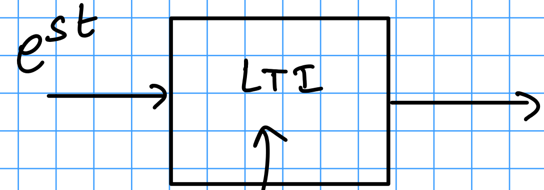


Steady state solutions to periodic inputs.



$$Y(t) = \int_{-\infty}^{\infty} e^{s(t-\tau)} h(\tau)\, d\tau$$

Impulse response $= h(t)$.

$$\therefore\ Y(t) = e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau}$$

Assuming this integral exists, $= H(s)$

$$\therefore\ e^{st} \longrightarrow H(s) e^{st}$$

e^{st} is an eigenfunction of LTI systems.

matrix A.

eigenvalue of the matrix is λ s.t.

$$Ax = \lambda x$$

Note $e^{st} u(t)$ is not an eigenfunction

If input $x(t) = e^{st} u(t)$

$$Y(t) = \int_{-\infty}^{\infty} e^{s(t-\tau)} u(t-\tau) h(\tau) d\tau$$

$$= e^{st} \underbrace{\int_{-\infty}^{t} h(\tau) e^{-s\tau} \, d\tau}_{H(s,t)}.$$

$\therefore$ o/p does not have the same form as the input.

eg.

$$h(t) = e^{-2t} u(t)$$

$$x(t) = e^{j\omega_0 t} u(t)$$

$$y(t) = e^{j\omega_0 t} \int_{0}^{t} e^{-(2+j\omega_0)\tau} \, d\tau.$$

$$= e^{j\omega_0 t} \underbrace{\left(\frac{1 - e^{-(2+j\omega_0)t}}{2 + j\omega_0} \right)}_{H(j\omega_0, t)}$$

$$= \frac{e^{j\omega_0 t}}{(2 + j\omega_0)} - \underbrace{\frac{e^{-2t}}{2 + j\omega_0}}_{\to 0 \text{ as } t \to \infty}.$$

Alternatively if

$$x(t) = e^{j\omega_0 t}, \quad h(t) = e^{-2t} u(t).$$

$$y(t) = \int_{0}^{\infty} e^{j\omega_0 (t-\tau)} e^{-2\tau} \, d\tau$$

$$= e^{j\omega_0 t} \cdot \frac{1}{2 + j\omega_0}.$$

Response to an "everlasting" exponential e^{st} is $H(s) e^{st}$ - often called sinusoidal steady state.

eg: $h(t) = u(t)$, $x(t) = e^{-(\sigma + j\omega_0)t}$.

$\sigma > 0$.

$$h(t) = u(t), \quad x(t) = e^{j\omega_0 t}$$

$$y(t) = \int_{-\infty}^{\infty} e^{j\omega_0(t-\tau)} u(\tau)\, d\tau$$

$$= e^{j\omega_0 t} \int_{0}^{\infty} e^{-j\omega_0 \tau}\, d\tau.$$

indeterminate; Integral does not exist.

If $h(t) = u(t)$

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau \longrightarrow \infty$$

e^{st}; 's' is any complex frequency. will result in an output

$H(s)e^{st}$ if $\int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau$ exists.

First we will look at periodic inputs.

$$x(t + T_0) = x(t) \quad \forall t.$$

Look at the signal $x(t) = e^{jk\omega_0 t}$.

Eigen func. prop. $e^{jk\omega_0 t} \longrightarrow H(jk\omega_0) e^{jk\omega_0 t}$.

where $H(jk\omega_0) = \int_{-\infty}^{\infty} h(\tau) e^{-jk\omega_0 \tau}\, d\tau$.

Since the system is linear,

$$\sum_k a_k e^{jk\omega_0 t} \longrightarrow \sum_k a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$x(t) = \sum_k a_k e^{jk\omega_0 t}$ is also periodic

$$e^{jk\omega_0 t} \; ; \; e^{jk\omega_0(t + 2\pi/\omega_0)}$$
$$= e^{jk\omega_0 t} \cdot e^{jk(2\pi)}$$
$$= e^{jk\omega_0 t}$$

$\Rightarrow \sum_k a_k e^{jk\omega_0 t}$ is also periodic with period $= \frac{2\pi}{\omega_0}$.

$x(t)$; ω_0, $2\omega_0$, $3\omega_0$ ....

Signal contains multiple frequencies, all of which are multiples of a fundamental frequency ω_0.

$e^{+j\omega_0 t}$, $e^{-j\omega_0 t}$; fundamental or 1st harmonic components.

$e^{j2\omega_0 t}$, $e^{-j2\omega_0 t}$; 2nd harmonic.

$e^{jk\omega_0 t}$, $e^{-jk\omega_0 t}$; k^{th} harmonic

Fourier series:

Any periodic signal can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

complex exponential.

Orthogonality.

$$\int_0^{T_0} e^{jk\omega_0 t} \cdot e^{-jl\omega_0 t}\, dt \qquad T_0 = \frac{2\pi}{\omega_0}$$

$$= \int_0^{T_0} e^{j(k-l)\omega_0 t}\, dt$$

$$= \left. \frac{e^{j(k-l)\omega_0 t}}{j(k-l)\omega_0} \right|_0^{T_0} \qquad k \neq l$$

$$= 0.$$

$k = l$; $\int_0^{T_0} e^{j(k-l)\omega_0 t}\, dt = T_0.$

$$\frac{1}{T_0}\int_0^{T_0} e^{j\omega_0 (k-l) t}\, dt = 1 \quad k = l$$
$$= 0 \quad k \neq l$$

Define "dot product"

$$f(t) \cdot g(t) = \frac{1}{T_0}\int_0^{T_0} f(t)\, g^*(t)\, dt$$

Each mode is orthogonal w.r.t this dot

product.

Can be used to find a_k:

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

$x(t)$ periodic

$$\Rightarrow \quad \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\omega_0 t} = a_k .$$