EE5120 Applied Linear Algebra Techniques for Electrical Engineers – Tutorial 2

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1. If P is an invertible matrix, prove that $rank(PA) = rank(A)$.

2. Let A be a $m \times n$ matrix. If $AB = 0$, show that $rank(A) + rank(B) \leq n$.

3. Show that $Ax = b$ has multiple solutions if and only if $b \in \; Col(A)$ and the dimension of $Null(A)$ is non-zero.

4. Find the basis for vector space $\{(x, y, z) : \; 2x + 3y + 4z = 0\}$

5. For the following matrices, find the basis vectors for all four fundamental subspaces (column space, row space, null space and left null space)

   a. $A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$

   b. $A = \begin{bmatrix} -3 & 2 & 3 \\ 9 & -6 & -9 \\ -2 & 4 & -2 \\ -7 & 8 & 2 \end{bmatrix}$

6. Consider the subspace of cubic polynomials, $p(x)$ such that $p(5) = p(7) = 0$.

   a. Show that it is a vector space.

   b. Find the basis for the vector space.

7. Check if the following set of vectors are linearly independent.

   a. $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \right\}$

   b. $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 13 \\ 12 \\ 28 \end{bmatrix} \right\}$

8. Let $\mathcal{A} = \{A_1, A_2, A_3\}$ and $\mathcal{B} = \{B_1, B_2, B_3\}$ be two sets of basis vectors for the vector space $\mathcal{V}$. Assume that $A_1 = 4B_1 - B_2$, $A_2 = -B_1 + B_2 + B_3$ and $A_3 = B_2 - 2B_3$. If the co-ordinate vector of v with respect to $\mathcal{A}$ is $\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$, find the co-ordinate vector v with respect to $\mathcal{B}$.

9. State whether the following statements are True or False and give reasons.

   (a) Let $S = \{v_1, v_2, \cdots, v_n\}$. If $\mathcal{V} = \text{span}\{S\}$, S is a basis for $\mathcal{V}$.

   (b) If $\{v_1, v_2, \cdots, v_n\}$ are a set linearly independent vectors in $\mathcal{V}$, they form a basis for $\mathcal{V}$.

   (c) Let $\{w_1, w_2, \cdots, w_m\}$ be linear combinations of the vectors $\{v_1, v_2, \cdots, v_n\}$, with $m > n$. If the vectors $\{v_1, v_2, \cdots, v_n\}$ are linearly independent, then $\{w_1, w_2, \cdots, w_m\}$ are also linearly independent.

   (d) If a matrix B is obtained from A by performing elementary row operations, $\text{rank}(B) = \text{rank}(A)$.