Electric Circuits and Networks **14-08-2018**

Lecture 8: Mesh Analysis

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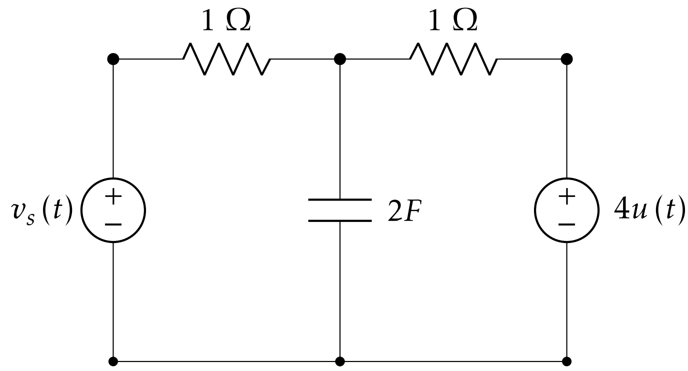
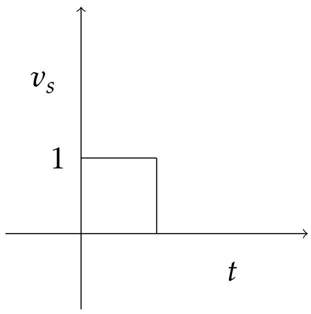
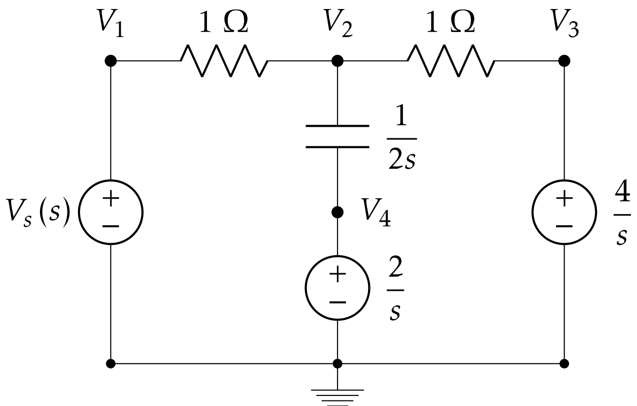
Nodal Analysis (cont...)
Example 1: Given $v_C(0^-) = 2V$



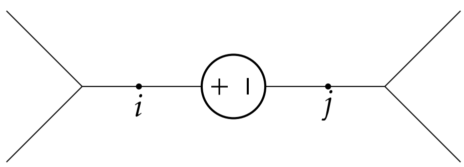


Using laplace transform the above circuit will be as follows:



Given $v_s(t) = u(t) - u(t-1)$. So, $V_s(s) = \dfrac{1-e^{-s}}{s}$
As we can observe, V_1, V_4, V_3 along with reference node form a supernode. So, we need to write KCL only at V_2.

$$V_2(s)(2+2s) - 1 \cdot V_1(s) - 1 \cdot V_3(s) - 2sV_4(s) = 0$$



If there is a voltage source between nodes i and j, write KCL for supernode. In the above example, $V_1, V_4, \&\ V_3$ are connected to reference node through voltage source so the supernodes are [(1)-(R)],[(4)-(R)] and [(3)-(R)] We do not write KCL for the reference node so only write one equation for node 2. Additional equations are

$$V_1 = \frac{1-e^{-s}}{s} \tag{1}$$

$$V_3 = \frac{4}{s} \tag{2}$$

$$V_4 = \frac{2}{s} \tag{3}$$

$$\tag{4}$$

Therefore,

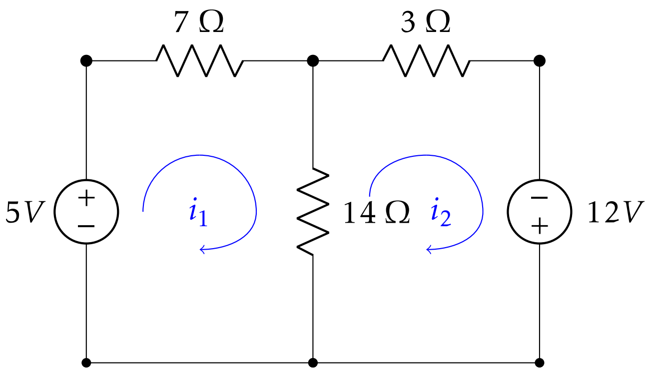
$$V_2(s) = \frac{1-e^{-s}}{2s(s+1)} + \frac{2}{s}$$

Find $v_2(t)$

Note: Remember that the voltage across capacitor is $V_2(s)$ not $V_2(s) - V_4(s)$

Mesh Analysis: Find loops that do not cross a branch (no loop inside the loop) → mesh. Mesh currents are unknowns (assume all mesh currents in clockwise direction)

Example 2:



$$\text{Mesh 1:} \quad -5 + 7i_1 + 14(i_1 - i_2) = 0$$

$$\text{Mesh 2:} \quad -12 + 3i_2 + 14(i_2 - i_1) = 0$$

or

$$(7+14)i_1 - 14i_2 = 5$$

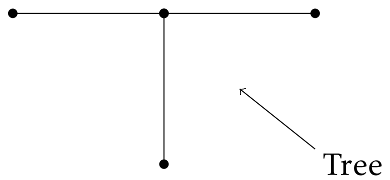
$$-14i_1 + (14+3)i_2 = 12$$

In the matrix form,

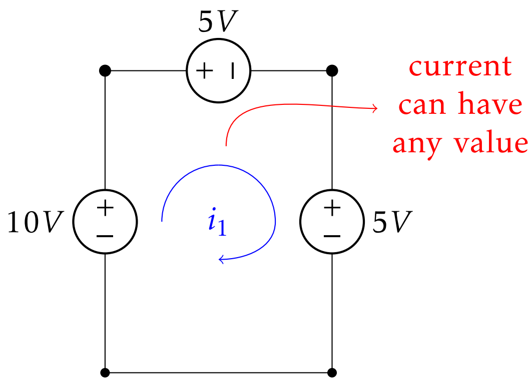
$$\begin{bmatrix} 21 & -14 \\ -14 & 17 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

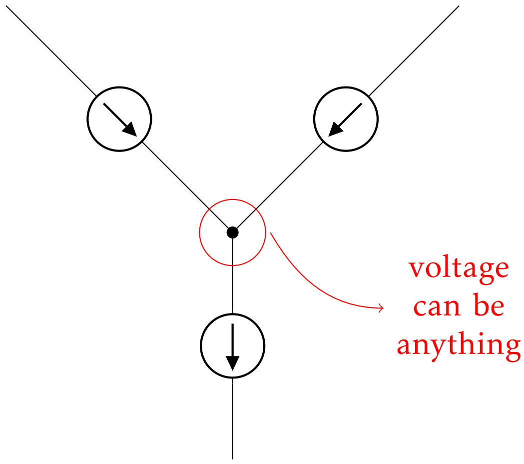
Invert the matrix and solve for $i_1 \& i_2$. Note that the diagonal entries are the sum of the impedances in the loop and the off-diagonal terms are the negative of the impedance between the two loops.

If we have a circuit with N nodes, b branches then the number of meshes will be equal to $b - (N-1)$. Explanation is given below:



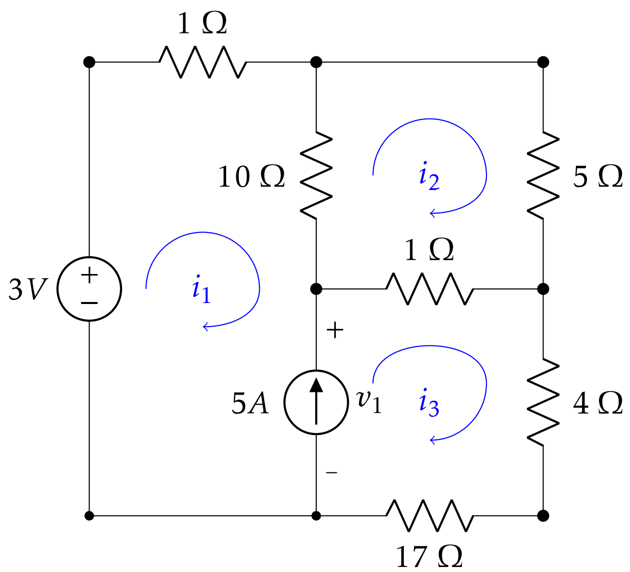
with 'N' nodes you can form a tree (no loops) with $N-1$ branches. Every additional branch you add will form a loop(mesh) with the exiting tree branches. The mesh current in the additional branch is independent of the currents in all other branches; Therefore, the equations are independent





In practice you should never do this.

Example 3:



If there is a current source, form a super mesh by adding the equations for two meshes. Assume voltage across the current source is v_1. The equations for mesh 1 and 3 can be written as

$$-3 + 10(i_1 - i_2) + v_1 = 0$$
$$1(i_3 - i_2) + 4i_3 + 17i_3 - v_1 = 0$$

The equation for supermesh is the sum of the above two equations.

$$10(i_2 - i_1) + 5i_2 + 1(i_2 - i_3) = 0$$

The additional equation needed can be obtained by writing the current source current as the difference of two mesh currents.

$$i_3 - i_1 = 5A$$

So we have,

$$\begin{aligned} 10i_1 - (10+1)i_2 + (17+4)i_3 &= 3 \\ -10i_1 + (10+5+1)i_2 - i_3 &= 0 \\ i_1 - i_3 &= -5 \end{aligned}$$

or,

$$\begin{aligned} 10i_1 - (11)i_2 + (21)i_3 &= 3 \\ -10i_1 + (16)i_2 - i_3 &= 0 \\ i_1 - i_3 &= -5 \end{aligned}$$

we can write above equations in matrix form

$$\begin{bmatrix} 10 & -11 & 21 \\ -10 & 16 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix}$$

Solve these equations for i_1, i_2 & i_3.