Electric Circuits and Networks	13-08-2018

Lecture 7: Nodal Analysis

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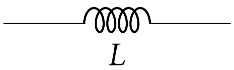
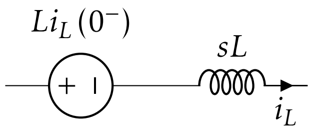
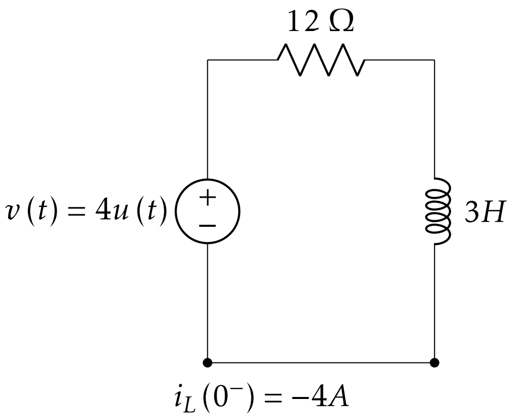
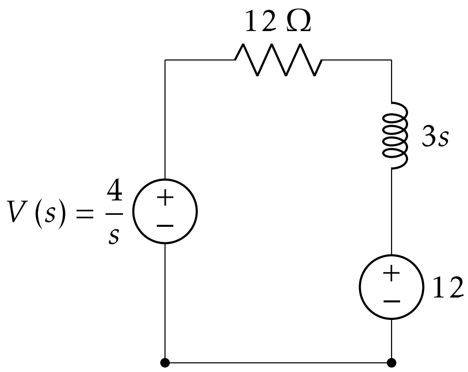
With initial condition





Example 1:





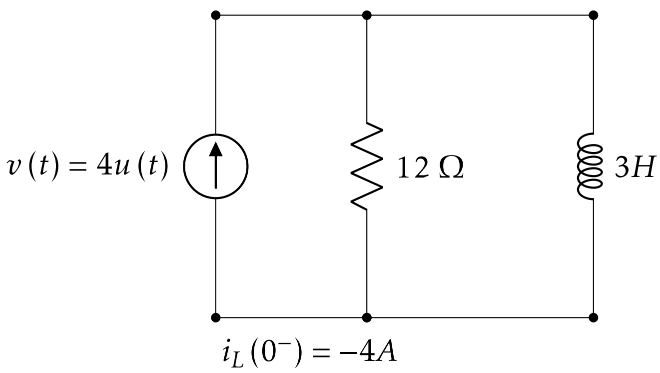
$$\frac{4}{s} = 12I(s) + 3sI(s) + 12$$

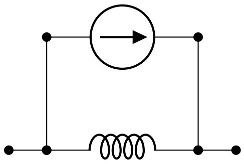
$$I(s) = \underbrace{\frac{4}{s}\frac{1}{4+s}}_{\text{Input}} - \underbrace{\frac{4}{4+s}}_{\text{Initial condition}}$$

$$i(t) = \underbrace{\frac{1}{3}\left(1 - e^{-4t}\right)u(t)}_{\text{zero state solution}} - \underbrace{4e^{-4t}}_{\text{zero input solution}}$$

So, the above circuit is linear w.r.t. input and state respectively, But as a complete system it is not linear

Exercise :



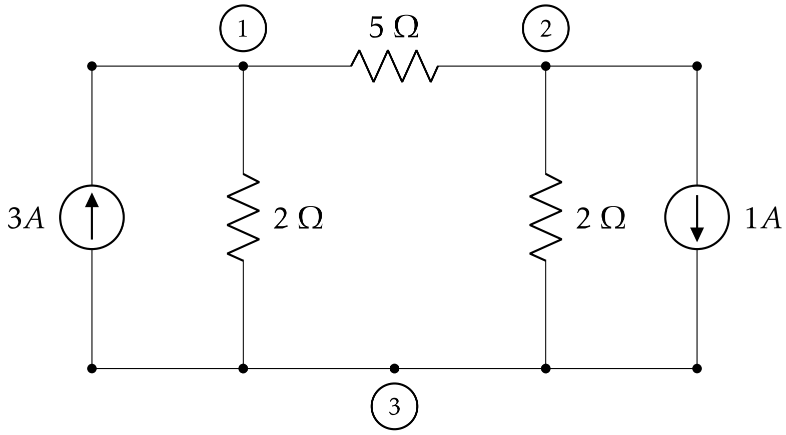


$$V(s) = -Li(0^-) + sLI(s)$$

$$I(s) = \frac{i(0^-)}{s} + \frac{1}{sL}V(s)$$

[**Hint:** It is more convenient to replace the inductor by an inductor and a current source in parallel]

Example 2:



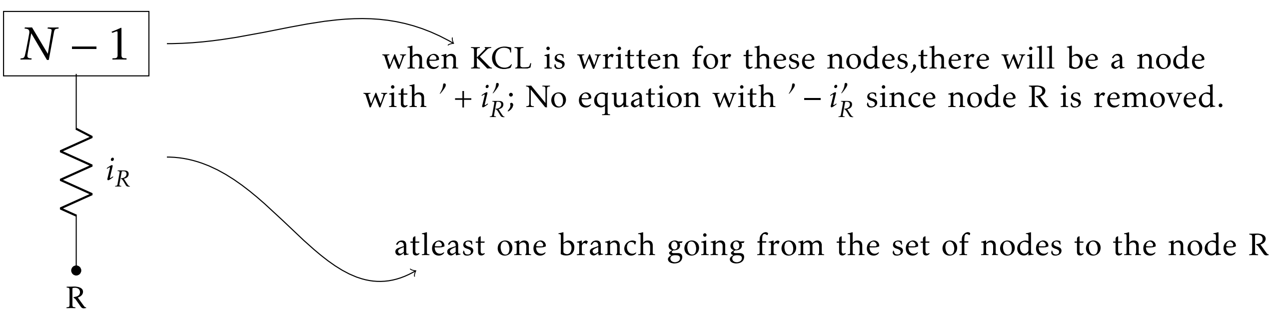
KCL

$$E_1: \quad i_1 + i_2 - 3 = 0 \tag{1}$$
$$E_2: \quad -i_2 + i_3 + 1 = 0 \tag{2}$$
$$E_3: \quad 3 - i_1 - i_3 - 1 = 0 \tag{3}$$

Notice that $\sum_{k=1}^{3} E_k = 0$ So, the equations are linearly dependent i.e. no new information is contained by E_3. Hence remove E_3

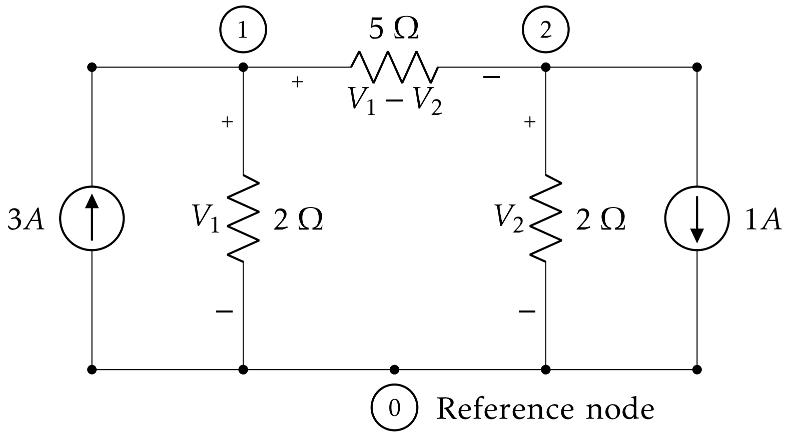
$$E_1 : i_1 + i_2 - 3 = 0$$
$$E_2 : -i_2 + i_3 + 1 = 0$$

One can check that E_2 can not be derived from E_1. This is not something specific to this circuit. In any circuit having N nodes there will be only $N-1$ linearly independent equation.



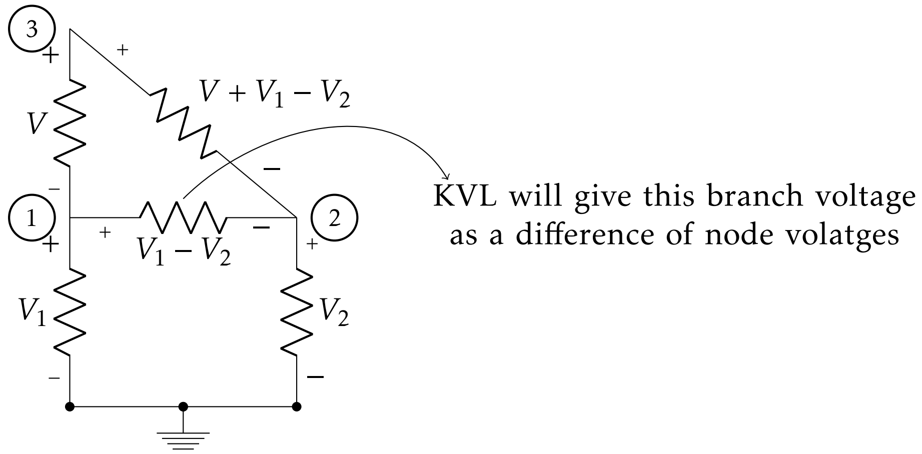
KCL, KVL, branch relation

$$E_1 : i_1 + i_2 - 3 = 0$$
$$E_2 : -i_2 + i_3 + 1 = 0$$



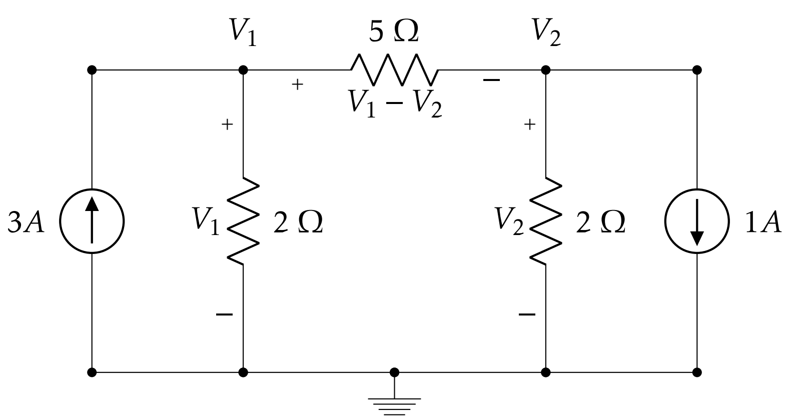
w.r.t to the reference node , volatge at node 1 is equal to V_1

Consider the following circuit in order to understand that by taking the voltage w.r.t. to reference node will automatically satisfy the KVL



node voltage at node 3 = $V_1 + V$.

Now come back to example 2, assign node voltages (Volatge of a node w.r.t. the reference node). KVL will be automatically satisfied.



At node 1

$$-3 + i_1 + i_2 = 0$$
$$i_1 = \frac{V_1}{2}, \quad i_2 = \frac{V_1 - V_2}{5}$$
$$\frac{V_1}{2} + \frac{V_1 - V_2}{5} = 3$$

Similarly,at node 2

$$\frac{V_2 - V_1}{5} + \frac{V_2}{2} = -1$$

Above two equations can be written in matrix form as follows:

$$\begin{bmatrix} 1/2 + 1/5 & -1/5 \\ -1/5 & 1/2 + 1/5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

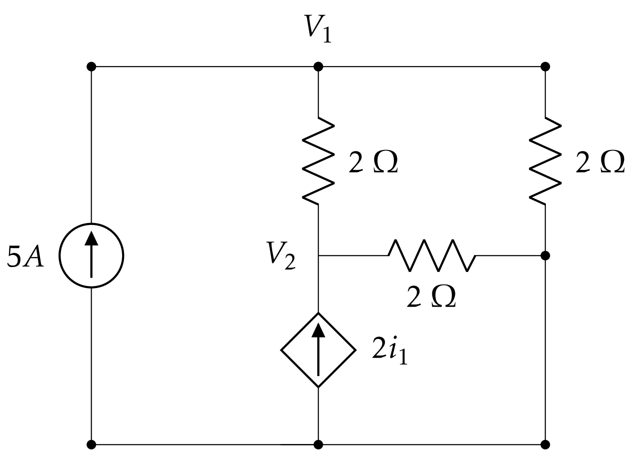
where

$$G = \begin{bmatrix} 1/2 + 1/5 & -1/5 \\ -1/5 & 1/2 + 1/5 \end{bmatrix}$$

is known as conductance (in general admittance) matrix. Note that the diagonal terms are the sum of the conductances incident at a node and the off-diagonal term is the negative of the conductance between the two nodes. Now solve for node voltages. This method is known as **Nodal analysis**

Note: Reference node can be any node but it makes sense to take the node with the largest number of branches, as reference node.

Example 3: circuit with dependent current source



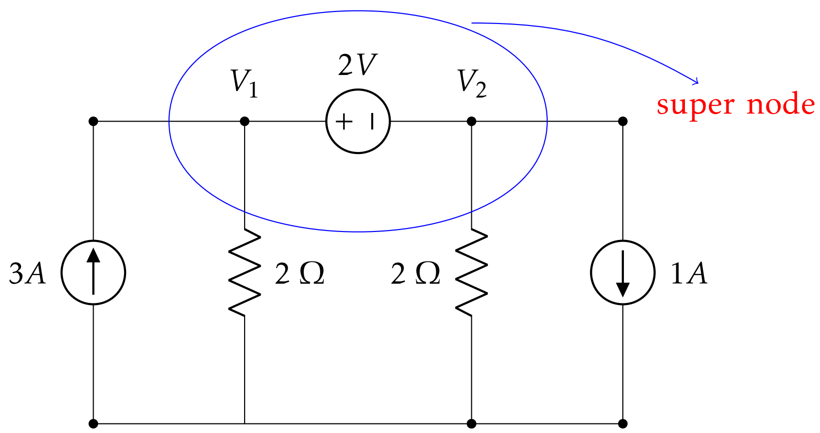
$$-5 + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0$$
$$\frac{V_2}{2} + \frac{V_2 - V_1}{2} - 2i_1 = 0$$

and

$$i_1 = \frac{V_1 - V_2}{2}$$

Write it in the matrix form and solve to get $V_1 \& V_2$

Example 4: circuit with voltage source in loop



Assume current through node 1 and 2 is i_v

$$-3+\frac{V_1}{2}+i_v=0$$
$$1+\frac{V_2}{2}-i_v=0$$

Add above equations then we get KCL at super node as

$$\frac{V_1}{2}+\frac{V_2}{2}=2$$

and the equation for the voltage source as

$$V_1-V_2=2$$

Solve for $V_1 \& V_2$

The equations can also be written using modified nodal analysis. Here the unknowns are all the node voltages and currents through voltage sources.

$$-3+\frac{V_1}{2}+i_v=0$$
$$1+\frac{V_2}{2}-i_v=0$$
$$V_1-V_2=2$$

In the matrix form, it can be wriiten as

$$\begin{bmatrix}0.5 & 0 & 1\\ 0 & 0.5 & -1\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}\mathrm{V}_1\\ \mathrm{V}_2\\ \mathrm{i}_V\end{bmatrix}=\begin{bmatrix}3\\ -1\\ 2\end{bmatrix}$$