Electric Circuits and Networks **01-08-2018**

Lecture 3: Component Models (Contd...)

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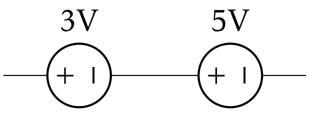
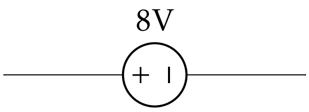
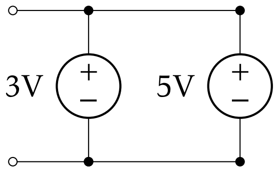
Series and Parallel connection of Ideal Sources

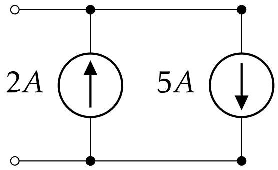


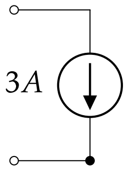


Series Connection: Same current

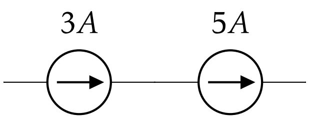


Ideal voltage sources can **not** be added in parallel.



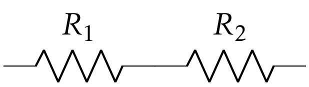


Parallel Connection: Same Voltage

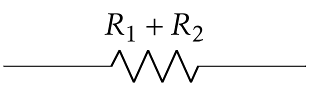


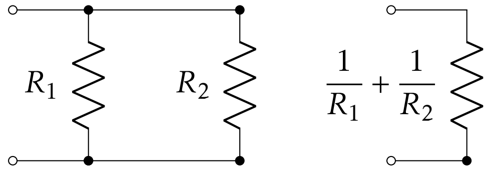
Ideal current sources can **not** be added in series.

Series and Parallel connection of Resistors



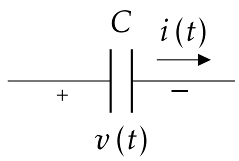
$$v(t) = (R_1 + R_2)\, i(t)$$





$$i(t) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v(t)$$

Capacitor



$q = CV$, where V is voltage drop across the capacitor

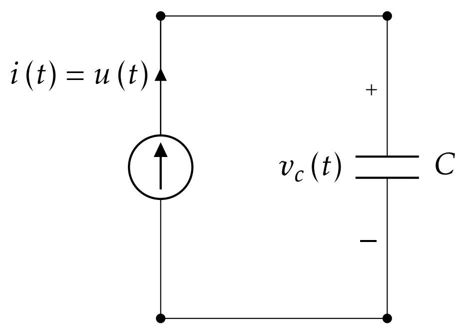
$$\begin{aligned} i(t) &= \frac{dq}{dt} = \frac{d}{dt}(CV) \\ &= C\frac{dV}{dt} + V\frac{dC}{dt} \end{aligned}$$

For a LTI capacitor, value of capaciatance is invariant of time $\frac{dC}{dt} = 0$

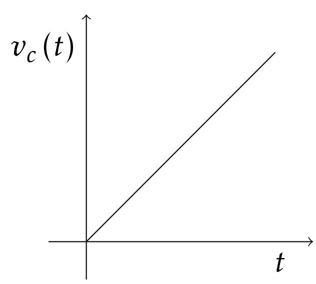
hence,

$$\boxed{i(t) = C\frac{dV}{dt}}$$

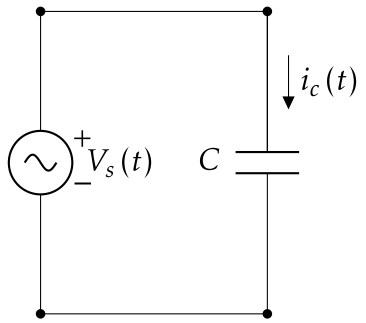
Example 1:



$$\begin{aligned} & i(t) = C\frac{dV}{dt} \\ \Rightarrow \quad & v_c(t) = v_c(0) + \frac{1}{C}\int_0^t i(t)\,dt \\ & v_c(t) = \frac{t}{C} \end{aligned}$$

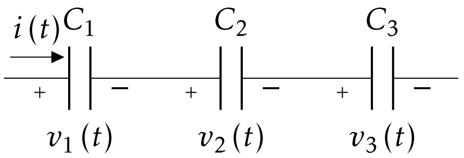


Example 2:



$$V_s(t) = Acos\omega t$$
$$i(t) = C\frac{dV}{dt}$$
$$= -\omega ACsin\omega t$$

Series and Parallel connection of Capacitors



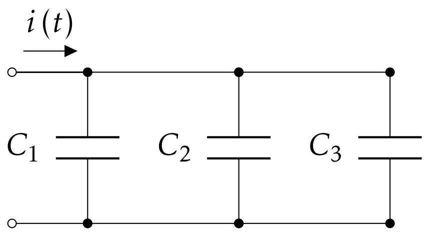
$$v_1(t) = v_1(0) + \frac{1}{C_1}\int_0^t i(t)dt$$
$$v_2(t) = v_2(0) + \frac{1}{C_2}\int_0^t i(t)dt$$
$$v_3(t) = v_3(0) + \frac{1}{C_3}\int_0^t i(t)dt$$
$$v(t) = v_1(t) + v_2(t) + v_3(t)$$
$$= \sum_{i=1}^{3} v_i(0) + \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_0^t i(t)dt$$

$So,$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$with,$

$$v_{eq}(0) = \sum_{i=1}^{3} v_i(0)$$



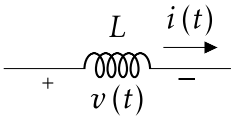
$$i(t) = C_1\frac{dV}{dt} + C_2\frac{dV}{dt} + C_3\frac{dV}{dt}$$
$$= (C_1 + C_2 + C_3)\frac{dV}{dt}$$

$So,$

$$C_{eq} = (C_1 + C_2 + C_3)$$

The initial voltage on all capacitors is the same.

Inductor



$\phi = Li$, where ϕ is magnetic flux and i is current through inductor

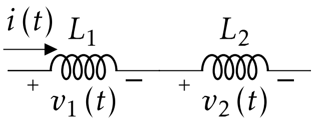
$$v(t) = \frac{d\phi}{dt} = \frac{d}{dt}(Li)$$
$$= L\frac{di}{dt} + i\frac{dL}{dt}$$

For a LTI capacitor, value of inductance is invariant of time $\quad \frac{dL}{dt} = 0$

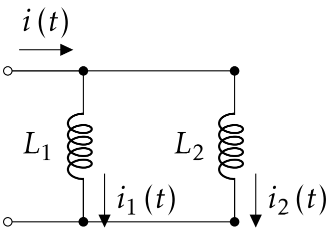
hence,

$$\boxed{v(t) = L\frac{di}{dt}}$$

Series and Parallel connection of Resistors

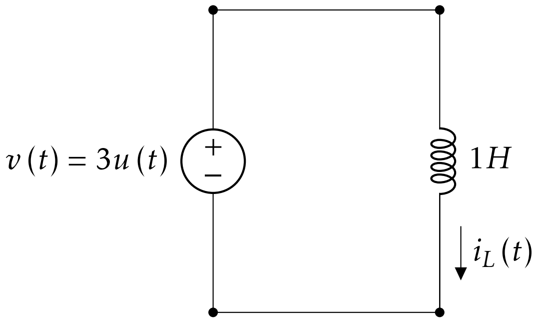


$$v(t) = v_1(t) + v_2(t)$$
$$= L_1\frac{di}{dt} + L_2\frac{di}{dt}$$
$$So,$$
$$L_{eq} = (L_1 + L_2)$$

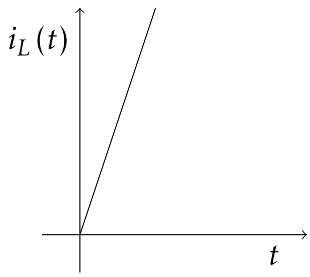


$$i(t) = i_1(t) + i_2(t)$$
$$= i_1(0) + \frac{1}{L_1}\int_0^t v(t)\,dt + i_2(0) + \frac{1}{L_2}\int_0^t v(t)\,dt$$
$$= \sum_{k=1}^{2} i_k(0) + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v(t)\,dt$$
$$So,$$
$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

Example 3:



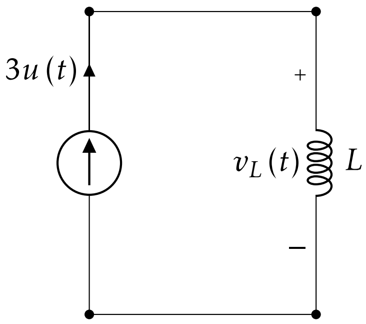
$$
\begin{aligned}
i_L(t) &= i_L(0) + \frac{1}{L_1}\int_0^t v(t)\,dt \\
&= i_L(0) + \int_0^t 3u(t)\,dt \\
&= i_L(0) + 3t
\end{aligned}
$$



Assume initial current is zero

$$i_L(t) = 3t$$

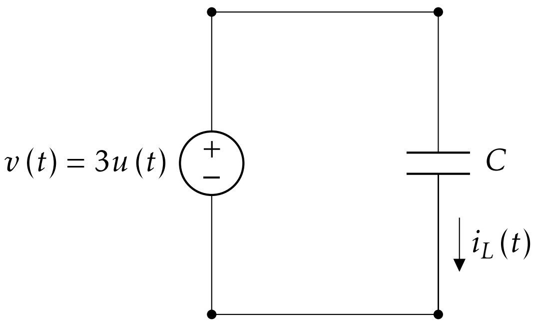
Example 4:



$$
\begin{aligned}
v_L(t) &= L\frac{di}{dt} \\
&= 3L\frac{du}{dt} \\
&= 3L\delta(t)
\end{aligned}
$$

A sudden change in current causes a impluse voltage across the inductor which is not practically possible,Therefore current through an inductor can not change instantaneously.

Example 5:



$$
\begin{aligned}
i_C(t) &= C\frac{dv}{dt} \\
&= 3C\frac{du}{dt} \\
&= 3C\delta(t)
\end{aligned}
$$

A sudden change in voltage causes a impluse current through the capacitor which is not practically possible,Therefore voltage across a capacitor can not change instantaneously.