**Electric Circuits and Networks** **xx-10-2018**

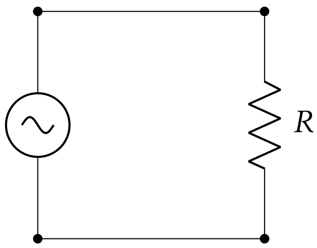
# Lecture 29: Power - Sinusoidal Steady State

*Lecturer: Dr. Vinita Vasudevan* *Scribe: Shashank Shekhar*

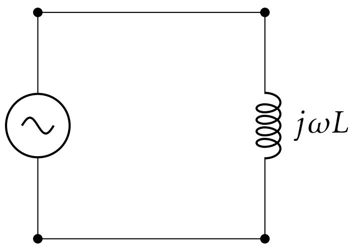
## Power - Sinusoidal Steady State

$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_I)$$

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_I)$$
$$= \frac{V_m I_m}{2}\{\cos(\theta_v - \theta_I) + \cos(2\omega t + \theta_v + \theta_I)\}$$
$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{V_m I_m}{2}\cos(\theta_v - \theta_I)$$
$$T = \frac{2\pi}{\omega}$$
$$\text{Let} \quad \theta_v - \theta_I = \alpha$$



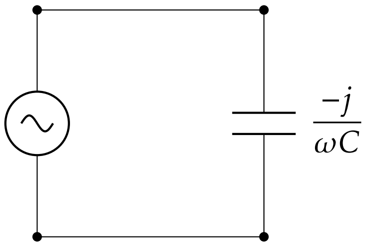

$$V = V_m \cos(\omega t + \theta_v)$$
$$I = \frac{V_m}{R}\cos(\omega t + \theta_v)$$
$$\alpha = 0; P_{avg} = \frac{V_m^2}{2R} = I_m^2 R$$




$$V = V_m \cos(\omega t + \theta_v)$$
$$i = \frac{V_m}{L}\int \cos(\omega t + \theta_v)$$
$$= \frac{V_m}{\omega L}\sin(\omega t + \theta_v)$$
$$\Rightarrow \theta_I = \theta_v - \pi/2 \Rightarrow P_{avg} = 0$$
$$p(t) = \frac{V_m^2}{2\omega L}\sin(2(\omega t + \theta_v))$$

Electrical energy is transferred back and forth between the load and the source

$$Z = j\omega L = \omega L \measuredangle \pi/2$$
$$I = \frac{V}{|Z|}\measuredangle(\theta_v - \pi/2)$$
$$\therefore\ \theta_v - \theta_I \rightarrow \text{Impedance angle}$$




$$Z = \frac{1}{\omega C}\measuredangle -\pi/2$$

$$P_{avg} = 0 \text{ as with the inductor}$$

$P_{avg}$ : Active power that is absorbed by the device and converted to the other forms of energy. Power transferred between the source and capacitor/inductor remains as electrical energy.

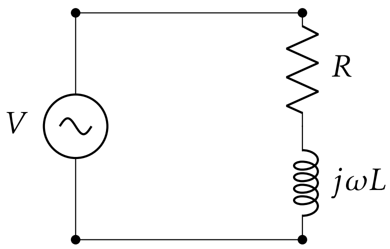
$$P_{avg} = \frac{V_m I_m}{2}\cos\alpha$$

where, $\cos\alpha$ is called the power factor.

Since $\alpha$ is the impedance angle, which varies from $-\pi/2$ to $\pi/2$; $\cos\alpha$ varies between 0 and 1.

* $\cos\alpha$ is the same whether the current leads voltage by $\alpha$ or lags by $\alpha$ - . To distinguish between the two types of loads, we have lagging power factor or leading power factor.

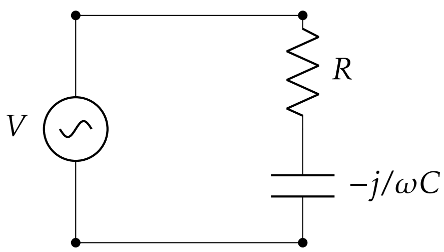
* Given a source voltage, for the same active power, we need a higher current if PF(power factor) is smaller.




$$I = \frac{V}{\sqrt{R^2+\omega^2L^2}}\measuredangle -\tan^{-1}\left(\frac{\omega L}{R}\right)+\theta_v$$

$$\theta_v - \theta_I = \tan^{-1}\left(\frac{\omega L}{R}\right) = \alpha$$

$$0 \leqslant \alpha \leqslant \pi/2$$




$$\alpha = \tan^{-1}\left(\frac{-1}{\omega CR}\right)$$

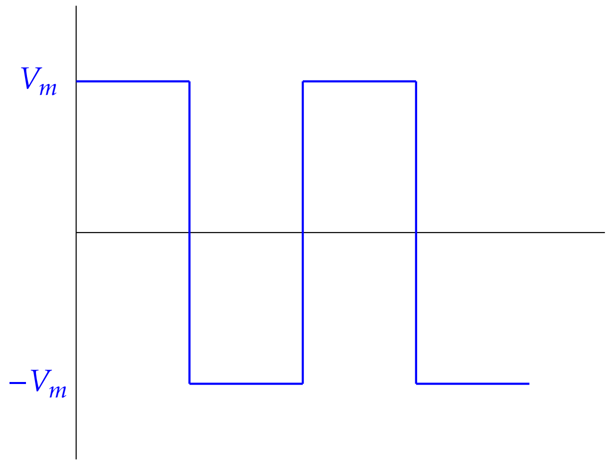
$$-\pi/2 \leqslant \alpha \leqslant 0$$

Root mean square value of a signal.

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \qquad T = \frac{2\pi}{\omega}$$

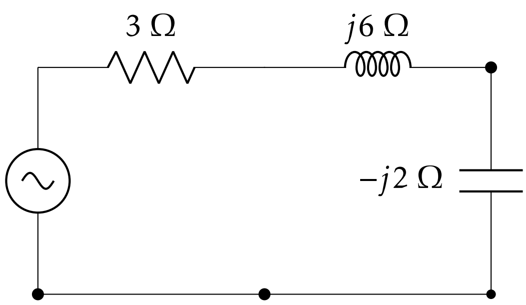
$$v(t) = V_m\cos(\omega t + \theta_v)$$

$$V_{eff} = \frac{V_m}{\sqrt{2}}$$




$$V_{eff} = V_m$$

**Example 1.**




$$\begin{aligned}
V_s &= 50\measuredangle-90^\circ \text{ rms}\\
Z &= 3+j4\\
&= 5\measuredangle\tan^{-1}\left(\frac{4}{3}\right)\\
I &= \frac{50}{5}\measuredangle-90^\circ-\tan^{-1}\left(\frac{4}{3}\right)\\
P_{avg} &= 500\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\\
&= 300 Watts
\end{aligned}$$

**Complex Power.**
RMS value of voltage and current phasors are $V_{eff}\measuredangle\theta_v$ and $I_{eff}\measuredangle\theta_I$ . Define the complex power as

$$\begin{aligned}
S &= VI^* = P+jQ\\
&= V_{eff}I_{eff}e^{j(\theta_v-\theta_I)}
\end{aligned}$$

where $(\theta_v-\theta_I)$ is angle of impedence

$P = Re\{S\} := V_{eff}I_{eff}\cos\alpha$ Average (active) Power, **units : Watts**

$Q = Im\{S\} = V_{eff}I_{eff}\sin\alpha$ Reactive Power, **units : VAR( Volt Ampere reactive)**

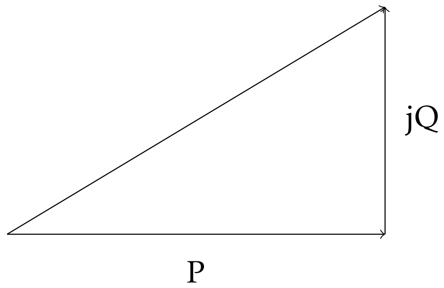
$$\begin{aligned}
S &= \sqrt{P^2+Q^2}\\
&= V_{eff}I_{eff}
\end{aligned}$$

Apparent power **Units: VA (Volt Ampere)**

$P$ is the same irrespective of whether current leads or lags.

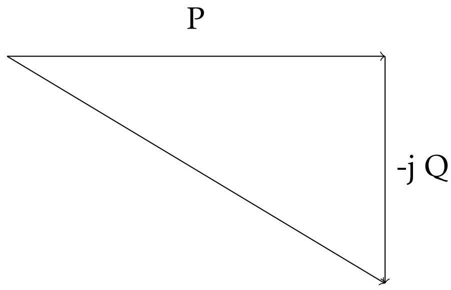
$$Q > 0 \qquad \text{if current lags } \{R- \ L\text{combination}\}$$
$$Q < 0 \qquad \text{if current leads } \{R- \ C\text{combination}\}$$

Most practical loads have a lagging power factor.




$$S = \sqrt{P^2 + Q^2}$$

$$\tan\alpha = \frac{Q}{P}$$




$$Q = \pm\sqrt{\frac{1}{PF^2} - 1}$$

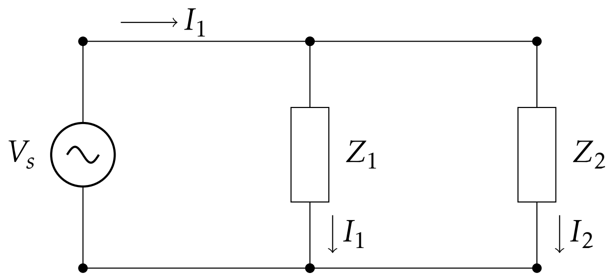
**"Conservation of Complex Power"**

Tellegans theorem

$$AI_b = 0$$
$$\Rightarrow AI_b^* = 0$$
$$A^T V_n = V_b$$
$$V^T I_b^* = V_n^T A I_b^* = 0$$

Both reactive and active power balance out in the network.

$$\sum_k V_k I_k^* = 0$$




$$I = I_1 + I_2$$

$Z_1$ : $50kW$ induction motor with lagging PF = 0.8. $V_s = 230Vrms$. What should $Z_2$ be so that PF = 0.95 ?

A low PF is not good as we require a large current for the same active power ($VI\cos\alpha$) Also there is a larger exchange of reactive power between the source and the load leading to transmission losses. We can increase the power factor by connecting another load. Make sure active power is the same in both cases.

$$
\begin{aligned}
S_1 : & P = 50kW \\
& Q = +50\sqrt{\frac{1}{PF^2} - 1} \;\; kVAR \\
& \quad = 37.5 \;\; kVAR
\end{aligned}
$$

We want

$$
\begin{aligned}
S_w &= 50 + j50\sqrt{\frac{1}{(0.95)^2} - 1} \;\; kVA \\
&= 50 + j16.43 \;\; kVA \\
&= S_1 + S_2 \\
\therefore \;\; S_2 &= j(16.43 - 37.5) \\
&= -j21.07 \;\; kVA \\
&= VI_2^* \\
\therefore \;\; I_2 &= j\frac{21.07 \times 10^3}{230} \\
&= j91.6\, A \\
Z_2 &= \frac{V}{I_2} = \frac{230}{91.6}\measuredangle - \pi/2 \\
&\quad = -j2.51\ \Omega \\
&\quad \text{(capacitive load)}
\end{aligned}
$$