Electric Circuits and Networks **10-10-2018**

Lecture 27: Sinusoidal Steady State (Contd...)

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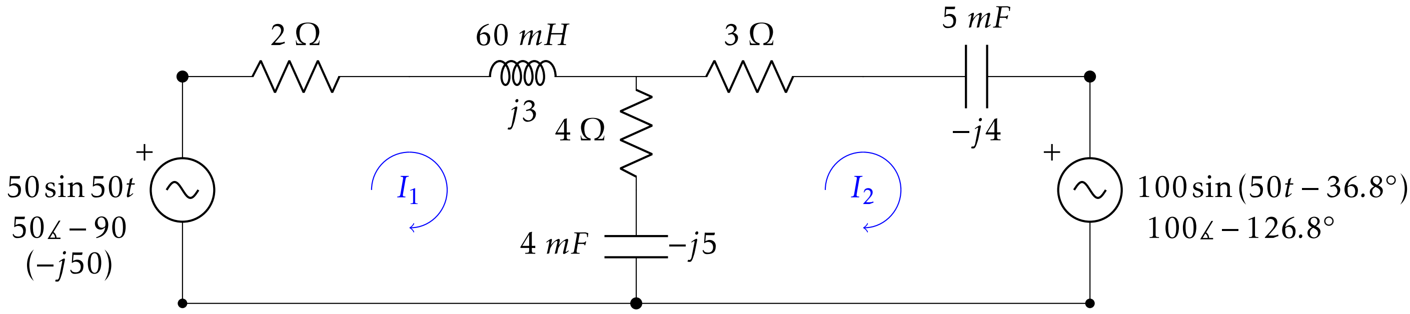
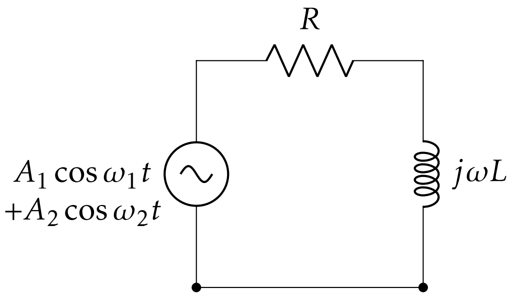
Example 1 : Superposition

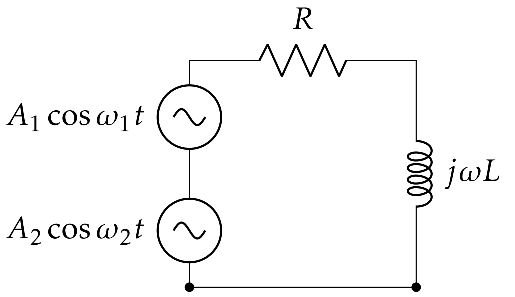


$$\begin{bmatrix} 6-j2 & -4+j5 \\ -4+j5 & 7-j9 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -j50 \\ -100\measuredangle -126.8 \end{bmatrix}$$

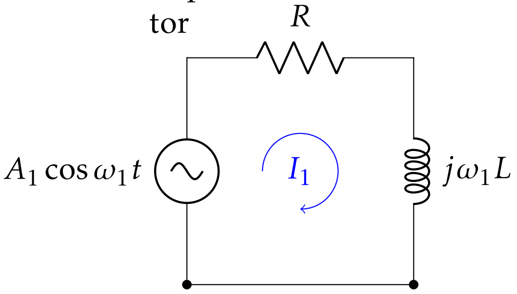
$$= \begin{bmatrix} -j50 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -100\measuredangle -126.8 \end{bmatrix}$$

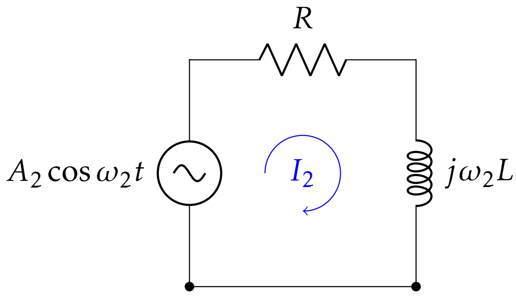
⇒ Superposition can be used to solve for the system response. Especially useful if you have sources with different frequencies.





Here the question is which ω value should we use for impedance of inductor





$$I_1 = \frac{A_1}{R + j\omega_1 L} \qquad I_2 = \frac{A_2}{R + j\omega_2 L}$$

Same frequency :

$$B_1 e^{j\omega t} + B_2 e^{j\omega t} = (B_1 + B_2)\, e^{j\omega t}$$

Adding the two signals is the same as adding the phasors and then multiplying by $e^{j\omega t}$

Different frequency :

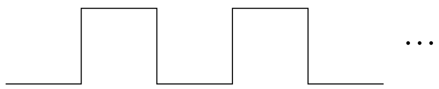
$$B_1 e^{j\omega_1 t} + B_2 e^{j\omega_2 t} \longrightarrow \text{Cannot add the phasors.}$$

Add signals in time domain after multiplying by $e^{j\omega t}$

Total response : $\dfrac{A_1}{R + j\omega_1 L} e^{j\omega_1 t} + \dfrac{A_2}{R + j\omega_2 L} e^{j\omega_2 t}$

Actual response : $Re\{I_1 e^{j\omega_1 t} + I_2 e^{j\omega_2 t}\}$

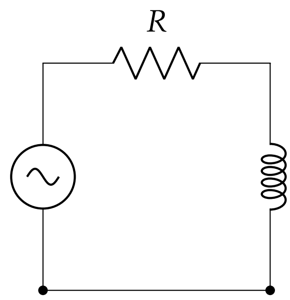
Periodic signal as Input

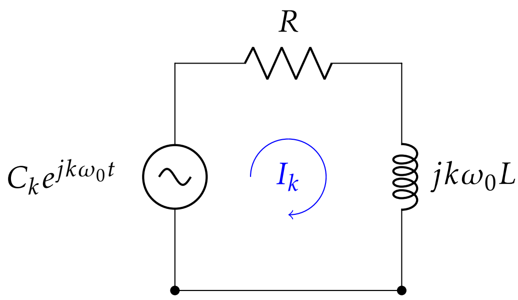


Periodic

$V(t) = \sum_k C_k e^{jk\omega_0 t}$

where, C_k is Phasor

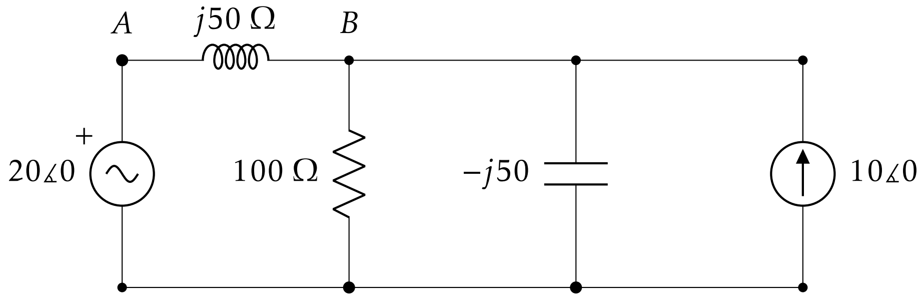


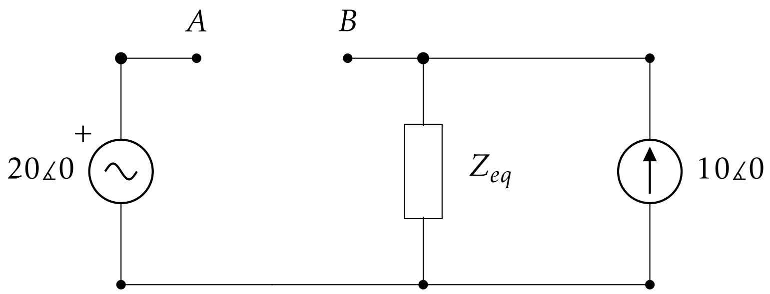


$$I_k = \frac{C_k}{R + jk\omega_0 L}$$

$$I(t) = \sum_k \frac{C_k}{R + jk\omega_0 L} e^{jk\omega_0 t}$$

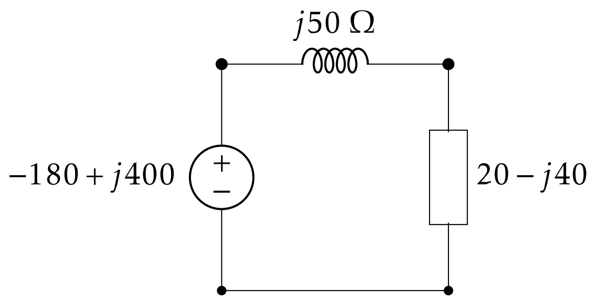
Example 2 : Find Thevenin's equivalent circuit as seen by inductor. **Note :** Proof of Thevenin's theorem depends only on substitution and superposition theorem; both of which can be used with phasors.





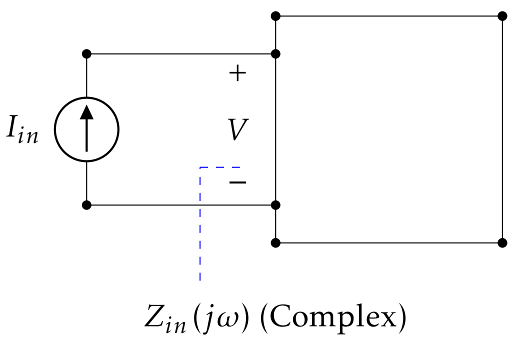
$$Z_{eq} = \frac{100(-j50)}{100-j50} = 20 - j40$$

$$V_{AB} = V_{oc} = 20\angle 0 - 10\angle 0 \left(Z_{eq}\right) = -180 + j400$$



$$\tan^{-1}\left(\frac{400}{-180}\right) = -65.77 + 180$$

Driving point impedance



$$V = I_{in} Z_{in}$$

Connect $I_{in}e^{j\omega t}$; response is $Ve^{j\omega t}$

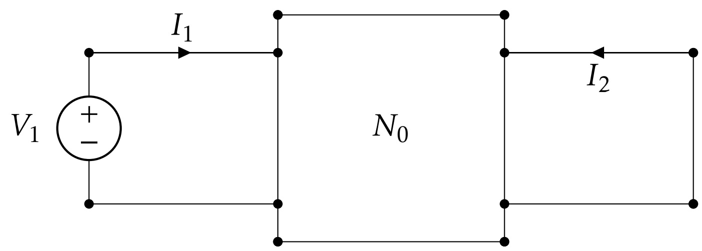
$$Ve^{j\omega t} = I_{in}Ze^{j\omega t}$$

$Z_{in} = \dfrac{V}{I_{in}}$ where V, I_{in} are phasors.

Z is complex and $Z = \dfrac{|V|}{|I_{in}|}e^{j(\theta_V - \theta_I)}$

Two port networks

For analysis in sinusoidal steady state, use phasors.



$$Y_{21} = \frac{I_2}{V_1}$$

$$Y_{11} = \frac{I_1}{V_1}$$

V_1, I_1 and I_2 are phasors.