Electric Circuits and Networks **9-10-2018**

Lecture 26: Sinusoidal Steady State (Contd...)

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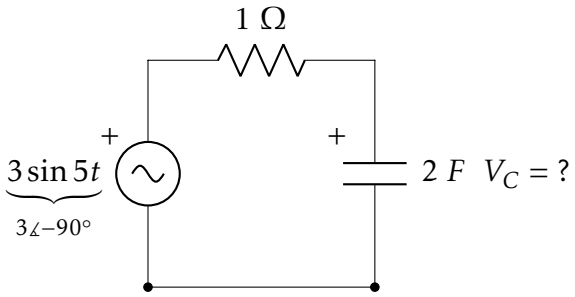
Sinusoidal Steady State

$$e^{j\omega t} \rightarrow H(j\omega)\, e^{j\omega t}$$

$$\text{Signal}: Re\{Ae^{j\theta} \cdot e^{j\omega t}\} = A\cos(\omega t + \theta)$$

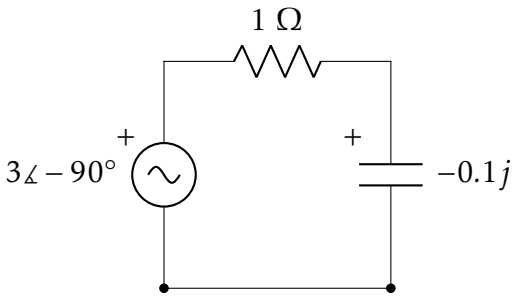
$$\text{Phasor}: Ae^{j\theta} \quad \text{(Complex amplitude)}$$

Example 1 : Find voltage across capacitor



$$\omega = 5\text{rad/s}$$

$$Z_C = \frac{1}{j\omega C} = -0.1j$$

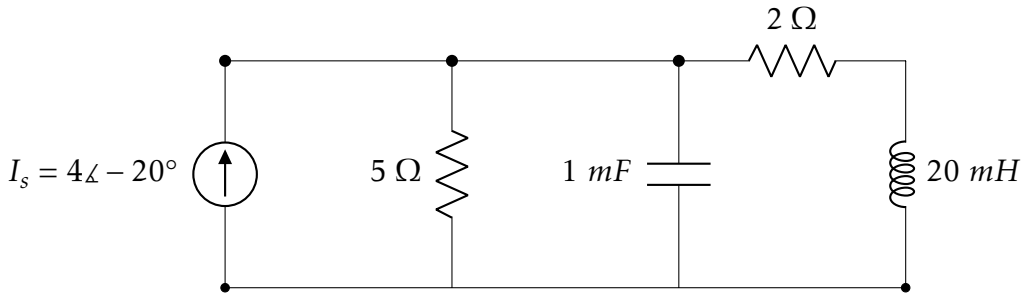


$$I = \frac{V_s}{1 - 0.1j}$$

$$V_C = 3\angle -90° \frac{-0.1j}{1 - 0.1j}$$

$$= 3\angle -90° \{\frac{-0.1j\,(1 + 0.1j)}{1.01}\}$$

Example 2 : Find current through inductor. $\omega = 100\text{rad/s}$



$$1\text{mF} \rightarrow -j10$$

$$20\text{mH} \rightarrow j2$$

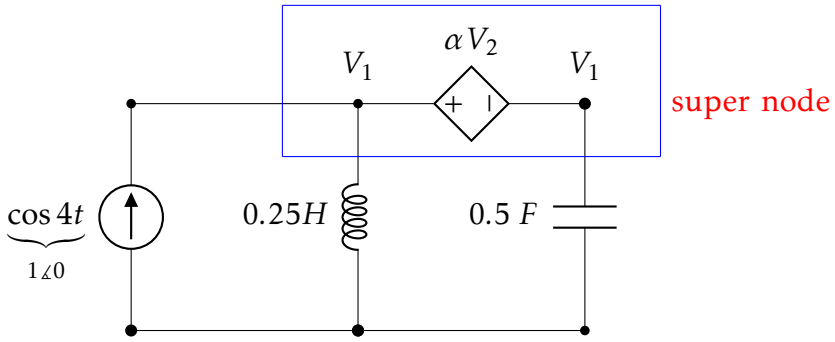
Parallel combination of 5Ω and $1\ mF$

$$Z_{eq} = \frac{5(-j10)}{5-j10} = 4-j2$$

$$\begin{aligned} I_L &= I_s \cdot \frac{4-j2}{2+j2+4-j2} \\ &= I_s \cdot \frac{2-j}{3} \\ &= \frac{4\sqrt{5}}{3} \angle \underbrace{\left(\tan^{-1}(-1/2) - 20^\circ\right)}_{\phi} \end{aligned}$$

$$i_L(t) = \frac{4\sqrt{5}}{3}\cos(100t+\phi)$$

Example 3 : Find current through capacitor using nodal analysis



$$\begin{aligned} V_1(-j) + V_2(j2) &= 1 \\ V_1 - V_2 &= 0.25V_2(j2) \end{aligned}$$

$$\begin{bmatrix} -j & j2 \\ 1 & -(1+0.5j) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

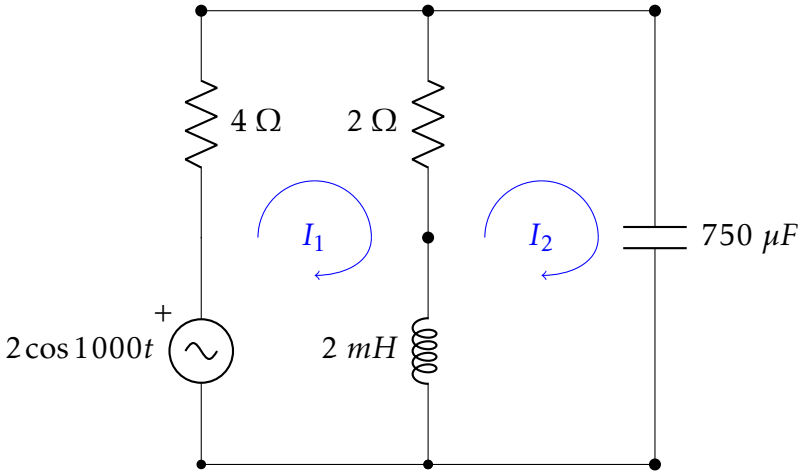
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} -(1+0.5j) & -j2 \\ -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} D &= j(1+0.5j) - j2 \\ &= -0.5 - j \end{aligned}$$

$$V_2 = \frac{-1}{-0.5-j} = \frac{1}{0.5+j} = 0.4-0.8j$$

$$I_C = (0.4-0.8j)(j2)$$

Example 4 : Find voltage across capacitor using mesh analysis.

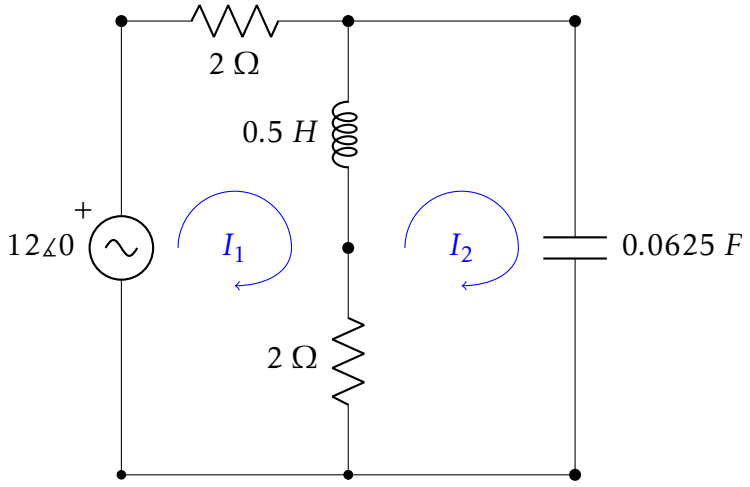


$$\begin{bmatrix} 6+j2 & -(2+j2) \\ -(2+j2) & 2+j2/3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

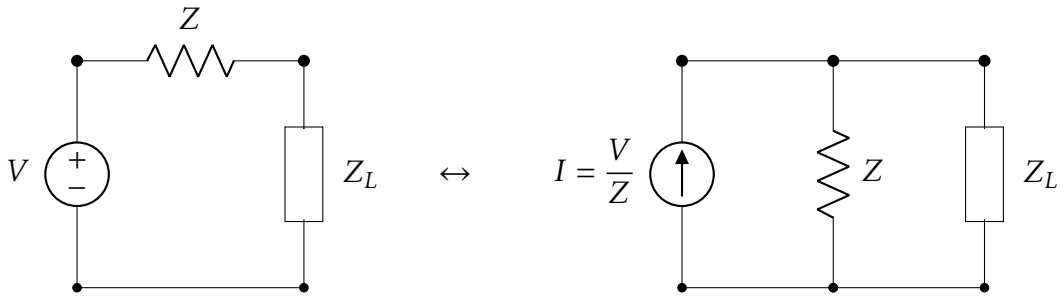
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} 2+j2/3 & (2+j2) \\ (2+j2) & 6+j2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{aligned} D &= \frac{32}{3} \\ I_2 &= \frac{12(1+j)}{32} \\ V_C &= -j\frac{16}{32}(1+j) \\ &= 0.5 - j0.5 \end{aligned}$$

Example 5 : Find current through capacitor using source transformation

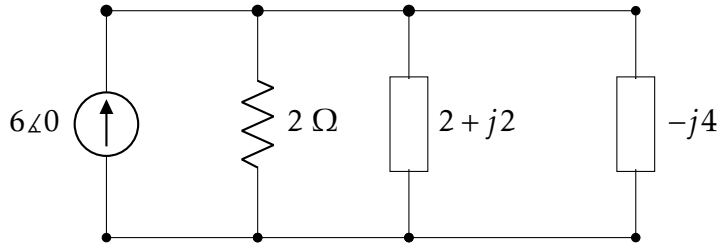


Use source transformation



$$I = \frac{V}{Z + Z_L}$$

$$I = \frac{V}{Z} \cdot \frac{Z}{Z + Z_L} = \frac{V}{Z + Z_L}$$



Use current division to find the phasor current I; then $i(t) = Re\{Ie^{j\omega t}\}$