Electric Circuits and Networks **24-09-2018**

Lecture 21: Z Parameter

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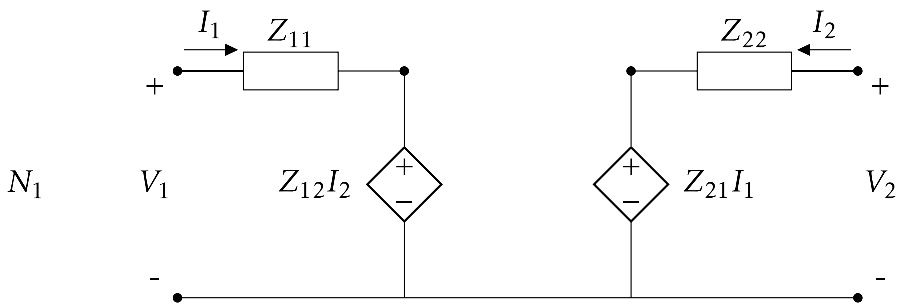
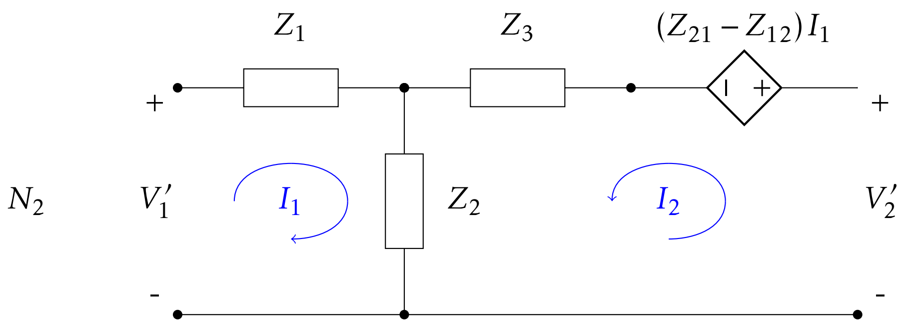
Z Parameters:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$





If we know the Y parameter of a network then we represent the network as a π or Δ network.Similarly using Z parameter we can represent the network as a T network. Find Z_1, Z_2 and Z_3 so that the two networks have the same characteristics at the two ports i.e if we connect current sources I_1 and I_2 at the ports, the corresponding voltages should be identical.

Port 1

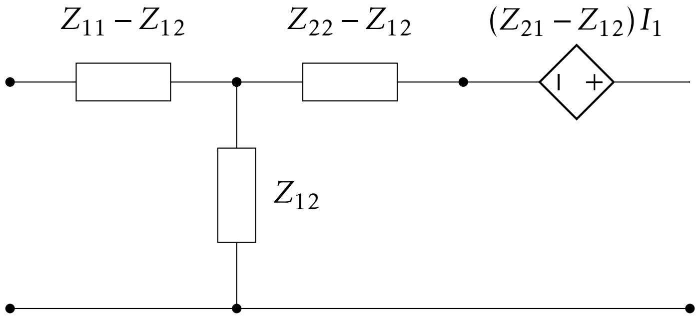
$$N_1 : V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$N_2 : V_1' = Z_1 I_1 + Z_2(I_1 + I_2)$$
$$= (Z_1 + Z_2)I_1 + Z_2 I_2$$
$$V_1 = V_1' \implies$$
$$Z_2 = Z_{12}$$
$$Z_1 = Z_{11} - Z_{12}$$

Port 2

$$N_1 : V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$N_2 : V_2' = Z_3 I_2 + Z_2(I_1 + I_2) + (Z_{21} - Z_{12})I_1$$
$$= Z_{12}I_1 + (Z_3 + Z_{12})I_2 + (Z_{21} - Z_{12})I_1$$

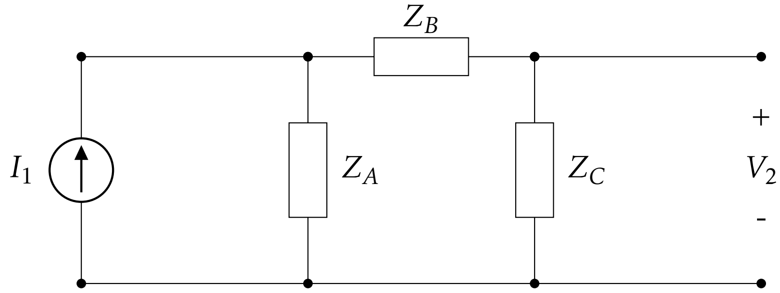
So

$$Z_3 = Z_{22} - Z_{12}$$



The above circuit is the equivalent T network for network N_1.

Example 1 : Find Z parameter of the below network



As we know that

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

After solving we will get

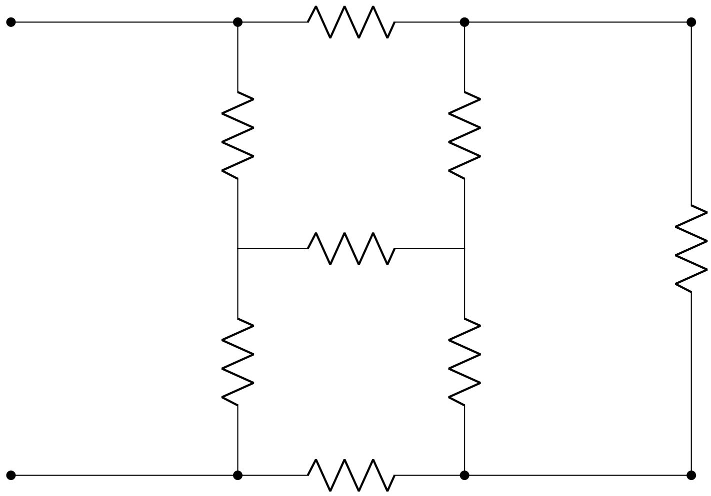
$$Z_{11} = \frac{Z_A (Z_B + Z_C)}{Z_A + Z_B + Z_C} \qquad Z_{21} = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$Z_{12} = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \qquad Z_{22} = \frac{Z_C (Z_A + Z_B)}{Z_A + Z_B + Z_C}$$

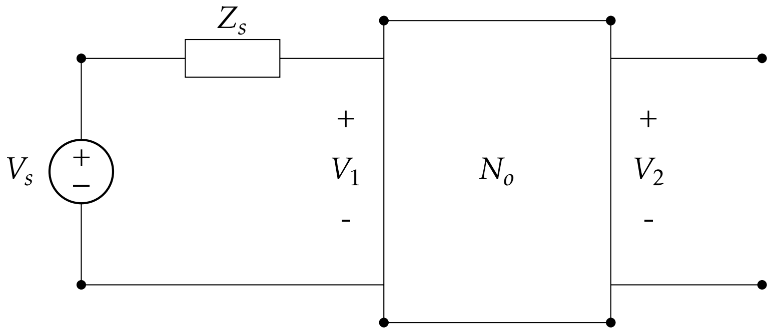
This can be converted to an equivalent T network. We get

$$Z_1 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$
$$Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$
$$Z_3 = \frac{Z_C Z_B}{Z_A + Z_B + Z_C}$$

Note that the network is reciprocal and $Z_{12} = Z_{21}$.
Exercise 1 : Find the equivalent R_{in} using $\Delta - T$ conversion.



Example 2 : Find Thevenin's equivalent looking at port 2. The Z parameters of N_o are known.



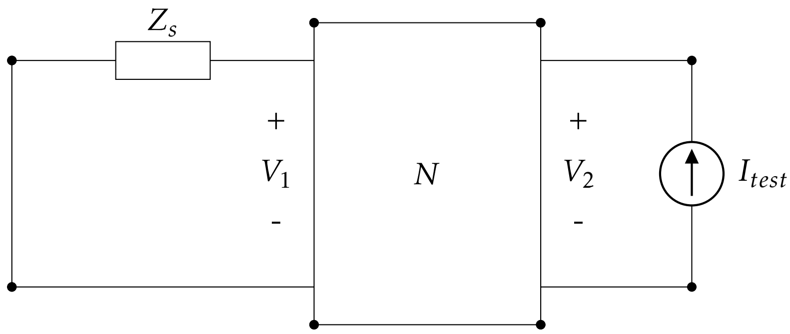
To find V_{oc} : Solve V_2 when $I_2 = 0$

$$V_1 = V_s - I_1 Z_s = Z_{11} I_1$$
$$V_2 = Z_{21} I_1$$

hence

$$V_{oc} = V_2 = \frac{Z_{21} V_s}{Z_{11} + Z_s}$$

To find Z_{th} : Connect a current source I_{test} at port 2 and evaluate $\frac{V_2}{I_{test}}$

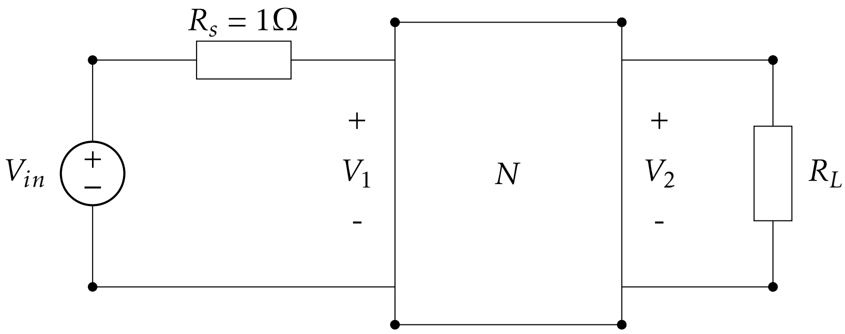


$$0 = (Z_{11} + Z_s) I_1 + Z_{12} (I_{test})$$

$$V_2 = Z_{21} I_1 + Z_{22} I_{test}$$

$$Z_{th} = \frac{V_2}{I_{test}} = Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11} + Z_s}$$

Exercise 2 : Given $Z = \begin{bmatrix} \frac{2}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{6}{s+1} \end{bmatrix}$ and circuit as given below,



Find zero input and zero state response, output = V_2 and $V_{in} = u(t)$
$\because$ Network is relaxed (i.e. N can not have initial condition/sources) So, zero input response (i.e. $V_{in} = 0$) will be = 0
For Zero State response: Solve following equations

$$V_1 = V_{in} - I_1 R_s = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = -I_2 R_L = Z_{21} I_1 + Z_{22} I_2$$