Electric Circuits and Networks **12-09-2018**

Lecture 18: Proof of Thevenin's theorem

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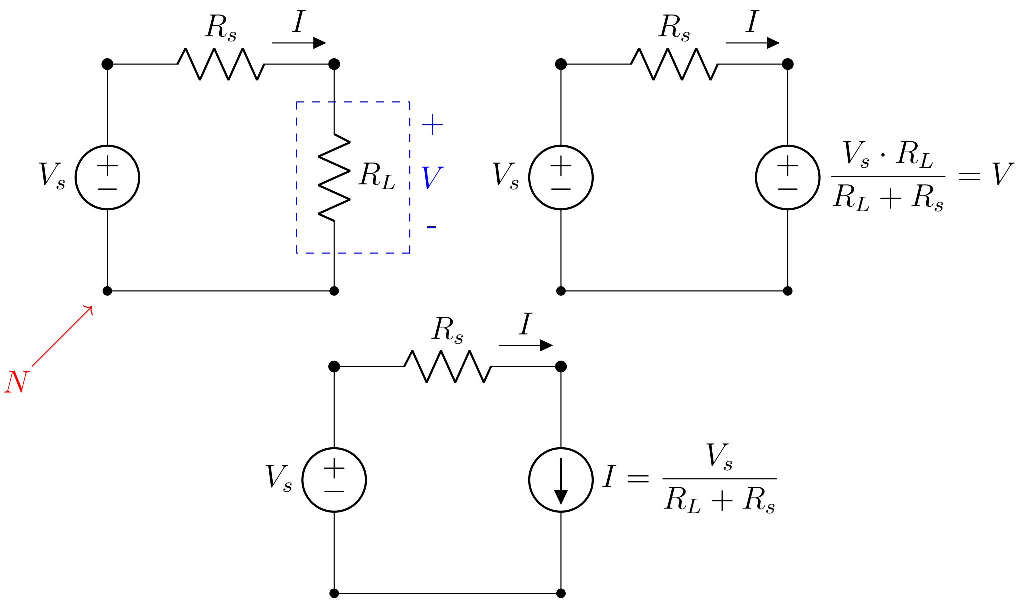
Substitution Theorem:

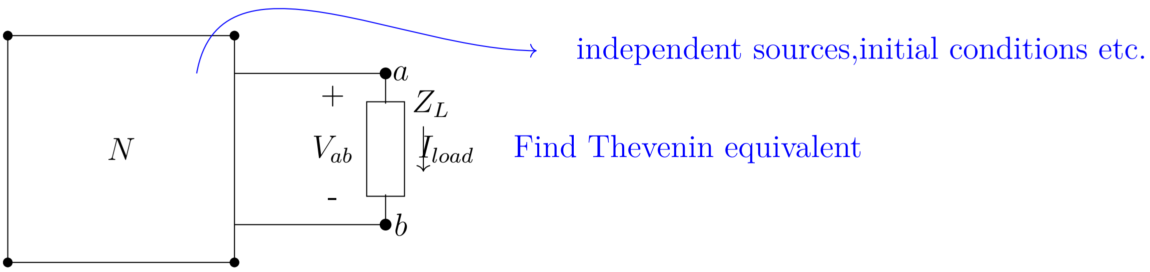


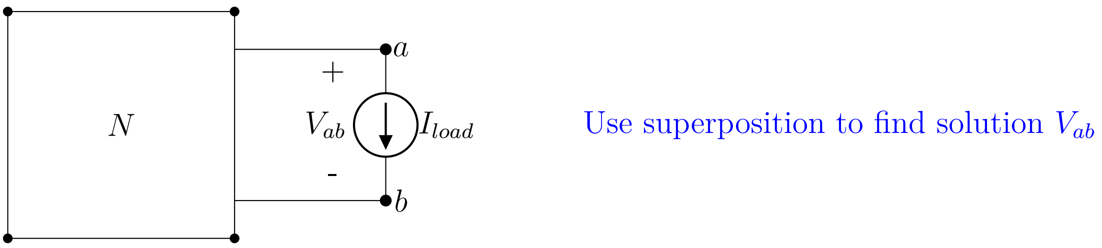
If R_L is replaced with a voltage source equal in value to the voltage across R_L in the original network, the other responses (current) remains the same. Equivalently, you can replace R_L with a current source equal in value to the current through R_L in the original network. The voltage across R_L = voltage across the current source in the modified network.

In general, solve for voltages, current in N. Assume we are doing nodal analysis and the solution to the node voltages are $\{V_1, V_2, \cdots V_k\}$; pick any branch and replace it with a voltage source with value equal to the actual branch voltage. For eg., if we have an impedance between node i and the reference node, replace it with a voltage source of value V_i. This is called as **modified network.** Solutions to N and modified network are identical. This is obvious: Since V_i is the actual solution obtained using nodal analysis, the currents in all other branches are unaffected. Hence the current through the voltage source in the modified network also remains the same. Caution: If replacement by a voltage source, gives two voltage sources in parallel, one cannot do this.

Proof of Thevenin's theorem

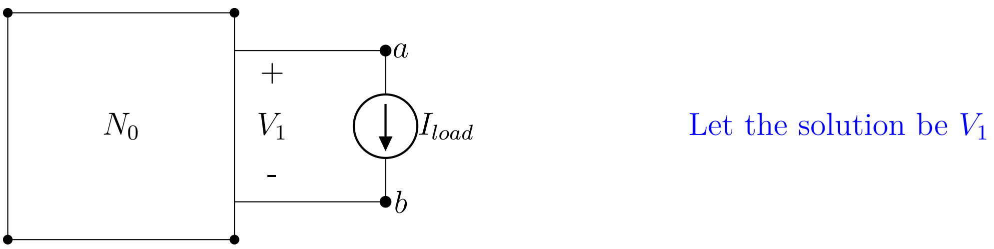
N is a network containing independent sources, initial conditions. We wish to replace it by its Thevenin equivalent across terminals a and b.





Note that $Z_L = \frac{V_{ab}}{I_{load}}$.

First, use substitution theorem and replace the load by a current source whose value is the current through the load (I_{load}). For this modified network, it is possible to find the solution using superposition. Set $I_{load} = 0$; (open circuit across terminals a and b). Activate one source at a time and find voltage across a & b i.e. V_{oc}. Set all sources in N to zero and activate I_{load}; We get the following circuit.



Network N_0 has no initial condition & no sources (short voltage sources and open current sources). If the driving point impedance is Z_{eq}, then $V_1 = -I_{load}Z_{eq}$.

By Superposition, total solution is

$$\begin{aligned} V_{ab} &= V_{oc} + V_1 \\ &= V_{oc} - I_{load}Z_{eq} \end{aligned}$$

Therefore,

$$\frac{V_{ab}}{I_{load}} = Z_L = \frac{V_{oc}}{I_{load}} - Z_{eq}$$

or

$$I_{load} = \frac{V_{oc}}{Z_L + Z_{eq}}$$

As far as the load is concerned, the network N can be replaced by a voltage source $= V_{oc}$ in series with an impedance Z_{eq}. This is true for any arbitrary load Z_L.