Electric Circuits and Networks	29-08-2018

Lecture 13: Natural Frequency and Bode Plot

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Natural Frequencies

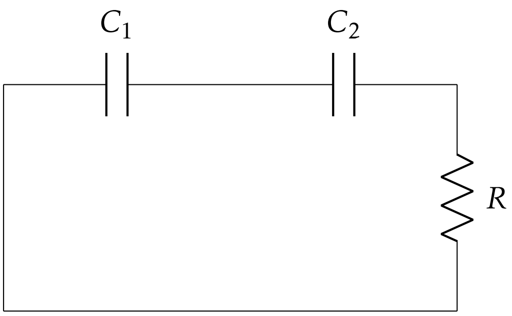
* No. of natural frequencies = # of independent initial conditions that can be assigned ≡ order of the circuit.

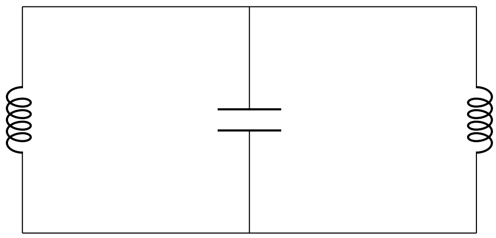
* can be obtained by solving for roots of the polynomial obtained by determinant of the node/mesh based network matrix.

* n natural frequencies $k_1e^{s_1t}+k_2e^{s_2t}+\cdots+k_ne^{s_nt}$ but it may happen that a frequency have multiplicity more than one.
  e.g. $\frac{1}{(s+a)^2} \rightarrow kte^{-at}$

* System is stable, poles in LHP/simple poles in $j\omega$ axis; transient will die out or you get solution of form $cos\omega_0t, sin\omega_0t, u(t)$

Exercise - Find the natural frequencies of the following circuits

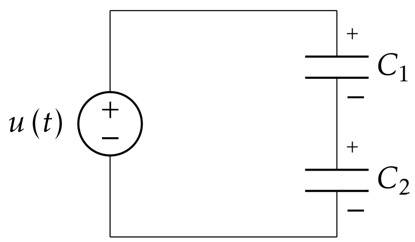




Hint: There could be additional poles at zero.

Laplace/Time domain analysis with loops of voltage sources and capacitors

Example 1 : Assume capacitors are initially uncharged.



To satisfy KVL, capacitors voltages must also be of the form $Ku(t)$ $\Rightarrow$ Impulsive current that charge both the capacitors.

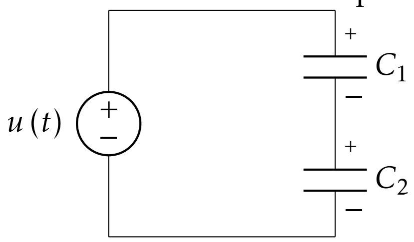
In the Laplace domain,

$$I(s)\left(\frac{1}{sC_1}+\frac{1}{sC_2}\right)=\frac{1}{s}$$

$$I(s)=\frac{C_1C_2}{C_1+C_2} \leftrightarrow i(t)=\frac{C_1C_2}{C_1+C_2}\delta(t)$$

$$V_{C_1}(s) = \frac{1}{sC_1}\frac{C_1C_2}{C_1+C_2} = \frac{1}{s}\frac{C_2}{C_1+C_2}$$
$$v_{C_1}(t) = \frac{C_2}{C_1+C_2}u(t)$$
$$v_{C_2}(t) = \frac{C_1}{C_1+C_2}u(t)$$

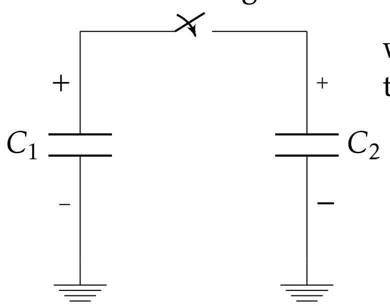
We can also solve this problem in the time domain using capacitor charge equation



$$C_1V_1 = C_2V_2$$
$$V_1 + V_2 = 1$$

Switch capacitor circuits

Assume initial voltage across C_1 is V_{C_1} and C_2 is initally uncharged. Final volage is V_f.



when switch is closed the charge in C_1 will be redistributed so that final voltage is the same.

Initial voltage across C_1 is $V_{C_1}(0)$
$Q = C_1V_{C_1}(0) = V_f(C_1+C_2)$

Bode Plots

* circuits is stable ($j\omega$ axis is in the ROC)

$\Rightarrow$ Given $H(s)$, can find $H(j\omega)$ Plot response to $e^{j\omega t}$ (Eigenfunction). Plot magnitude and phase response.

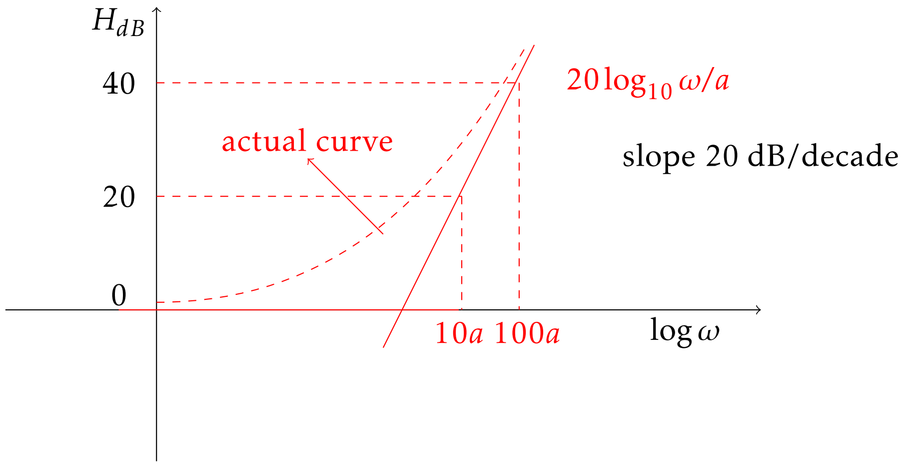
$$e^{j\omega t} \to \underbrace{H(j\omega)}e^{j\omega t}$$
$$|H(j\omega)| \;\&\; \measuredangle H(j\omega)$$

Example 1: $H(s) = 1 + s/a$

$$H(j\omega) = 1 + j\omega/a$$
$$|H(j\omega)| = \sqrt{1+\frac{w^2}{a^2}}, \quad \measuredangle H(j\omega) = \arctan(\omega/a)$$
$$H_{dB} = 20\log_{10}|H(j\omega)| \quad decibel$$
$$= 10\log_{10}|H(j\omega)|^2$$

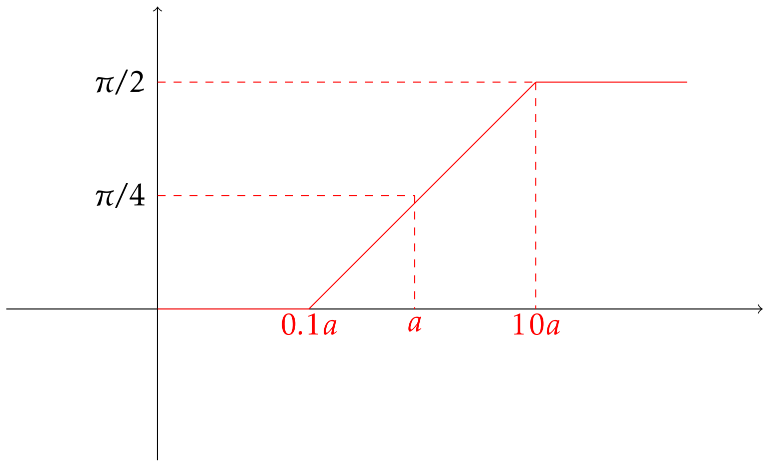
$$|H(j\omega_2)| = 10|H(j\omega_1)|$$
$$H_{1_{dB}} - H_{2_{dB}} = 20\log_{10}\frac{|H(j\omega_2)|}{|H(j\omega_1)|}$$
$$= 20dB$$

$$
\begin{aligned}
|H(j\omega_2)| = 2|H(j\omega_1)| \Rightarrow \ 6dB\text{change} \\
|H(j\omega)|^2 = 1 + \frac{w^2}{a^2} \\
H_{dB} = 10\log_{10}\left(1 + \frac{w^2}{a^2}\right) \\
H_{dB} \approx 10\log_{10} 1 = 0 \quad \omega \ll a; \\
H_{dB} \approx 20\log_{10} \omega/a \quad \omega \gg a
\end{aligned}
$$



$$\measuredangle H(j\omega) = \arctan(\omega/a)$$

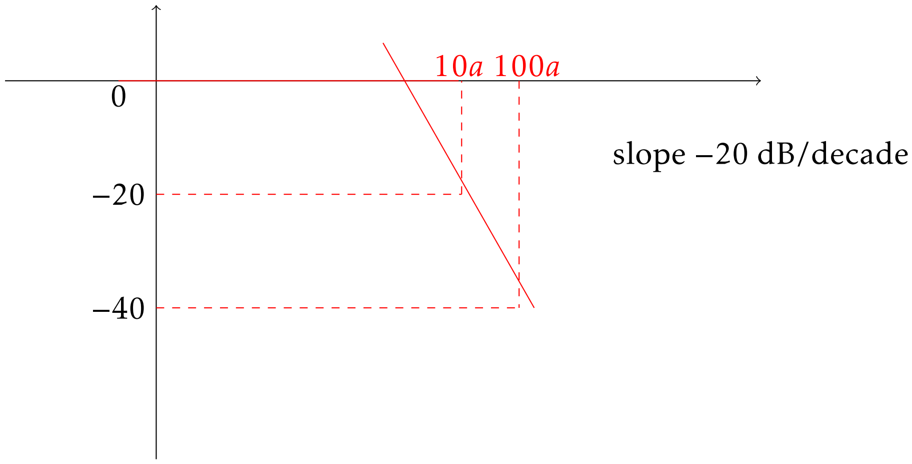
$$
\begin{aligned}
\omega \ll a \quad &\longrightarrow \quad \measuredangle H(j\omega) \approx 0 \\
\omega = a \quad &\longrightarrow \quad \measuredangle H(j\omega) = \pi/4 \\
\omega \gg a \quad &\longrightarrow \quad \measuredangle H(j\omega) \approx \pi/2
\end{aligned}
$$



Example 2: $H(s) = \dfrac{1}{1 + s/a}$

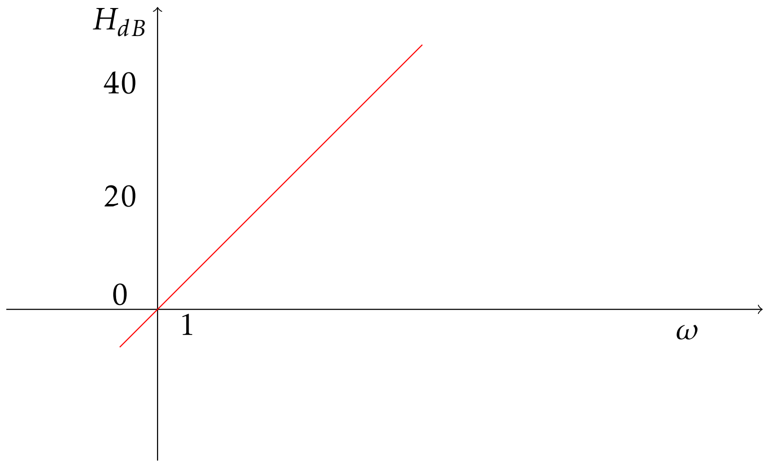
$$|H(j\omega)| = \frac{1}{\sqrt{1 + \frac{w^2}{a^2}}}, \quad \measuredangle H(j\omega) = -\arctan(\omega/a)$$

$$H_{dB} = -10\log_{10}\left(1+\frac{w^2}{a^2}\right)$$



Example 3: $H(s) = s$

$$|H(j\omega)| = \omega, \quad \measuredangle H(j\omega) = \pi/2$$



Exercise $H(s) = \dfrac{1}{(1+s/10)(1+s/100)}$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2/100}\cdot\sqrt{1+\omega^2/10^6}}$$