

Tree Architectures: Carry look-ahead.

$$\begin{aligned}(\underline{g_1}, \underline{p_1}) \cdot (g_0, p_0) &= (\underline{g_1 + p_1 g_0}, \underline{p_1 p_0}) \\ &= (g_{1:0}, p_{1:0})\end{aligned}$$

$$\begin{aligned}(g_2, p_2) \cdot (g_{1:0}, p_{1:0}) &= (g_{2:0}, p_{2:0}) \\ &= (g_2 + p_2 g_1 + p_2 p_1 g_0, p_2 p_1 p_0)\end{aligned}$$

$$(g_{1:0}, p_{1:0}) (\underline{c_{-1}}, 0) = c_1 \quad (g_1, p_1) \cdot (c_0, 0) = c_1$$

$$(g_{2:1}, p_{2:1}) (c_0, 0) = c_2$$

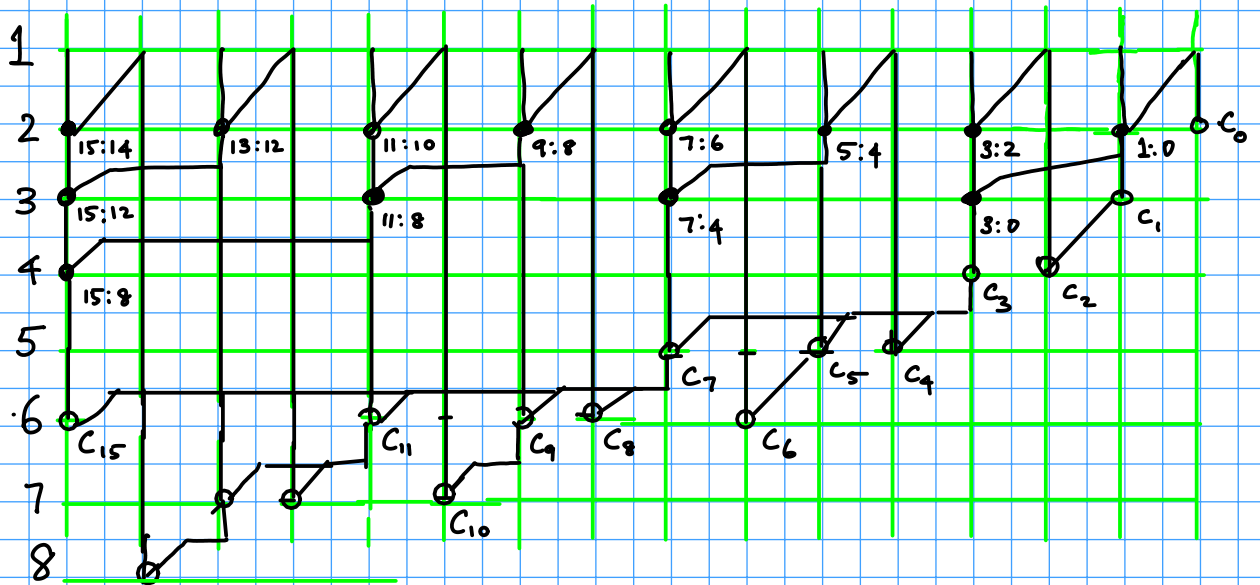
$$\rightarrow (g_1 + p_1 g_0, p_1 p_0) (c_{-1}, 0) = g_1 + p_1 g_0 + p_1 p_0 c_{-1} = c_1$$

$$(g_{7:4}, p_{7:4}) \cdot (c_{-1}, 0) \times$$

$$(g_{7:4}, p_{7:4}) \cdot (c_3, 0) = c_7$$

$$\bullet \longrightarrow o/p - (g, p)$$

$$\circ \longrightarrow i/p - \text{Carry}$$



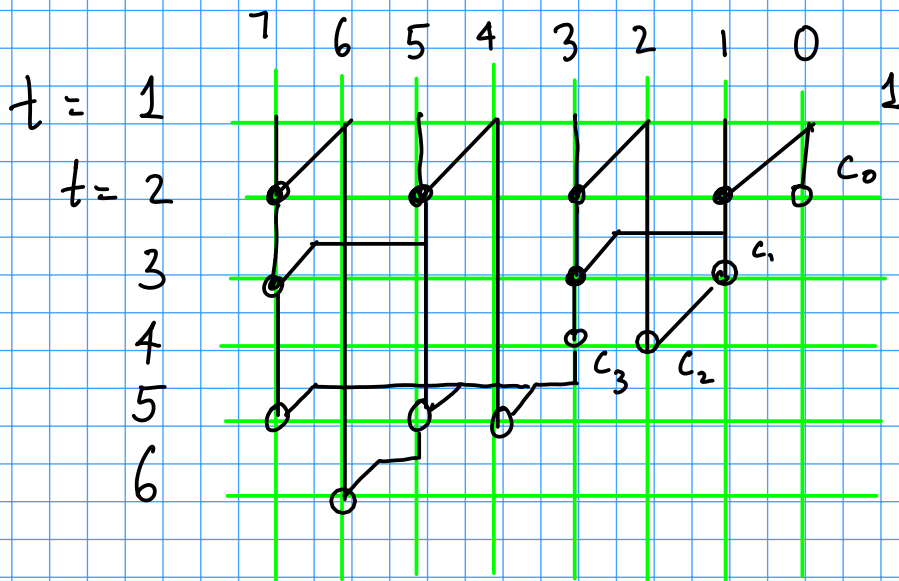
Operator
"dot" $\bullet \rightarrow$

$$(G_2, P_2) \bullet (G_1, P_1)$$

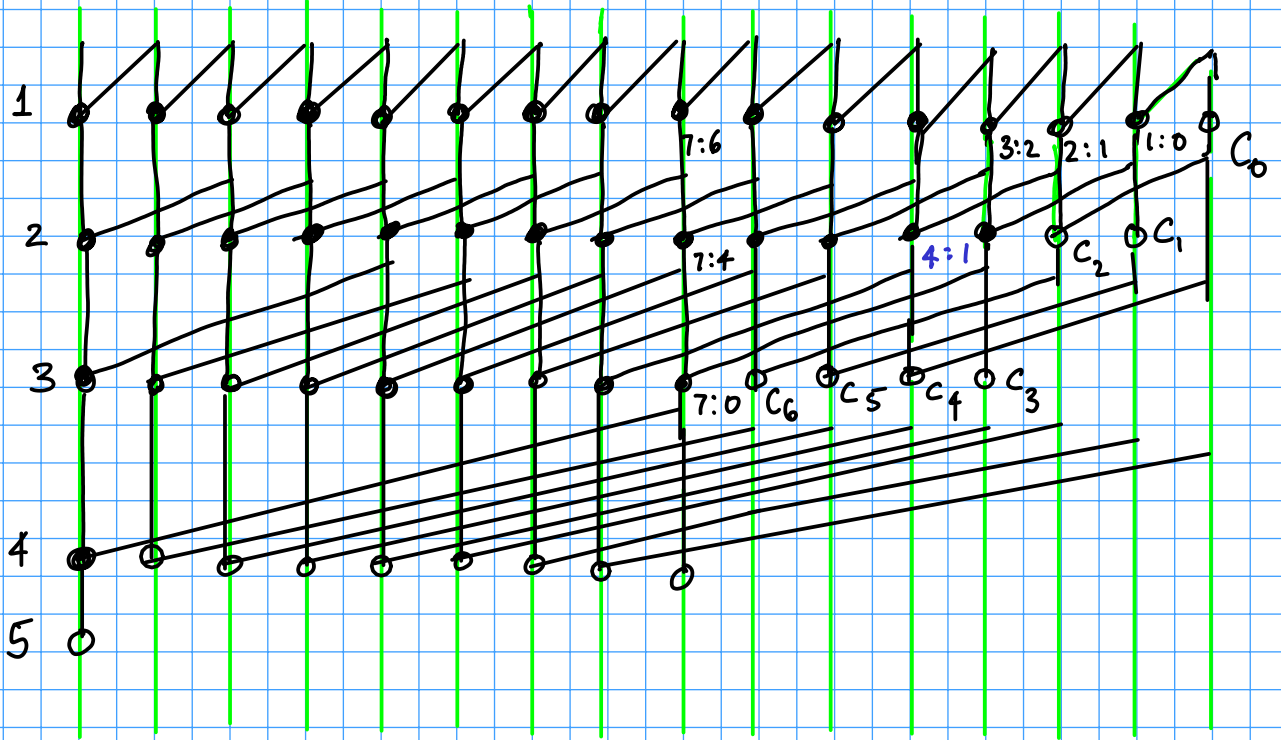
$$= (G_2 + P_2 G_1, P_2 P_1)$$

$$0 \rightarrow (G, P) \bullet (C_i, 0)$$

$$C_{i+1} = G + P C_i$$



Delay ↑ses from 6 to 8 when # of bits doubles.



$$(g_5:2, P_5:2) = (g_5 + P_5 g_4 + P_5 P_4 g_3 + P_5 P_4 P_3 g_2, P_5 P_4 P_3 P_2)$$

Delay $\sim \log_2 N$

$$g_0 + P_0 C_{-1}$$

$$g_1 + P_1 C_0$$

Trade-off:

$$g_2 + P_2 C_1 = g_2 + P_2 g_1 + \underbrace{(P_2 P_1)}_{\text{complexity}} C_0$$

Hardware, fanout, time
 ↑
 no. of gates complexity of the gate

After getting sum in the reqd. time, can optimize non-critical paths for power (replace by high- V_T gates or lower supply voltage)

More complex blocks. Assume

$$\begin{cases} C_0 = g_0 + p_0 C_{-1} \\ C_1 = g_1 + p_1 g_0 + p_1 p_0 C_{-1} \\ C_2 = g_2 + p_2 g_1 + p_2 p_1 g_0 + p_2 p_1 p_0 C_{-1} \\ C_3 = (g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 g_0, p_3 p_2 p_1 p_0) \end{cases}$$

Max^m of
4 series transistors
• (C₋₁, 0)

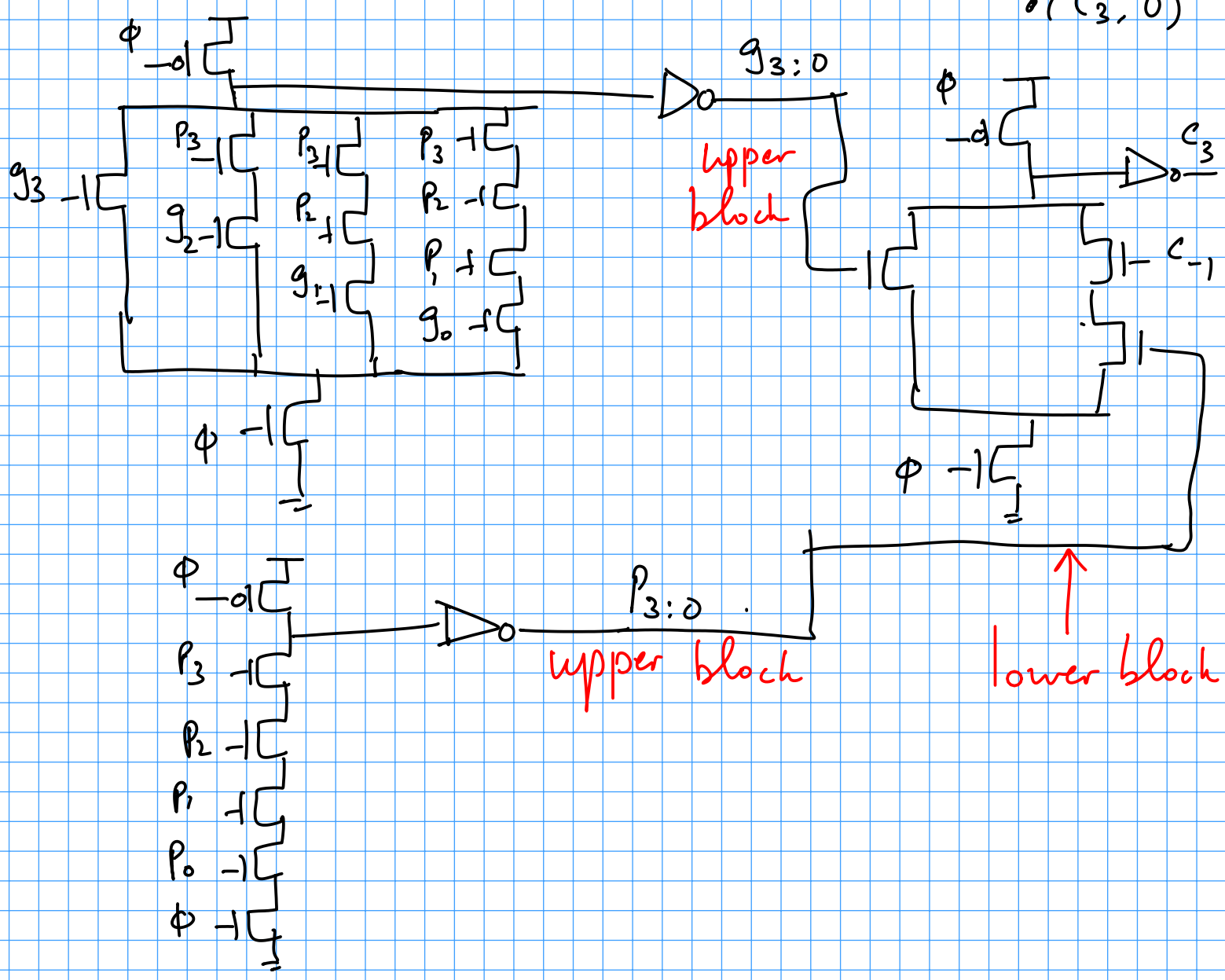
$$C_4 = g_4 + p_4 C_3$$

$$C_5 = g_5 + p_5 g_4 + p_5 p_4 C_3$$

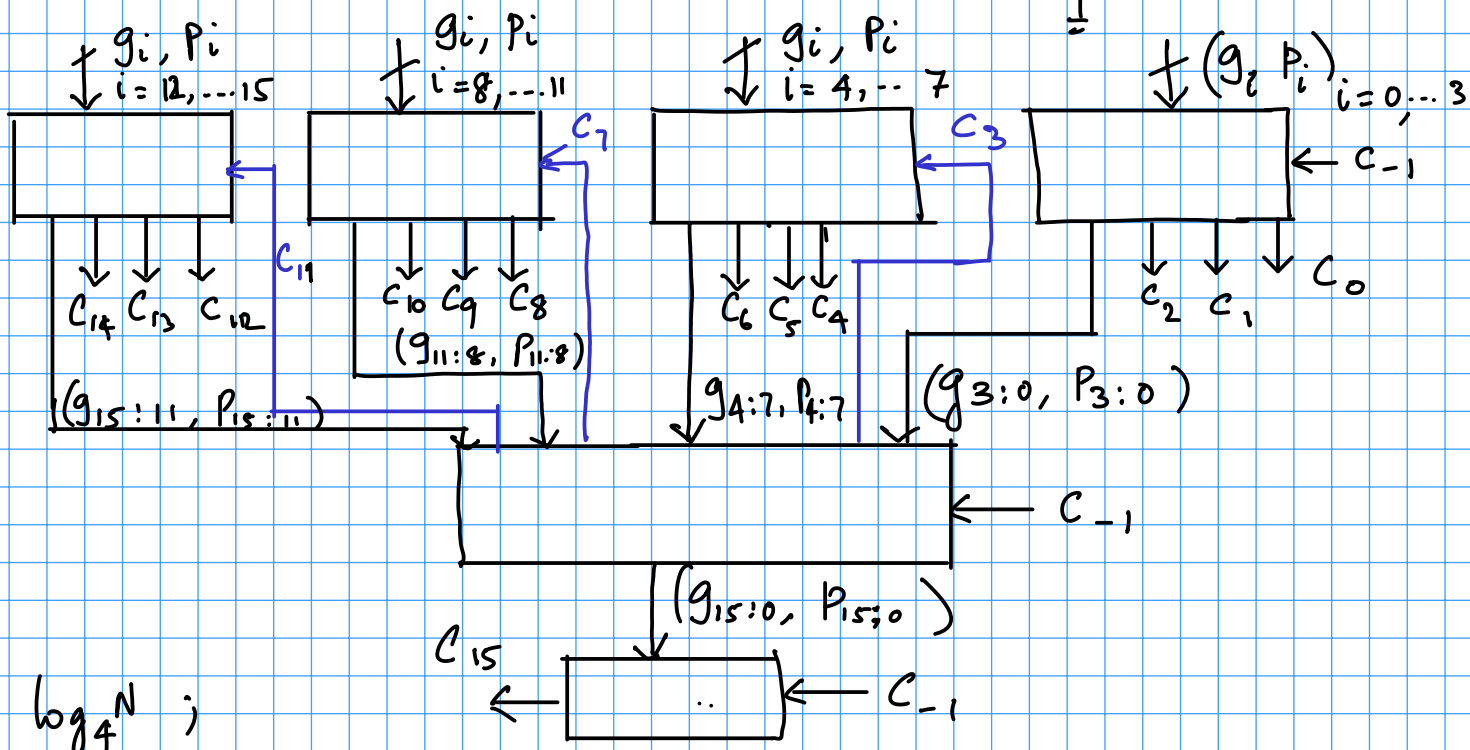
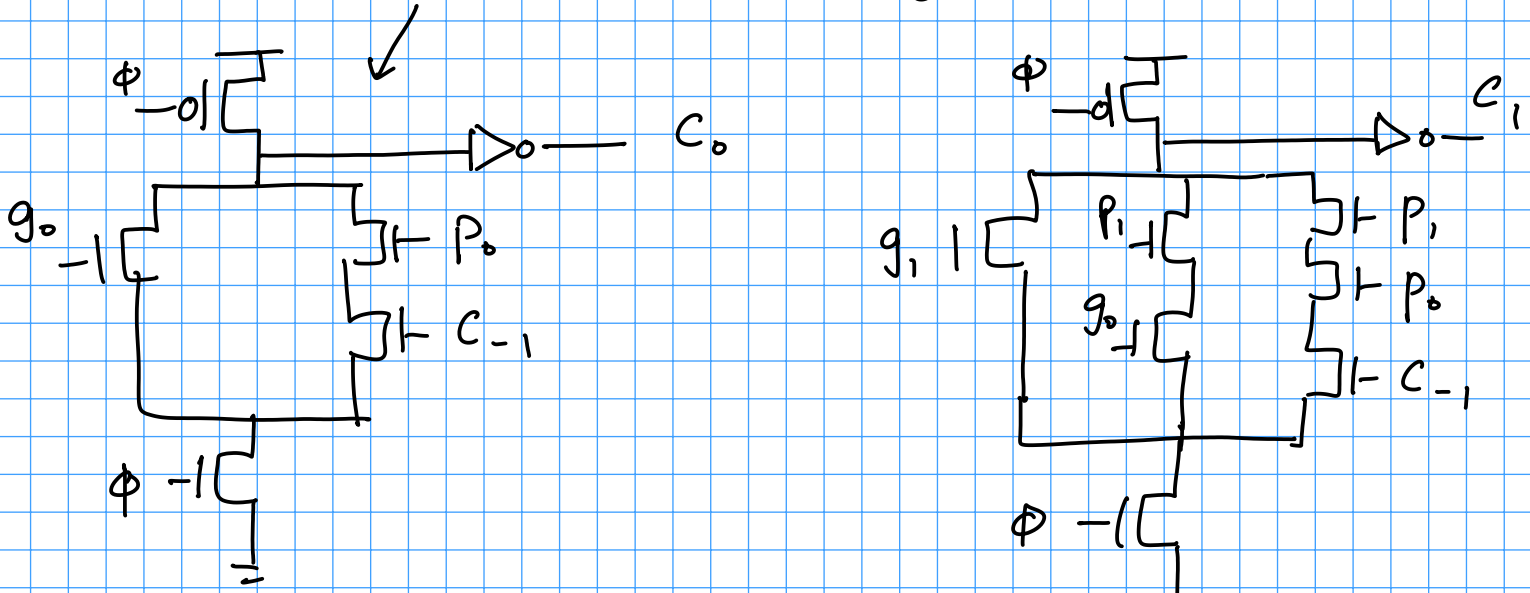
$$\vdots$$

$$C_7 = (g_7 + p_7 g_6 + \dots, p_7 p_6 p_5 p_4)$$

• (C₃, 0)



$\bullet \rightarrow (G_{3:0}, P_{3:0}), (G_{7:4}, P_{7:4})$
 $o \rightarrow o/p$ is a carry



$\log_4 N$;
 each block is more complex.
 Block carry look ahead.