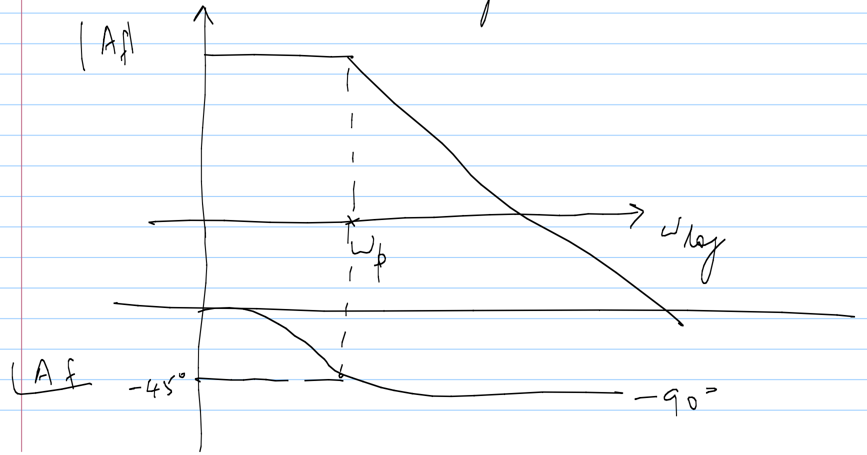
13/8/14

lec 9

1) LHP poles
2) Unconditionally stable



2-pole system

$$A(s) = \frac{A_0}{(1 + s/\omega_p)^2}$$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \frac{2s}{A_0 f \omega_p} + \frac{s^2}{\omega_p^2 A_0 f}}$$

general form of 2nd order system:

1) $s^2 + 2\zeta\omega_n s + \omega_n^2$

2) $\dfrac{s^2}{\omega_0^2} + \dfrac{s}{\omega_0 Q} + 1$

* $\left(\dfrac{-b}{2a}\right)$ is negative ⇒ LHP poles

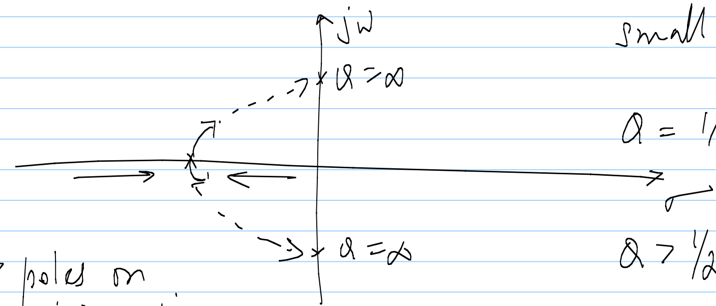
* $\angle G = -180°$ only @ $\omega = \infty$
⇒ technically unconditionally stable

$$\omega_0 = \omega_p \sqrt{A_0 f}$$

$$Q = \frac{\sqrt{A_0 f}}{2}$$

$$\left(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1\right) = D(s)$$

$$\frac{s}{\omega_0} = \frac{-1}{2Q} \pm j\sqrt{1 - \frac{1}{4Q^2}}$$

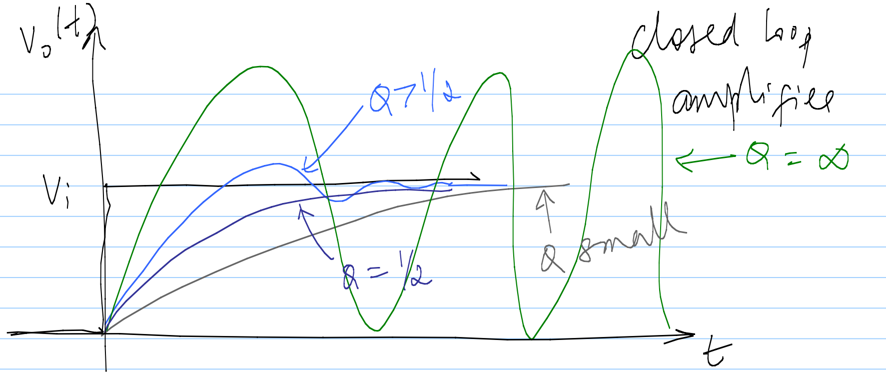


small Q → real LHP poles

Q = 1/2 → 2 equal real poles

Q > 1/2 ⇒ complex conj. poles

Q = ∞ ⇒ poles on jω axis



$$Q = \frac{\sqrt{A_0 f}}{2}$$

We want large $A_0 f \Rightarrow$ large Q

3rd order System

$$A(s) = \frac{A_0}{\left(1 + s/\omega_p\right)^3}$$

$$= \frac{A_0}{1 + \frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}}$$

$$CLG(s) = \frac{1}{f} \cdot \frac{A_0 f}{1 + A_0 f} \cdot \frac{1}{1 + \left[\frac{3s}{\omega_p} + \frac{3s^2}{\omega_p^2} + \frac{s^3}{\omega_p^3}\right]\frac{1}{(1 + A_0 f)}}$$

$$D(s) = 1 + \frac{3s}{\omega_p(1 + A_0 f)} + \frac{3s^2}{\omega_p^2(1 + A_0 f)} + \frac{s^3}{\omega_p^3(1 + A_0 f)}$$

$$D\left(\frac{s}{\omega_p}\right) = 1 + \frac{3s}{1 + A_0 f} + \frac{3s^2}{1 + A_0 f} + \frac{s^3}{1 + A_0 f}$$

Roots of $(1 + A_0 f) + 3s + 3s^2 + s^3 = 0$

$$(1 + s)^3 + A_0 f = 0$$

$$s = -1 + \left(-A_0 f\right)^{1/3}$$

example $A_0 f = 8$

$$\left.\begin{aligned} s_1 &= -1 - 2 = -3 \\ s_2 &= -1 - 2e^{-j2\pi/3} \\ s_3 &= -1 - 2e^{+j2\pi/3} \end{aligned}\right\} \quad \begin{aligned} s &= -3\omega_p \\ &= +j\sqrt{3}\,\omega_p \\ &= -j\sqrt{3}\,\omega_p \end{aligned}$$