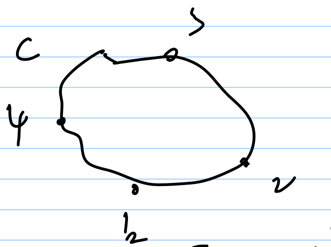
28/4/15

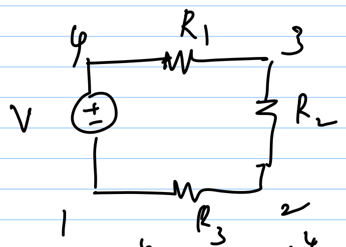
Lec 43

Magnetic Circuits

KVL : $\oint \bar{E}\cdot d\bar{l} = 0$ ← no changing flux within the ckt

→ Small size
→ low freq.





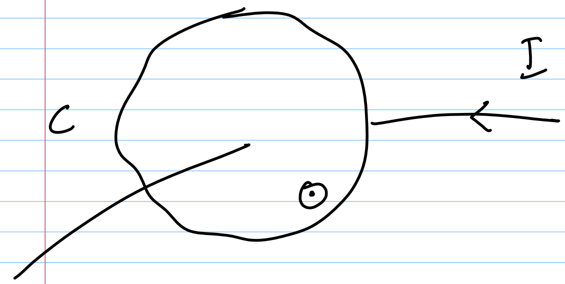
KVL

$V_{12} + V_{23} + V_{34} = V_{14}$

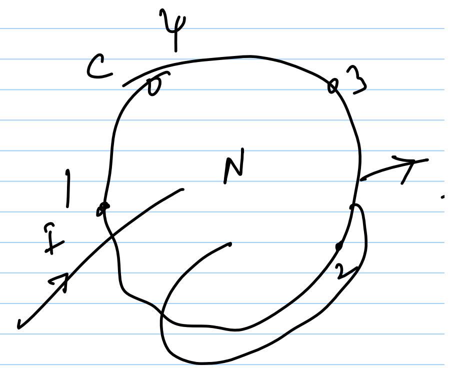
EMF ← $\int_1^2 \bar{E}\cdot d\bar{l} + \int_2^3 + \int_3^4 = \int_1^4 \bar{E}\cdot d\bar{l}$

For magnetic

$C \oint \bar{H}\cdot d\bar{l} = I_{tot.}$ ← Ampere's law



$I_{tot} = I$



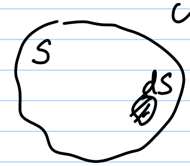
$I_{tot.} = N \cdot I$

$$\underbrace{\int_1^2 \bar{H}\cdot d\bar{l}} + \int_2^3 (\,) + \int_3^4 (\,) + \int_4^1 (\,) = NI$$

MMF = magneto motive force $\mathcal{F}$

$$\mathcal{F}_{12} + \mathcal{F}_{23} + \mathcal{F}_{34} + \mathcal{F}_{41} = NI$$

KCL



closed surface, no charge accumulation

$\nabla \cdot \bar{J} = -\cancel{\dfrac{d\rho}{dt}} \to 0$

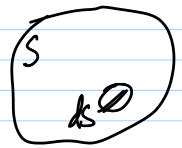
$\int_S \bar{J}\cdot d\bar{s} = 0$

total current entering (leaving) the surface $= 0$

Magnetics

$$\nabla \cdot \bar{B} = 0$$

$(d\phi)$ flux $= \bar{B} \cdot \bar{ds}$



flux density

$\bar{H}$ = mag. field

Total flux entering / leaving surface $= 0$

$\phi \longleftrightarrow I$

linear magnetic material

$B = \mu H = \mu_r \mu_0 H$

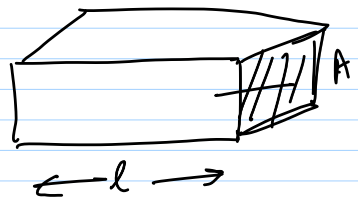
relative perm. (for μ_r)

permeability of free space (for μ_0)

Resistor R : $V = IR$ $\left(I = \dfrac{V}{R} \right)$

$R = \dfrac{\rho l}{A}$ $\left\{ I = \dfrac{\sigma A}{l} \cdot V \right\}$

resistivity (for ρ) conductivity (for σ)



$\phi = B \cdot A$

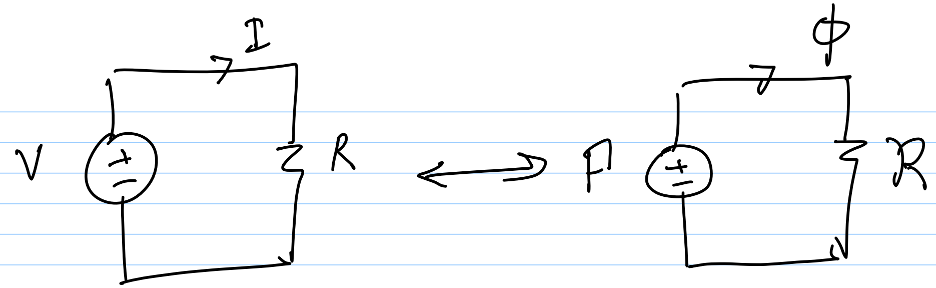
$F = H \cdot l$

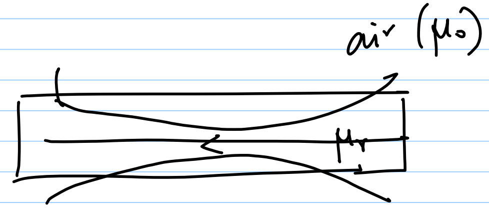
$$\frac{\phi}{A} = \mu_r \mu_0 \frac{F}{l}$$

$$\underset{\text{(current)}}{\phi} = \left(\mu_r \mu_0 \frac{A}{l} \right) \cdot \underset{\text{(voltage)}}{F}$$

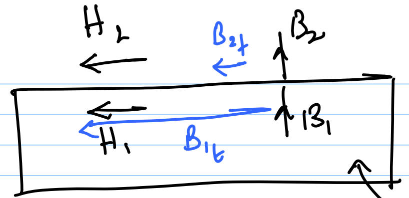
P = permeance

R = reluctance $= \dfrac{1}{P}$



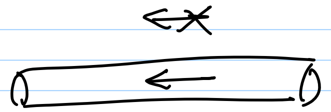


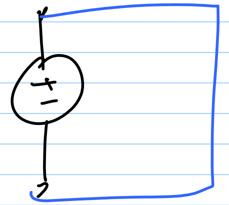
$B = \mu H$

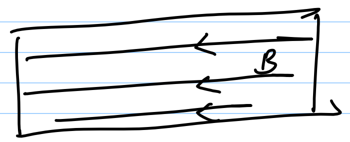


$\mu_r >> 1$

$\approx 1000s$



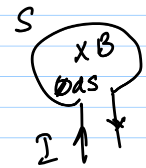




ϕ : Wb ; B : T (Wb/m^2)

H : $\frac{A}{m}$. turns ; $\mathcal{F}$ = A. turns

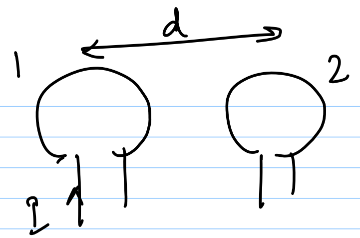
$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{A.\text{turns}}{Wb}$$



$$\int_S \overline{B} \cdot \overline{ds} = \phi$$

$$\phi = (L) I$$

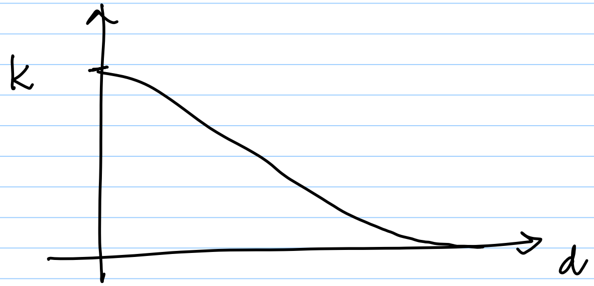
self inductance



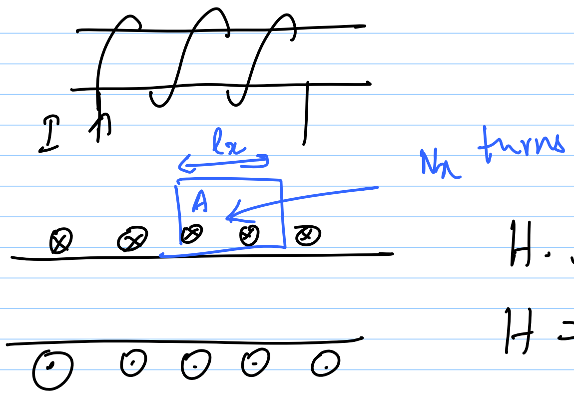
$$\int_{S_2} \overline{B} \cdot \overline{ds} = \phi_{21}$$

'k'

$$\frac{\phi_{21}}{I_1} = M$$ (mutual inductance between coils 1 & 2)



Solenoid (air core)



$$H \cdot l_x = N_x \cdot I$$

$$H = \frac{N_x}{l_x} \cdot I$$

$$B = \mu_0 \frac{N_x}{l_x} \cdot I \ ;$$

flux density $\phi = B \cdot A = \mu_0 A \frac{N_x}{l_x} \cdot I$

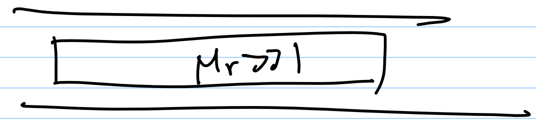
Total flux linkage

$$\psi = N\phi$$

$\frac{N_x}{l_x}$ = # of turns / unit length.

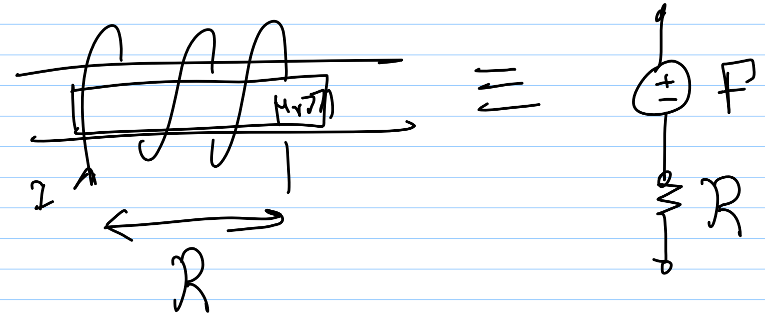
$$\psi = \mu_0 A N \cdot \frac{N_x}{l_x} \cdot I$$

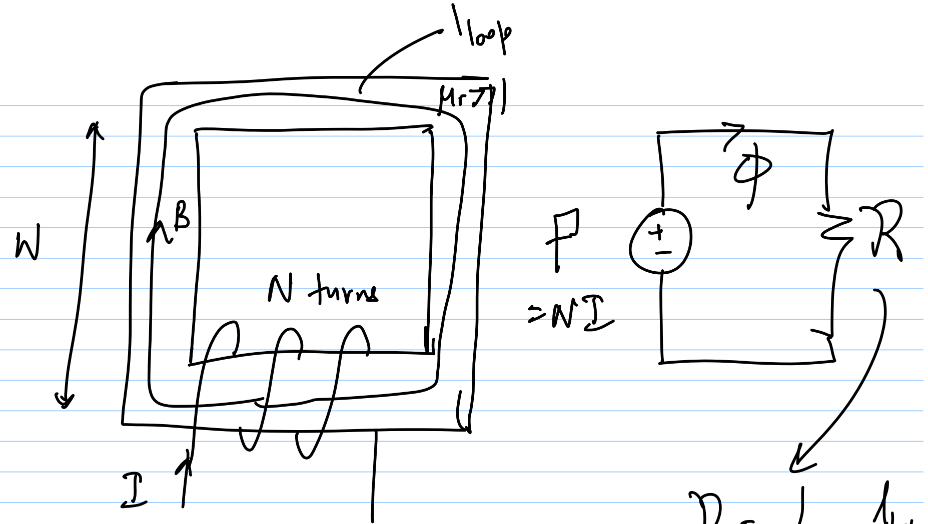
← L of solenoid



$(\mu_r \mu_0 \cdots)$

$$F = NI$$





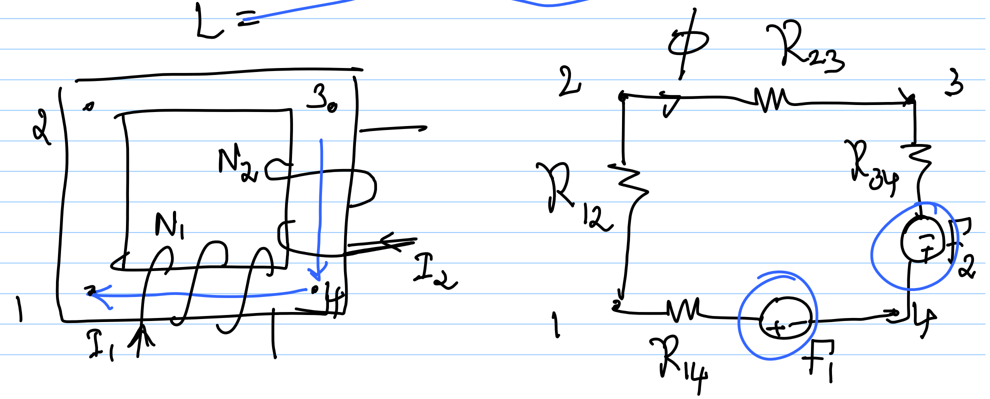
$$R = \frac{1}{\mu_r \mu_0} \cdot \frac{l_{loop}}{A}$$

$$\phi = \frac{NI}{R} = \mu_0 \mu_r \frac{A}{l_{loop}} \cdot NI$$

flux linkage $\psi = N\phi$

$$= \mu_0 \mu_r \frac{A}{l_{loop}} \cdot N^2 I$$

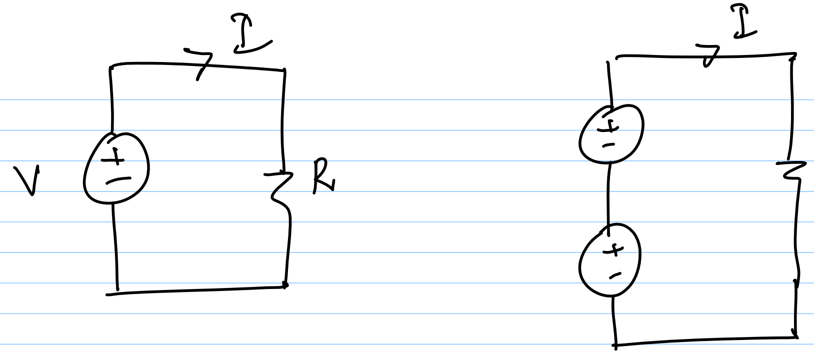
L =

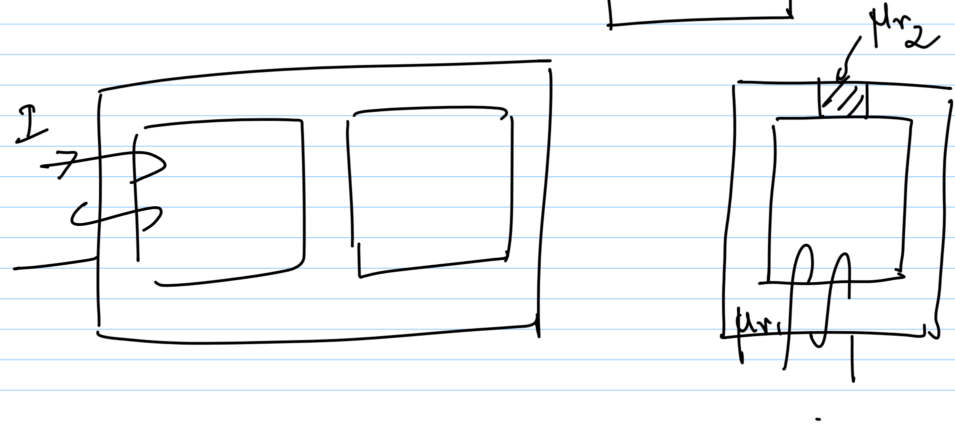


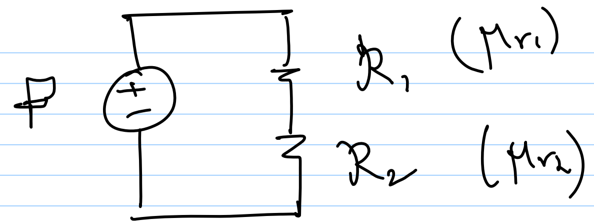
$$L_1 = \mu_0 \mu_r \frac{A}{l_{loop}} N_1^2$$

$$L_2 = \mu_0 \mu_r \frac{A}{l_{loop}} \cdot N_2^2$$

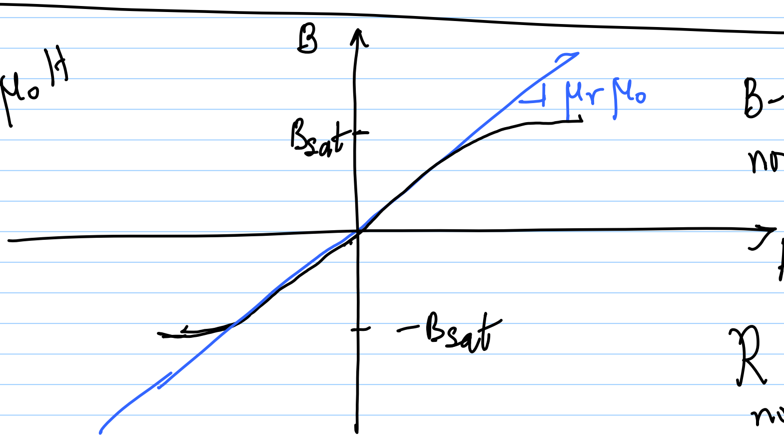
$$M = \mu_0 \mu_r \frac{A}{l_{loop}} N_1 N_2$$





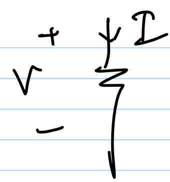


$B = \mu_r \mu_0 H$



B-H nonlinearity

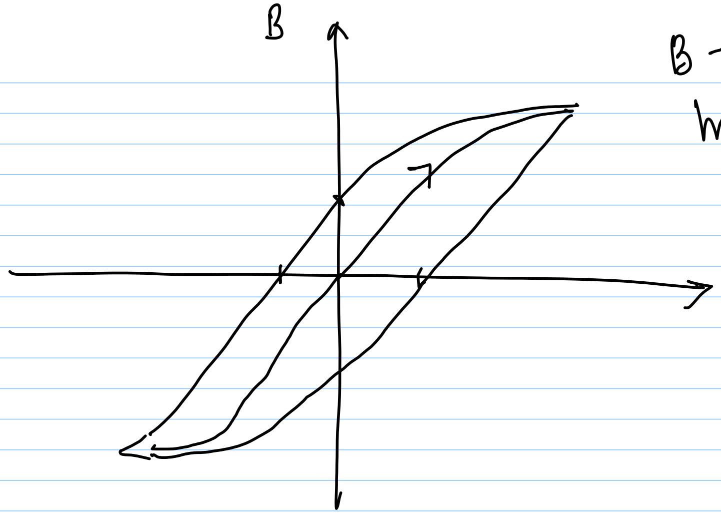
$\mathcal{R}$ is nonlinear



$$I = \frac{V}{R}$$

$$I = f(V)$$

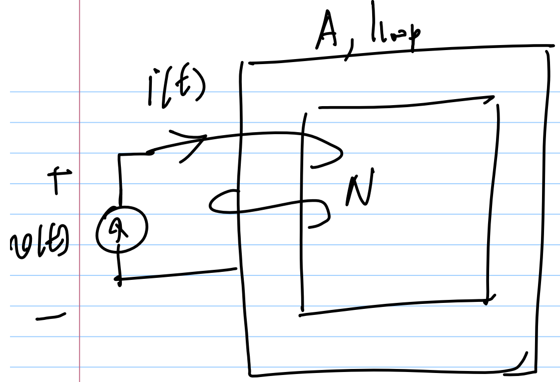
$$R = \frac{1}{f'(V)}$$



B-H curve
hysteresis

$$p(t) = v(t) \cdot i(t)$$

$$E = \int p(t)\,dt = \int v(t)\, i(t)\,dt$$



$$H = \frac{NI}{l_{loop}}$$

$$v(t) = \frac{d}{dt}\psi$$

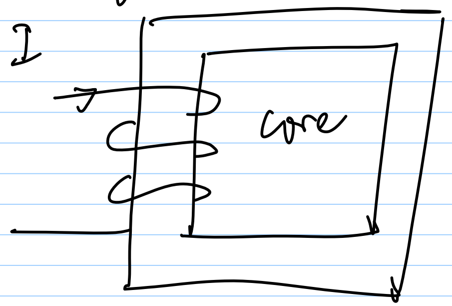
$$= N\frac{d\phi}{dt} = NA\frac{dB}{dt}$$

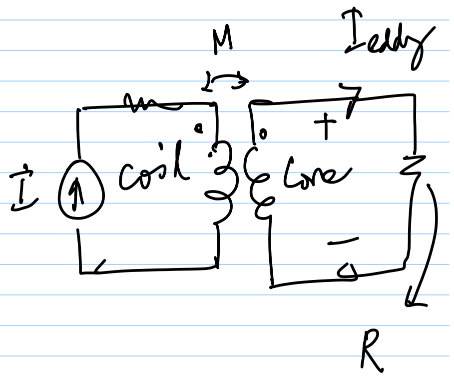
$$v(t) = NA\frac{dB}{dt}$$

$$E = \int \cancel{N} A \frac{dB}{dt} \cdot H \cdot \frac{l_{loop}}{\cancel{N}} \cdot dt$$

$$E = A \cdot l_{loop} \int H\, dB$$

Eddy current losses





(Eddy current losses)