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A mesh reconfiguration scheme for speeding up Monte Carlo simulations of electromagnetic scattering by random rough surfaces

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ABSTRACT

Traditional methods of Monte Carlo simulations of random rough surface scattering that use the finite element method involve the generation of multiple meshes for the purpose of taking ensemble averages. We propose a mesh reconfiguration scheme that instead uses a single master mesh. The main idea is to locally modify only the air–surface interface region in the mesh for each instance of a random rough surface. This method achieves a four fold improvement in computation time without any loss of accuracy.

1. The finite element method and Monte Carlo simulations

The finite element method (FEM) is a numerical technique used for the solution of partial differential equations with specified boundary conditions. In particular, we implement a two-dimensional vector-element based finite-element method to compute the electromagnetic backscatter from random rough one-dimensional surfaces [1].

There are two computational loads in such a computation. The first is the usual FEM itself, where the backscatter is calculated for a given rough surface within the computational domain. The presence of a statistically random object, such as a random rough surface, adds the next computational load; it is not sufficient to calculate the backscatter from a single instance of a rough surface, but rather an ensemble average over many different instances of rough surfaces is needed – a Monte Carlo process – in order to accurately capture the statistical properties of the rough surface.

The usual method of accomplishing this ensemble averaging is to generate a finite element mesh for each instance of the rough soil, and then to run the FEM on each mesh. We find in our work that the computational time required to generate a mesh can be many times more than the time required to solve the FEM for a given mesh. This is driven by the need to build a mesh for each instance of the scattering problem though much of the geometry is common from instance to instance. Given that at least forty instances of rough surfaces are needed in order for the backscatter ensemble to converge with a standard deviation less than 1 dB, a very large amount of time is spent in generating meshes that differ only in the regions around the air–surface interface. In this paper, we describe an elegant workaround to this brute force technique, where, we accomplish the entire Monte Carlo process by using a single mesh and achieve a four-fold improvement in computational run time. It exploits the fact that only specific regions of the mesh need to be modified, and it is accomplished in a simple manner within the existing FEM code starting from the original mesh. Hence there is no need to link expensive meshing algorithms or libraries to the existing FEM code.

2. Faster computation by using a single mesh

The broad idea in using a single mesh is as follows. A single mesh (not specific to any rough surface) is created (one time cost); the coordinates of a new rough surface are generated. These coordinates are superimposed onto the mesh, and a smooth mesh contour corresponding to this rough surface is produced by moving the mesh nodes nearest to the generated coordinates. With the surface thus defined, the FEM proceeds as usual and the backscatter is computed for this instance of rough surface.

In subsequent iterations, the mesh is restored to its original state and the above procedure is repeated from the step of rough surface coordinate generation. Thus, the computationally laborious tasks of generating a new mesh for the rough surface and inputting it into the FEM program are done away with.

The broad idea described above is now expanded in a more precise algorithm.

1. An important length scale to take into consideration before creating the single mesh is the spatial sampling period of the

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Table 1
Comparison of parameters for the two solution methods when considering 100 rough surface instances.

	Mesh creation time	Avg. no. of elements per mesh	FEM time per instance	Total time
Multiple-mesh	10 h 34 min	1,030,000	1 min 07 s	12 h 19 min
Single-mesh	8 min	1183,000	1 min 41 s	2 h 50 min

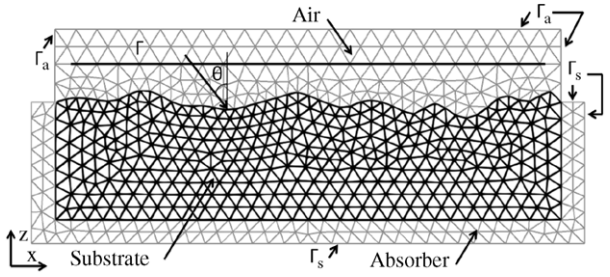

Fig. 1. Schematic of the computational domain for the 2D finite element method [1] (not to scale), showing an air–surface interface onto which an electromagnetic wave is incident at an angle θ from the vertical. The domain boundaries in air and substrate are represented by Γ_a and Γ_s, respectively, and the surface equivalence principle is applied on the contour Γ for calculating electromagnetic scattering. The air, substrate, and adiabatic-absorber [8] regions are also shown in the schematic. The horizontal extent of the air region is chosen to be 70λ, the vertical extent of air is 2.5λ, and that of the substrate is 3.5λ, and the absorbing strip has a uniform thickness of λ, where λ is the wavelength of the incident radiation.

rough surface; the average edge length for a mesh element in the region of the air–surface interface must be at least less than this sampling period. When using first-order vector edge elements in the electromagnetic calculation, the edge length must be less than $\lambda_m/20$ (where λ_m is the wavelength in the material) for numerical convergence of the FEM solution [2]. Similarly, the spatial sampling period of the rough surface, Δx, must be such that $\Delta x < l/10$, where l is the surface correlation length, in order to accurately represent a random rough surface with either exponential or Gaussian correlation statistics [3]. To simulate correlation lengths up to $\lambda/4$, we choose $\Delta x = \lambda/40$, where λ is the wavelength in air. With these length scales taken into consideration a single finite element mesh is created using the Delaunay triangulation (DT) algorithm, implemented in Sandia National Laboratories, CUBIT program [4].

2. Using schemes described previously by other researchers [5,6] the coordinates of a rough surface with specified statistics are generated. The extremities of the surface are clamped at the mean value of the surface height.
3. For each rough surface coordinate, the nearest mesh node is found. For this purpose, an associate data structure such as a vertex or edge adjacency list for the mesh is required. The mesh node coordinates are updated with the coordinates of the rough surface and a record of this change is made. The connectivity of this node to other nodes remains unchanged.
4. For each pair of adjacent rough surface coordinates, say A, B (which have now become mesh nodes), the shortest path on the mesh is computed using the A^* algorithm [7]. The criteria used in step 1 ensure that this path is composed of at least one edge. The following two cases arise:
   (a) If the path has only one edge, it is added to the rough surface list.
   (b) If the path has say, N (>1) edges, then $N-1$ points are linearly interpolated between A and B. For each of these $N-1$ points, step 3 is repeated, i.e. the nearest mesh node is found, and if this node has not previously been visited by this algorithm, it is moved to the location under consideration and a record of this change is made. Say there are M such new mesh nodes.

   Finally, between each pair of adjacent nodes picked from the set that consists of A, the M nodes, and B, the A^* algorithm is applied to find the shortest path, and all these $M+1$ paths are added to the rough surface list.
5. Now that the entire rough surface list is generated, it is possible to demarcate air and substrate elements and hence assign the dielectric constant of each element. With the rough substrate thus defined, the usual FEM procedure takes over and the electromagnetic backscatter for this rough surface instance is computed.
6. To repeat the computation for a new rough surface, the recorded changes are undone (restoring the mesh to its original state at the end of step 1), the dielectric property of each element is reset, and the algorithm continues from step 2.

This algorithm continues till the standard deviation of the random variable corresponding to the ensemble averaged backscatter falls below a preset threshold. If there have been P rough surface instances, then the ensemble averaged backscatter (a real quantity), x_P, is given as $x_P = \Sigma_1^P x_i/P$, where x_i is the backscatter for the ith instance. If sufficient trials are taken and the surface statistics are not pathological (for instance corresponding to a diffraction grating), the central limit theorem gives that the above process will converge to a Gaussian distribution of second moment $P\sigma^2$, where $\sigma = \Sigma_1^P (x_i - x_P)^2/P$. When an average of this distribution over P instances is taken, the associated standard deviation goes as $\sqrt{P\sigma^2}/P$. In our work, we find that to get this number below 1 dB requires P to be at least 40.

Admittedly, moving mesh nodes in the manner of step 3 and 4b may cause the mesh to lose its Delaunay property. This means that no point in a DT is inside the circumcircle of any triangle of the DT. This property is required for mathematical convergence of the FEM solution [2]. However, this issue affects only about 0.4% of the mesh elements; a typical mesh contains about 1183,000 elements and only about 3000 are on either side of the rough surface. Indeed, we find no appreciable loss in accuracy on this account.

3. Results

We report that the results from simulations run on a single core of an Intel Core i7 3.4 GHz processor. Fig. 1 shows the computational domain under consideration. Table 1 compares the processor times between the multiple-mesh and the single-mesh method. Clearly, the biggest saving is in the mesh creation time. The FEM time per instance tends to be slightly higher in the case of the single mesh for two reasons—firstly, the time for rough surface generation is part of the FEM program in the case of the single mesh, and secondly, the single mesh has a slightly higher density to cater to the requirement specified in step 1 of the algorithm, which leads to an increase in the FEM matrix dimension.

It is clear that using a single mesh along with our algorithm leads to a significant improvement in computation time. Further, we note that there is no quantifiable degradation in accuracy in the new method. Fig. 2 compares for the two methods, the ensemble averaged bi-static radar scattering cross-section, defined as the normalized scattering intensity as a function of scattering angle for a fixed incidence angle, θ_i [1]. This ensemble average is taken over 100 instances and in both cases, the associated standard deviation for the backscatter (i.e. when the scattering angle is $\pi + \theta_i$) is on the order of 0.7 dB. It must be noted that independent of the mesh

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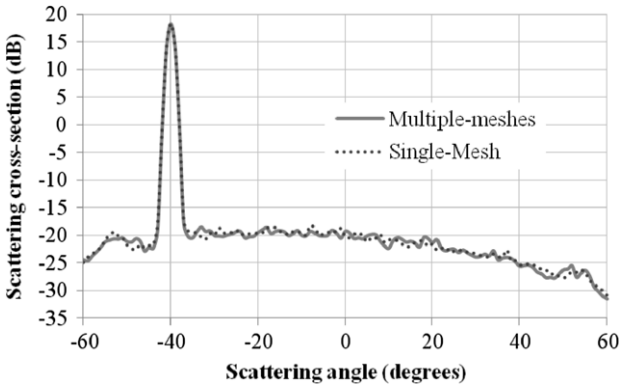

Fig. 2. Ensemble averaged bi-static radar scattering cross-section derived from the multiple-mesh and single-mesh method for transverse magnetic polarization. The relevant simulation parameters are mentioned in the caption of Fig. 1. Specifically, $\theta = 40$ and $\lambda = 0.24$ m. The random rough surface has Gaussian correlation statistics with $kh = 0.1$, and $l = 16.2$ h, where $k = 2\pi/\lambda$, h is the surface r.m.s height and l is the surface correlation length.

method, each time a finite set of trials is conducted, the resulting ensemble average will differ slightly, converging in the limit of a very large number of trials. Thus the two curves shown in Fig. 2 will not be identical.

In summary, we present a novel mesh reconfiguration algorithm that improves the computation time by a factor of four over existing methods for the calculation of ensemble averaged electromagnetic backscatter from random rough surfaces.

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