Polarization response of a cloud of rough cylinders

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Abstract — **We present an analytical model to compute the field scattered from a cloud of finite rough cylinders. The scheme presented in this paper allows the calculation of the ensemble averaged covariance matrix of the scattered field in terms of the correlation function of the cylinder roughness and thus obviates the need for a full-fledged monte-carlo simulation. Using our analytical model, we compare the back-scattered polarisation response of a cloud of rough cylinders to that of a cloud of smooth cylinders – it is found that depending on the cylinder radius and orientation, the surface roughness can have a significant impact on this response. The analytical model presented in this paper allows for an easy and computationally efficient inclusion of cylinder surface roughness into the existing forest models.**

1 INTRODUCTION

Modelling the scattering properties of vegetation patches and forest covers is important in the context of remote-sensing since they are a significant contributor to the radar cross-section. These patches can often be modelled as an ensemble of randomly oriented cylinders over a smooth or rough surface. One of the major obstacles to developing a computationally efficient model is the statistical variation in the properties of such scatterers – the orientation of the cylinders is random and can only be specified upto a probability distribution. Moreover, the surface of such cylinders is typically rough, with the roughness becoming significant at high radar frequencies.

An exact computation of the scattering properties of such rough scatterers involves performing a full Monte-carlo simulation – this becomes computationally intractable for realistic models. There have been many attempts to approximately model such scatterers – including treating the surface roughness as a periodic corrugation over the smooth cylinder [1] and modelling the scattered fields in the geometric optics regime [2]. Perturbative methods have also been applied to analyzing electromagnetic scattering from azimuthally rough conducting cylinders [3, 4, 5] and cylinders with an impedance boundary condition [6].

A three dimensional perturbative method for computing the electromagnetic fields from an infinitely long homogenous rough cylinder has recently been proposed [7]. This method enables the computation of the statistical averages of the scattered electromagnetic fields and scattering cross-sections in terms of the surface correlation function, thereby obviating the need to perform a full Monte Carlo simulation. In this paper we extend this method to approximately compute the electromagnetic fields from a finite rough-cylinder [8] and a cloud of finite rough-cylinders. Using the developed model, we analyze the impact of surface roughness on the scattering properties of a cloud of finite rough cylinders and demarcate regimes for the cylinder radius and orientation wherein the surface roughness is of considerable effect and needs to included in the scattered field calculations.

This paper is organised as follows – the analytical model is outlined in Section 2 and the impact of surface roughness is numerically studied in Section 3 to demarcate regimes wherein the cylindrical scatterers can be approximated as smooth without considerable loss of accuracy.

2 ANALYTICAL MODEL

The scatterer geometry is shown in Fig. 1. Throughout this section, (r, ϕ, z) denote the coordinates in cylindrical coordinate system and (ρ, θ, β) denote the coordinates in the spherical coordinate system. We also use two sets of coordinate axes (Fig. 1a), $x - y - z$ which is a fixed set of coordinate axes and $x' - y' - z'$ which is a coordinate axis with z' parallel to the cylinder axis (directed along (θ_c, β_c) in the unprimed coordinates) and the $x' - y'$ axis chosen so as to ensure that the incident wave-vector $\mathbf{k}_{\text{inc}}$ lies in the $x' - z'$ plane at an angle α' from the z' axis. Priming a coordinate is understood to denote the latter coordinate system. A time dependance of $\exp(-j\omega t)$ is assumed and suppressed throughout this section. The incident electromagnetic field is a plane wave with electric field $\mathbf{E}_{\text{inc}}$:

$$\begin{aligned}\mathbf{E}_{\text{inc}} &= \boldsymbol{\mathcal{E}}_{\text{inc}} \exp(j\mathbf{k}_{\text{inc}} \cdot \mathbf{r}) \\ &= \boldsymbol{\mathcal{E}}_{\text{inc}} \exp[jk_0(x' \sin\alpha' + z' \cos\alpha')] \qquad (1)\end{aligned}$$

where $\boldsymbol{\mathcal{E}} = \mathcal{E}_1^{\text{inc}}(\hat{\mathbf{z}}' \sin\alpha' - \hat{\mathbf{x}}' \cos\alpha') + \mathcal{E}_2^{\text{inc}}\hat{\mathbf{y}}'$ is the polarization of the transmitting antenna. The cylinder is assumed to be azimuthally rough, with roughness described by a stochastic process $h(\phi')$ which is the surface's radial height over the mean radius a. The length of the cylinder is denoted by L and it is assumed that the cylinder is homogenous.

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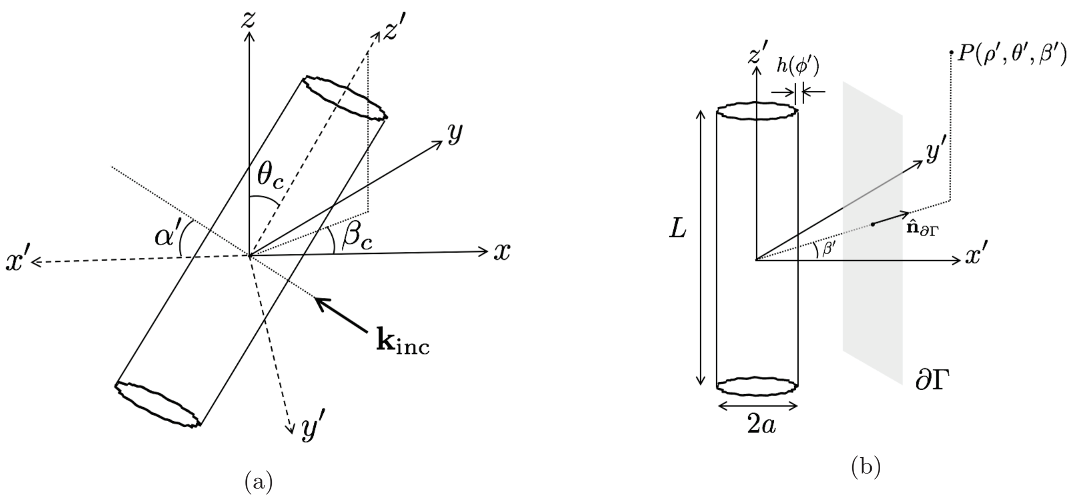


Figure 1: Schematic of the scatterer geometry. (a) shows the orientation of the coordinate systems used in the paper and (b) shows the effective surface $\partial\Gamma$ used for computing the electromagnetic far field at point P for a finite cylinder.

To compute the scattered fields at (ρ', θ', β') (assumed to be in the scatterer's far field), we equivalently compute the fields radiated by an effective electrical current $\mathbf{J}_{\text{eff}} = \hat{\mathbf{n}}_{\partial\Gamma} \times \mathbf{H}_{\text{sca}}\big|_{\partial\Gamma}$ and magnetic current $\mathbf{K}_{\text{eff}} = \mathbf{E}_{\text{sca}}\big|_{\partial\Gamma} \times \hat{\mathbf{n}}_{\partial\Gamma}$ at a plane surface $\partial\Gamma$ (Fig. 1b), where $(\mathbf{E}_{\text{sca}}, \mathbf{H}_{\text{sca}})$ are the scattered electromagnetic fields and $\hat{\mathbf{n}}_{\partial\Gamma}$ is the normal to $\partial\Gamma$. Following the analysis done by Hulst [8] in his treatment of a finite smooth cylinder, we approximate the scattered fields on $\partial\Gamma$ by the scattered fields for an infinite rough cylinder multiplied by a rectangular window in z':

$$(\mathbf{E}_{\text{sca}}, \mathbf{H}_{\text{sca}})\big|_{\partial\Gamma} \approx (\mathbf{E}_{\text{sca}}^{(\infty)}, \mathbf{H}_{\text{sca}}^{(\infty)})\big|_{\partial\Gamma} \mathcal{R}(z'/L) \qquad (2)$$

where $(\mathbf{E}_{\text{sca}}^{(\infty)}, \mathbf{H}_{\text{sca}}^{(\infty)})$ are the scattered electromagnetic fields from an infinitely long rough cylinder and $\mathcal{R}(u) = 1$ for $|u| < 1/2$ and 0 otherwise. Since the cylinder is assumed to be only azimuthally rough, the scattered fields have the following functional form:

$$\begin{bmatrix} \mathbf{E}_{\text{sca}}^{(\infty)} \\ \mathbf{H}_{\text{sca}}^{(\infty)} \end{bmatrix} \approx \mathcal{F}(r', z') \sum_{\substack{p \in \{h,v\} \\ q \in \{x,y,z\}}} \hat{\mathbf{q}}' \mathcal{E}_p^{\text{inc}} \begin{bmatrix} S_{q,p}(\phi') \\ G_{q,p}(\phi')/\eta_0 \end{bmatrix} \qquad (3)$$

where $\mathcal{F}(r', z') = \sqrt{2} \exp[j\{k_0(r' \sin\alpha' + z' \cos\alpha') - \pi/4\}]/\sqrt{\pi k_0 r' \sin\alpha'}$ and $S_{q,p}(\phi'), G_{q,p}(\phi') \ \forall \ q \in \{x, y, z\}, p \in \{1, 2\}$ govern the azimuthal dependence of the electromagnetic fields and depend on $h(\phi')$ through:

$$\begin{bmatrix} S_{q,p}(\phi') \\ G_{q,p}(\phi') \end{bmatrix} = \begin{bmatrix} S_{q,p}^{(0)}(\phi') \\ G_{q,p}^{(0)}(\phi') \end{bmatrix} + \sum_{n=-\infty}^{\infty} \begin{bmatrix} S_{q,p;n}^{(1)}(\phi') \\ G_{q,p;n}(\phi') \end{bmatrix} h_n + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \begin{bmatrix} S_{q,p;n,m}^{(2)}(\phi') \\ G_{q,p;n,m}^{(2)}(\phi') \end{bmatrix} h_n h_m \qquad (4)$$

where $h_n = \int_0^{2\pi} h(\phi') \exp(-jn\phi') d\phi'/2\pi$ and their coefficients can be computed from the perturbative solution for an infinitely long cylinder [7] .

Computing the scattering fields $(\mathbf{E}_{\text{sca}}, \mathbf{H}_{\text{sca}})$ now simply involves computing the fields radiated by the effective current densities $(\mathbf{J}_{\text{eff}}, \mathbf{K}_{\text{eff}})$ [10]. Closed form expressions for the scattered fields can be obtained if (a) the azimuthal dependence in the effective currents due to $S_{q,p}(\phi'), G_{q,p}(\phi') \ \forall \ q \in \{x, y, z\}, p \in \{1, 2\}$ is ignored and they are approximated by their values at the foot of perpendicular on $\partial\Gamma$ from the origin and (b) $\mathcal{F}(r', z')$ is approximated by a gaussian along $\partial\Gamma$. Using these approximations we obtain the following expressions for the scattered fields:

$$E_{\rho'}^{\text{sca}} = 0 \qquad (5a)$$

$$E_{\theta'}^{\text{sca}} = \sum_{p \in \{h,v\}} \Big[\{G_{x,p}(\beta') \sin\beta' - G_{y,p}(\beta') \cos\beta'\} \times \sin\theta' + S_{z,p}(\beta') \Big] \frac{2\mathcal{V}(\rho', \theta')}{\sin\alpha'} \mathcal{E}_p \qquad (5b)$$

$$E_{\beta'}^{\mathrm{sca}} = \sum_{p\in\{h,v\}} \Bigg[\{S_{x,p}(\beta')\sin\beta' - S_{y,p}(\beta')\cos\beta'\}$$

$$\times\cos\theta' - G_{z,p}(\beta')\Bigg]\frac{2\mathcal{V}(\rho',\theta')}{\sin\alpha'}\mathcal{E}_p \qquad (5c)$$

where:

$$\mathcal{V}(\rho',\theta') = \frac{jL\exp(jk_0\rho')}{4\pi\rho'}\mathrm{sinc}\left[\frac{k_0L}{2}(\cos\theta' - \cos\alpha')\right] \qquad (6)$$

Interpreting $E_{\theta'}^{\mathrm{sca}}$ and $E_{\beta'}^{\mathrm{sca}}$ as the two orthogonal components of the scattered electric field components ($\mathcal{E}_1^{\mathrm{sca}}$) and vertical ($\mathcal{E}_2^{\mathrm{sca}}$) scattered components respectively, it is easy to compute the scattering matrix $\mathbf{S}$ defined via $[\mathcal{E}_1^{\mathrm{sca}}, \mathcal{E}_2^{\mathrm{sca}}]^T = \mathbf{S}(\theta,\beta;\mathbf{k}_{\mathrm{inc}})[\mathcal{E}_1^{\mathrm{inc}}, \mathcal{E}_2^{\mathrm{inc}}]^T$ in the unprimed coordinates from Eq. 5 and by transforming the primed to unprimed coordinates.

The quantity of interest while characterizing the scattering properties of an ensemble of scatterers (such as a cloud of rough cylinders) is the ensemble average of the covariance matrix [9], which is defined by $\mathbf{C} = [S_{11}, S_{12}, S_{21}, S_{22}]^\dagger[S_{11}, S_{12}, S_{21}, S_{22}]$, where S_{ij} $i,j \in \{1,2\}$ are the elements of the scattering matrix $\mathbf{S}$. The covariance matrix has to be averaged not only over the surface roughness $h(\phi')$ but also over the cylinder orientation (θ_c, β_c). In our calculations, we capture the randomness in cylinder orientation through an explicit probability distribution function $p(\theta_c, \beta_c)$ over which the covariance matrix is averaged. However, since the scattering matrix is evaluated only upto second order in $h(\phi')$, the ensemble average of the covariance matrix can be expressed entirely in terms of the surface autocorrelation function $R(\Phi) = \langle h(\phi' + \Phi)h(\phi')\rangle$ using $\langle h_n^* h_m\rangle = R_n\delta_{n,m}$ and $\langle h_n h_m\rangle = R_n\delta_{n,-m}$ where $R_n = \int_0^{2\pi} R(\Phi)\exp(-jn\Phi)\, d\Phi/2\pi$.

3 IMPACT OF SURFACE ROUGHNESS

To quantify the impact of surface roughness on the scattering properties of a cloud of rough cylinders, we compute the back-scattered power received in a monostatic radar measurement. By comparing this power with that obtained for a cloud of smooth cylinder, we identify regimes in the cylinder radii and orientation wherein it is important to include surface roughness while analyzing the scattering properties of such a system.

The statistics of the surface roughness is modelled by a gaussian autocorrelation function:

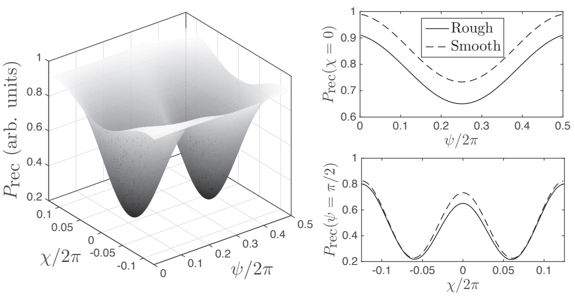
$$R(\Phi) = h_0^2\exp(-a^2\Phi^2/l^2)\ \forall\ \Phi \in [-\pi, \pi) \qquad (7)$$

where h_0 is the root-mean-square (RMS) surface roughness and l is the correlation length. While h_0 governs the absolute magnitude of the surface roughness, l is an estimate of the distance beyond which roughness at two points on the surface become uncorrelated. The perturbative solution is approximately valid for $h_0 < 0.25a$ and $l > 0.1a$ [7] – in all our calculations, we assume $h_0 = 0.25a$ and $l = 0.1a$ so as to simulate the roughest surface within the validity of the perturbative solution. The randomness in the cylinder orientation is captured by the cosine-square distribution function:

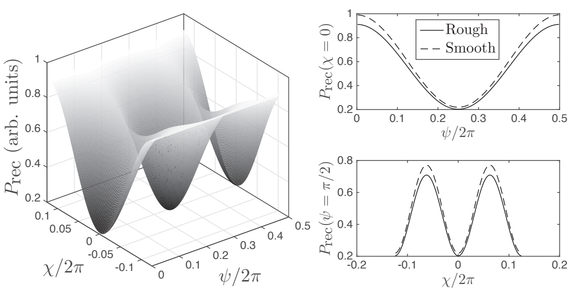
$$p(\theta_c, \beta_c) = \frac{1}{2\pi}\frac{(\cos\theta_c)^{2N}}{\int_0^\pi(\cos\theta_c)^{2N}\, d\theta_c} \qquad (8)$$

The index N governs the orientation of the cylinders – for large N the cylinders are nearly vertically oriented.

The back-scattered received power (P_{rec}) can be calculated using the stokes vectors corresponding to the receiving and transmitting antennas and the ensemble average of the covariance matrix [9]. Fig. 2 shows P_{rec} as a function of the transmitting antenna ellipticity angle χ and orientation angle ψ for both copolarized and crosspolarized receiving antenna – a significant deviation between the response of a cloud of rough and smooth cylinders is observed.

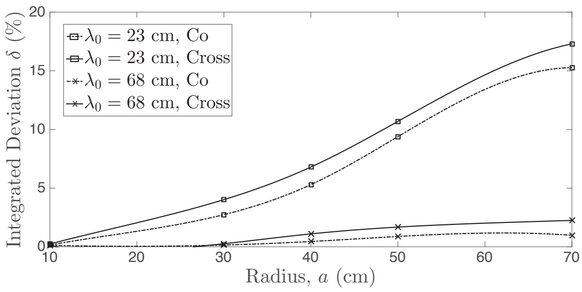


(a) Copolarized

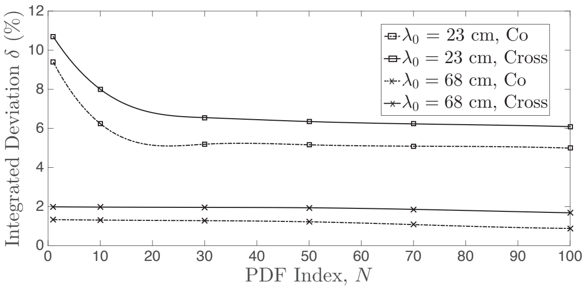


(b) Crosspolarized

Figure 2: Variation of received power P_{rec} (arbitrary units) with the transmitting antenna orientation ψ and ellipticity χ. In all simulations, $a = 50$ cm, $L = 100$ cm, $\lambda_0 = 23$ cm, $N = 10$ and cylinder dielectric constant $\epsilon_r = 20 + 7.0j$.



(a) δ vs. a



(b) δ vs. N

Figure 3: Variation of integrated deviation with cylinder radius a and the index N. $L = 100$ cm and cylinder dielectric constant $\epsilon_r = 20 + 7.0j$ is assumed in all the simulations, $N = 1$ in (a) and $a = 50$ cm in (b).

This difference can be quantified by computed the integrated deviation δ:

$$\delta = \frac{\int_0^{\pi} |P_{\text{rec}}(\psi, \chi = 0) - P_{\text{rec}}^{(0)}(\psi, \chi = 0)| d\psi}{\int_0^{\pi} P_{\text{rec}}^{(0)}(\psi, \chi = 0) d\psi} \quad (9)$$

where the $P_{\text{rec}}^{(0)}$ is the back-scattered power received from a cloud of smooth cylinders. Fig. 3a shows the variation of the integrated deviation δ with the cylinder radius and index N (which governs the cylinder orientation). Clearly, the deviation increases with an increase in the mean radius a at the same radar wavelength – consequently, simulation of the response of such a system to smaller radar wavelenghts necessitates the need for including surface roughness in the model. This is specially important for radar scattering measurements with S-band radars ($\lambda_0 \sim 6$ cm).

From Fig. 3b it can be seen that the impact of surface roughness becomes more pronounced with an increase in the randomness in the orientation of the cylinders – it is therefore more important to include cylinder roughness in the analysis of the 'canopy' layer rather than the 'trunk layer' (modulo the radius of the cylinders in the two layers). It was also found that the cylinder length has little impact on δ and thus is not an important factor in deciding whether or not to include surface roughness in the simulation model.

In conclusion, the analytical model developed in this paper allows for an efficient inclusion of surface roughness in the existing forest models. The approximate regimes demarcated in this paper are expected to be useful guidelines for inclusion of surface roughness while developing realistic models to simulate radar scattering from forests and vegetation patches.

Acknowledgments

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