Optimization

- Nocedal & Wright → Textbook.

Introduction

Key elements → Motivation from nature → Be efficient!

1. Objective function → Something to be maximized or minimized.
2. Variables of the problem.
3. Nature of the variables – eg. constrained / unconstrained.

→ Problem model.

What after this?

(1) Solve it → Choosing the right algorithm.

↔ tradeoff between complexity model v/s algorithm

(2) Check for optimality.

Types of Problems

(1) Unconstrained v/s Constrained.

Box constraint

$lb \leq x \leq ub$

$f(x) \geq 0$

(2) Continuous v/s Discrete.

↳ most of engg.

↳ eg. chess

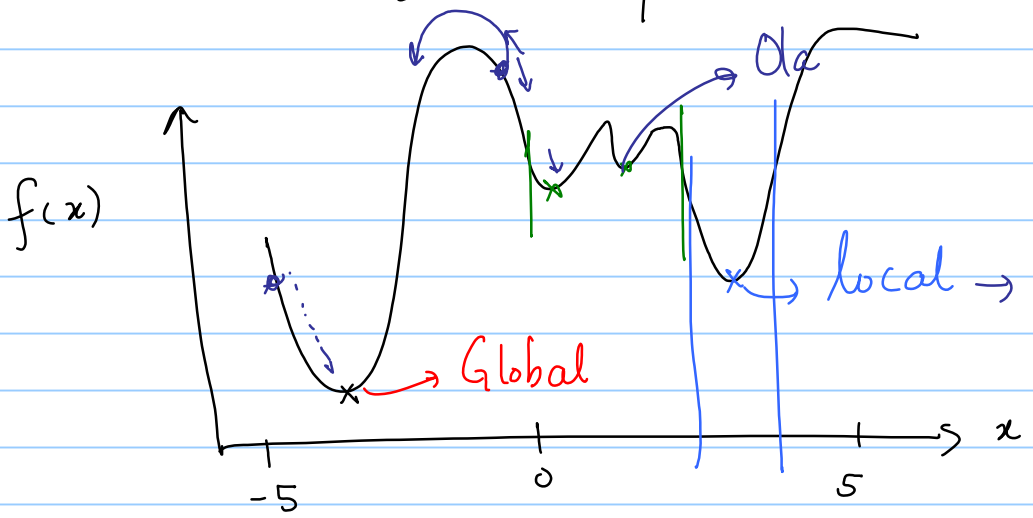
P1 $\phi(x)$ → cont's valued

P2 $\phi(x)$ s.t. $x \in \{0, 1\}$

$\phi(x)$, $x \in \mathbb{R}^{10000}$

s.t. $x(x-1) = 0$

③ Local v/s Global Optimization



$\min f(x)$

← non linear

→ Simulated annealing

→ Evolutionary algo.

e.g. Due to Joe keller. → How to live longer?

Given: HR → person at rest 80 bpm

HR → person exercising 120 bpm

↳ Everyone is born with a fixed no of heart beats.

What fraction should we exercise?

Say, x is the fraction

How many HB's gone? $f(x) = 120x + 80(1-x)$

$x = 0$!

Modeling error.

More $\underbrace{\text{I exercise}}_{x}$ → resting HR ↓

x is small $\rightarrow$ $g(x) \rightarrow 80$ Couch potato

x is large $\rightarrow$ $g(x) \rightarrow 50$

$g(x) = 50 + 30e^{-100x}$ ← art

Model → $f(x) = 120x + g(x)(1-x)$ ← New Obj fn.

Minimize → Set $f'(x) = 0 \rightarrow x \approx 53.7$ mins/day. s.t. $0 \leq x \leq 1$

$f''(x) > 0$ Check ✓ FDA approved.

— x —