↳ Case of complex valued A.
(and Hermitian).

1) $x^H A x \rightarrow$ real
2) $\lambda \rightarrow$ real
3) x_i are distinct.

→ All eigvectors are orthogonal to each other.

Complex valued → U
"Unitary matrix"

$$U^H U = I = U U^H$$

1) $\|x\|^2$ & $\|Ux\|^2$

$$(Ux)^H (Ux)$$

$$= x^H U^H U x = \|x\|^2$$

doesn't alter the length.

(2) $Ux = \lambda x$

$$\| Ux \| = \| \lambda x \|$$

$$\| x \| = |\lambda| \, \| x \|$$

$$\Rightarrow \quad |\lambda| = 1.$$

$$\rightsquigarrow \quad \cancel{\lambda = \pm 1}$$

(3) Eig vecs of distinct eig vals are orthogonal.

$$Ux_1 = \lambda_1 x_1 \,, \quad Ux_2 = \lambda_2 x_2$$

$$x_1^H x_2 = \underbrace{x_1^H U^H}\underbrace{U x_2} = (Ux_1)^H (Ux_2)$$

$$= \lambda_1^* \lambda_2 \, x_1^H x_2$$

$$(1 - \lambda_1^* \lambda_2) \, x_1^H x_2 = 0$$

$\lambda_1 \neq \lambda_2 \leftarrow$

$\lambda_1 \lambda_1^* = 1$

$\therefore \lambda_2 \lambda_1^* \neq 1 \therefore \Rightarrow x_1^H x_2 = 0$

Similarity Transforms.

Say A can be diagonalized,

$$S^{-1} A S = \Lambda \quad -(1)$$

"The basis of eigvectors diagonalizes A"

↳ generalize this to

$$M^{-1} A M = B$$

(M^{-1} exists)

If happens, A & B are called similar

Uses:

1. Recursion $\underline{u_n} = A\,\underline{u_{n-1}}$ ①

Working with $v = Mu$

$\Rightarrow u = M^{-1}v$ ③

Subs ③ into ①

$$M^{-1}v_n = A M^{-1} v_{n-1}$$

$$v_n = \underbrace{M A M^{-1}}_{B} v_{n-1}$$

2. Eig values of A, B.

$$M^{-1}AM = B \text{ (given)}$$

$$Ax = \lambda x$$

$$A = MBM^{-1}$$

$$MBM^{-1}x = \lambda x$$

$$\underbrace{B}\,\underbrace{M^{-1}x} = \underbrace{\lambda}\,\underbrace{M^{-1}x}$$

$\therefore$ λ is also eigvalue

$M^{-1}x$ is its eigvector.

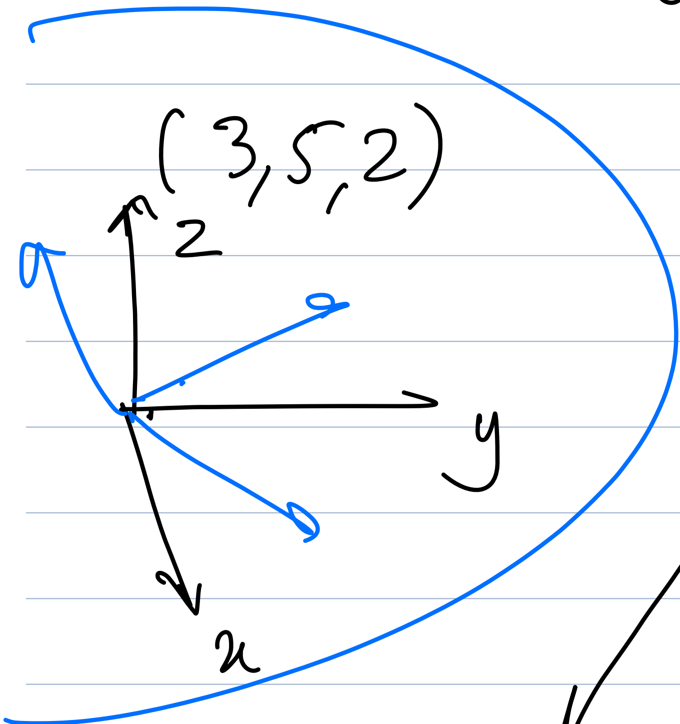
(3) Change of variables [COV]

Basis of $\mathbb{C}^n$ #1

$\{x_1, x_2, \ldots, x_n\}$

Basis of $\mathbb{C}^n$ #2

$\{y_1, y_2, \ldots, y_n\}$



$$X = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \\ | & | & & | \end{bmatrix}$$

$$y_i = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \begin{bmatrix} M_{1i} \\ M_{2i} \\ \vdots \\ M_{ni} \end{bmatrix}$$

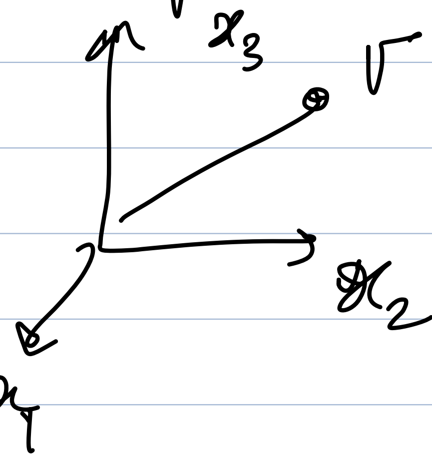
$$\begin{bmatrix} y_i & y_k \\ | & | \end{bmatrix} = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \begin{bmatrix} M_{1i} & M_{1k} \\ \vdots & \vdots \\ M_{ni} & M_{nk} \end{bmatrix}$$

$$Y = XM$$

Some matrix
"coeffs of LC"

Representation of a point.

$$v = [x_1 \cdots x_n]\begin{bmatrix}\alpha_1\\ \vdots \\ \alpha_n\end{bmatrix}$$



$$v = X\alpha$$

Also

$$= [y_1 \cdots y_n]\begin{bmatrix}\beta_1\\ \vdots \\ \beta_n\end{bmatrix} = Y\beta$$

$$v = \underbrace{Y\beta} = X\underbrace{M\beta}$$

For basis Y β's are repr

For basis X $M\beta$ are the

repr. reprs.

$Y = XM$ — BASIS

$\beta \longleftrightarrow M\beta$ — COEFFS