$$u_{n+1} = A^{n+1} u_0 \rightarrow U_n = A^n u_0$$

Assuming A is diagonalizable:

$$A = S \Lambda S^{-1}$$

$$u_N = S \Lambda^N \underbrace{S^{-1} u_0}_{c}$$

$$= \underbrace{\begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix}} \underbrace{\begin{bmatrix} \lambda_1^n & & \\ & \ddots & \\ & & \lambda_n^N \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}}$$

(Col pic)

$$\begin{bmatrix} c_1 \lambda_1^N \\ \vdots \\ c_n \lambda_n^N \end{bmatrix}$$

$$\boxed{u_N = c_1 \lambda_1^N x_1 + \cdots + c_n \lambda_n^N x_n.}$$

① Initial Conds (C_i's)

② Eigvalues.

↳ What if u_0 was an eigvec

$$u_N = A^N u_0 = \lambda_i^N u_0$$

↳ N large?

→ 3 possibilities

1. Stable if $|\lambda_i| < 1 \ \forall i$
2. Neutraly Stable if $|\lambda_i| = 1$ for some i, $|\lambda_i| < 1$ for others
3. Unstable if any $|\lambda_i| > 1$

→ MARKOV MATRICES

Real	Complex
$A^T = A$	Hermitian (1)

symmetric

$(A^T)_{ij} = (A)_{ji}$

$A^H = A$

$A_{ij} = A^*_{ji}$

② Skew Hermitian

$A_{ij} = -A^*_{ji}$

diagonal elements ⇒ Real ①

Purely imag ②

Properties of Hermitian matrices

$A^H = A$

① $x^H A x$ for any $x \in \mathbb{C}^n$

$$(x^H A x)^H = x^H A^H x$$

$$= x^H A x$$

a no & it's complex conjugate

$\Rightarrow$ Purely real.

② Eigenvalues.

$$Ax = \lambda x$$

$$\underbrace{x^H A x}_{\text{purely real}} = \lambda \underbrace{x^H x}_{\|x\|^2 \geq 0}$$

$\Rightarrow$ λ is purely real.

③ Eigvects of distinct eigValues.

$$Ax_1 = \lambda_1 x_1 \quad , \quad Ax_2 = \lambda_2 x_2$$

$$x_1 \overset{?}{\longleftrightarrow} x_2$$

$$\lambda_1 \neq \lambda_2$$

$$x_1^H A x_2 = \lambda_2 x_1^H x_2$$

$$\downarrow$$

$$x_1^H A^H x_2$$

$$= (Ax_1)^H x_2 = (\lambda_1 x_1)^H x_2$$

$$= \lambda_1 x_1^H x_2$$

$$(\lambda_2 - \lambda_1) x_1^H x_2 = 0$$

$$\Rightarrow \quad x_1^H x_2 = 0$$

i.e. orthogonal.

$\Rightarrow$ Corollary:

① When eigs are distinct, S matrix that diagonalizes A is orthogonal.

② Say $A \in \mathbb{R}^{n \times n}$, symmetric

$$Ax = \lambda x$$

$$(A - \lambda I)x = 0$$

$\underbrace{\text{real} \quad \text{real}}_{\text{real.}}$ Must be real $\mathbb{R}^n$

$S = Q$, $S^{-1} = Q^T$

$$A = S \Lambda S^{-1}$$

$$\boxed{A = Q \Lambda Q^T}$$

Spectral theorem

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} x_1 & - \\ x_2 & - \\ \vdots & \\ x_n & - \end{bmatrix}$$

$$A = \left(\lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T + \cdots + \lambda_n x_n x_n^T \right)$$

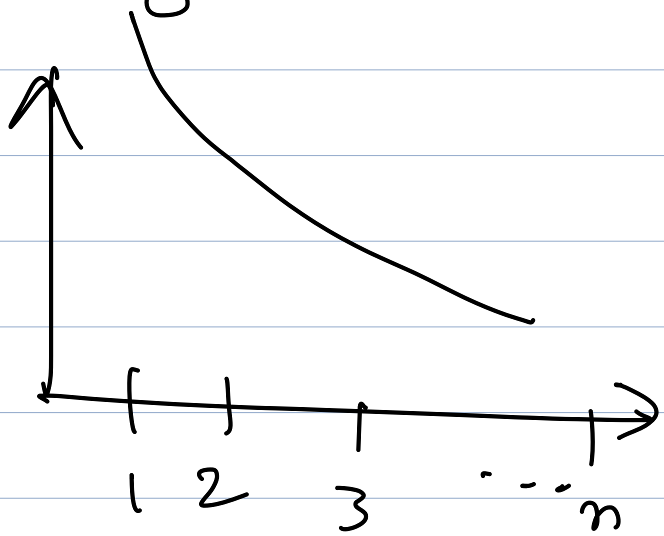
outer product Notation.

Rank 1

$\therefore$ Sum of Rank 1 matrices.

$A x \;=\; \lambda\, x$

$A x_3 = \lambda_3 x_3$



$$A \approx \lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T .$$

if $\lambda_3 << \lambda_1, \lambda_2$

Spl case of real sym matrix: eigvecs are orthogonal even if eigs are repeated.