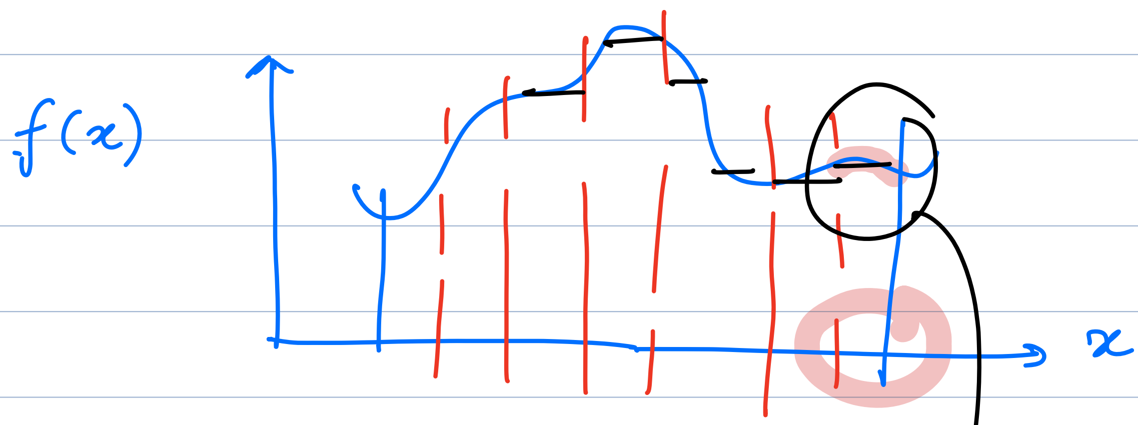
Polynomial Approximations.



over here use simple approx

eg. define $p_0(z) = 1$, $p_1(z) = z$, $p_2(z) = z^2$ over $(0, 1)$

$$\begin{bmatrix} p_0(z) & p_1(z) & p_2(z) & \cdots \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \end{bmatrix} = f(z)$$

we want

given

$$p_0(z)\, x_0 + p_1(z)\, x_1 + p_2(z)\, x_2 = f(z)$$

$$\begin{bmatrix} (p_0, p_0) & (p_0, p_1) & (p_0, p_2) \\ (p_1, p_0) & (p_1, p_1) & (p_1, p_2) \\ (p_2, p_0) & (p_2, p_1) & (p_2, p_2) \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (p_0, f) \\ (p_1, f) \\ (p_2, f) \end{bmatrix}$$

$A_{ij} = \frac{1}{i+j+1}$. $\downarrow$ Solve it.

Hilbert matrix

$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \rightarrow \text{Hilbert matrix}$$

$R_3 \rightarrow R_3 - \frac{2}{3} R_2$ $\quad [0 \quad 1/36 \quad 1/30]$

Even for a 10×10 system, round off error becomes too much $\therefore$ theoretically ✓
numerically ✗.

$\therefore$ Soln! Apply GS to functions.

1) $q_0(z) = \dfrac{p_0(z)}{\sqrt{(p_0, p_0)}} = 1$

2) $q_1'(z) = p_1(z) - (p_1, q_0)\, q_0(z)$

$= z - (z, 1) \times 1$

$= z - 1/2$

$\therefore q_1(z) = \dfrac{z - 1/2}{\sqrt{(z - 1/2, z - 1/2)}} = 2\sqrt{2}\left(z - \frac{1}{2}\right)$

3) $q_2(z) = p_2(z) - (p_2, q_0)\, q_0 - (p_2, q_1)\, q_1$

$= z^2 - (z^2, 1) \times 1 - (z^2, q_1)\, q_1$

[quad(z)]

These are Legendre polynomials

$\therefore\ f(z) = x_0 q_0(z) + x_1 q_1(z) + x_2 q_2(z)$

$$x_i = \frac{(q_i(z), f(z))}{(q_i(z), q_i(z))}$$

— x —.

Determinants

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $|A| = ad - bc.$

3 properties:

1) $|I_n| = 1$ $\qquad \mathbb{1}_n$

2) Det changes sign by row exchange.

$A' = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$ $\quad |A'| = bc - ad$

$= -|A|.$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ permutation matrix

$PA \to$ row

$AP \to$ cols.

$|P| = -1$

3) Det. depends <u>linearly</u> on the first row.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} a' & b' \\ c & d \end{pmatrix}$$

$$\boxed{C = \begin{pmatrix} a+a' & b+b' \\ c & d \end{pmatrix}}$$

$|C|, |B|, |A|$?

$$|C| = (a+a')d - (b+b')c = |A| + |B|$$

$$|A+B| \neq |A| + |B|$$

Next. $t \in \mathbb{R}$

$$A' = \begin{bmatrix} ta & tb \\ c & d \end{bmatrix}, \quad |A'| = t|A|$$

$$|tA| \neq t\,|A|$$

$$\begin{bmatrix} ta & tb \\ tc & td \end{bmatrix}$$