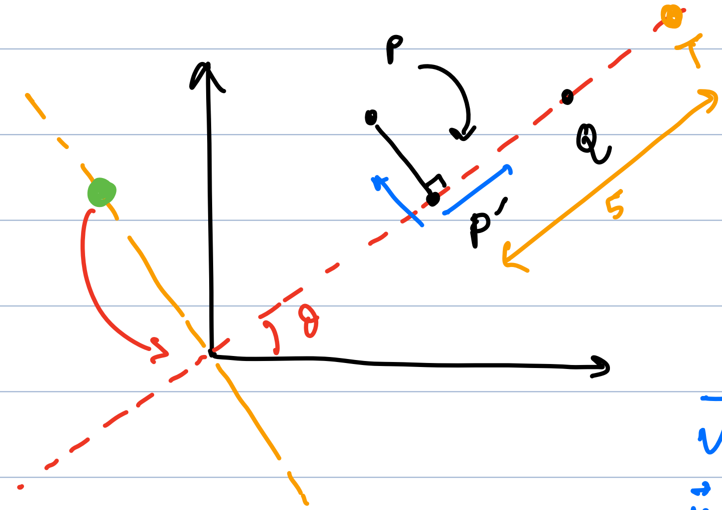
Matrix (from linear transf) contd.

Is this matrix unique?

No

1) Projection



orig basis: $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

Another basis

$\vec{v}_1 = P'Q$

$\vec{v}_2 = PP'$

Any vector $\vec{v}$ in $\mathbb{R}^2$ is to be expressed in terms of $\vec{v}_1, \vec{v}_2$.

assume P' is origin

$\vec{T} = (5)\,\vec{v}_1 + (0)\,\vec{v}_2$

In orig basis

$\vec{T} = (\quad)\,\hat{x} + (\quad)\,\hat{y}$

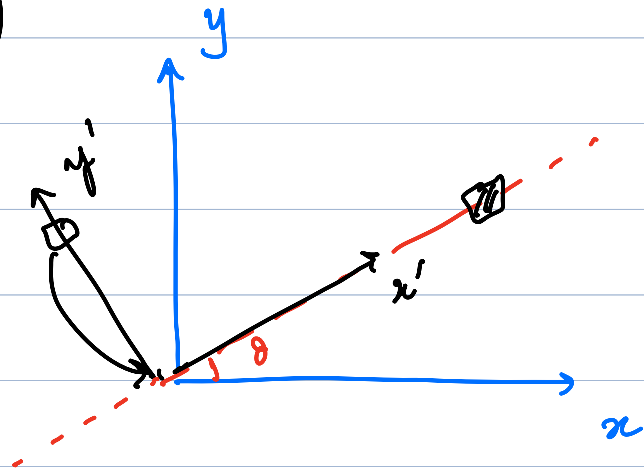
What is projection matrix in new basis?

$\begin{pmatrix}1\\0\end{pmatrix}$

i/p = $P'Q \rightarrow$ P $\rightarrow$ o/p? Q

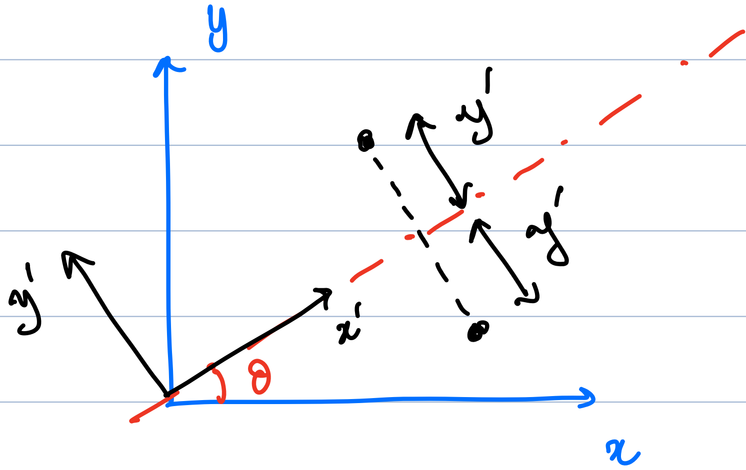
$PP' \rightarrow$ P $\rightarrow$ o/p?

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



2) Reflection

In x'-y' basis



$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Aside: Eigenvalue problem: $Ax = \lambda x$

Linear transf $\longrightarrow$ canonical basis $\longrightarrow$ general matrix A.

$\downarrow$

eigenvectors as basis

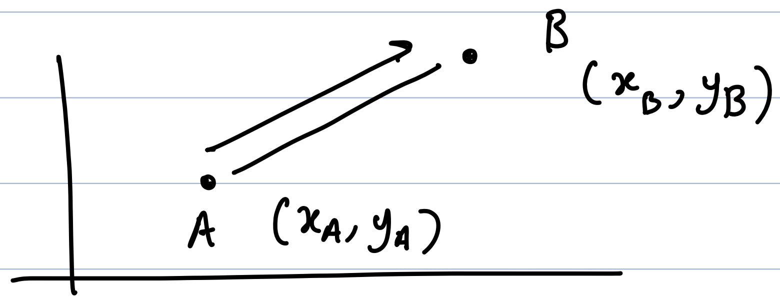
$\downarrow$

diagonal matrix A

(eig values on diagonal).

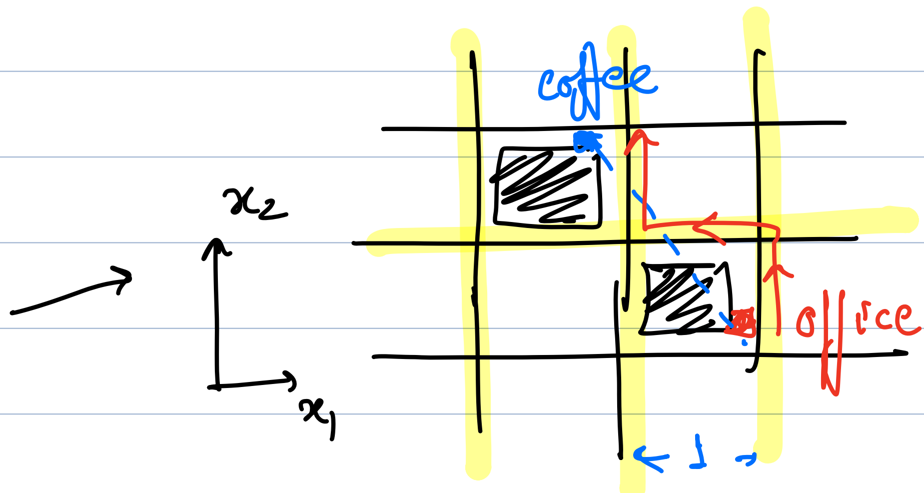
III Orthogonality.

"Length" of a vector?



$$l_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}.$$

Eucledean distance.



"Norm" $\equiv$ distance. $\quad p: V \longrightarrow \mathbb{R}$

eg. $\vec{BA} \xrightarrow{p} l_{AB}$

Properties:

1) $p(a\vec{v}) = |a|\, p(\vec{v}) \qquad a \in \mathbb{R}$
2) $p(\vec{v}) \geq 0$
3) $p(v) = 0$ only when $\bar{v} = 0$
4) $p(u+v) \leq p(u) + p(v) \qquad u, v \in V$

We say 'p' is a legitimate "norm."

$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p} \quad \text{for } p \geq 1.$$

Lp-norm.

e.g.

1) $p = 1 \longrightarrow \|x\|_1 = \sum_{i=1}^{n} |x|_i$ $\quad L_1$

Manhattan distance.

2) $p = 2 \longrightarrow \|x\|_2 = \sqrt{\sum |x_i|^2}$ $\quad L_2$

Eucledean norm.

3) $p \to \infty \to \|x\|_\infty = \max_i |x_i|$ $\quad L_\infty$

$$\|x\|_2^2 = \begin{cases} x^T x & \text{if } x \in \mathbb{R}^n \\ x^H x & \text{if } x \in \mathbb{C}^n \end{cases}$$

↳ Set of mutually orthogonal vectors $\{v_1, \ldots, v_n\}$
s.t. $\quad v_i^T v_j = \|v_i\|^2 \delta_{ij} \quad \forall \, i,j \leq n.$

Prove that they are linearly indep^n.

write $\sum^n c_i v_i = 0$ If all $c_i = 0$

$i=1$ then lin. indep.

$$v_k^T \left(\sum_{i=1}^{n} c_i v_i \right) = 0$$

$$c_k \underbrace{\|v_k\|^2} = 0$$

$$\Rightarrow \quad c_k = 0 \quad , \quad k = 1, \ldots, n$$

∴ They are linearly indepn.

$$\|A\|_F = \sqrt{\sum_{i,j} |A_{ij}|^2}$$ Frobenius.

$L_0 \rightarrow$ Interesting
Not a norm
Measures no of non zero entries in a vector.

$$x = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 2 \end{pmatrix} \qquad \|x\|_0 = 3.$$

$0^0 \rightarrow 0$
$\cancel{2}\ \alpha^0 \rightarrow 1$
$\alpha \neq 0$

N.

$$\|x\|_p^p = \underbrace{\left(\sum |x_i|^p \right)}$$