Linear Transformations

$\int_0^t$

$\frac{d}{dt}$

$p(t) \rightarrow \boxed{I} \rightarrow \boxed{D} \rightarrow ? \quad p(t)$ ✓

$\boxed{D} \rightarrow \boxed{I} \rightarrow ? \quad \cancel{p(t)}$

$$p: \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad I: A_{intg} \quad D: A_{dif.}$$

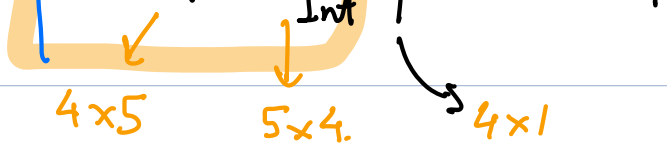
Integration: $\boxed{P_3} \rightarrow P_4$

$$A_{intg}: \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{pmatrix}_{5\times 4} \downarrow$$

Differentiation: $P_4 \rightarrow P_3$

$$A_{dif}: \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}_{4\times 5}$$

$A_{dif} \times A_{\int} \; p = p$



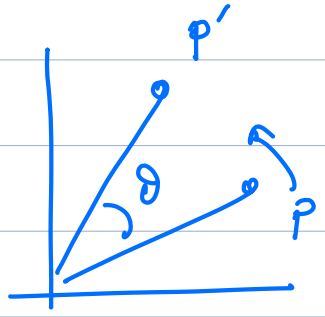
$$\begin{pmatrix} 1 & 0 & & \\ & 1 & & \\ 0 & & 1 & \\ & & & 1 \end{pmatrix}_{4\times 4}$$

Can we say: Diff is the left inverse of Integration. ? ✓

$$A_{dif} \cdot A_{int} = I$$

— x —

Some more e.g. Rotations / Projections / Reflections.

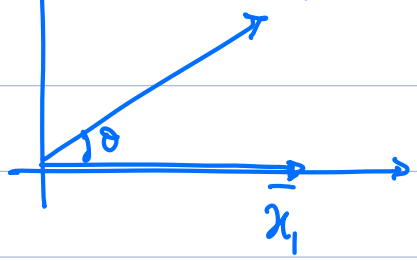


⇒ They must be repr by matrix transf.

Rotation:

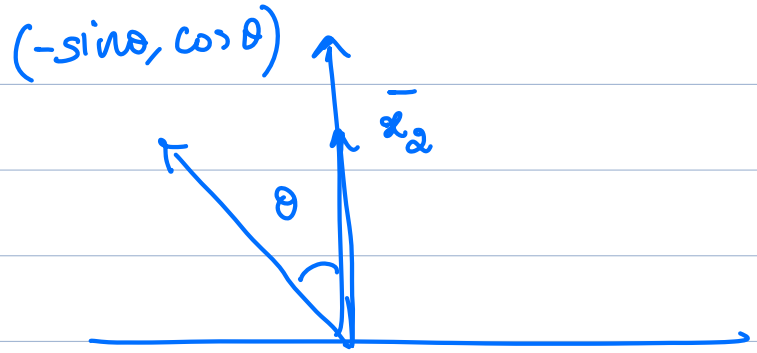
1) Basis vectors ? $x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\vec{x}_1$:



$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\vec{x}_2$



Canonical basis: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\vec{x}_1$ $\vec{x}_2$ ...

$$U \xrightarrow{B} V \xrightarrow{A} W$$

$v = Bu$ $\qquad$ $w = Av$

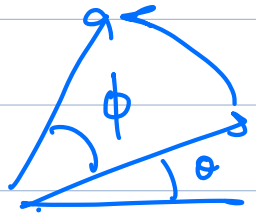
$\hookrightarrow w = A \cdot B u.$

$$\Rightarrow \quad U \xrightarrow{A \cdot B} W$$

$$u \xrightarrow{R_\theta} v \xrightarrow{R_\phi} w$$

$$w = R_\phi R_\theta u = \begin{pmatrix} \cos\phi & -\sin\phi \\ \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \end{pmatrix}$$

$$\begin{pmatrix} \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \sin\theta & \cos\theta \end{pmatrix}$$

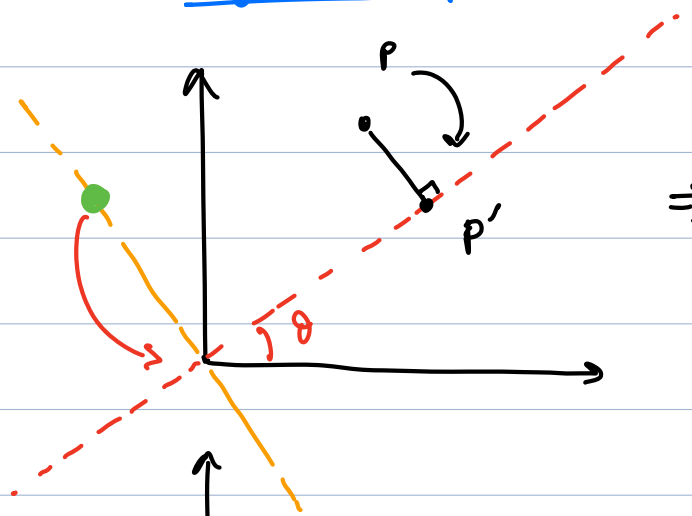


$$= \begin{pmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\,) & \cos(\,) \end{pmatrix}$$

$$= \boxed{R_{\theta+\phi}}$$

2) Projection

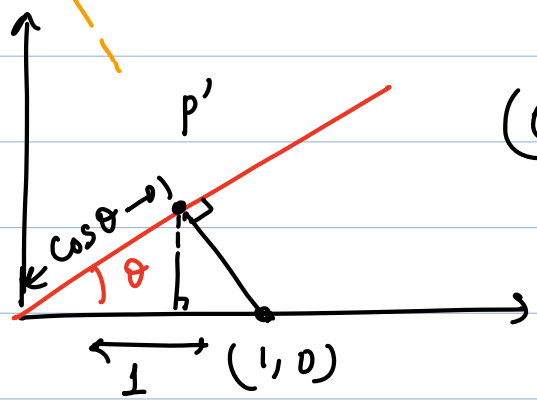
P ?



$$\Rightarrow P = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$$

a)

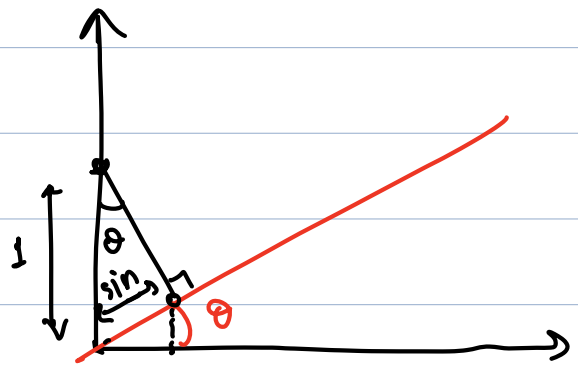
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$(\cos^2\theta, \cos\theta\sin\theta)$$

b)

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$(\sin\theta\cos\theta, \sin^2\theta)$$

a) Null space of P? A line $\perp^r$ to red line thru $\vec{0}$

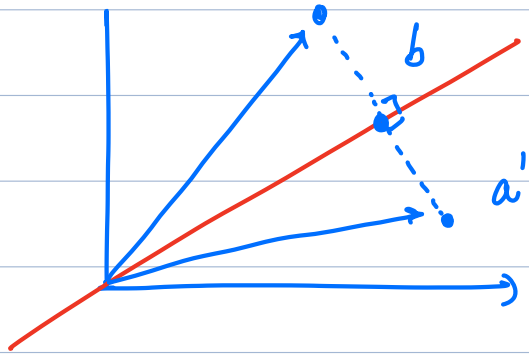
b) Is it invertible? $\times$

c) If I project twice, I get the same pt.

$$P^2 = P$$

3) Reflection transf.

a



$b = Pa$

$Pa' = b$

$\frac{1}{2}(a + a') = b$

$a' = 2b - a = (\underbrace{2P - I}_{H})\, a$

H reflection operator.

HW. $\left(Ha' \overset{?}{=} a. \right)$

$\leftarrow \infty \rightarrow$