ELL212 - Tutorial 8, Sem II 2015-16

Instructor: Uday Khankhoje

1) Consider a planar interface between two media; refractive index n_1 on the left, and n_2 on the right, with $n_1 > n_2$, and an electromagnetic plane wave incident on the interface from the left.
   a) Show that beyond a critical angle of incidence, the wave in the second medium will be *evanescent* (a rapidly attenuating field which transports no energy).
   b) Calculate the reflection coefficient for polarization perpendicular to the plane of incidence, and show that you get 100% reflection beyond the critical angle.
   c) Derive the explicit forms of the electric and magnetic fields in the second medium.
   d) Calculate the Poynting vector in the second medium and show that on average, no energy is transmitted in the direction perpendicular to the interface.
   e) Discuss a few practical applications of the above discussed effect.

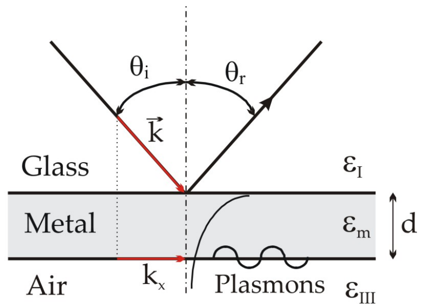

Fig. 1: Kretschmann confuguration for exciting SP

2) Surface plasmon (SP) is an Electromagnetic wave (occurring typically near and at optical frequencies) that propagates along the interface of a metal and dielectric, while decaying evanescently along the direction transverse to the interface (assume this direction to be y-axis). The existence of this surface wave is attributed to the negative dielectric constant of metals. Figure 1 shows a very popular arrangement used to excite SP at a metal-air interface. The glass prism gives rise to a field profile in the metal decaying evanescently along the y -direction due to total internal reflection. Assume:
$$k_x = k_{SP} = k_0\sqrt{\frac{\epsilon_0\epsilon_m}{\epsilon_0+\epsilon_m}},$$
where k_0 =propagation constant in air and $\epsilon_m < 0$.
Why is total internal reflection (or the prism) needed to excite SP? For a given frequency ω, what should be the value of the angle of incidence θ_i in order to excite SP at the metal-air interface? //[Note: This problem is based on the problem 9.39 of Griffith's, 4 ed.]
3) (a) Prove that there is no Brewster's angle for an EM plane wave that is polarized in a plane perpendicular to the plane of incidence. For definiteness: assume that Snell's laws are given: ($\theta_i = \theta_r$, $n_1 \sin\theta_i = n_2 \sin\theta_t$), an interface along the $x - y$ plane separating two non-conducting, non-magnetic media, the $x - z$ plane as the plane of incidence, and the usual EM boundary conditions: $\epsilon_1 E_1^{\perp} = \epsilon_1 E_2^{\perp}$, $\vec{E}_1^{\|} = \vec{E}_2^{\|}$, $B_1^{\perp} = B_2^{\perp}$, $\vec{H}_1^{\|} = \vec{H}_2^{\|}$.

   (b) Imagine racing a car on a track which is lined on the left and right with shiny highly reflective sheets placed vertically, and a road that is black as soot (and hence not very reflective). Along which axis (horizontal or vertical) should the polarization axis of your sun-glasses be aligned? Answer the question and give a qualitative explanation (no math, but diagrams encouraged) in no more than a few sentences. Make sure your answer is logically consistent. Hint: Assume that incoming light falling on all surfaces consists equally of light polarized horizontally and vertically.
4) * Prove the uniqueness theorem.