Tutorial 7 - Solutions

1) $\vec{E}(\vec{r},t) = \vec{A}_e(\vec{r}) \cdot \cos(\omega t - \varphi_e(\vec{r}))$

$\vec{H}(\vec{r},t) = \vec{A}_h(\vec{r}) \cos(\omega t - \varphi_h(\vec{r}))$

a) Complex envelope of a monochromatic vector field $\vec{v}(\vec{r},t)$ is given by

$$\vec{v}(\vec{r},t) = \mathrm{Re}\{ \underbrace{\tilde{v}(\vec{r})}_{\text{complex envelope}} \cdot e^{i\omega t} \}$$

Now, from above eq$^{(n)}$

$$\vec{E}(\vec{r},t) = \mathrm{Re}\{ \underbrace{\vec{A}_e(\vec{r}) e^{-i\varphi_e(\vec{r})}}_{\tilde{E}(\vec{r})} \cdot e^{i\omega t} \}$$

$$\vec{H}(\vec{r},t) = \mathrm{Re}\{ \underbrace{\vec{A}_h(\vec{r}) e^{-i\varphi_h(\vec{r})}}_{\tilde{H}(\vec{r})} \cdot e^{i\omega t} \}$$

b) In terms of complex envelope $\tilde{E}(\vec{r})$, $\tilde{H}(\vec{r})$

$$\vec{E}(\vec{r},t) = \mathrm{Re}\{ \tilde{E}(\vec{r}) \cdot e^{i\omega t} \} = \frac{\tilde{E}(\vec{r}) \cdot e^{i\omega t} + \tilde{E}^*(\vec{r}) e^{-i\omega t}}{2}$$

$$\vec{H}(\vec{r},t) = \mathrm{Re}\{ \tilde{H}(\vec{r}) \cdot e^{i\omega t} \} = \frac{\tilde{H}(\vec{r}) \cdot e^{i\omega t} + \tilde{H}^*(\vec{r}) e^{-i\omega t}}{2}$$

$$\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t) \times \vec{H}(\vec{r},t)$$

$$= \frac{1}{4}\Big[\tilde{E}(\vec{r}) \times \tilde{H}^*(\vec{r}) + \tilde{E}^*(\vec{r}) \times \tilde{H}(\vec{r}) + \tilde{E}(\vec{r}) \times \tilde{H}(\vec{r}) \cdot e^{2i\omega t} + \tilde{E}^*(\vec{r}) \times \tilde{H}^*(\vec{r}) e^{-2i\omega t} \Big]$$

$$\Rightarrow \langle S(\vec{r},t) \rangle = \tilde{S}(\vec{r}) = \frac{\tilde{E} \times \tilde{H}^*(\vec{r}) + \tilde{E}^*(\vec{r}) \times \tilde{H}(\vec{r})}{2 \times 2} \quad \left(\text{used } \langle e^{\pm 2i\omega t} \rangle = 0 \right)$$

$$= \frac{1}{2} \mathrm{Re}\{ \tilde{E}(\vec{r}) \times \tilde{H}^*(\vec{r}) \}.$$

Scanned by CamScanner

2) a) $\tilde{E}_f(x) = \hat{z} E_0 e^{-ik_0 x}$ $\tilde{E}_b(x) = \hat{z} e^{-i\varphi_0} E_0 e^{ik_0 x}$

Net complex envelope

$$\tilde{E}(x) = \hat{z} E_0 \left[e^{-ik_0 x} + e^{-i\varphi_0} e^{ik_0 x} \right]$$

$$= \hat{z} E_0 e^{-i\varphi_0/2} \left[e^{-i(k_0 x - \varphi_0/2)} + e^{i(k_0 x - \varphi_0/2)} \right]$$

$$= 2\hat{z} E_0 e^{-i\varphi_0/2} \cos\left(k_0 x - \frac{\varphi_0}{2}\right)$$

The real electric field:

$$\vec{E}(x,t) = \mathrm{Re}\{ \tilde{E}(x) \cdot e^{i\omega t} \}$$

$$\boxed{\vec{E}(x,t) = 2 E_0 \cos\left(\omega t - \frac{\varphi_0}{2}\right) \cos\left(k_0 x - \frac{\varphi_0}{2}\right) \hat{z}.}$$

To compute $\vec{H}(x,t)$, use Faraday's law:

$$\nabla \times \vec{E}(x,t) = -\mu_0 \frac{\partial \vec{H}(x,t)}{\partial t}$$

Now

$$\nabla \times \vec{E}(x,t) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & 2E_0 \cos\left(k_0 x - \frac{\varphi_0}{2}\right) \cos\left(\omega t - \frac{\varphi_0}{2}\right) \end{vmatrix}$$

$$= \hat{y}\, 2 E_0 k_0 \sin\left(k_0 x - \frac{\varphi_0}{2}\right) \cos\left(\omega t - \frac{\varphi_0}{2}\right).$$

$$\Rightarrow \boxed{\vec{H}(x,t) = -\frac{2 E_0 k_0}{\omega \mu_0} \hat{y} \sin\left(k_0 x - \frac{\varphi_0}{2}\right) \sin\left(\omega t - \frac{\varphi_0}{2}\right).}$$

b) Poynting vector

$$\vec{S}(x,t) = \vec{E}(x,t) \times \vec{H}(x,t)$$

$$= \frac{4 E_0^2 k_0}{\omega \mu_0} \hat{x} \sin\left(k_0 x - \frac{\varphi_0}{2}\right) \cos\left(k_0 x - \frac{\varphi_0}{2}\right) \cdot \sin\left(\omega t - \frac{\varphi_0}{2}\right) \cos\left(\omega t - \frac{\varphi_0}{2}\right)$$

Scanned by CamScanner

$$= \frac{E_0^2 k_0}{\omega \mu_0} \sin(2k_0 x - \varphi_0) \sin(2\omega t - \varphi_0).$$

Hence

$$\langle \vec{S}(x,t) \rangle = 0 \quad \left(\text{since } \left\langle \sin\left(2\omega t - \frac{\varphi_0}{2}\right) \right\rangle = 0 \right)$$

Physically this is expected since the forward and backward travelling waves ~~ca~~ have equal amplitudes and hence carry the same power per unit area – thus bidirectional flow of power results in 0 net power transfer. However, it should be noted that the instantaneous power transfer is not 0, only average power transfer is 0. (i.e. $\vec{S}(x,t) \neq 0$, $\langle \vec{S}(x,t) \rangle = 0$).

3) In conductor

$$k^2 = \omega^2 \mu_0 \epsilon_0 \left[\epsilon - \frac{i\sigma}{\omega \epsilon_0} \right] \quad \text{——— (1)}$$

and a forward propagating wave has a complex envelope (time variation $\sim e^{+i\omega t}$).

$$\tilde{\vec{E}}(x, \cdot) = E_0 \hat{z} \, e^{-ikx} \quad \text{——— (2)}$$

From (1), k is complex. Also, let $k = k_R - i k_I$ (note that $k_I > 0$, else (2) would imply an electric field which grows exponentially with x along propagation direction). Thus

$$\tilde{\vec{E}}(x) = \hat{z} E_0 e^{-ik_R x} e^{-k_I x}$$

and

$$\vec{E}(x,t) = \hat{z} E_0 e^{-k_I x} \cos(k_R x - \omega t).$$

The electrical energy per unit volume is given by

$$u_E(x,t) = \frac{1}{2} \epsilon_0 \epsilon E^2 = \frac{1}{2} \epsilon_0 \epsilon E_0^2 e^{-2k_I x} \cos^2(k_R x - \omega t).$$

Scanned by CamScanner

Thus
$$\boxed{\langle u_E(x,t)\rangle = \frac{1}{4}\epsilon_0 \epsilon E_0^2 e^{-2k_I x}}$$

To compute magnetic field, we first calculate $\tilde{H}(x)$ using Faraday's law (you can also directly calculate $\vec{H}(x,t)$, but this is an alternative procedure which might be somewhat simpler).

$$\nabla \times \tilde{E}(x) = -i\omega\mu_0 \tilde{H}(x)$$

$$\Rightarrow \tilde{H}(x) = \frac{i}{\omega\mu_0}\underbrace{\nabla\times\tilde{E}(x) = -\frac{i}{\omega\mu_0}\frac{\partial\tilde{E}}{\partial x}\cdot\hat{y}}_{\text{(Prove this)}}$$

$$= \frac{i}{\omega\mu_0}(k_I + i k_R)\cdot E_0 e^{-k_I x} e^{-ik_R x}\hat{y}$$

Thus
$$\vec{H}(x,t) = \mathrm{Re}\left[\tilde{H}(x)\cdot e^{i\omega t}\right]$$

$$= \mathrm{Re}\left[\frac{i}{\omega\mu_0}(k_I + i k_R) E_0 e^{-k_I x} e^{-ik_R x}\hat{y}\right]$$

$$= \frac{E_0\left[k_I^2 + k_R^2\right]^{1/2}}{\omega\mu_0}\cos\left(-k_R x + \omega t + \frac{\pi}{2} + \tan^{-1}\frac{k_R}{k_I}\right) e^{-k_I x}\cdot\hat{y}$$

Hence
$$u_H(x,t) = \frac{1}{2}\mu_0 H^2 = \frac{E_0^2(k_I^2 + k_R^2)}{2\omega^2\mu_0} e^{-2k_I x}\cos^2\left(\omega t - k_R x + \frac{\pi}{2} + \tan^{-1}\frac{k_R}{k_I}\right)$$

$$\Rightarrow \boxed{\langle u_H(x,t)\rangle = \frac{E_0^2}{4\omega^2\mu_0}(k_I^2 + k_R^2)\, e^{-2k_I x}}$$

Scanned by CamScanner

To compare the two, consider the ratio :

$$\frac{\langle u_H(x,t)\rangle}{\langle u_E(x,t)\rangle} = \left[\frac{k_J^2 + k_R^2}{\omega^2 \mu_0 \epsilon_0 \epsilon}\right]$$

Now, for large σ, we have from Eq. ①

$$k^2 = (k_R - i k_J)^2 \simeq - \frac{\omega^2 \mu_0 \epsilon_0}{\omega \epsilon_0} \cdot i \frac{\sigma}{1}$$

$$= + \omega \mu_0 \sigma \, e^{-i\pi/2}$$

$$\Rightarrow k_R - i k_J = \sqrt{\omega \mu_0 \sigma} \, e^{-i\pi/4}$$

$$= \sqrt{\frac{\omega \mu_0 \sigma}{2}} (1 - i)$$

$$\Rightarrow k_R = k_J = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$$

and

$$\boxed{\frac{\langle u_H\rangle}{\langle u_E\rangle} = \frac{\omega^2 \mu_0 \sigma}{\omega^2 \mu_0 \epsilon_0 \epsilon} = \frac{\sigma}{\omega \epsilon_0 \epsilon} >> 1}$$

therefore magnetic energy dominates in this media.

4) a) $\vec{E}(x=0,t) = \hat{z} E_0 e^{i\omega_0 t} e^{-t^2/\tau_0^2} = \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} d\omega$

$$\Rightarrow \tilde{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{z} E_0 e^{i\omega_0 t} e^{-t^2/\tau_0^2} e^{-i\omega t} dt$$

$$= \frac{E_0 \hat{z}}{2\pi} \sqrt{\frac{\pi}{1/\tau_0^2}} \, e^{-\frac{\tau_0^2 (\omega - \omega_0)^2}{4}}$$

(use the standard integral: $\int_{-\infty}^{\infty} e^{-(ax^2+bx)} dx = \sqrt{\frac{\pi}{a}} \cdot e^{b^2/4a}$)

$$\boxed{\tilde{E}(\omega) = \frac{1}{2\sqrt{\pi}} E_0 \tau \hat{z} \, e^{-\frac{\tau_0^2}{4}(\omega - \omega_0)^2}}$$

Scanned by CamScanner

b) $$\vec{E}(x,t) = \int_{-\infty}^{\infty} \tilde{E}(\omega)\cdot e^{i(\omega t - k(\omega)x)}\, d\omega$$

$$= \int_{-\infty}^{\infty} \frac{E_0 \tau_0 \hat{z}}{2\sqrt{\pi}}\, e^{-\frac{\tau_0^2}{4}(\omega-\omega_0)^2}\cdot e^{i\omega t}\, e^{-i\left\{\frac{\omega_0}{v_p} + \frac{\omega-\omega_0}{v_g} + \frac{\alpha}{2}(\omega-\omega_0)^2\right\}x}\, d\omega$$

Making change of variables $\omega - \omega_0 = \Omega$

$$\vec{E}(x,t) = \frac{E_0 \tau_0 \hat{z}}{2\sqrt{\pi}}\left[\int_{-\infty}^{\infty} d\Omega\cdot e^{-\frac{\tau_0^2\Omega^2}{4}}\, e^{i\Omega\left(t-\frac{x}{v_g}\right)}\, e^{-i\frac{\alpha\Omega^2 x}{2}}\, d\Omega\right] \times e^{i\left(\omega_0 t - \frac{\omega_0 x}{v_p}\right)}$$

$$= \frac{\hat{z} E_0 \tau_0}{2\sqrt{\pi}}\, e^{i\omega_0\left(t-\frac{x}{v_p}\right)}\left[\int_{-\infty}^{\infty} d\Omega\cdot e^{-\left(\frac{\tau_0^2}{4} + \frac{i\alpha x}{2}\right)\Omega^2}\, e^{i\Omega\left(t-\frac{x}{v_g}\right)}\right]$$

$$= \frac{\hat{z} E_0 \tau_0}{2\sqrt{\pi}}\, e^{i\omega_0\left(t-\frac{x}{v_p}\right)}\left[\frac{\pi}{\frac{\tau_0^2}{4} + i\frac{\alpha x}{2}}\right]^{1/2} e^{-\left\{\frac{(t - x/v_g)^2}{\tau_0^2 + 2i\alpha x}\right\}}$$

(using: $\int_{-\infty}^{\infty} e^{-(ax^2+bx)}\, dx = e^{\frac{b^2}{4a}}\cdot\sqrt{\frac{\pi}{a}}$)

$$\boxed{\vec{E}(x,t) = \frac{\hat{z} E_0}{\left[1 + \frac{2i\alpha x}{\tau_0^2}\right]^{1/2}}\, e^{-\frac{(t - x/v_g)^2}{\tau_0^2 + 2i\alpha x}}\, e^{i\omega_0\left(t - \frac{x}{v_p}\right)}}$$

c) The amplitude of electric field:

$$|\vec{E}\cdot\hat{z}| = \frac{E_0}{\left[1 + \frac{4\alpha^2 x^2}{\tau_0^4}\right]^{1/4}}\left| e^{-\frac{\left(t-\frac{x}{v_g}\right)^2}{\tau_0^2 + 2i\alpha x}}\right|$$

$$= \frac{E_0}{\left[1 + \frac{4\alpha^2 x^2}{\tau_0^4}\right]^{1/2}}\cdot e^{-\frac{(t - x/v_g)^2}{\tau_0^4 + 4\alpha^2 x^2}\cdot\tau_0^2}\cdot\left| e^{\frac{2i\alpha x}{\tau_0^4 + 4\alpha^2 x^2}\left(t-\frac{x}{v_g}\right)^2}\right|$$

Scanned by CamScanner

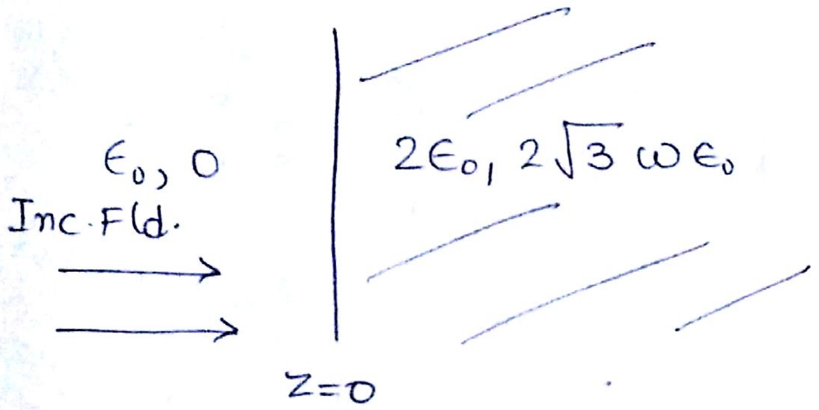
$$|\vec{E}\cdot\hat{z}| = \frac{E_0}{\left(1+\frac{4\alpha^2x^2}{\tau_0^4}\right)^{1/4}}\, e^{-\frac{(t-x/v_g)^2}{(\tau_0^2+4\alpha^2x^2/\tau_0^2)}}$$

Thus
$$\boxed{\tau(x) = \sqrt{\tau_0^2 + \frac{4\alpha^2x^2}{\tau_0^2}} > \tau_0}$$

and hence the electric feild pulse broadens with ~~[illegible]~~. propagation.

For large x, $\tau(x) \simeq \frac{2\alpha x}{\tau_0} \Rightarrow \boxed{\frac{\Delta\tau}{\Delta x} = \frac{2\alpha}{\tau_0}}$

5) a).



Since incident field is in vacuum

$$k = \frac{\omega}{c}$$

The complex electric field:

$$\tilde{E}_{inc} = \hat{x} E_0 e^{-ik_0 z}$$

$$\Rightarrow \tilde{H}_{inc} = \frac{k_0 E_0}{\omega\mu_0}\,\hat{y}\, e^{-ik_0 z}$$

$$\Rightarrow \vec{H}_{inc} = \frac{k_0 E_0}{\omega\mu_0}\cos(\omega t - k_0 z)\,\hat{y}$$

Scanned by CamScanner

We will solve problem in terms of complex fields.

b) let

$$\tilde{E}_{refl} = \Gamma \cdot E_0 e^{+ik_0 z} \hat{y} \text{ (reflected backward travelling wave).}$$

$$\tilde{E}_{tran} = \tau E_0 e^{-ik_c z} \hat{y} \text{ (transmitted forward travelling wave).}$$

where Γ is reflection coefficient, τ is transmission coefficient and k_c is wave-number / prop. constant in the conductor.

$$k_c^2 = \omega^2 \mu_0 \epsilon_0 \left[\epsilon_r - \frac{i\sigma}{\omega \epsilon_0}\right]$$

$$= \frac{\omega^2}{c^2}\left[2 - i2\sqrt{3}\right]$$

$$= \frac{\omega^2}{c^2} \cdot 4\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = \frac{4\omega^2}{c^2} e^{-i\pi/3}$$

$$\Rightarrow k_c = \frac{2\omega}{c} e^{-i\pi/6} = \frac{2\omega}{c}\left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right].$$

Hence,

$$\tilde{E}_{refl} = \Gamma E_0 e^{ik_0 z} \hat{y}$$

$$\tilde{E}_{tran} = \tau E_0 e^{-ik_0\sqrt{3}z} e^{-k_0 z} \hat{y} \quad \left(k_0 = \frac{\omega}{c}\right)$$

Imposing continuity of parallel $\vec{E}$ at $z=0$,

$$1 + \Gamma = \tau \quad \text{— (1)}$$

Now, compute magnetic fields:

$$\tilde{H}_{inc} = \frac{k_0 E_0}{\omega \mu_0} e^{ik_0 z} \hat{y} \qquad \tilde{H}_{refl} = -\frac{k_0 E_0 \Gamma}{\omega \mu_0} e^{-ik_0 z} \hat{y}$$

$$\tilde{H}_{tran} = \frac{(k_0\sqrt{3} - ik_0)}{\omega \mu_0} E_0 \tau e^{-ik_0\sqrt{3}z} e^{-k_0 z} \hat{y}$$

Imposing cont^n of parallel $\vec{H}$ at $z=0$,

Scanned by CamScanner

$$\frac{k_0 \cancel{E_0}}{\cancel{\omega} \mu_0} - \frac{k_0 \cancel{E_0} \Gamma}{\cancel{\omega \mu_0}} = \frac{k_0 \cancel{E_0}}{\cancel{\omega} \mu_0} \tau(\sqrt{3} - i).$$

$$\Rightarrow \quad 1 - \Gamma = \tau(\sqrt{3} - i) \quad — (2)$$

Solve (1) and (2) to obtain Γ and τ.

$$\tau = \frac{2}{(1 + \sqrt{3} - i)} \simeq 0.687 \angle 20.1^\circ$$

$$\Gamma = \frac{(1 + i - \sqrt{3})}{(1 + \sqrt{3} - i)} \simeq 0.426 \angle 14.6^\circ$$

Thus real reflected field:

$$\vec{E}_{refl} = 0.687\, E_0 \cos(\omega t + k_0 z + 20.1^\circ)\, \hat{y}$$

real transmitted field

$$\vec{E}_{tran} = 0.426\, E_0 \cos(\omega t - k_0 \sqrt{3} z + 14.6^\circ) \cdot e^{-k_0 z} \hat{y}$$

6) a) clearly, field in II would be 0 since there is metal in that region. In region III, the field would also be 0 since the EM wave could not propagate through metal. We explicitly compute field in region I - the same procedure can be followed to rigorously show that the field in region III is 0. Again, we work with complex fields.

$$\vec{E}_{inc} = E_0(8\hat{x} + 6\hat{y} + 5\hat{z}) \sin\left[\omega t - \frac{\omega}{5c}(3x - 4y)\right]$$

$$\Rightarrow \tilde{E}_{inc} = -i E_0 (8\hat{x} + 6\hat{y} + 5\hat{z})\, e^{i \frac{\omega}{5c}(3x - 4y)}.$$

reflected wave would again be a plane wave. Let us assume

$$\tilde{E}_{refl} = E_0(a_x \hat{x} + a_y \hat{y} + a_z \hat{z})\, e^{i(k_x x + k_y y)}.$$

Scanned by CamScanner

Since reflected wave should propagate away from the interface at $x=0$, $k_x > 0$. Also, $k_x = \sqrt{k_0^2 - k_y^2}$.

At $x=0$, parallel components of $\vec{E}$ vanish. Thus

$$\hat{z}\cdot\left(\vec{E}_{inc} + \vec{E}_{refl}\right)\Big|_{x=0} = 0 \qquad \text{—— ①}$$

$$\hat{y}\cdot\left(\vec{E}_{inc} + \vec{E}_{refl}\right)\Big|_{x=0} = 0 \qquad \text{—— ②}$$

From ①,

$$\left[-i5e^{i\frac{4\omega y}{5c}} + a_z e^{ik_y y}\right] = 0$$

$$\Rightarrow \quad a_z = 5i, \quad k_y = \frac{4\omega}{5c}$$

From ②

$$\left[-i6e^{i\frac{4\omega y}{5c}} + a_y e^{ik_y y}\right] = 0$$

$$\Rightarrow \quad a_y = 6i, \quad k_y = \frac{4\omega}{5c}.$$

To calculate a_x, we use the fact that $\nabla\cdot\vec{E}_{refl} = 0$

$$\Rightarrow \frac{\partial(\tilde{E}_{refl}\cdot\hat{x})}{\partial x} + \frac{\partial(\tilde{E}_{refl}\cdot\hat{y})}{\partial y} + \frac{\partial(\tilde{E}_{refl}\cdot\hat{z})}{\partial z} = 0$$

$$\Rightarrow ik_x a_x + ik_y a_y = 0$$

$$\Rightarrow a_x = -\frac{k_y}{k_x}\cdot a_y = -\frac{k_y}{\left[k_0^2 - k_y^2\right]^{1/2}}\cdot a_y$$

$$= -\frac{4\omega/5c}{\left[\frac{\omega^2}{c^2} - \frac{16\omega^2}{25c^2}\right]}\cdot(6i) = -8i$$

Thus

$$\tilde{E}_{refl} = i\left[-8\hat{x} + 6\hat{y} + 5\hat{z}\right]E_0 e^{+i\frac{\omega}{5c}(3x+4y)}$$

$$\Rightarrow \vec{E}_{refl} = -\left[-8\hat{x} + 6\hat{y} + 5\hat{z}\right]E_0 \sin\left(\omega t + \frac{3\omega x}{5c} + \frac{4\omega y}{5c}\right)$$

Scanned by CamScanner

Hence, total field in region I-

$$\vec{E} = \vec{E}_{inc} + \vec{E}_{refl.}$$

$$= 8\hat{x} E_0 \left\{ \sin\left(\omega t - \frac{3\omega x}{5c} + \frac{4\omega y}{5c}\right) + \sin\left(\omega t + \frac{3\omega x}{5c} + \frac{4\omega y}{5c}\right) \right\}$$

$$+ (6\hat{y} + 5\hat{z}) \cdot E_0 \left\{ \sin\left(\omega t - \frac{3\omega x}{5c} + \frac{4\omega y}{5c}\right) - \sin\left(\omega t + \frac{3\omega x}{5c} + \frac{4\omega y}{5c}\right) \right\}$$

$$\boxed{\vec{E} = 16\hat{x} E_0 \sin\left(\omega t + \frac{4\omega y}{5c}\right) \cos\left(\frac{3\omega x}{5c}\right) - (12\hat{y} + 10\hat{z}) E_0 \sin\left(\frac{3\omega x}{5c}\right) \cos\left(\omega t + \frac{4\omega y}{5c}\right)}$$

Note that this resembles a standing wave in x- a consequence of the metal being perfect and hence reflectivity being 1.

b) For a near perfect conductors, fields would exponentially de cay inside instead of exactly zero. In this case

$$k_c^2 = \frac{\omega^2}{c^2}\left[\epsilon_r - \frac{i\sigma}{\omega\epsilon_0}\right]$$

$$= \frac{\omega^2}{c^2} \cdot [1 - 2i]$$

$$= \frac{\omega^2}{c^2} \sqrt{5} \cdot e^{-i\, 63.4^\circ}$$

$$\Rightarrow k_c = \frac{\omega}{c} \cdot 5^{1/4} e^{-i\, 31.7^\circ}$$

$$= \frac{\omega}{c}(1.27 - i\, 0.79)$$

Thus, field magnitude varies as $\sim \exp\left(-\frac{0.79\,\omega x}{c}\right)$. from conductor surface.

Scanned by CamScanner

If d is large enough to ensure

$$\exp\left(-\frac{0.79\,\omega d}{c}\right) \simeq \exp(-5) \simeq 0$$

no significant fields would be detected in region III.

$$\Rightarrow d \sim \left[\frac{5\,c}{0.79\,\omega}\right] \simeq \lambda_0$$

where

$\lambda_0 = \dfrac{2\pi c}{\omega}$ is the free space wavelength

Scanned by CamScanner