# EEL207 Tutorial 6 Solutions: 2015-16, Sem II

Instructor: Uday Khankhoje

**P.1:** Use the complex electric field vector $\tilde{E}$ to simplify computations

$$\tilde{E} = A\frac{\sin\theta}{r}\left(1+\frac{j}{kr}\right)\exp(j(kr-\omega t))\hat{e}_\phi$$

a) Use $\nabla^2\vec{v} = \nabla(\nabla\cdot\vec{v}) - \nabla\times(\nabla\times\vec{v})$ to compute $\nabla^2\tilde{E}$ and prove that if $\tilde{E}$ satisfies the wave equation then so does $\tilde{E}^*$ and hence $\vec{E}$. Also $v = \frac{\omega}{k}$. Using $\nabla\times\tilde{E} = -j\omega\mu_0\tilde{H}$ to compute $\tilde{H}$. The final expression is

$$\tilde{H} = \frac{A}{j\omega\mu_0 r^2}\left[2\cos\theta\left(1+\frac{j}{kr}\right)\hat{e}_r - \sin\theta\left(jkr + j - \frac{1}{kr}\right)\hat{e}_\theta\right]\exp(j(kr-\omega t))$$

The real magnetic field vector can be calculated using $\vec{H} = \mathrm{Re}(\tilde{H})$. For large $r$

$$\tilde{E} \simeq A\frac{\sin\theta}{r}\exp(j(kr-\omega t))\hat{e}_\phi$$
$$\tilde{H} \simeq -A\frac{k\sin\theta}{r\omega\mu_0}\exp(j(kr-\omega t))\hat{e}_\theta$$

b) Use $\langle\ \vec{S}\ \rangle = 1/2\ \mathrm{Re}(\tilde{E}\times\tilde{H}^*)$. For large $r$

$$\langle\ \vec{S}\ \rangle = \frac{kA^2}{2\omega\mu_0}\frac{\sin^2\theta}{r^2}\hat{e}_r$$

c) The power $P$ is given by

$$\begin{aligned}P &= \int_0^{2\pi}\int_0^{\pi}\langle\ \vec{S}\ \rangle\cdot\hat{e}_r R^2\sin\theta\mathrm{d}\theta\mathrm{d}\phi\\ &= \frac{kA^2}{2\omega\mu_0}\int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}\sin^3\theta\mathrm{d}\theta\\ &= \frac{4\pi A^2 k}{3\omega\mu_0}\end{aligned}$$

**P.2:** The electric field vector can be rewritten as

$$\tilde{E} = E_0(\hat{e}_x + \alpha\hat{e}_y)\exp\left(j\omega t - j\left(\frac{3\omega x}{c} - j\frac{\omega y}{c}\right)\right)$$

a) Thus

$$\tilde{k} = \frac{3\omega}{c}\hat{e}_x - \frac{j\omega}{c}\hat{e}_y$$

Also

$$|\tilde{k}|^2 = \frac{\omega^2}{c^2}\epsilon_r = \frac{8\omega^2}{c^2} \implies \epsilon_r = 8$$

Using $\tilde{k}\cdot\tilde{E} = 0$ we obtain $\alpha = -3j$. Also $\vec{E} = \mathrm{Re}(\tilde{E})$.

$$\vec{E} = E_0\exp\left(-\frac{3\omega y}{c}\right)\left[\hat{e}_x\cos\left(\omega t - \frac{3\omega x}{c}) + 3\hat{e}_y\sin\left(\omega t - \frac{3\omega x}{c}\right)\right]$$

b) To calculate $\tilde{H}$, use $\tilde{H} = \frac{1}{\omega\mu_0}\tilde{k}\times\tilde{E}$ and $\vec{H} = \mathrm{Re}(\tilde{H})$.

$$\tilde{H} = -\frac{8jE_0}{c\mu_0}\exp\left(-\frac{\omega y}{c}\right)\exp\left(j\omega t - j\frac{3\omega x}{c}\right)\hat{e}_z$$
$$\vec{H} = \frac{8E_0}{c\mu_0}\exp\left(-\frac{\omega y}{c}\right)\sin\left(j\omega t - j\frac{3\omega x}{c}\right)\hat{e}_z$$

c) The time averaged Poynting vector is

$$\langle\ \vec{S}\ \rangle = \frac{12E_0^2}{c\mu_0}\exp\left(-\frac{2\omega y}{c}\right)\hat{e}_x$$