Homework 2: EEL760 Antenna Theory & Design

Important details:
(1) Released to class: 24.09.2015. Due on: 5pm 05.10.2015 in my dept. mailbox. 10% penalty of max marks for each day of delay.
(2) You are *free to discuss* with your friends and look at course material. But, the submitted work should be *entirely original*. On the cover sheet, please mention who you discussed the problem set with. Don't worry, there are no negative points for discussing the problems! But remember the distinction between discussing and copying – the former is welcome, the latter results in an instant fail!
(3) State all *intermediate steps* clearly, making reasonable assumptions as you go along. The solution should be self-contained and should make sense to any general scientist reading it. If you need to get any additional information to solve the problem, be sure to add a reference.
(4) Please write *neatly* and *concisely*. If you can submit a LaTeX typed document, even better!
(5) Do *not* rewrite the questions in your submission.
Please follow all the above instructions

1) Consider a current source (for simplicity, a Hertz dipole) aligned at $45°$ from the vertical direction. We are interested in finding the radiation pattern due to such an antenna when placed some distance above a perfectly conducting ground plane.
(a) Mention two ways of solving this electromagnetic problem, with appropriate figures for each method.
(b) Explain how these methods are conceptually equivalent. This explanation should be valid in the near-zone of the antenna as well. Please give clear mathematical reasons, not verbose essays.

2) Consider a small loop (SL) of current with area S placed on the $x - y$ plane, centred at $(0, 0, 0)$ and a Hertz dipole (HD) placed at $(d, 0, 0)$, both having current I_0 in them.
(a) Can we use the concept of pattern multiplication to find the far field? Give reasons.
(b) Write an expression for the electric field, $\vec{E}(r, \theta, \phi)$, in the far-zone.
(c) Is beam-forming possible in this case? i.e., can we change the angular location of beam lobes and side-lobes by changing the current excitation (amplitude or phase) of the individual elements? Explain.
(d) Now imagine an array of SL and HD antennas. Each SL is placed at even integer locations, i.e. $0, 2d, 4d, \ldots, 2(N-1)d$ and each HD is placed at odd integer locations $d, 3d, 5d, \ldots, (2N-1)d$. Answer questions (a), (b), (c) for this configuration.

3) Consider a 5 element linear array that is uniformly spaced with distance = $\lambda/8$, uniform current amplitudes, and with constant current phase difference between consecutive elements as $\alpha = 3\pi/4$.
(a) Derive and plot the normalized array factor.
(b) By graphical means, sketch the radiation pattern, showing (1) the visible region, and (2) all lobes and nulls. For (2), also explicitly write the amplitude of the lobes, angular locations of lobes and nulls.
(c) Does the radiation pattern resemble a broadside pattern, or an end-fire pattern? Explain.

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4) In the Galerkin method (i.e. testing functions are the same as the expansion basis function of the unknown function), starting with the operator equation of the form $\mathcal{L}f(r) = g(r)$, we arrived at a matrix representation as $L\mathbf{a} = \mathbf{b}$. Here, $f(r) = \Sigma_{i=1}^{n} a_i f_i(r)$, with basis functions $f_i(r)$, column vectors are $\mathbf{a} = [a_1 \ldots a_n]^T$ and $\mathbf{b} = [\langle f_1(r), g(r)\rangle \ldots \langle f_n(r), g(r)\rangle]^T$, and the matrix elements are $L_{mn} = \langle f_m(r), \mathcal{L}f_n(r)\rangle$.
Now, suppose that there exists another vector space whose basis vectors, $\{h_1(r), \ldots, h_n(r)\}$, are the eigenfunctions of the operator $\mathcal{L}$ (not necessarily orthogonal, i.e. $\langle h_i, h_j\rangle \neq 0$ in general). Assume that $f(r)$ is representable in this basis.
(a) Is it possible to have a matrix representation of $\mathcal{L}$ to always be diagonal? Be sure to catch all the conditions.

Let us now move to the discrete world, where every continuous function is replaced by an equivalent column vector, i.e. $f_i(r) \rightarrow \mathbf{f}_i$. Equivalently, the operator also gets discretized. Thus, $\mathbf{f} = a_1\mathbf{f}_1 + \ldots + a_n\mathbf{f}_n = F\mathbf{a}$, where the kth column of F is $\mathbf{f}_k$. Also, $\langle \mathbf{p}, \mathbf{q}\rangle = \mathbf{p}^T\mathbf{q}$.
(b) By expressing $\mathbf{f}$ in the $\mathbf{h}$ basis, arrive at an alternate expression for L_{mn} in terms of the Gramian matrix (look it up) of $\mathbf{h}$. Is there some advantage of this representation?
(c) Derive (not just state) the eigen decomposition of (any) matrix L (assuming that L can be diagonalized). Using this representation of L, arrive at the same result as (b).
(d) [BONUS] Make a connection between (c) and the singular value decomposition of the operator $\mathcal{L}$.

5) (a) Summarize the steps in the computation of the field scattered by a 2D dielectric object.
(b) We are interested in finding the scattered field in the far-zone, where certain approximations can be made to the Green's functions. Derive an analytical expression for the field under these assumptions.