Band structure calculation for dielectric and metallic photonic crystal (EEL766)

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Actually...

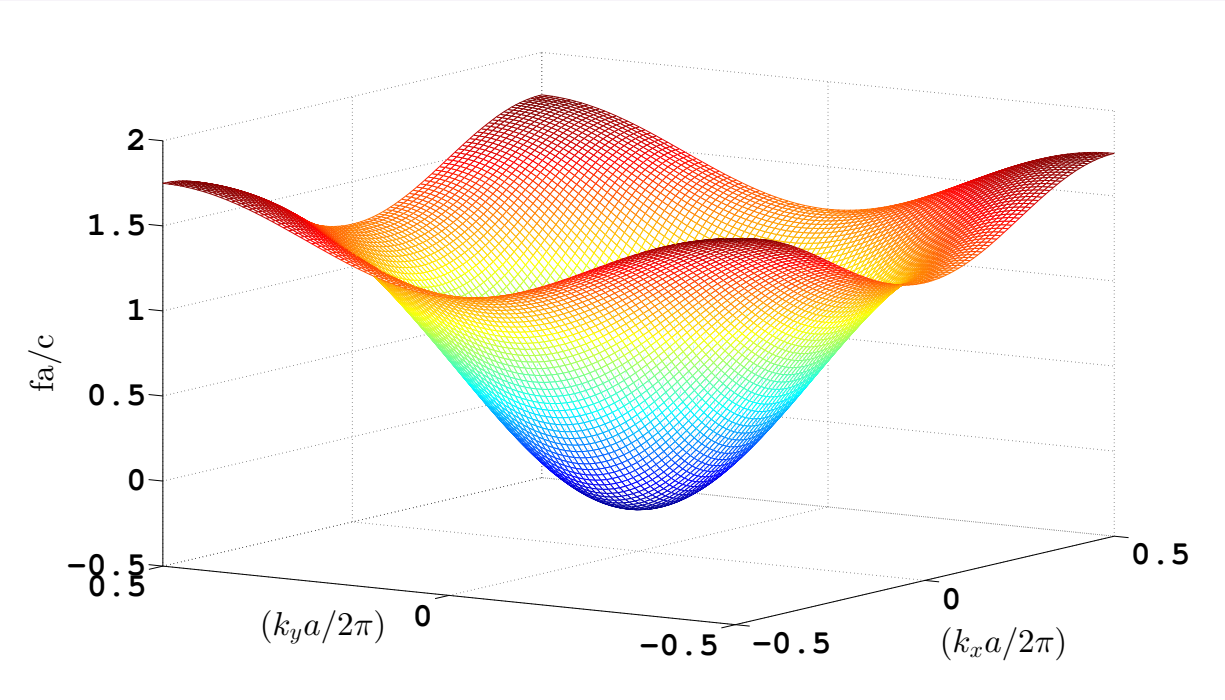


Figure: A typical band-diagram

However...

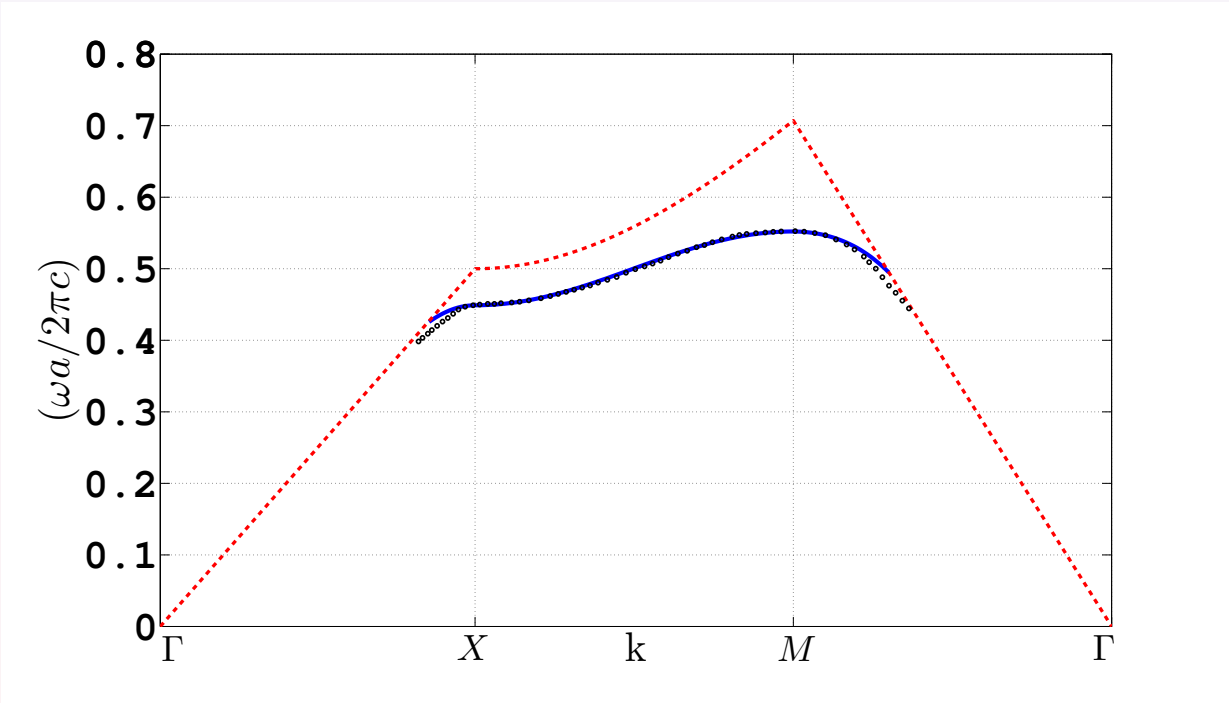
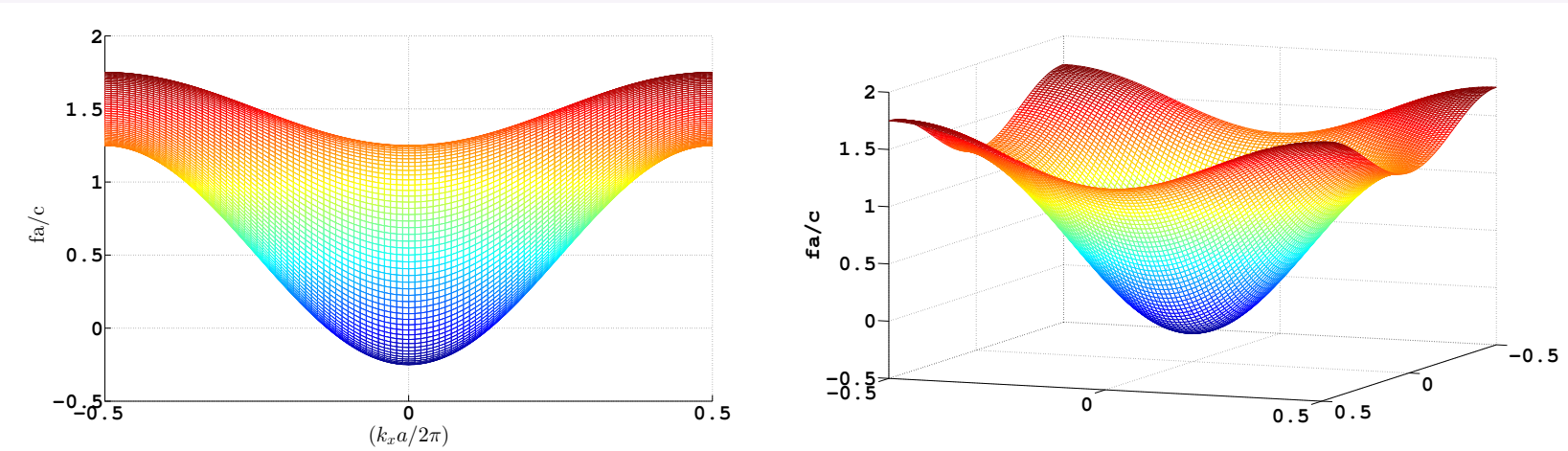


Figure: This is what we plot! Ignore the black circles for the time being. The blue line shows the band-diagram and the red dotted line is light-line.

Keep walking along the line and get back home



But does it work all the time??

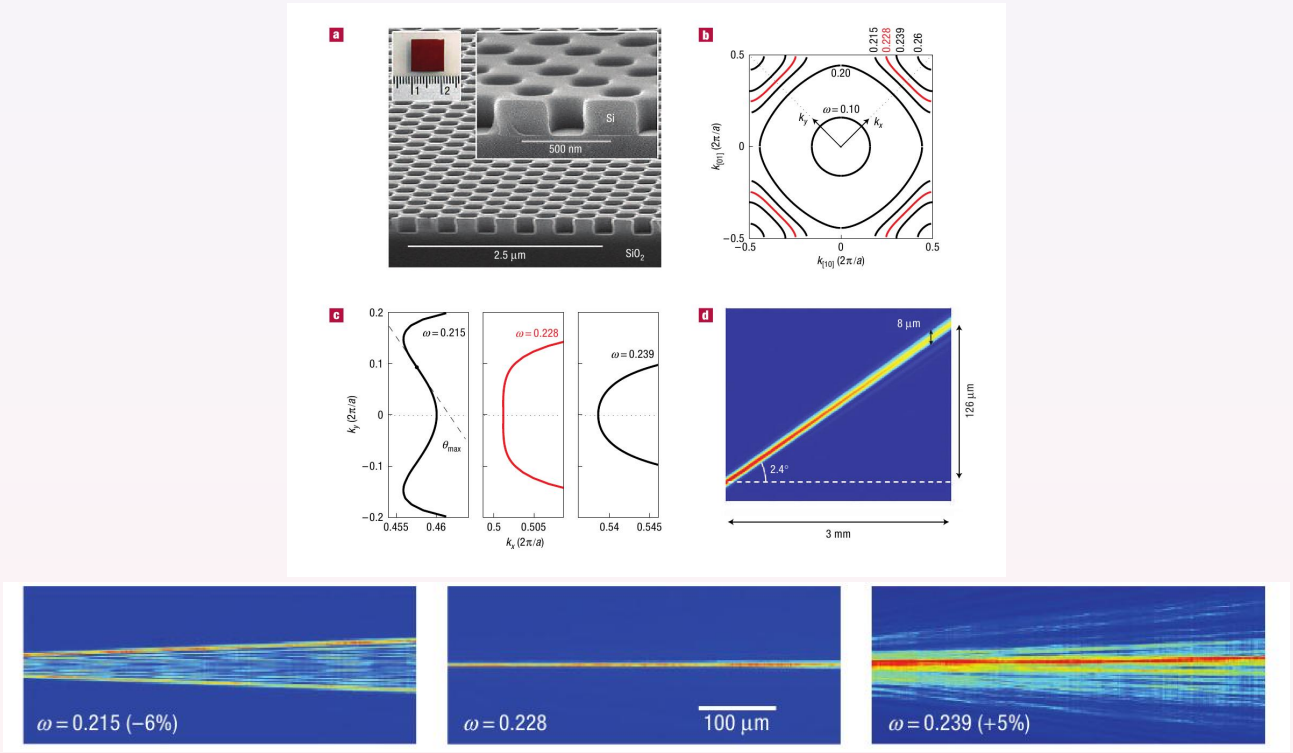
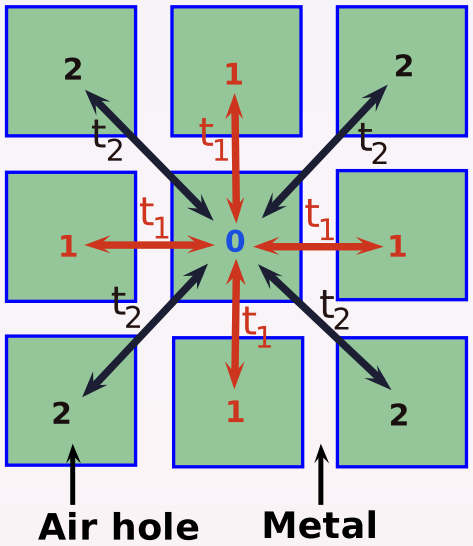


Figure: Super-collimation in PhC. $a = 350$ nm, $r = 0.3a$ [P. T. Rakich et al., *Nature Materials* **5**, 93, 2006].

Interaction process in tight binding model



Schematic of the interaction process between the scattered field from the lattice sites: interaction only up to second nearest neighbor is shown. The interaction path/ the arrows are drawn along the direction of the lattice vector. The numbers on the neighboring holes denote their distances from the 0^{th} hole. Here, t_1 and t_2 denote the hopping parameters.

The work presented from this slide onwards was done *@Photonics Lab, Dept. of Electrical Engineering*, IIT Delhi, under supervision of *Dr. Kushal Shah.*

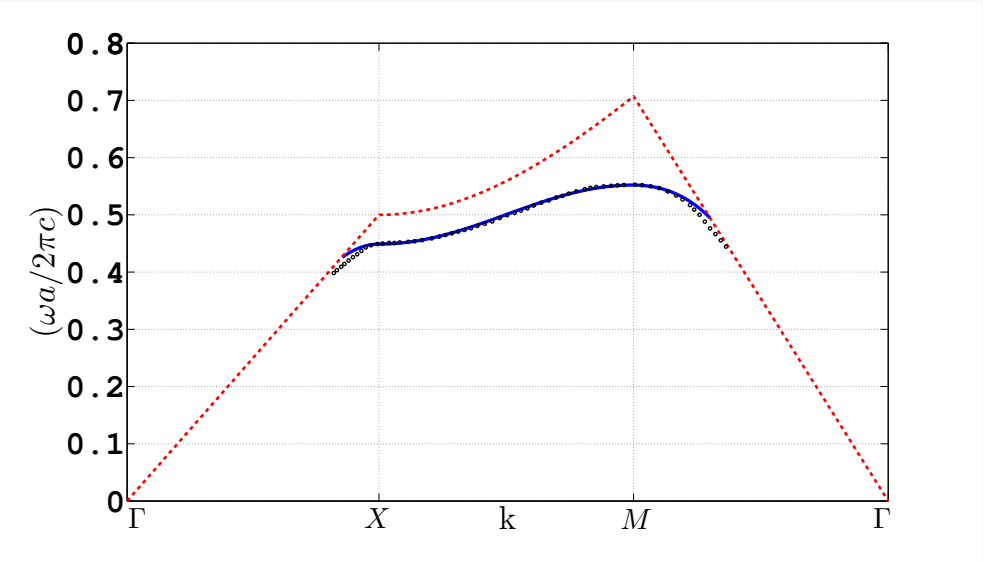
Final equations

- $\omega_k = t_0 - 2t_1\left[cos(k_x a) + cos(k_y a)\right] - 4t_2 cos(k_x a) cos(k_y a)$
- At Γ $(k_x a = k_y a = 0)$, X $(k_x a = \pi, k_y a = 0)$ and M $(k_x a = k_y a = \pi)$:

$$\begin{aligned} t_0 - \omega_\Gamma &= 4(t_1 + t_2) \quad : \Gamma & (1)\\ \omega_X &= t_0 + 4t_2 \quad : X & (2)\\ \omega_M &= t_0 + 4t_1 - 4t_2 \quad : M & (3) \end{aligned}$$

- $t_0 = \frac{1}{2}\left[\omega_X + \left(\frac{\omega_\Gamma + \omega_M}{2}\right)\right]$
- $t_1 = \frac{1}{8}\left(\omega_M - \omega_\Gamma\right)$
- $t_2 = \frac{1}{8}\left[\omega_X - \left(\frac{\omega_\Gamma + \omega_M}{2}\right)\right]$

Comaparison of tight binding model with numerical results: first band



High frequency limit band diagram of surface wave on a 2D array of square holes of infinite depth drilled on PEC. For each hole, the each side, $b = 0.875a$. Blue/solid line: dispersion diagram obtained using our theory, black circles: dispersion diagram obtained by FDTD (refer to M. Qiu, *Opt. Express* **13** 7583, 2005) . Red/dashed line: light line.

- $\omega_\Gamma = 0$, $\omega_X = 0.448931$ and $\omega_M = 0.55256$ (from M. Qiu, *Opt. Express* **13** 7583, 2005).
- $t_0 = 0.3625$, $t_1 = 0.0691$ and $t_2 = 0.0216$

Super-collimation

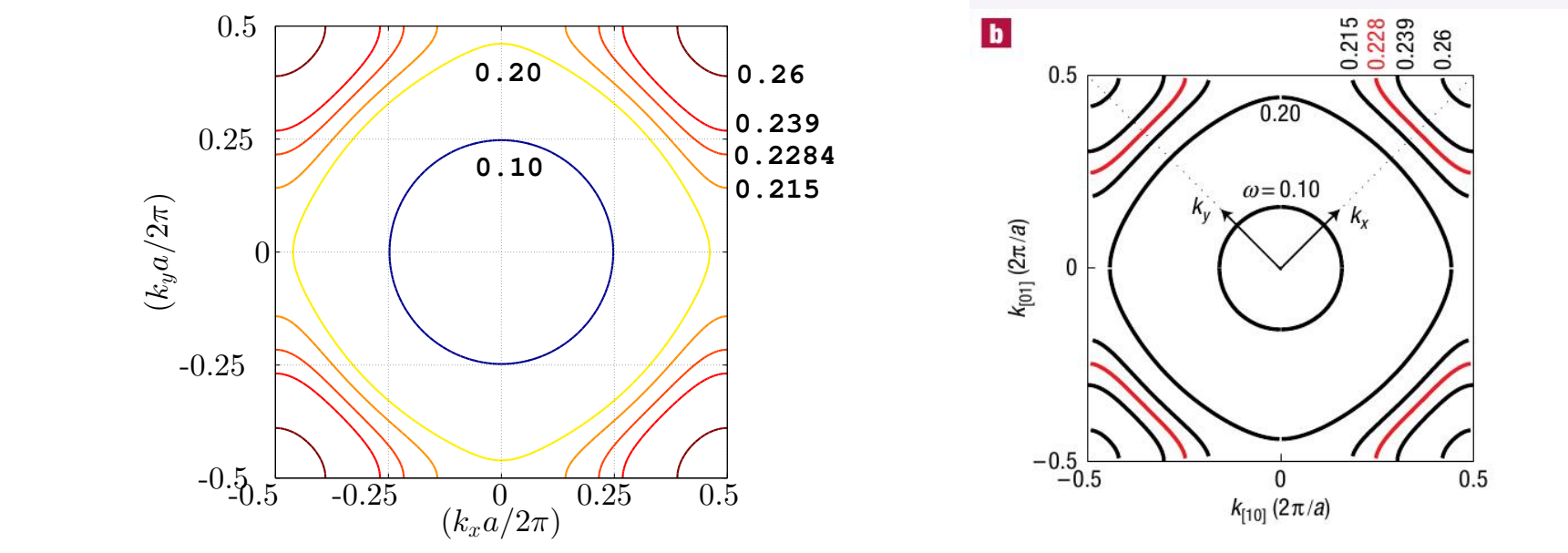
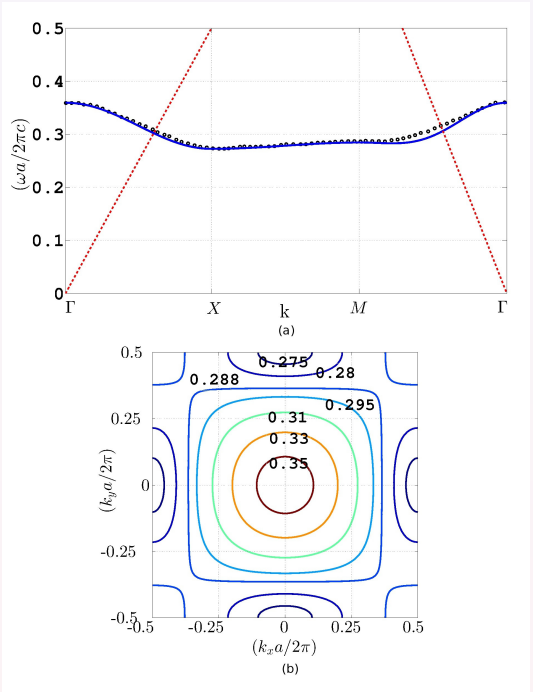


Figure: Contour plot for the first band of the super-collimating photonic crystal. Left: predicted by tight binding model. Right: FDTD simulation [P. T. Rakich et al., *Nature Materials* **5**, 93, 2006].

Comaparison of tight binding model with numerical results: second band



Second band of dielectric PhC (dimension is same as that of the super-collimating structure, except that the SiO_2 undercladding is absent here). The black circles correspond to the FDTD results obtained by J. Witzens et al. (*IEEE J. of Selected topics in Quantum Elec.* **8** 1246, 2002). FDTD results predicted a flat contour at $0.295(c/a)$.

- $\omega_\Gamma = 0.3594$, $\omega_X = 0.2733175$ and $\omega_M = 0.284436$.
- The hopping parameters : $t_0 = 0.2991$, $t_1 = -0.0086$ and $t_2 = -0.0064$.

References

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