Photonic Crystals

(or how to slow, trap, bend, split, and do other funky things to light)

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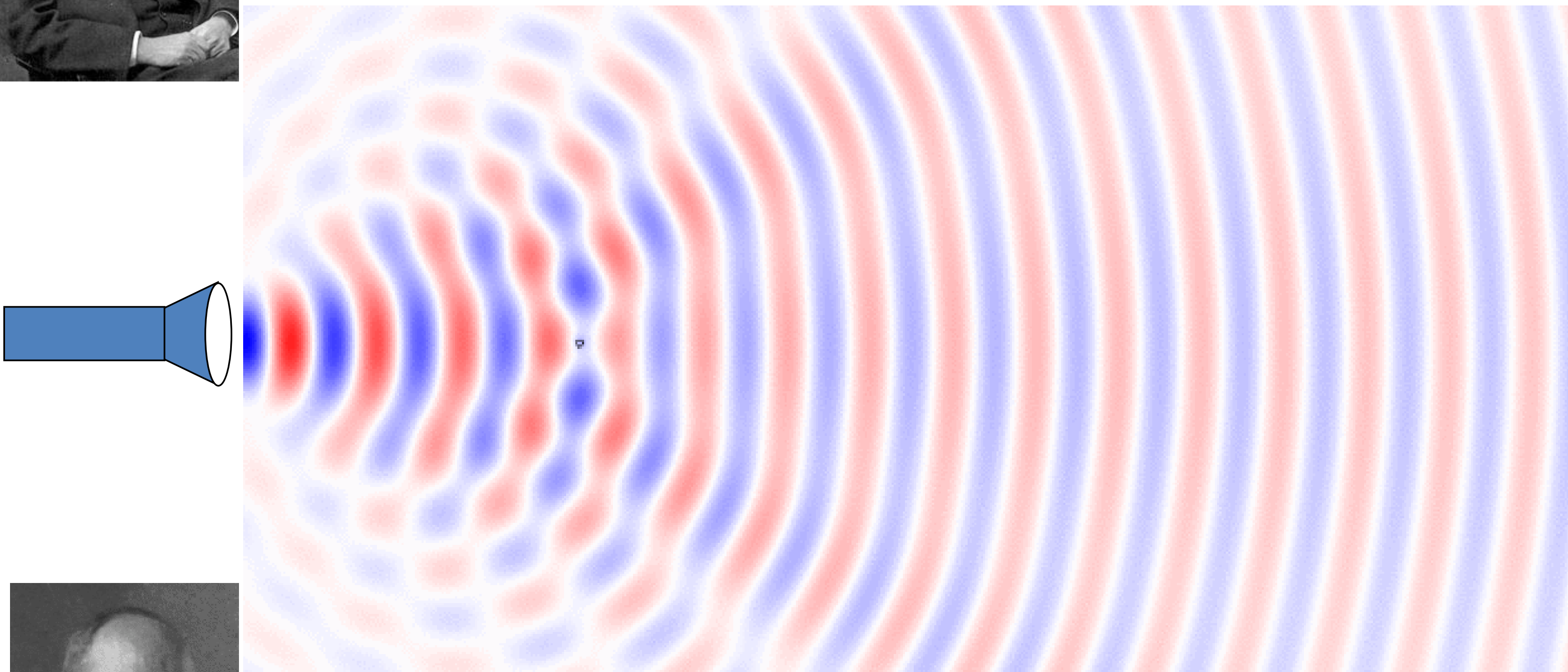
[Based on material made generous made available by S G Johnson, MIT, at http://ab-initio.mit.edu/photons/]

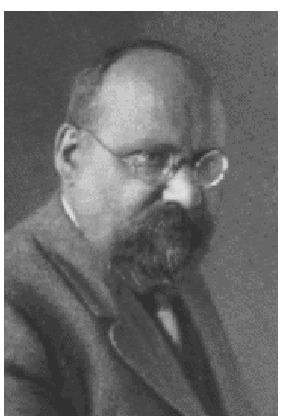
small particles:
Lord Rayleigh (1871)
why the sky is blue

...Waves Can Scatter

here: a little circular speck of silicon



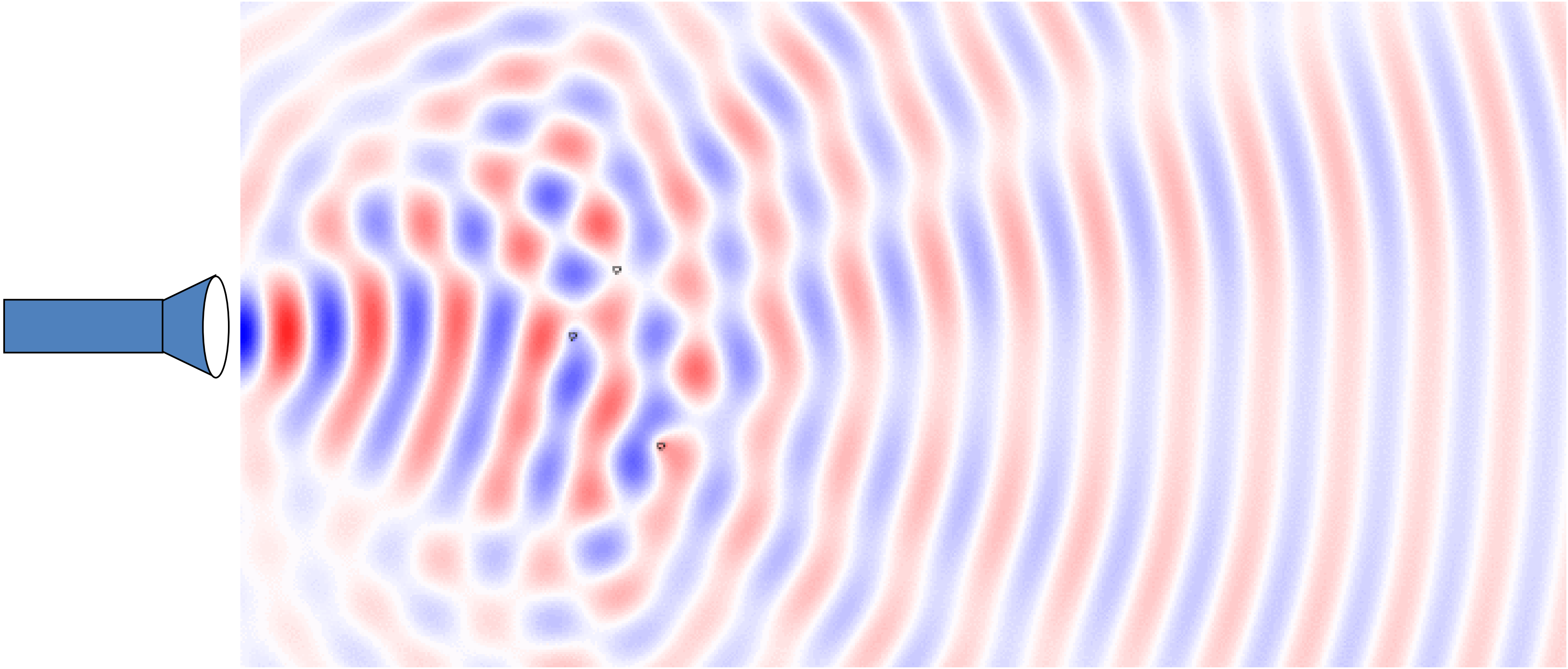
checkerboard pattern: interference of waves
traveling in different directions



scattering by spheres:
solved by Gustave Mie (1908)

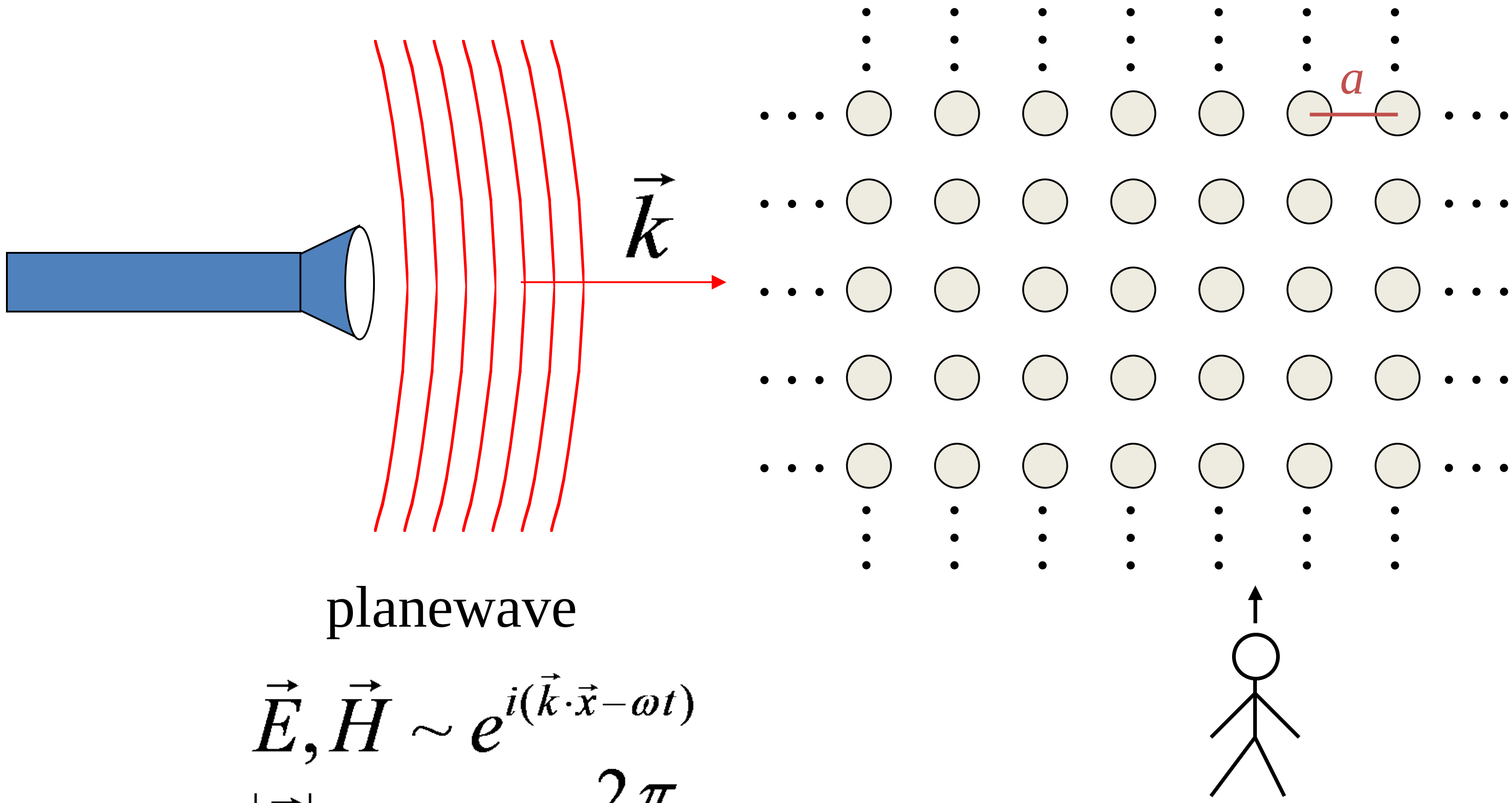
Multiple Scattering is Just Messier?

here: scattering off three specks of silicon



can be solved on a computer, but not terribly interesting…

To Begin: A Cartoon in 2d



planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

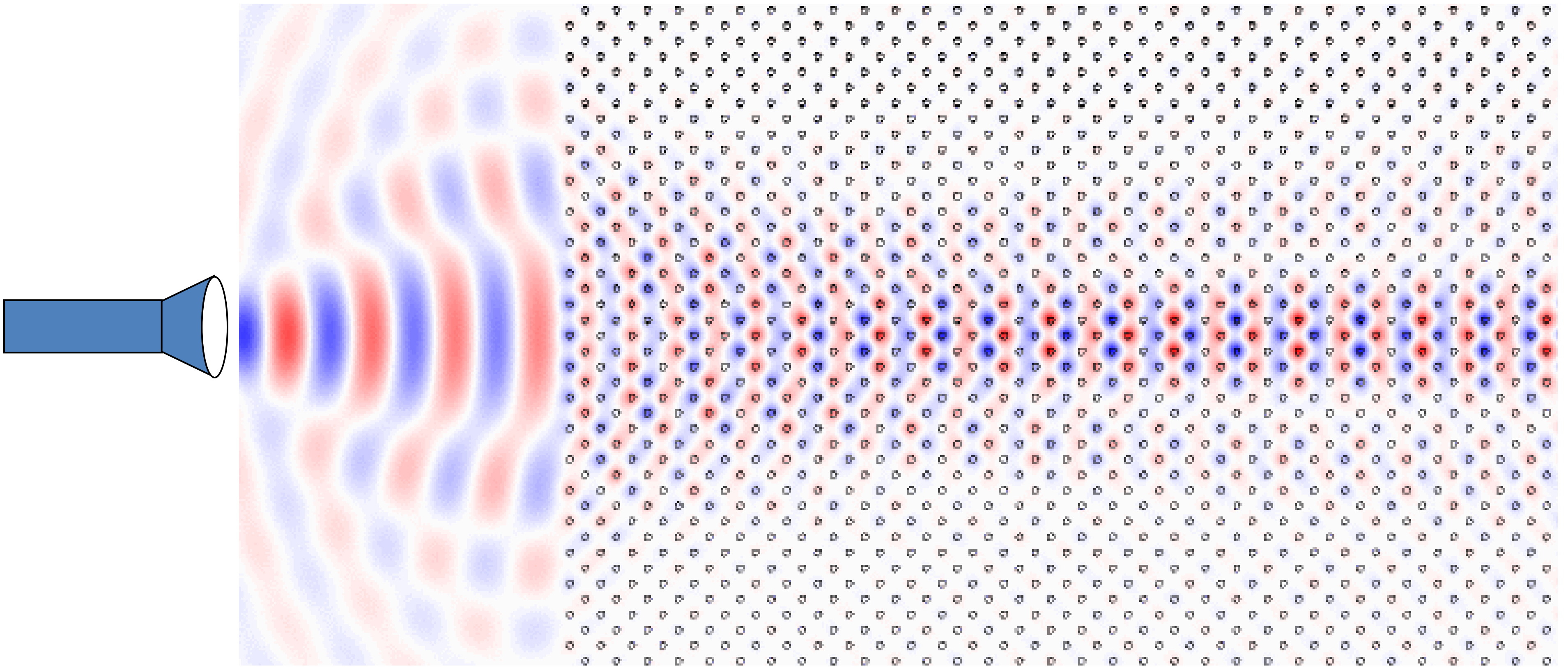
for most λ, beam(s) propagate through crystal without scattering (scattering cancels coherently)

...but for some λ (~ 2a), no light can propagate: a photonic band gap

An even bigger mess?
zillons of scatterers

Blech, light will just scatter like crazy and go all over the place … how boring!

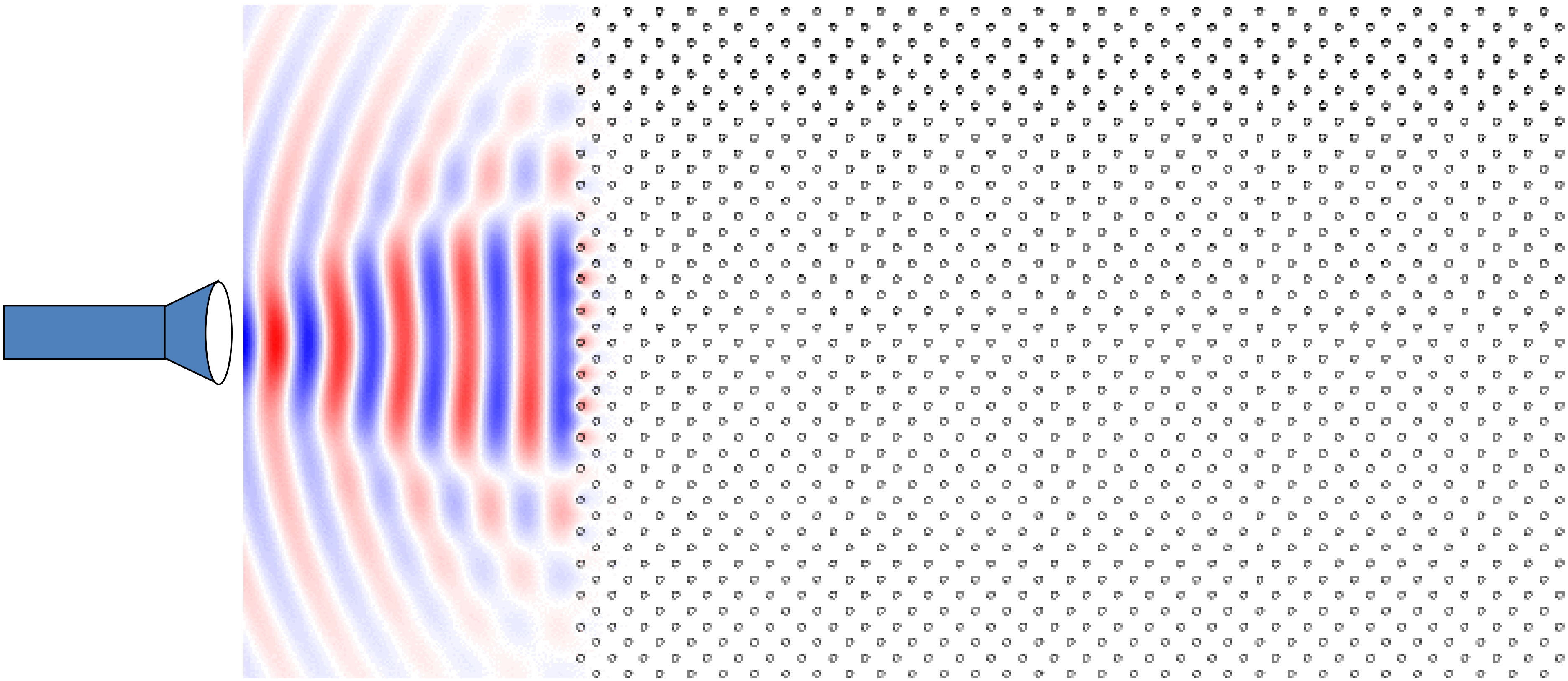
Not so messy, not so boring…



the light seems to form several *coherent beams*
that propagate *without scattering*
… and almost *without diffraction* (*supercollimation*)

A slight change? Shrink ⎣ by 20%

an "optical insulator" (photonic bandgap)

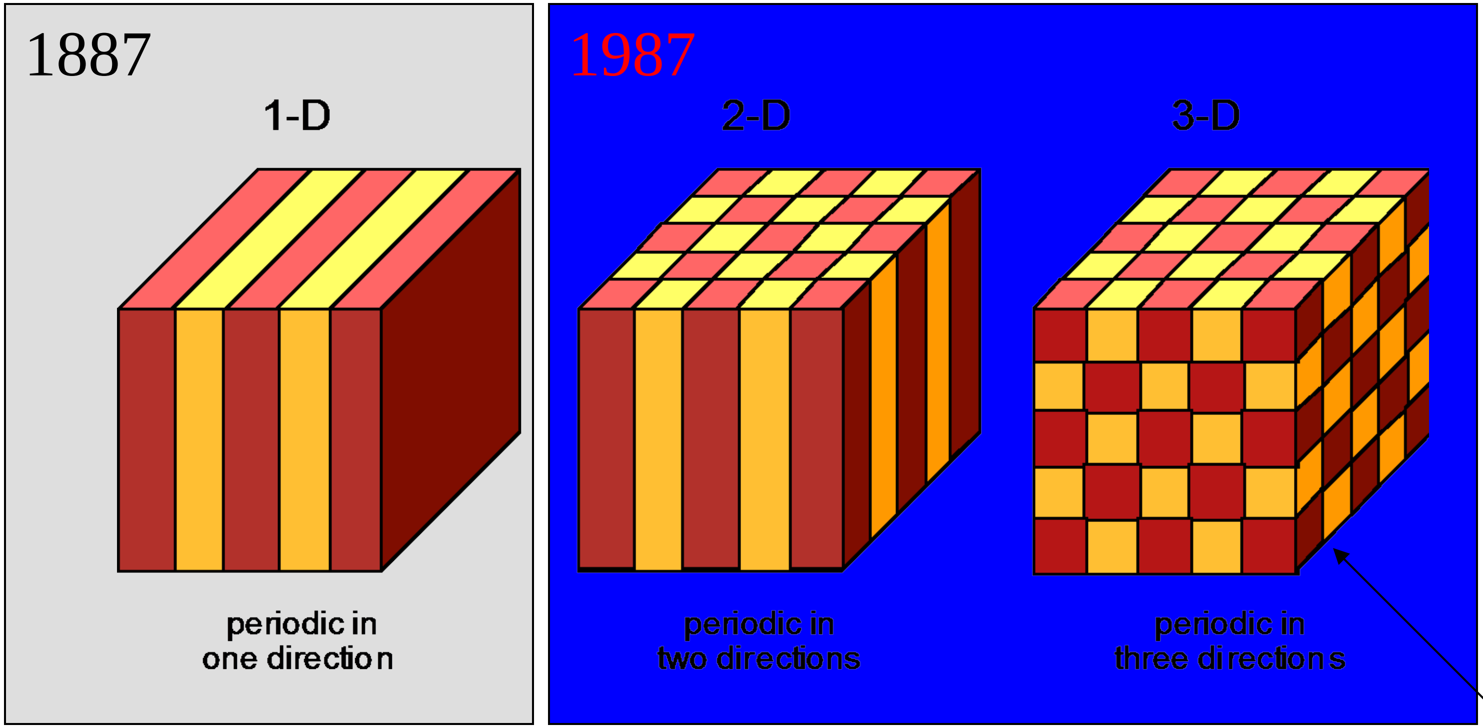


light cannot penetrate the structure at this wavelength!

all of the scattering destructively interferes

Photonic Crystals

periodic electromagnetic media



(need a more complex topology)

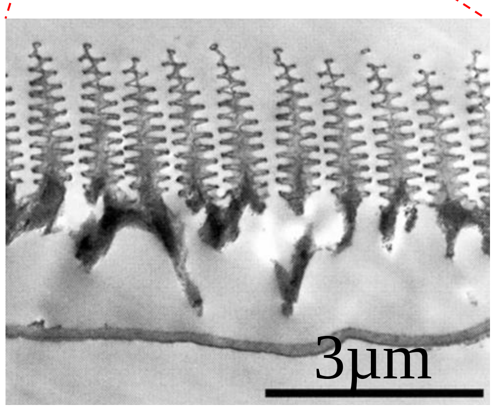
with photonic band gaps: "optical insulators"

Photonic Crystals in Nature

Morpho rhetenor butterfly

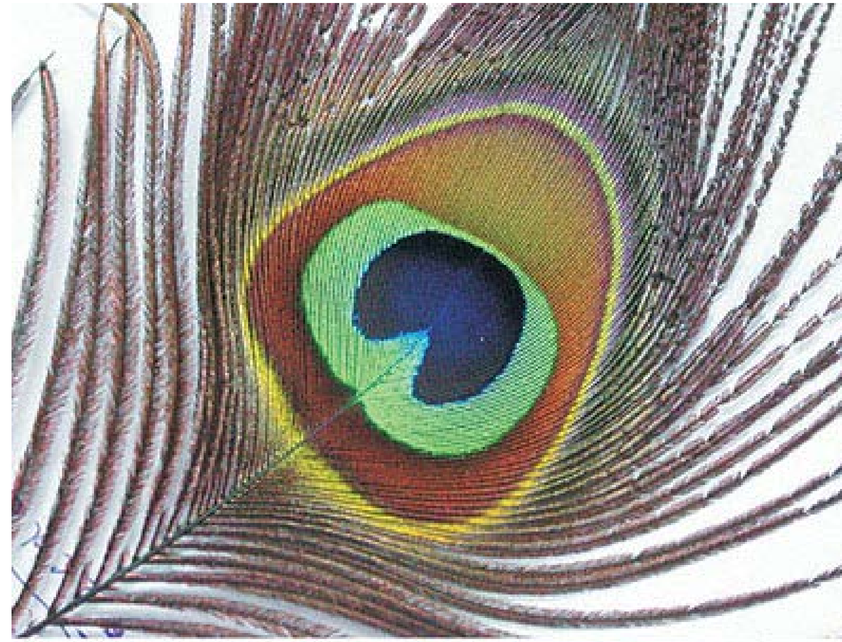
wing scale:

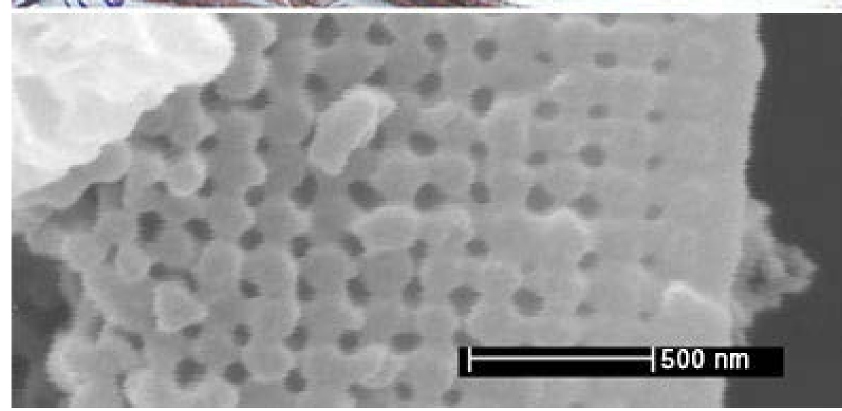
[P. Vukosic *et al.*, *Proc. Roy. Soc: Bio. Sci.* **266**, 1403 (1999)]



[also: B. Gralak *et al.*, *Opt. Express* **9**, 567 (2001)]

Peacock feather





[J. Zi *et al*, *Proc. Nat. Acad. Sci. USA*, **100**, 12576 (2003)]

[figs: Blau, *Physics Today* **57**, 18 (2004)]

How do we solve for modes?

As usual, start with our favourite:

$$\nabla \times \vec{E} = -j\omega\mu\,\vec{H}$$

$$\nabla \times \vec{H} = j\omega\epsilon_0\epsilon_r\,\vec{E}$$

We eliminate one variable and end up with a eigen value problem!

$$\boxed{\nabla \times \left(\frac{1}{\epsilon_r}\nabla\right.} \times \left.\vec{H}\right) = \boxed{\left(\frac{\omega}{c}\right)}^2 \vec{H}$$

Hermitian Eigenproblems

$$\nabla \times \frac{1}{\varepsilon} \nabla \times \vec{H} = \left(\frac{\omega}{c}\right)^2 \vec{H}$$

+ constraint

$$\nabla \cdot \vec{H} = 0$$

eigen-operator — eigen-value — eigen-state

Hermitian for real (lossless) ε

→ well-known properties from linear algebra:

ω are real (lossless)
eigen-states are orthogonal
eigen-states are complete (give all solutions)*

* Technically, completeness requires slightly more than just Hermitian-ness.

Periodic Hermitian Eigenproblems

[G. Floquet, "Sur les équations différentielles linéaries à coefficients périodiques," *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
[F. Bloch, "Über die quantenmechanik der electronen in kristallgittern," *Z. Physik* **52**, 555–600 (1928).]

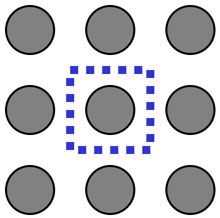
if eigen-operator is periodic, then Bloch-Floquet theorem applies:

can choose: $$\vec{H}(\vec{x},t) = e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}\vec{H}_{\vec{k}}(\vec{x})$$

planewave

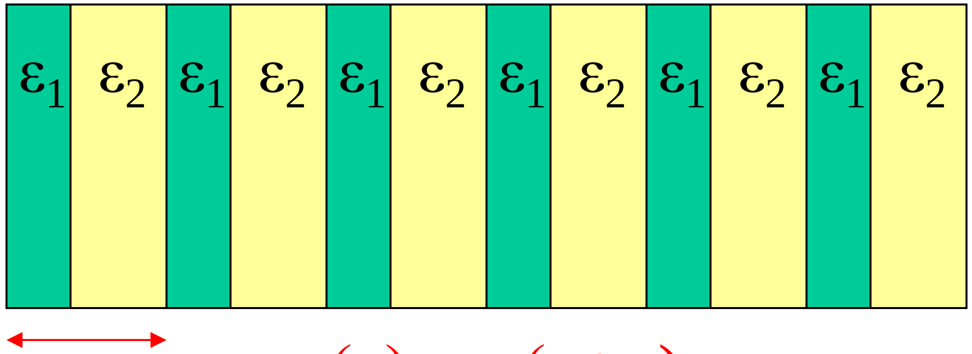
periodic "envelope"

Corollary: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete $\omega_n(\mathbf{k})$



Periodic Hermitian Eigenproblems in 1d

$$H(x) = e^{ikx} H_k(x)$$



$\varepsilon(x) = \varepsilon(x+a)$

Consider $k+2\pi/a$: $$e^{i\left(k+\frac{2\pi}{a}\right)x} H_{k+\frac{2\pi}{a}}(x) = e^{ikx}\left[e^{i\frac{2\pi}{a}x} H_{k+\frac{2\pi}{a}}(x) \right]$$

periodic!
satisfies same equation as H_k
$= H_k$

k is periodic:

$k + 2\pi/a$ equivalent to k

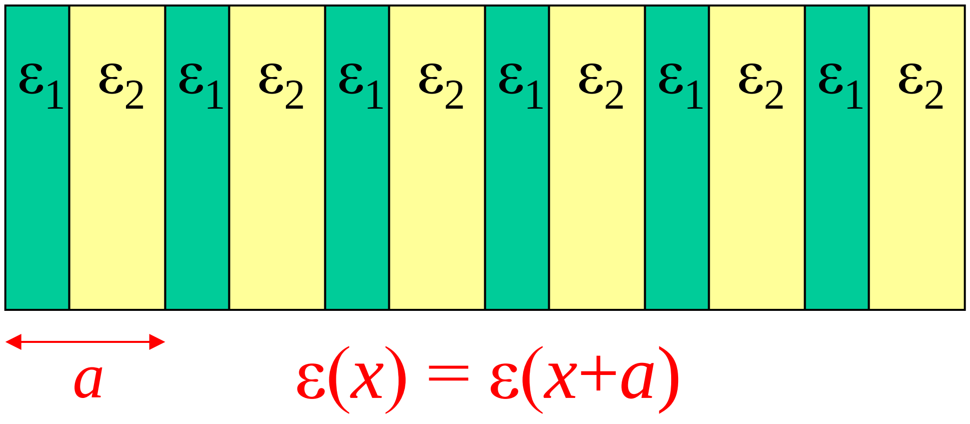
"quasi-phase-matching"

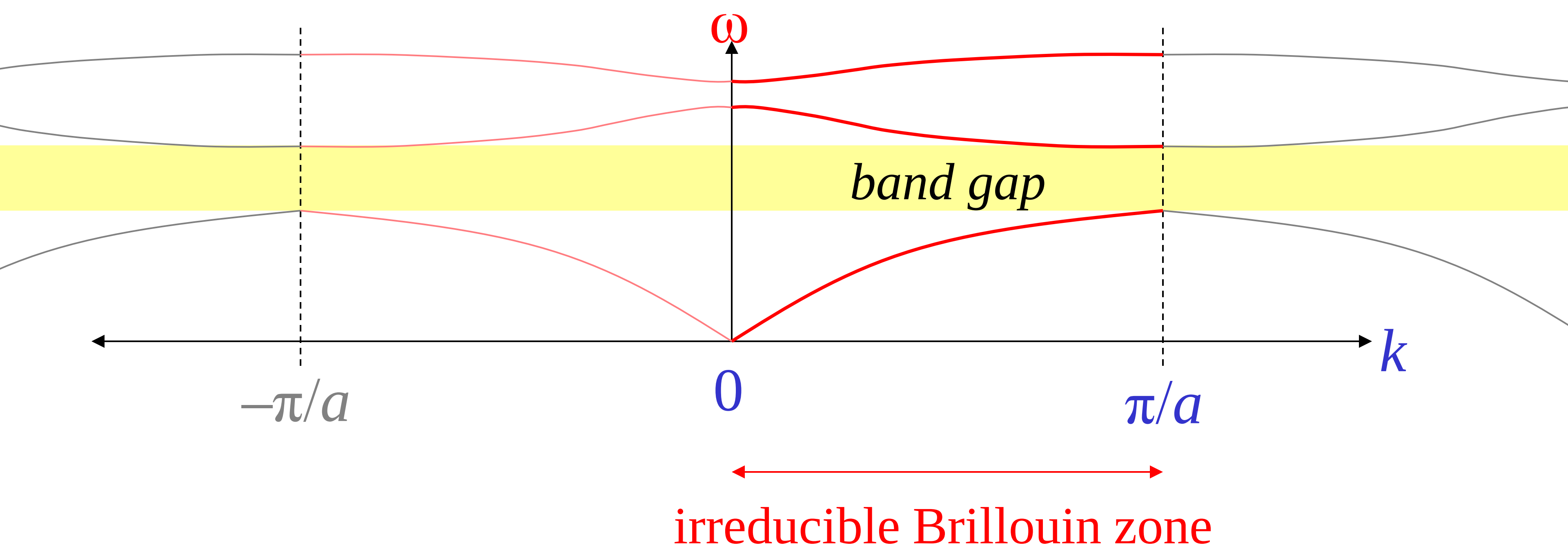
Periodic Hermitian Eigenproblems in 1d

k is periodic:

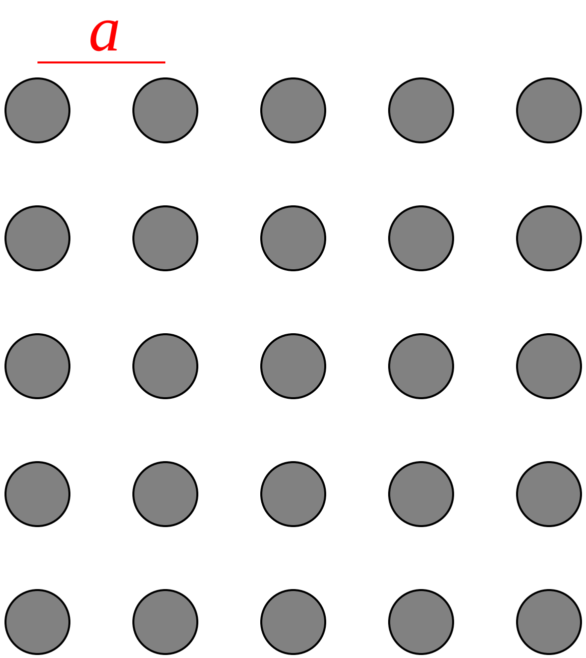
k + 2π/*a* equivalent to *k*

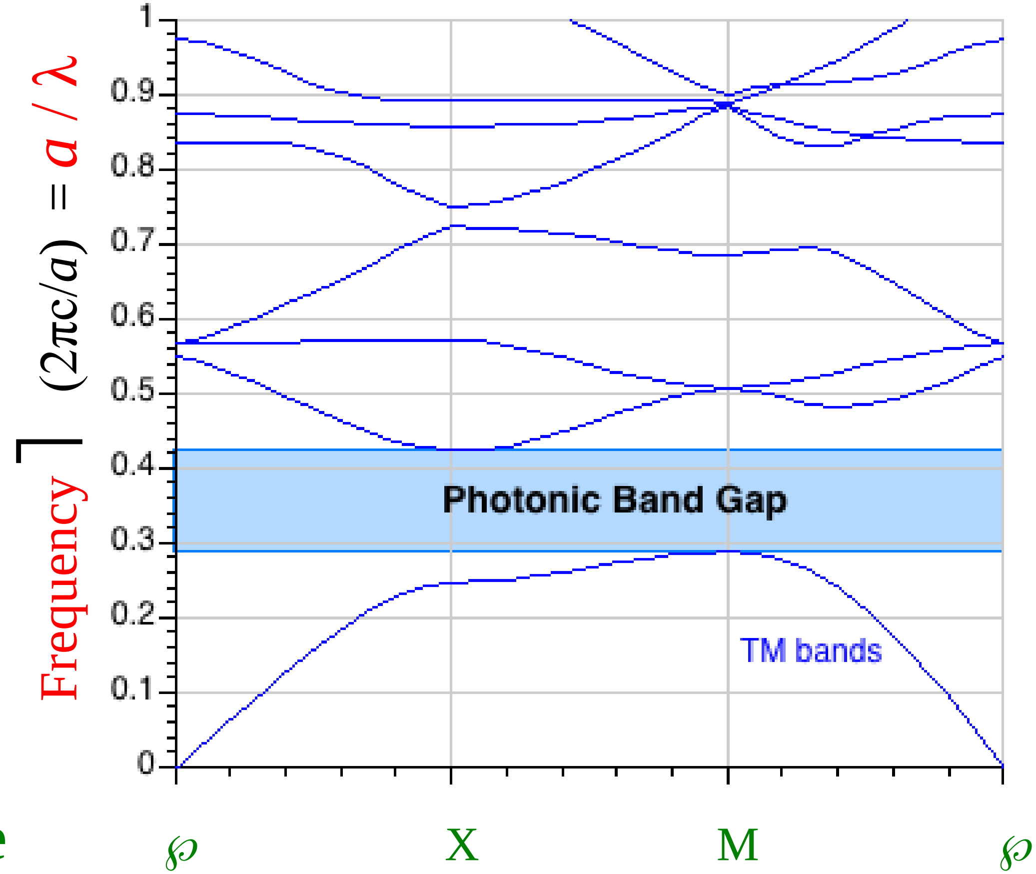
"quasi-phase-matching"



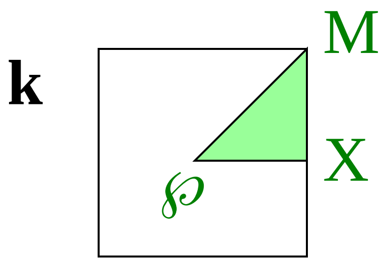


2d periodicity, Σ=12:1

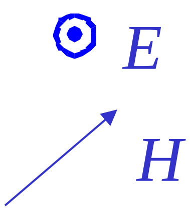




irreducible Brillouin zone

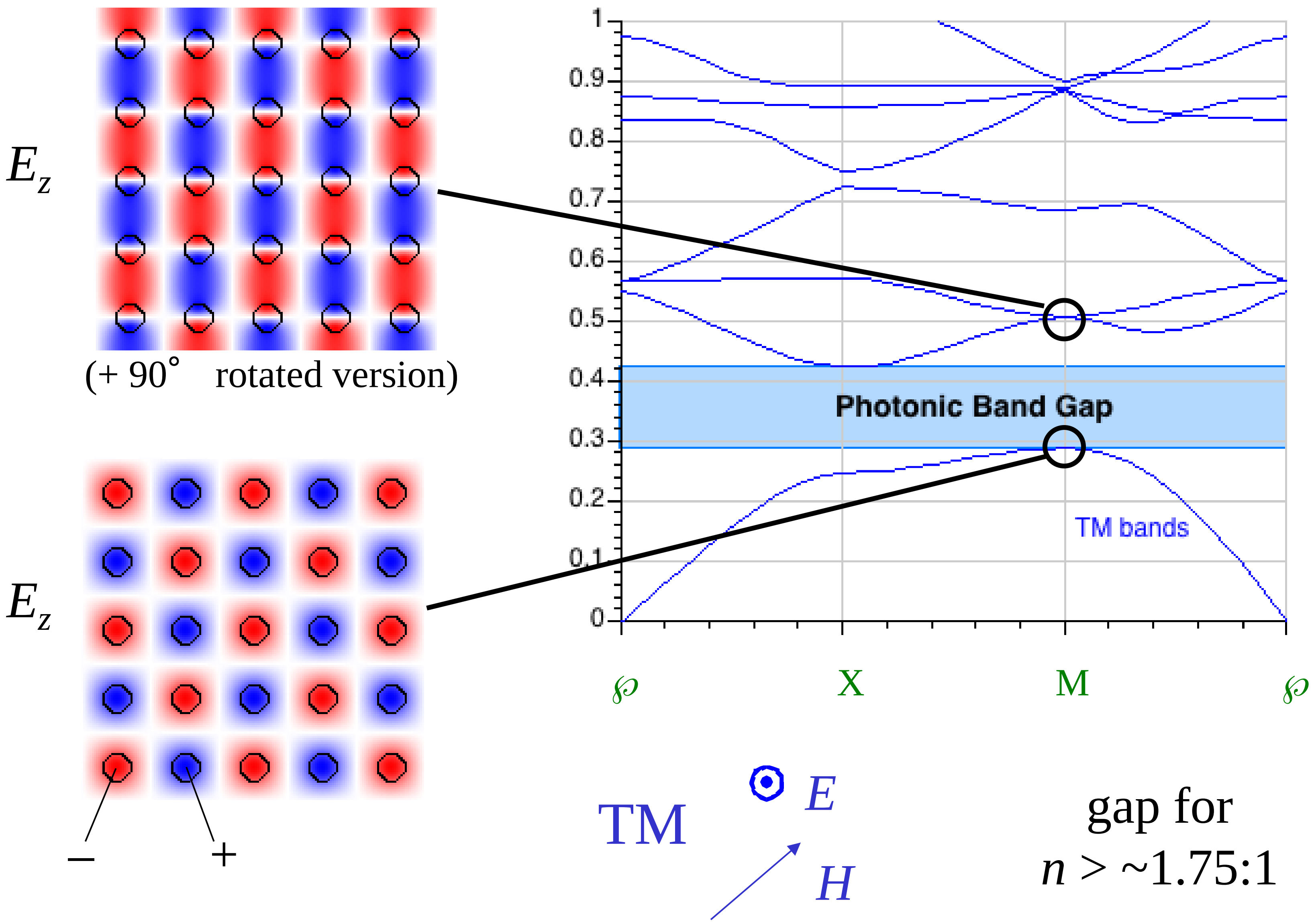


TM



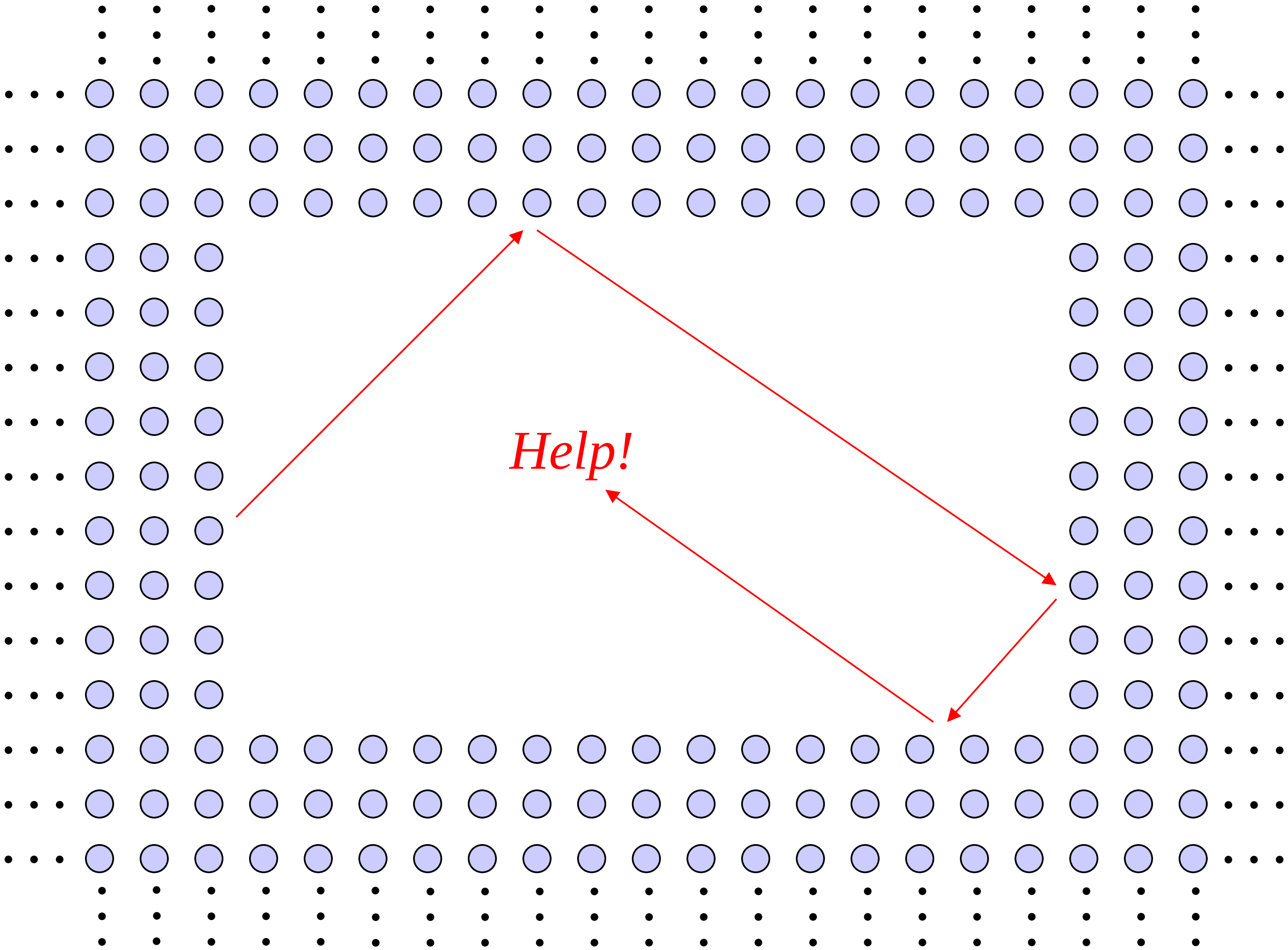
gap for
n > ~1.75:1

2d periodicity, Σ=12:1

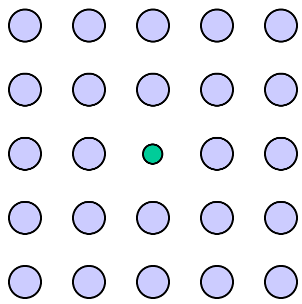


gap for
n > ~1.75:1

Cavity Modes

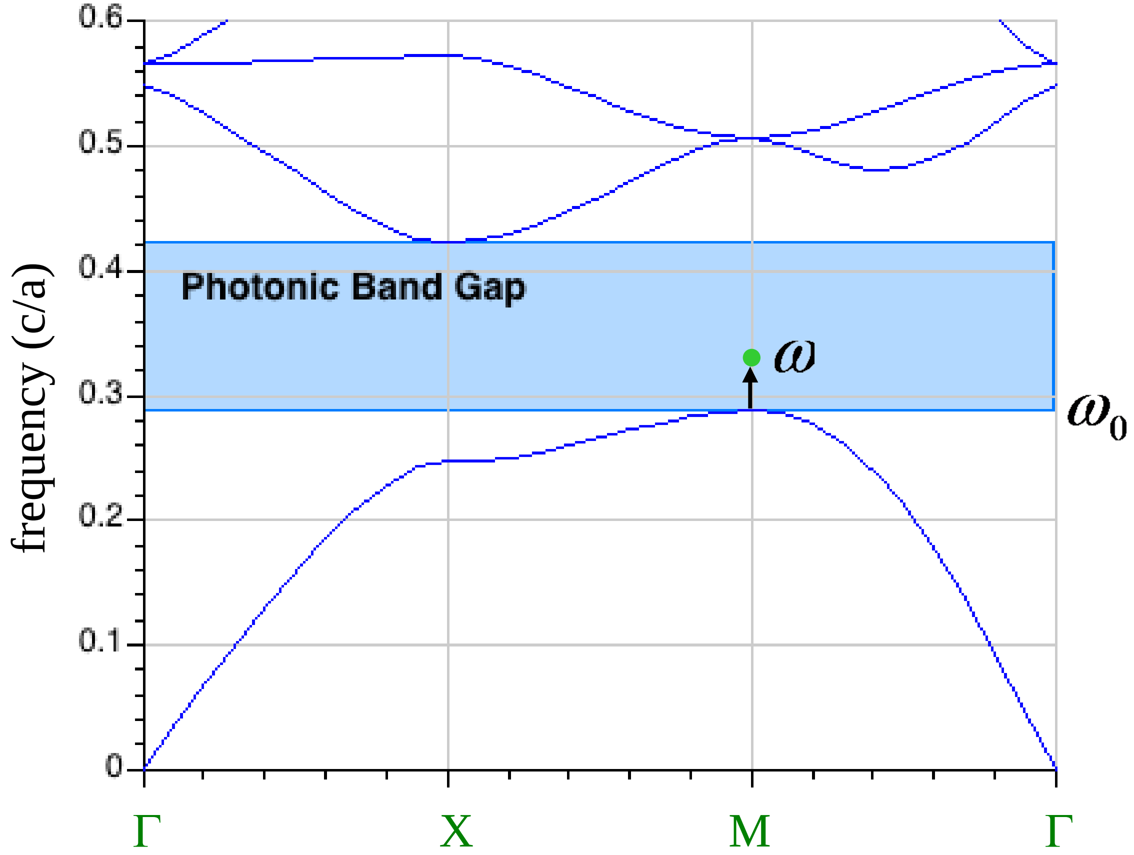


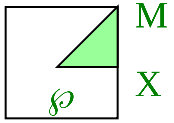
Single-Mode Cavity



A *point defect* can push up a single mode from the band edge

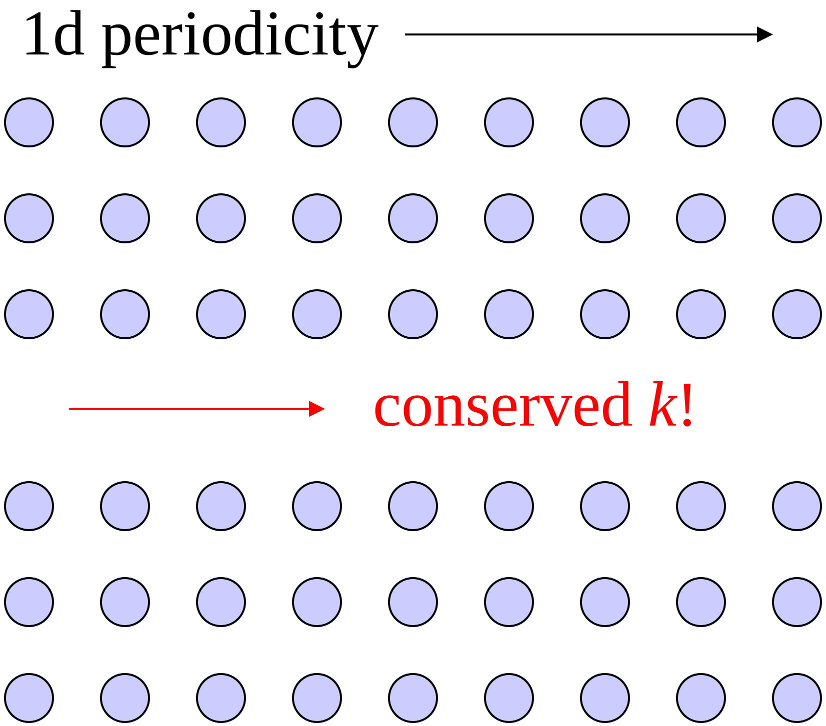
Bulk Crystal Band Diagram

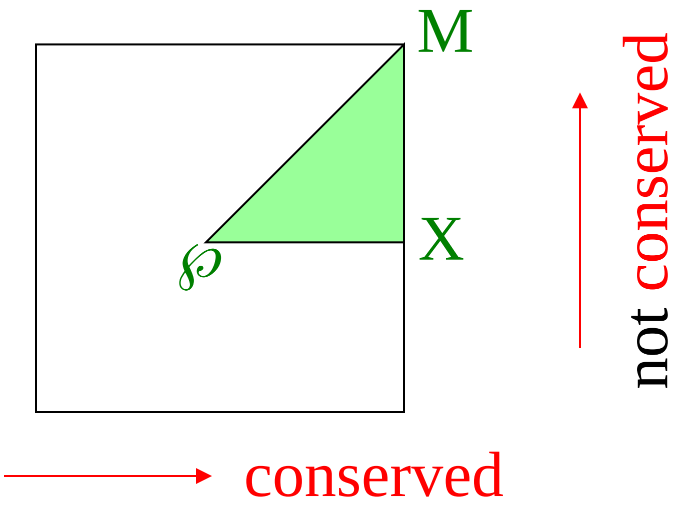




(k not conserved)

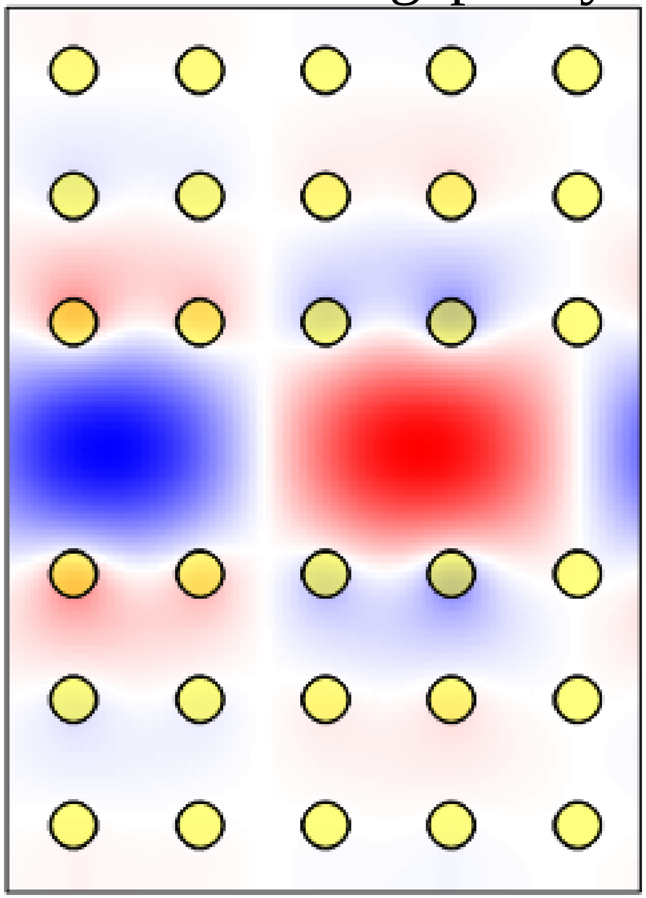
Photonic Crystal Waveguides



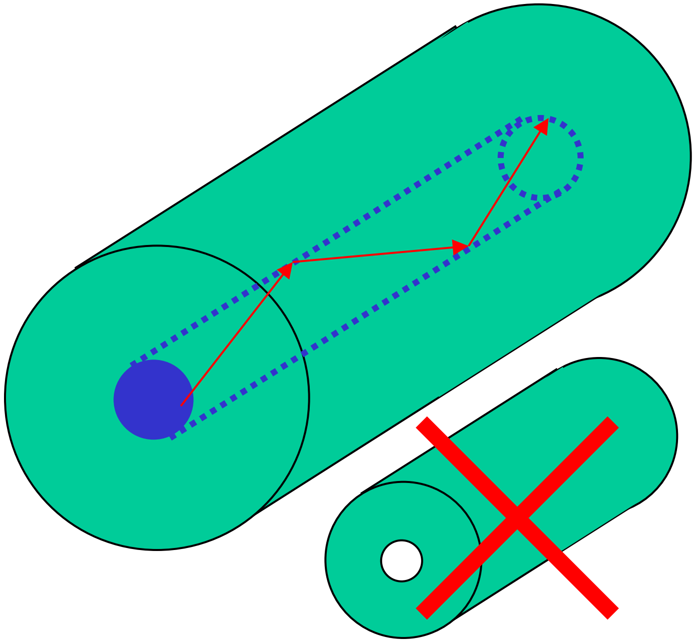


Guiding Light in Air!

mechanism is gap only



vs. standard optical fiber:
"total internal reflection"
— requires *higher*-index core



no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

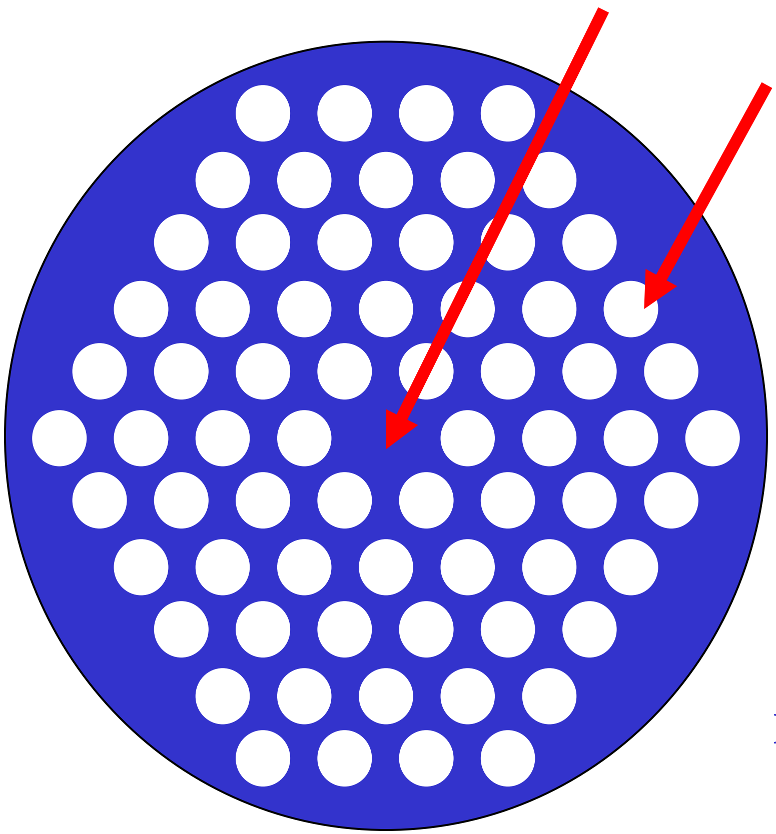
Breaking the Glass Ceiling II:

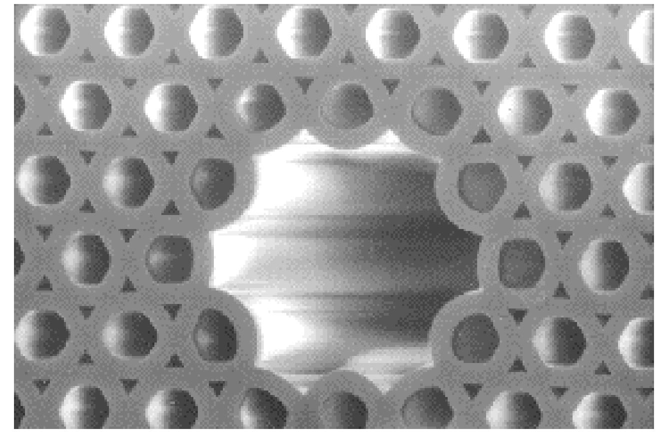
Solid-core Holey Fibers

silica

solid
core

holey cladding
forms
effective
low-index
material



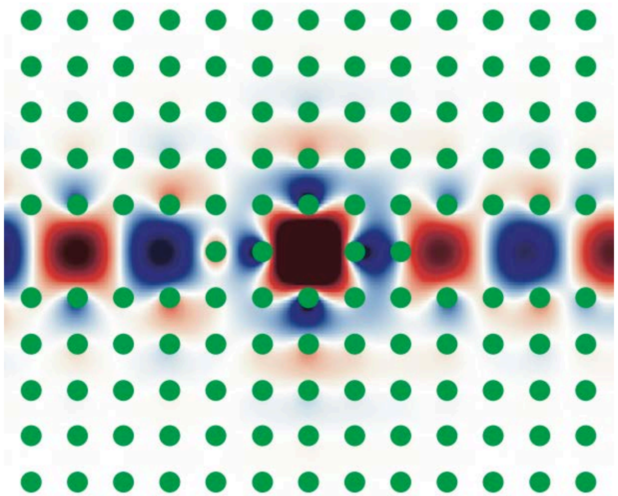


[R. F. Cregan *et al.*, *Science* **285**, 1537 (1999)]

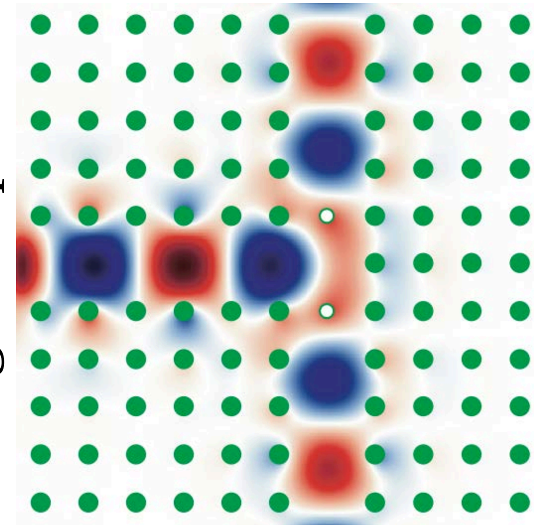
strong confinement = enhanced nonlinearities, birefringence, …

[J. C. Knight *et al.*, *Opt. Lett.* **21**, 1547 (1996)]

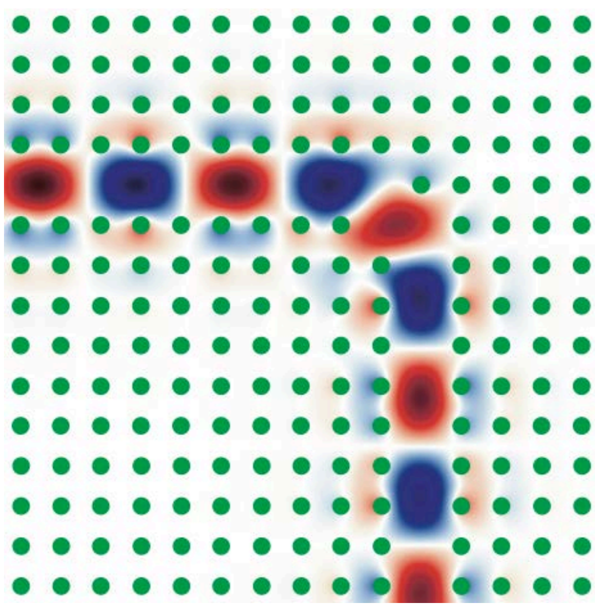
"1d" Waveguides + Cavities = Devices



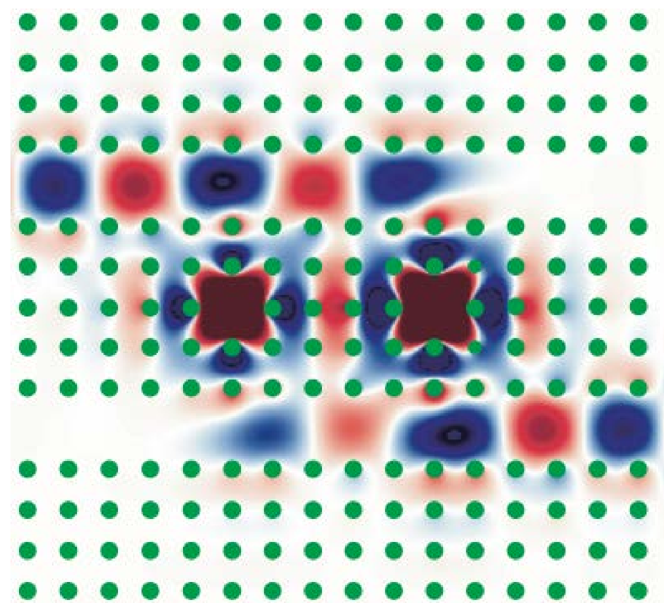
resonant filters



waveguide splitters

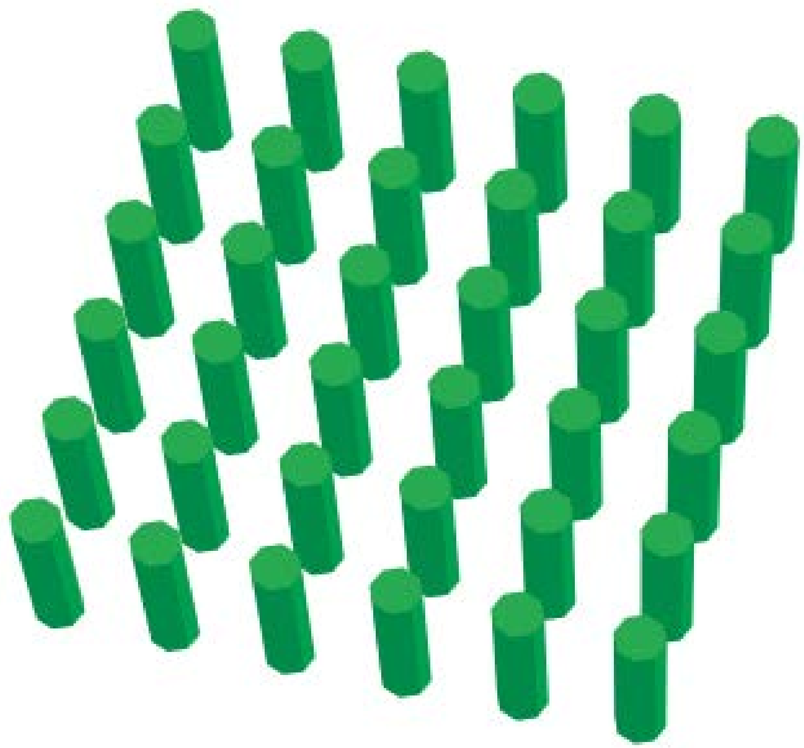
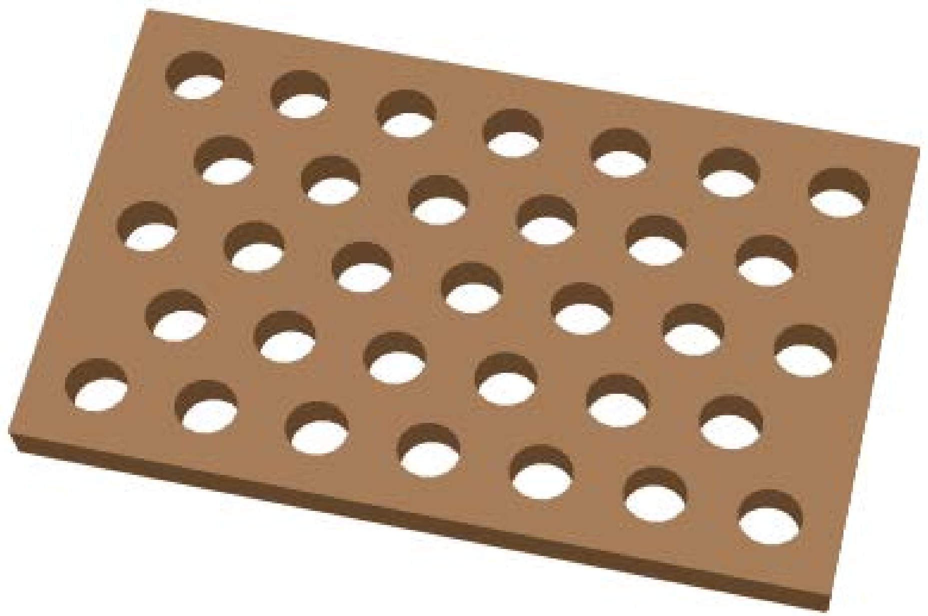


high-transmission sharp bends



channel-drop filters

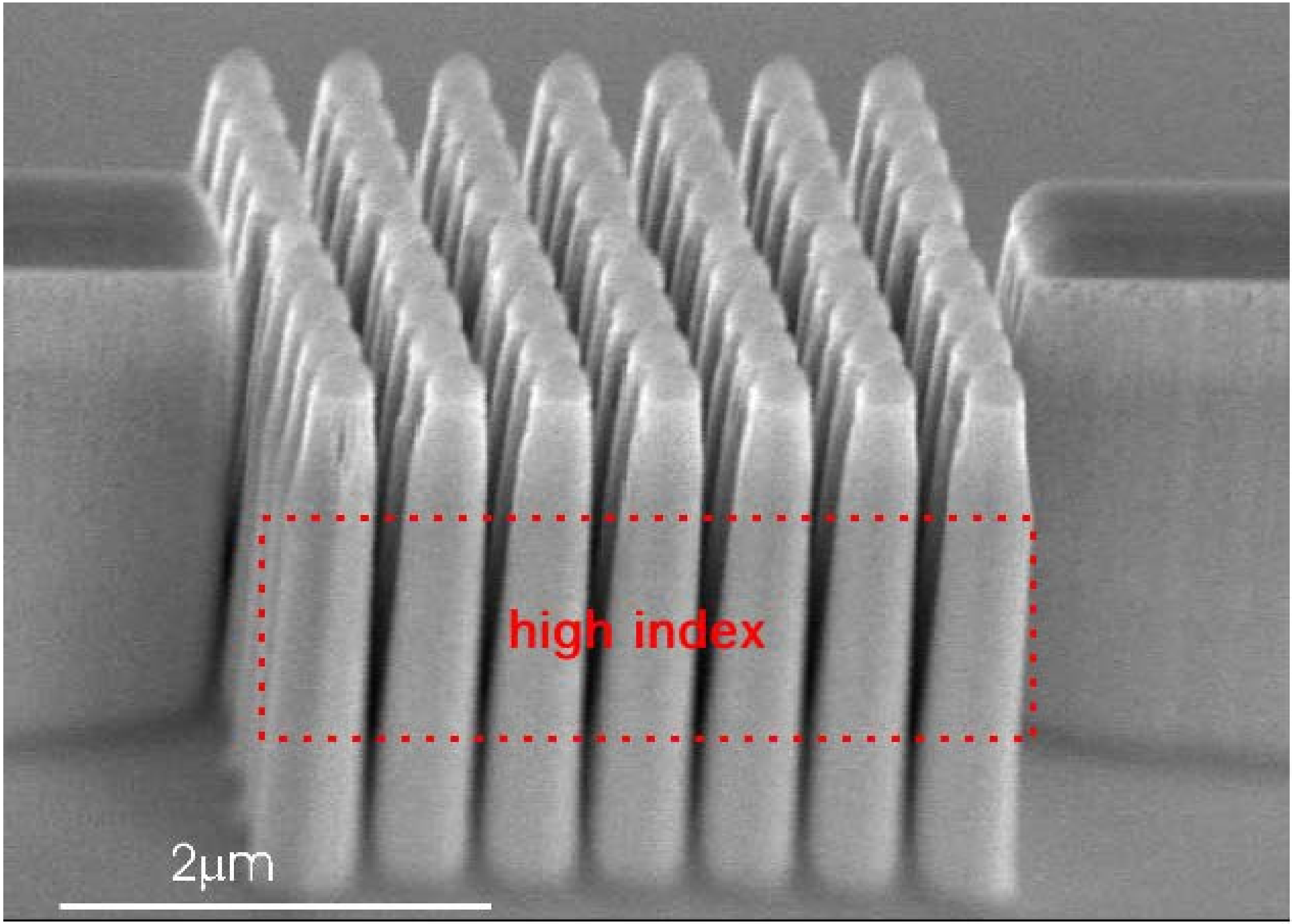
Photonic-Crystal Slabs

2d photonic bandgap + vertical index guiding

[J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light*, 2nd edition, chapter 8]

Extruded Rod Substrate



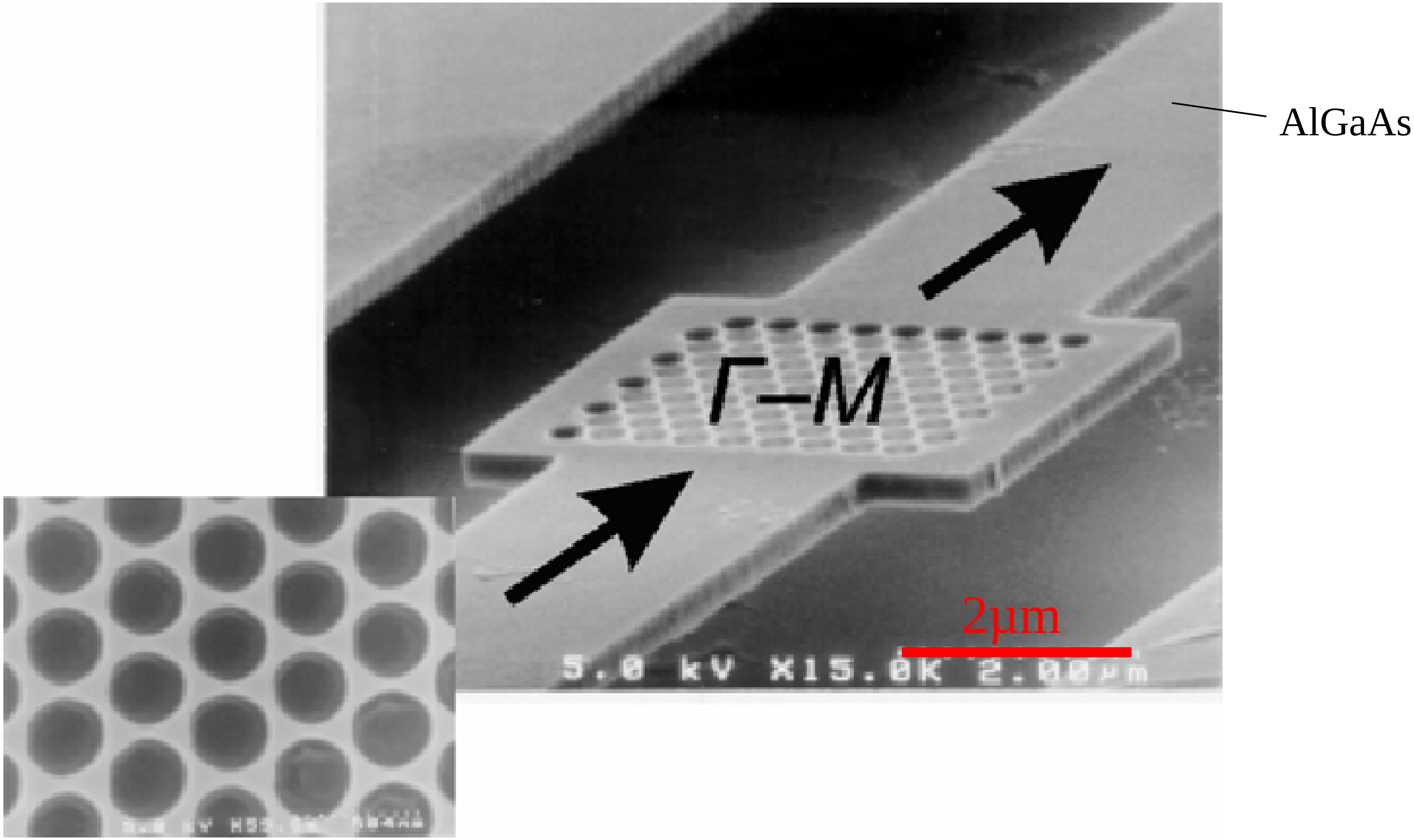
S. Assefa, L. A. Kolodziejski

(GaAs on AlO_x)

[S. Assefa *et al.*, *APL* **85**, 6110 (2004).]

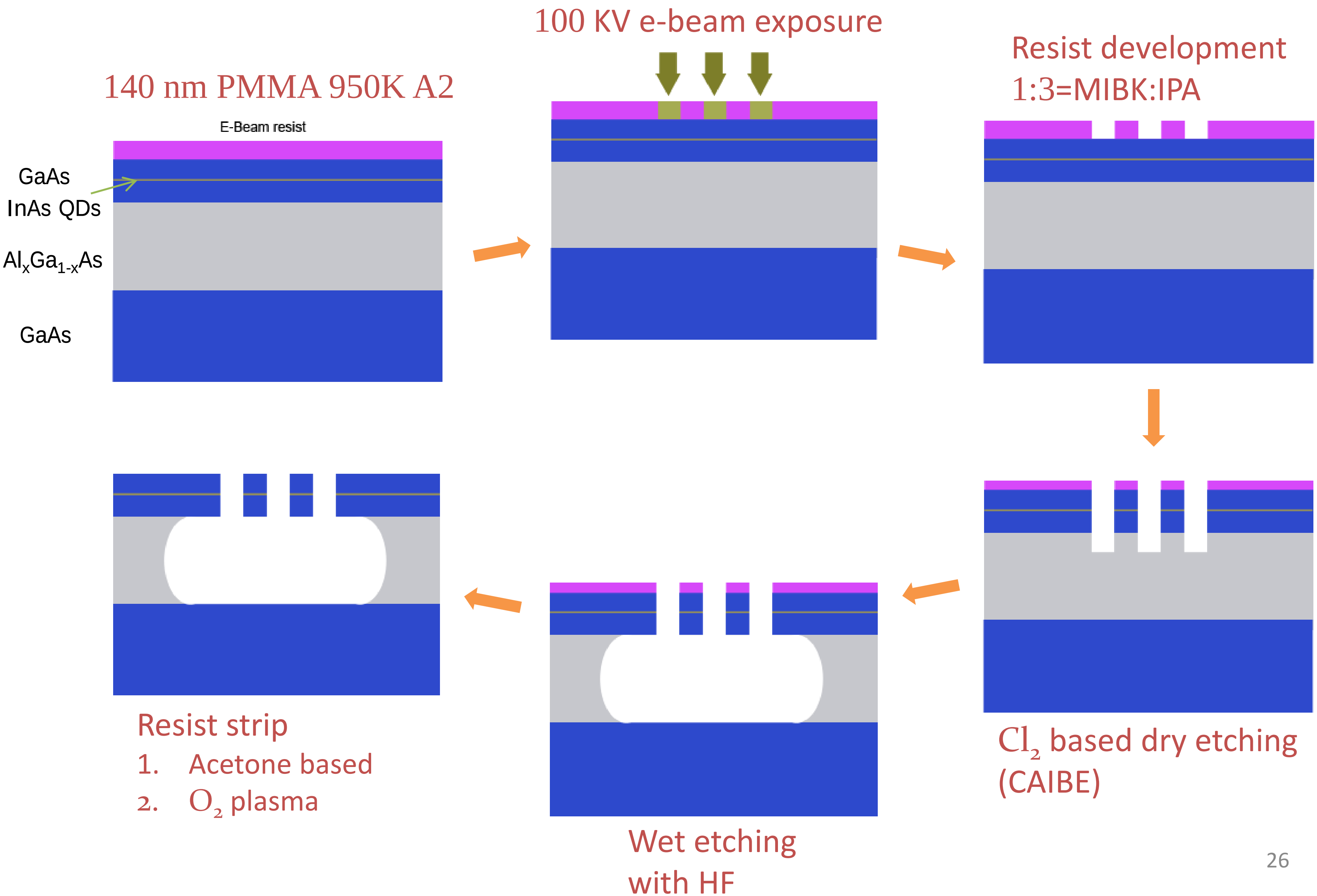
Air-membrane Slabs

who needs a substrate?



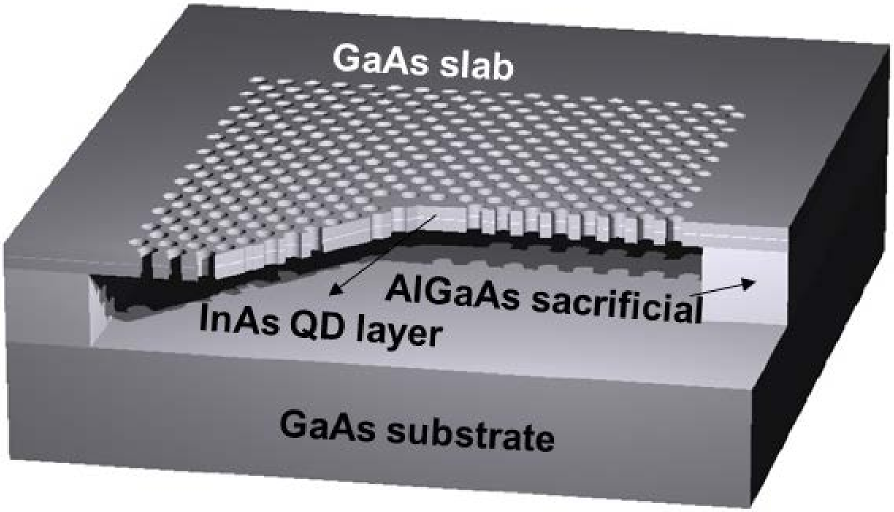
[N. Carlsson *et al.*, *Opt. Quantum Elec.* **34**, 123 (2002)]

2D Photonic Crystal Slab Fabrication

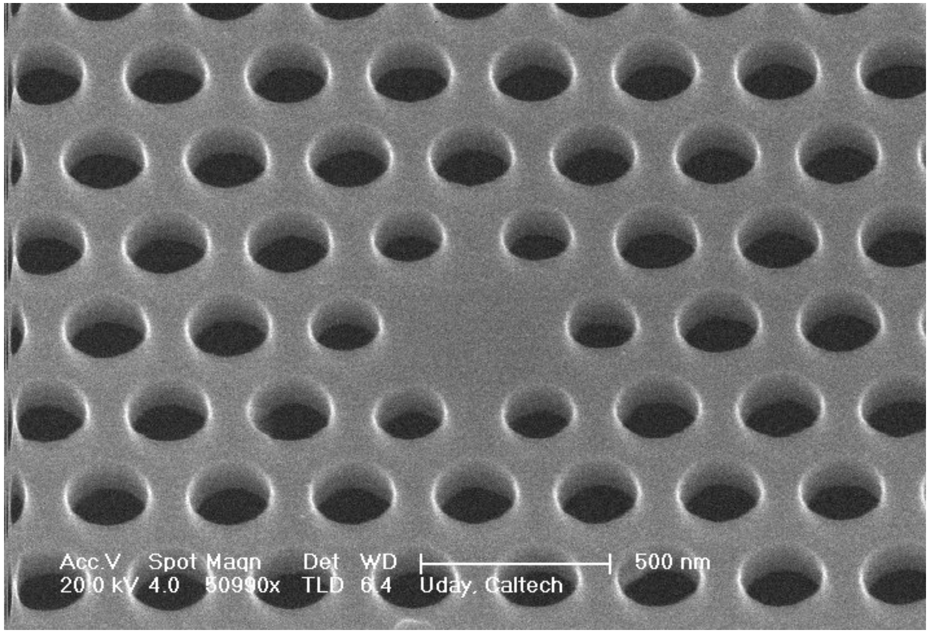


Device Fabrication

Schematic of a L3 cavity ➡



[Cr: S-H. Kim]



⬅ SEM of a L1 cavity
(top view)