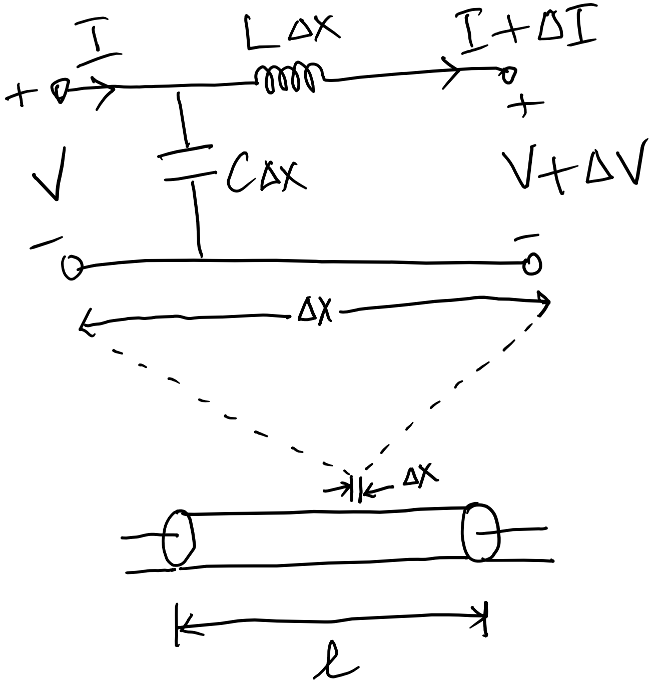
EE2025

Lecture 3/4

Sudharsanan Srinivasan

Telegraph equations



$$\Delta V = -L\Delta x \frac{\partial I}{\partial t} \Rightarrow \frac{\Delta V}{\Delta x} = -L\frac{\partial I}{\partial t}$$

$$\Delta I = -C\Delta x \frac{\partial V}{\partial t} \qquad \frac{\Delta I}{\Delta x} = -C\frac{\partial V}{\partial t}$$

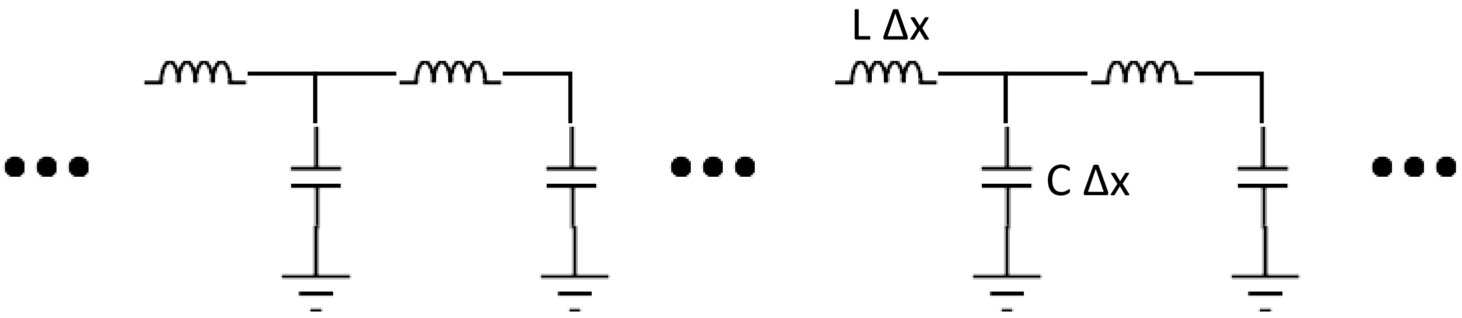
In the limit $\Delta x \to 0$

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$$

Telegraph Equations

Impedance and delay

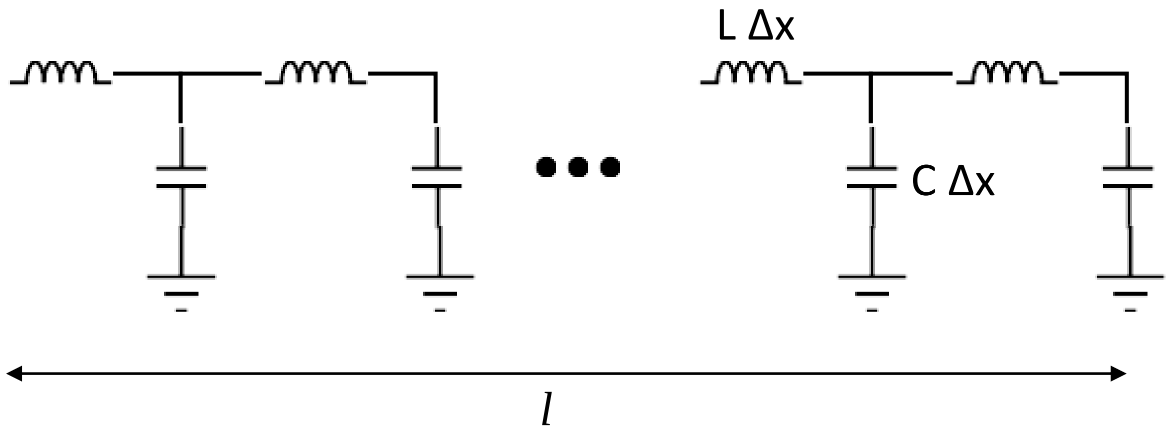


$$V(x,t) = V^{+}(t - {}^{x}/_{v}) + V^{-}(t + {}^{x}/_{v})$$

$$I(x,t) = \frac{V^{+}(t - {}^{x}/_{v})}{Z_0} - \frac{V^{-}(t + {}^{x}/_{v})}{Z_0}$$

where $Z_0 = \sqrt{\frac{L}{C}}$ *and* $v = \frac{1}{\sqrt{LC}}$

Impedance and delay

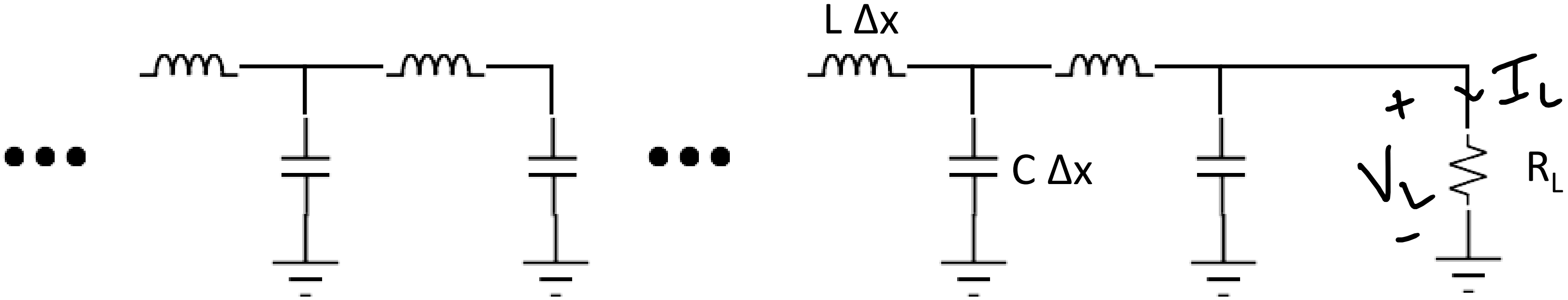


$$C_{length} = \frac{\tau}{Z_0}$$

$$\tau = \frac{l}{v} = \text{"speed of light delay"}$$

$$L_{length} = \tau Z_0$$

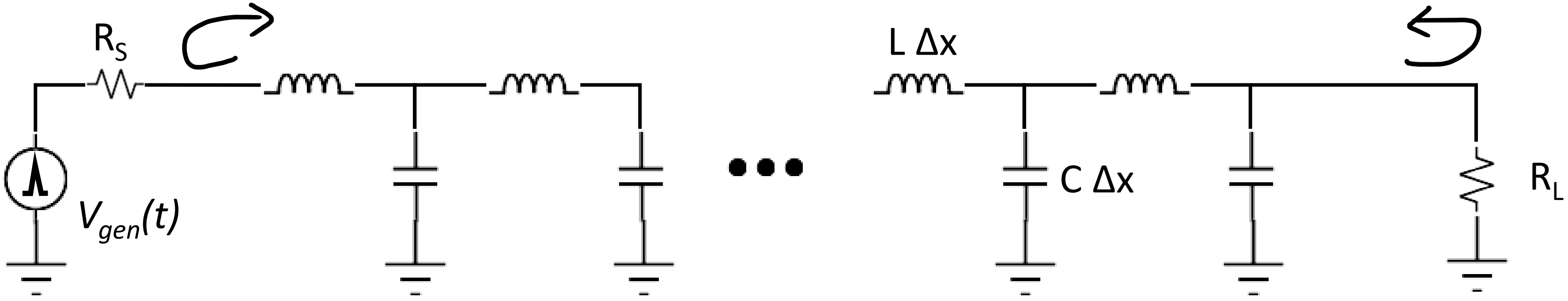
Reflection coefficient



$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{V^-}{V^+}$$

$$\tau_L = \frac{2R_L}{R_L + Z_0} = 1 + \Gamma_L = \frac{V_L}{V^+}$$

Reflection coefficient



At end of line:

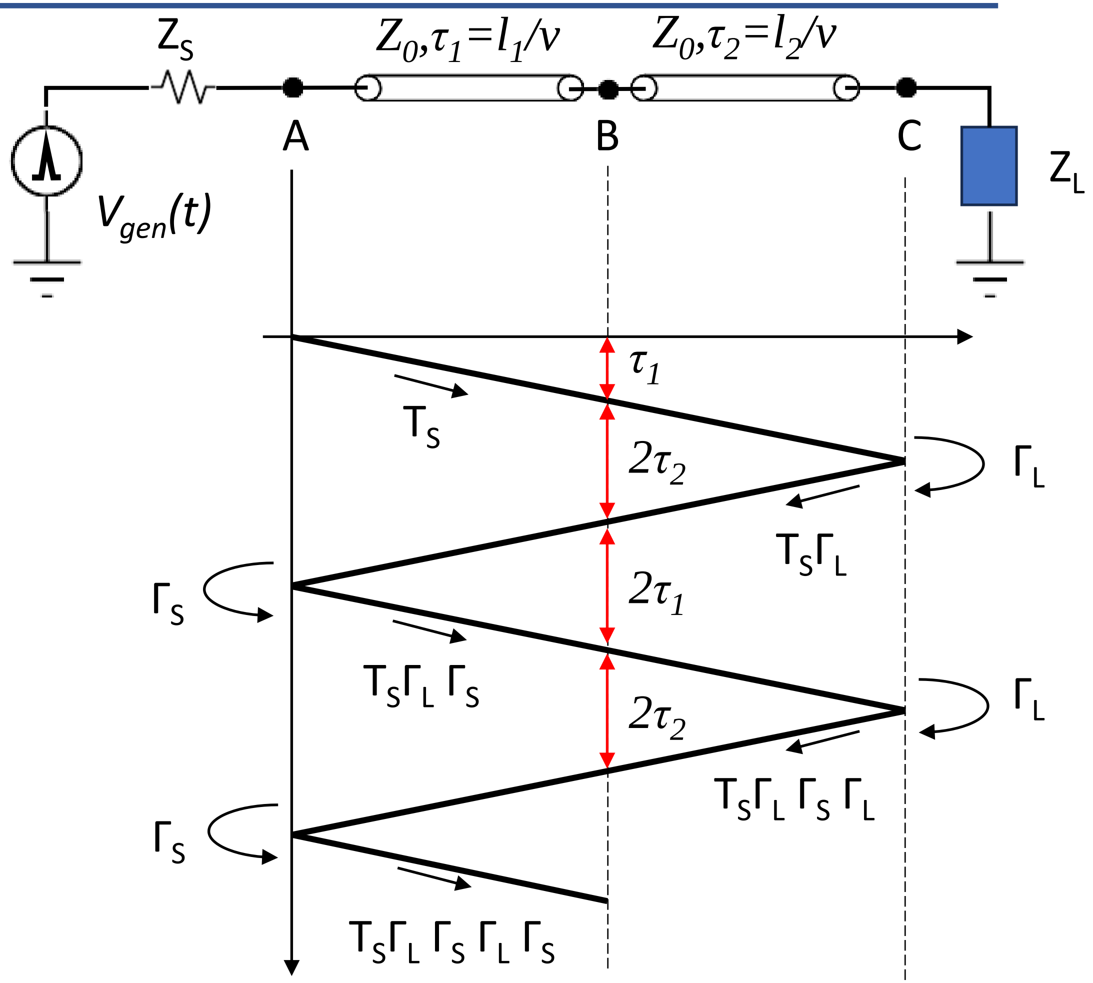
$$V^- = \Gamma_L V^+$$

At beginning of line:

$$V^+ = \Gamma_S V^- + T_S V_{gen} \qquad T_S = \frac{Z_0}{Z_0 + R_S}$$

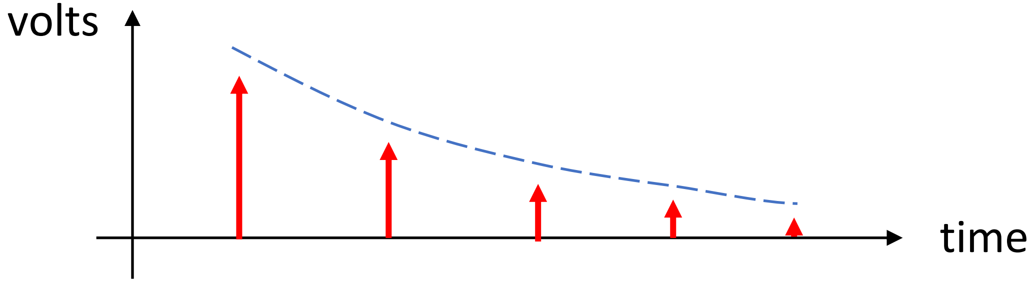
T-lines in the time domain

Lattice/Echo diagrams



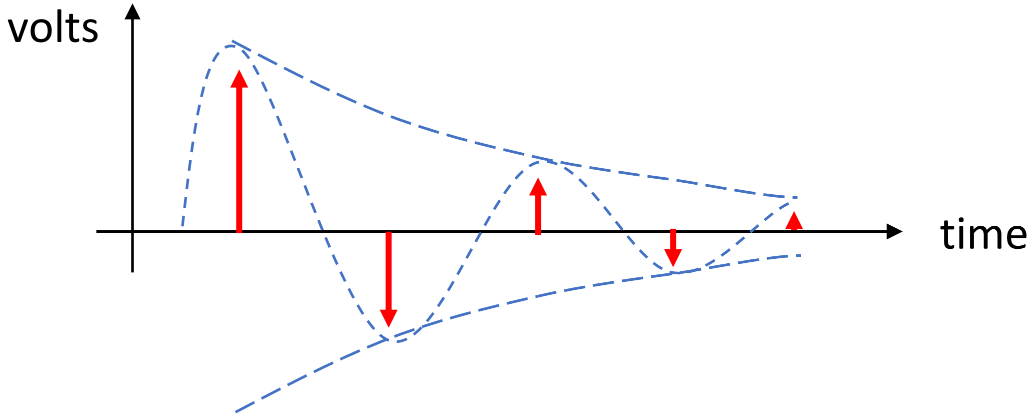
Two outcomes

If $\Gamma_L \Gamma_S$ is positive, impulse response decays exponentially

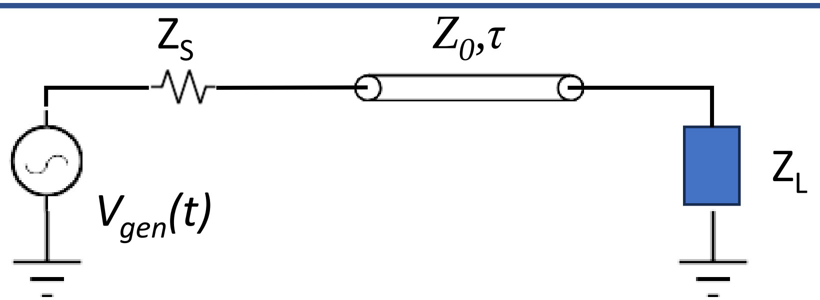


If $\Gamma_L \Gamma_S$ is negative, impulse response alternates sign → ringing

Similar to RLC ringing, but why?



T-lines in the frequency domain



$V_{gen}(t) = \mathrm{Re}\,[V_0 e^{j\omega t}]$

Why would we want to do this analysis?

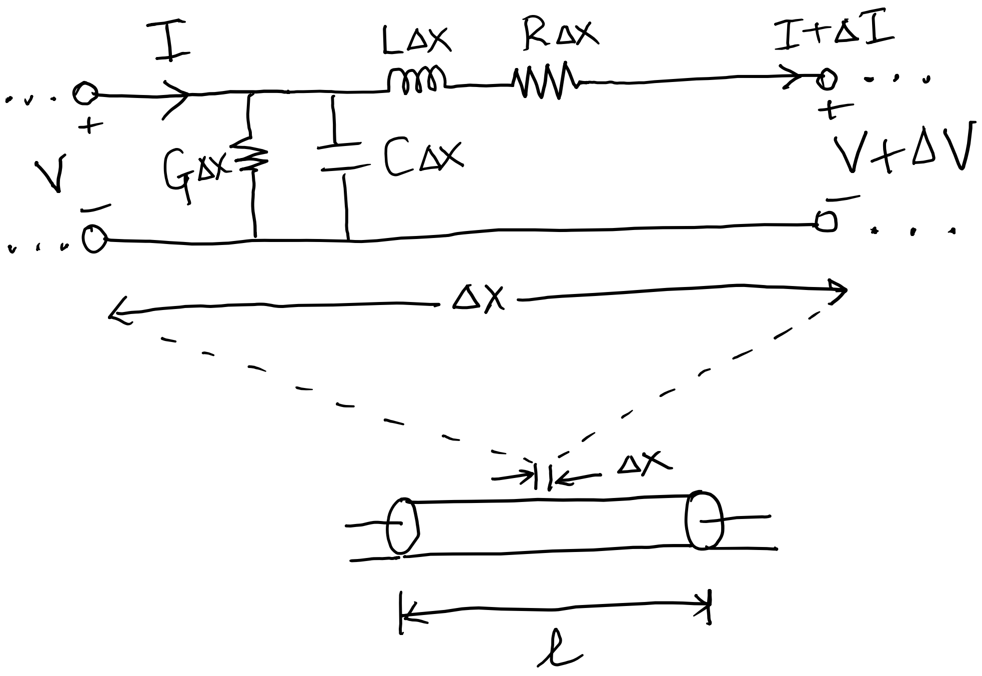
Time-domain analysis:

- Intuitive and clear: pulses bouncing back and forth. Very difficult with reactive (L, C) load or generator impedances

Frequency-domain analysis:

- Less intuitive.
- Easy with reactive (L, C) load or generator impedances
  - (1) standing waves
  - (2) Smith Chart

Generalized distributed model



$$\Delta V = -(R + j\omega L) I \, \Delta x$$

$$\Delta I = -(G + j\omega C) V \, \Delta x$$

$$\frac{d^2 V}{dx^2} = \gamma^2 V$$

$$\frac{d^2 I}{dx^2} = \gamma^2 I$$

where

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\gamma = \alpha + j\beta$$

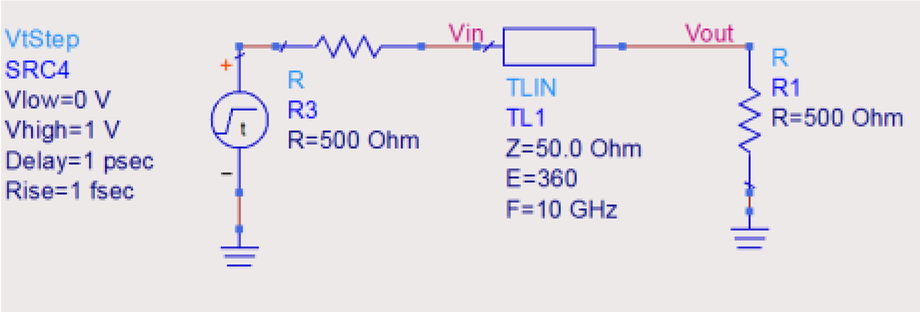
$$V = V^+ e^{-\gamma x} + V^- e^{\gamma x}$$

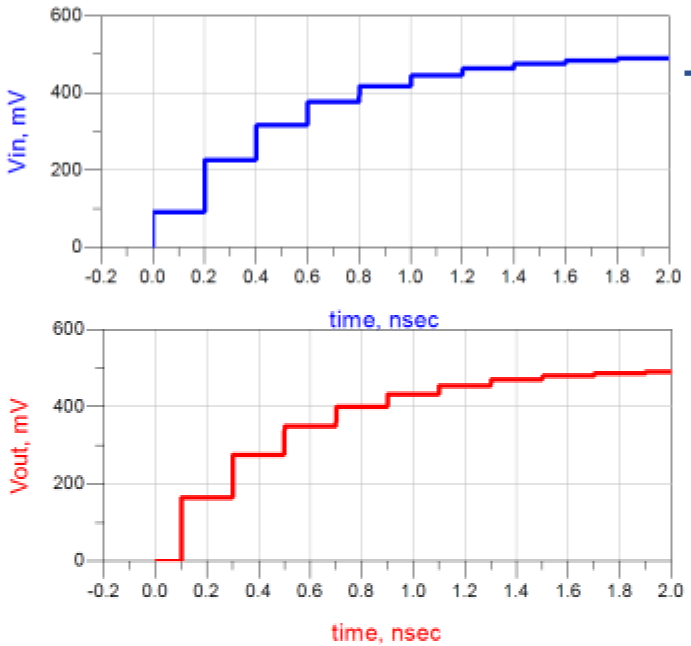
$$I = I^+ e^{-\gamma x} + I^- e^{\gamma x}$$

Extra

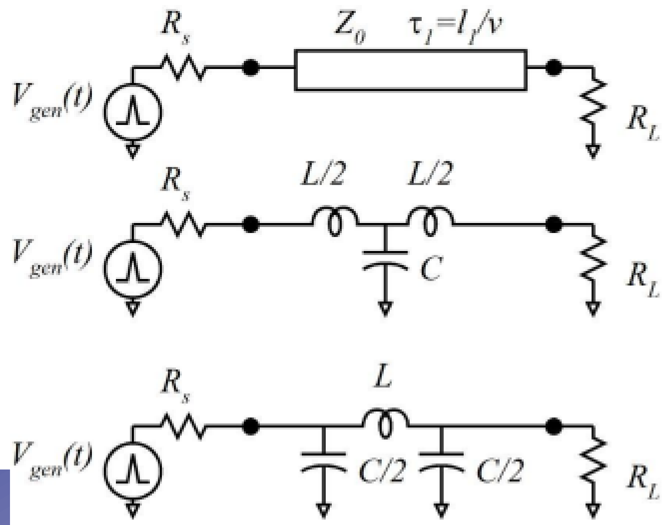
Back-up on digital lines

Low Z_0

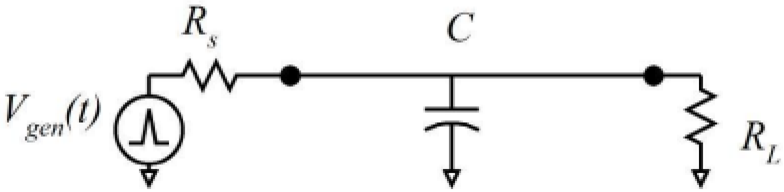




$$L = Z_0\tau \,, C = \tau / Z_0$$



Approximate model

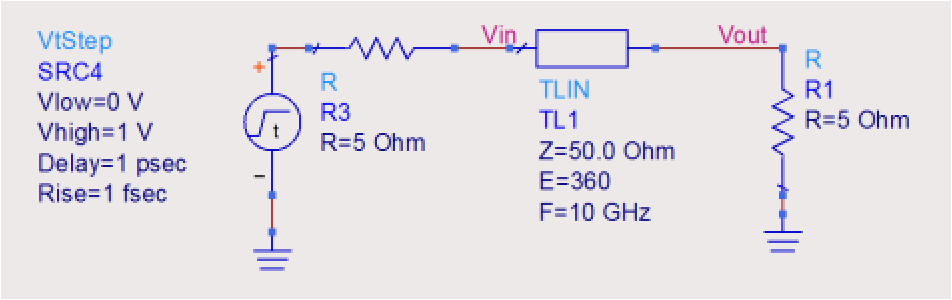


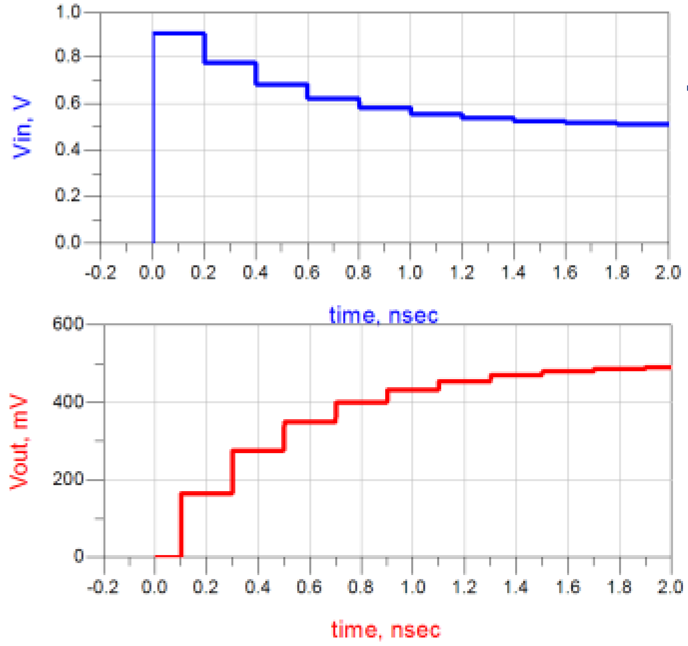
$$L/(R_L + R_s) << (R_L \parallel R_s)C$$

$\rightarrow$ neglect inductor

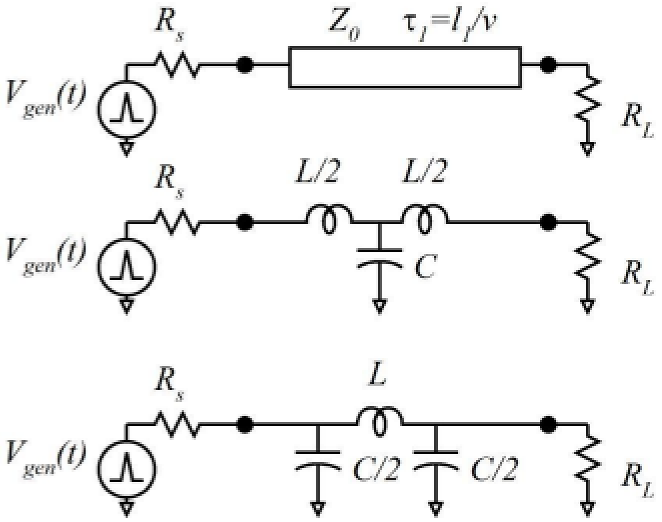
RC circuit $\rightarrow$ charging.

High Z_0

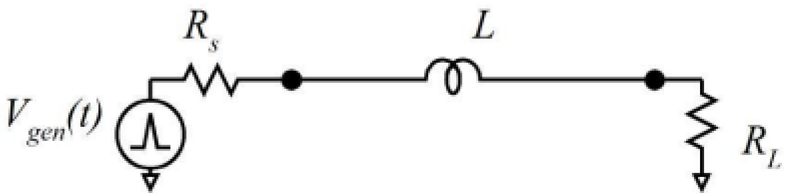




$$L = Z_0 \tau \, , C = \tau / Z_0$$



Approximate model

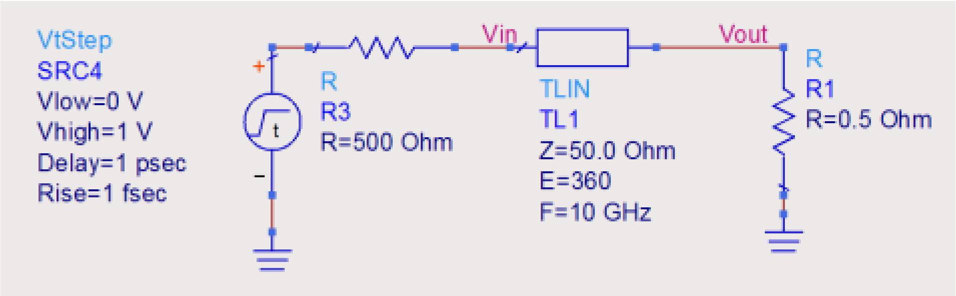


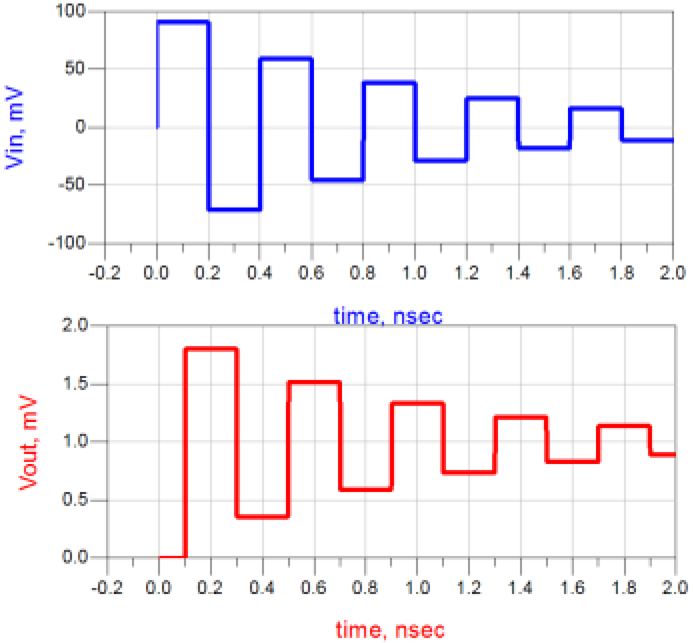
$$L / (R_L + R_s) >> (R_L \| R_s) C$$

$\rightarrow$ neglect capacitor

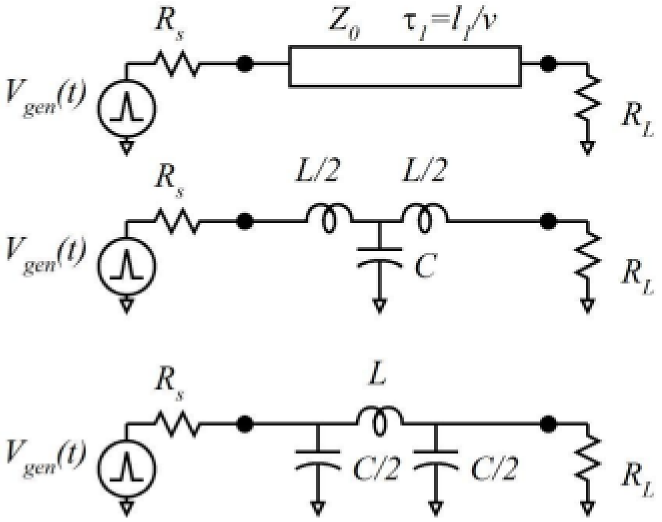
RL circuit $\rightarrow$ charging.

$R_L << R_S$

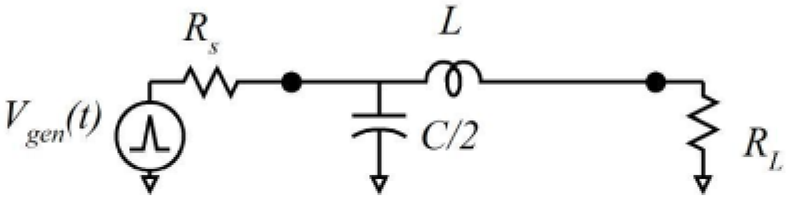




$$L = Z_0\tau \,, C = \tau / Z_0$$



Approximate model



$$R_L C/2 << R_S C/2$$

$\rightarrow$ neglect 2nd capacitor

RLC circuit $\rightarrow$ ringing