# Two-Way Relaying: Protocols and Performance

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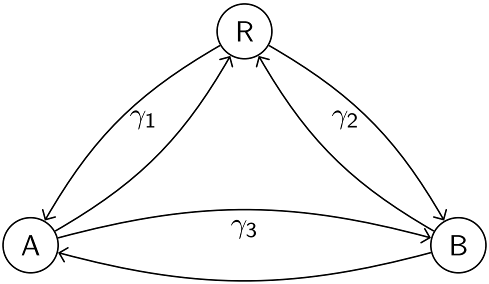
# Wireless Bidirectional (Two-Way) Relaying



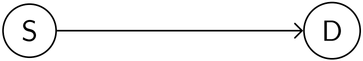

- Nodes $A$ and $B$ want to communicate with each other
- Half-duplex nodes, power-constrained
- Gaussian links, Reciprocal, SNRs: $\gamma_1,\ \gamma_2,\ \gamma_3$
- Rate region: Set of achievable $(R_a, R_b)$

# Outline

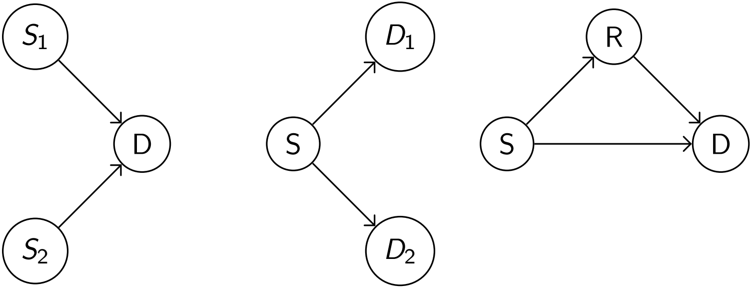
- Introduction
  - Multihop multiflow problems
  - Interference, relaying, network coding
- Two-way relaying without direct link
  - Relaying protocols and achievable rate regions
  - Outer bounds
  - Code designs
- Two-way relaying with direct link
  - Relaying protocols and achievable rate regions
  - Outer bounds

# Multi-terminal wireless communication

- Point-to-point communication
  - Capacity – eg. $C(SNR) = \log(1 + SNR)$
  - Practical code designs

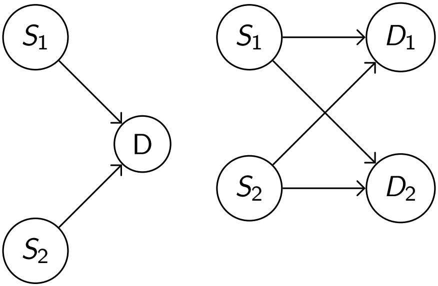



- Multi-terminal wireless communication
  - Capacity region known only for some scenarios
  - Not many practical code designs



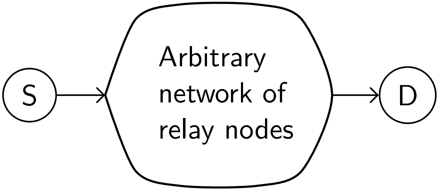
# Single-hop Multiple-flow problems

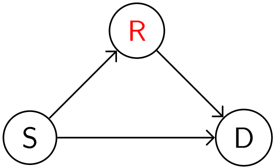
- Interference



# Multi-hop Single-flow problems

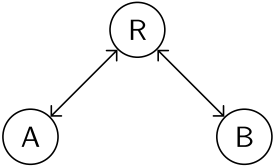
- Relaying: forwarding of flow, scheduling (half-duplex)
- Interference

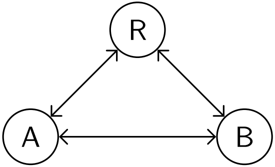


# Multi-hop Multi-flow problems

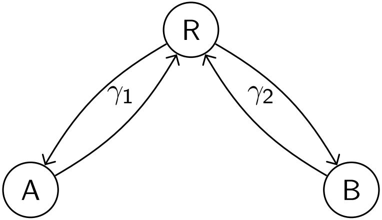
- Relaying, interference, network coding
- Simple examples



Two-way relaying without direct link

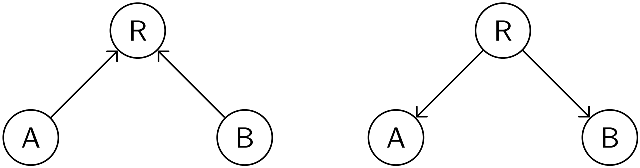
# Two-way relaying without direct link




- Goal: Nodes A and B exchange information through relay
- Half-duplex wireless nodes
- Gaussian links, known channel coefficients
- What are the possible rate pairs ($R_a$, $R_b$)?
  - Rate region

# Components of a Relaying Protocol

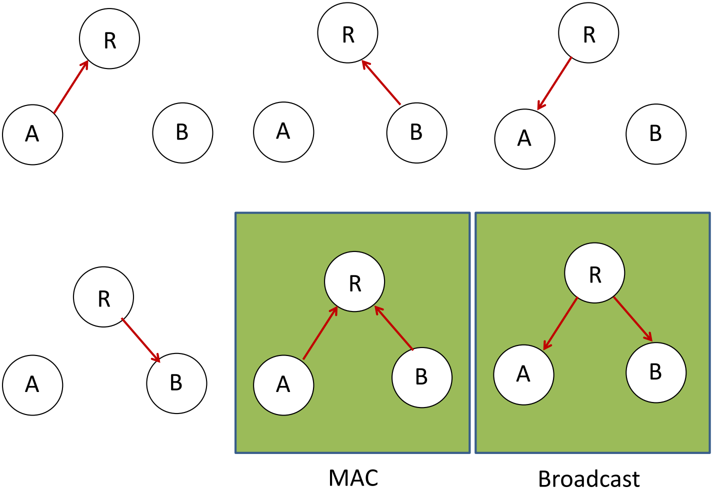
- Sequence of states



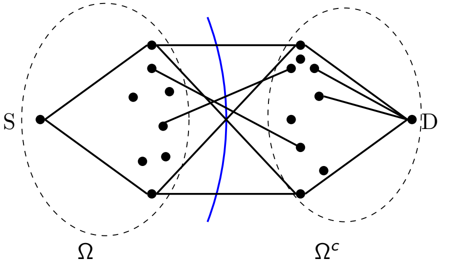

- Coding scheme in each state
  - Encoding at the transmitters
  - Decoding/Processing at the receivers
    - Processing at the Relay – Amplify, Decode, Estimate, Compress, etc.
    - Side information across states

# Network States

- States: Multiple Access (MAC), Broadcast



# Outer Bounds: Cut-Set Bound

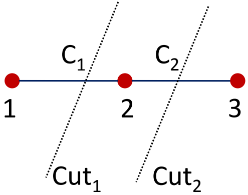



- Full Duplex Network[1]: $R \leq \min_{\Omega}[\text{Cut capacity}]$
- Half Duplex Network [2]: $R \leq \sup_{\lambda_k} \min_{\Omega} \sum_{k=1}^{\mathscr{M}} \lambda_k[\text{Cut capacity in state } k]$

1 T. M. Cover, J. A. Thomas, Elements of Information Theory, John Wiley, 2004.

2 M. Khojestepour, A. Sabharwal, B. Aazhang, "Bounds on achievable rates for general multiterminal networks with practical constraints", IPSN, pp. 146-161, 2003

# Two-hop relay network: Half-Duplex Cut-Set Bound




| State | $\text{Cut}_1$ | $\text{Cut}_2$ |
|---|---|---|
| $S_0$ (00) | 0 | 0 |
| $S_1$ (01) | 0 | $C_2$ |
| $S_2$ (10) | $C_1$ | 0 |
| $S_3$ (11) | 0 | $C_2$ |

- Enough to consider $S_1$ and $S_2$ $(\lambda_1 + \lambda_2 = 1)$

$$C_{HD} = \max_{\lambda_1,\lambda_2} \min(\lambda_2 C_1, \lambda_1 C_2)$$

- $\lambda_1 C_2 = \lambda_2 C_1$
  $\Rightarrow C_{HD} = \dfrac{C_1 C_2}{C_1 + C_2}$
- $C_{FD} = \min(C_1, C_2)$

# Simple Outer Bounds

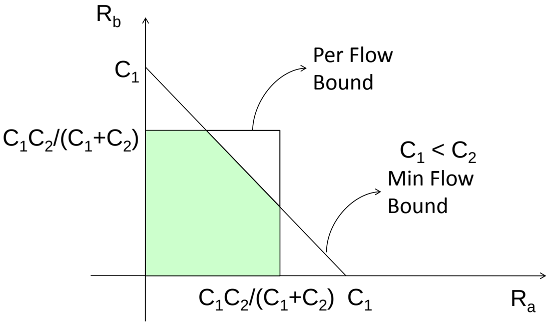
- Per flow bound
  - Cut-set bound[1]

$$R_a \leq \frac{C_1 C_2}{C_1 + C_2}$$

$$R_b \leq \frac{C_1 C_2}{C_1 + C_2}$$

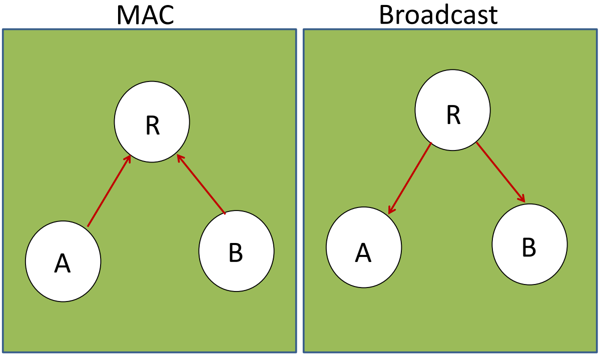
- Min flow bound

$$R_a + R_b \leq \min(C_1, C_2)$$



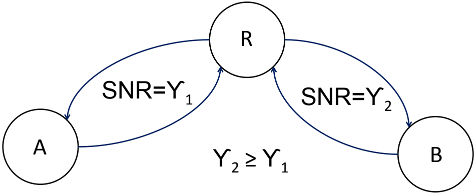

[1] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, Bounds on achievable rates for general multi-terminal networks with practical constraints, in In Proc. of 2nd International Workshop on Information Processing (IPSN),Palo Alto, CA, Apr. 2003, pp. 146-161.

# Relaying Schemes




- Two phase protocols
  - MAC phase followed by Broadcast phase
- Need to choose coding scheme for each phase
  - What should the relay node R do?
  - What should nodes A and B do?

# Amplify and Forward



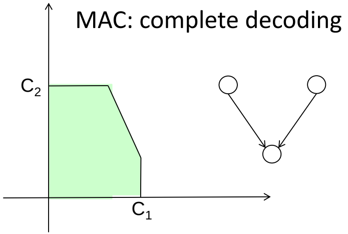

- Relay node B amplifies signal received in MAC phase

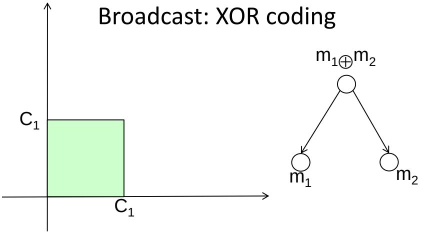
$$y_R = h_{AR}x_A + h_{BR}x_B + n_R$$

- $R_b = C\left(\frac{\gamma_1\gamma_2}{2\gamma_1 + \gamma_2 + 1}\right)$, $R_a = C\left(\frac{\gamma_1\gamma_2}{\gamma_1 + 2\gamma_2 + 1}\right)$
- Favors the flow with better source-relay link
  - If $\gamma_2 > \gamma_1$, $R_b > R_a$
- Significant noise amplification

# Decode and Forward

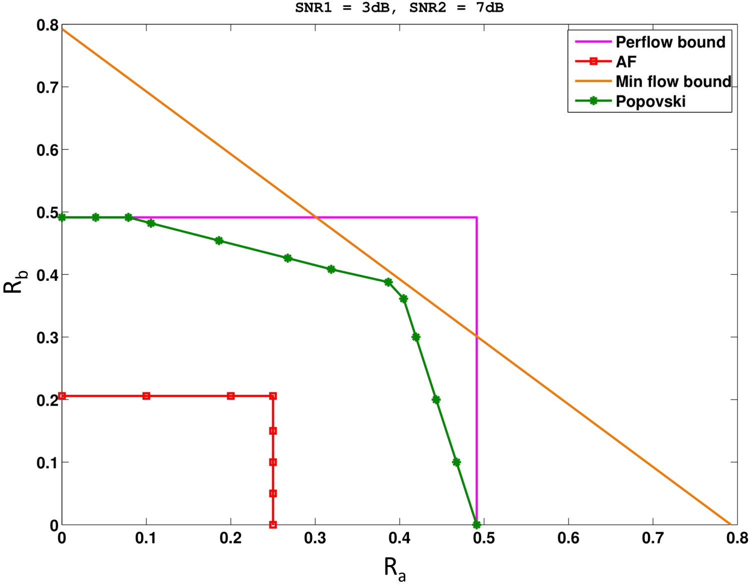
- MAC
  - Decode both messages $m_1$ and $m_2$
- Broadcast
  - Transmit $m_1 \oplus m_2$
  - Network coding
  - Use lower rate $C_1$



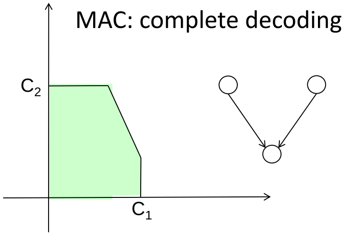

P. Popovski and H. Yomo, Physical network coding in two-way wireless relay channels, in IEEE International Conference on Communications, 2007. ICC 07., 24-28 June 2007, pp. 707-712.

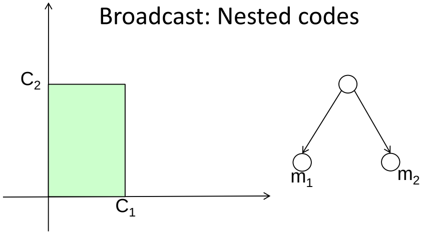
# Comparison of Rate Regions



# Broadcast with Side Information

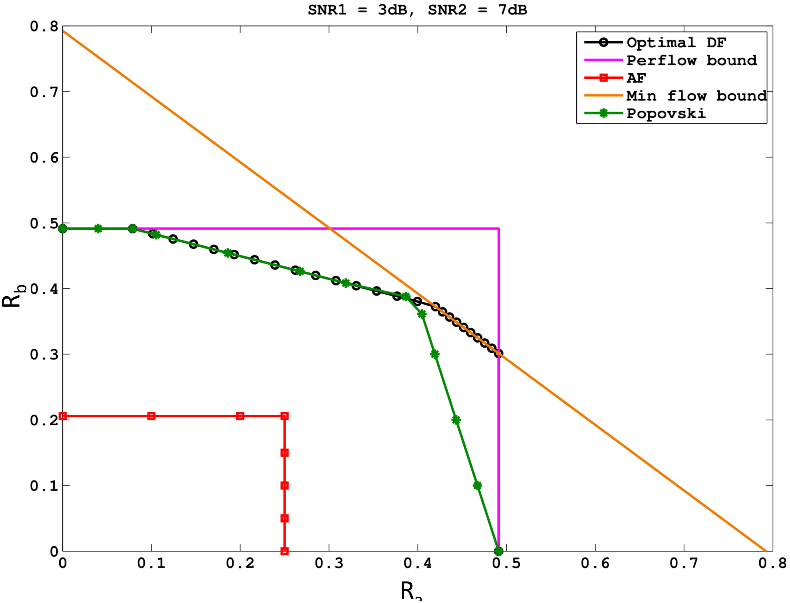
- MAC
  - Decode both messages $m_1$ and $m_2$
- Broadcast
  - Use nested codes
  - Use side information at receiver completely
- Optimal DF




Y. Wu, "Broadcasting when Receivers Know Some Messages A Priori," IEEE International Symposium on Information Theory, 2007. ISIT 2007, pp.1141-1145, 24-29 June 2007.

# Comparison of Rate Regions



# Compute and Forward

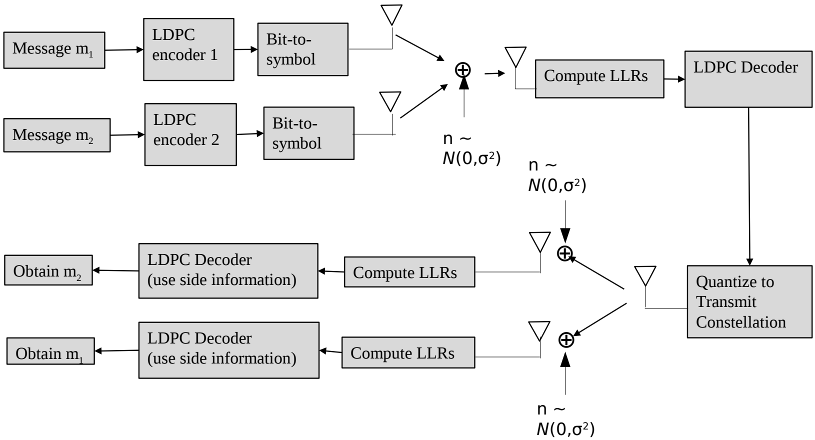
- Messages from A and B need not be decoded at relay
  - Only XOR of messages needed in XOR coding scheme
- Decode a function of the two messages

# Code Designs: Summary

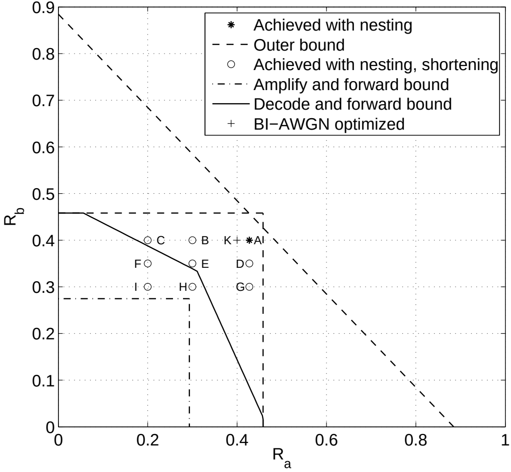
- Designs
  - Lattice Coding
    - Wilson, Narayanan, Pfister, & Sprintson 2010
  - Repeat-Accumulate Codes
    - Zhang & Liew 2009
    - Focus on symmetric channels/rates
    - XOR functions
- Our design
  - MAC phase
    - Estimate the LLR of XOR of codewords
    - Use nested LDPC codes
    - Quantize and transmit
  - Broadcast phase
    - Transmit XOR estimate of codewords
    - Decode using side information

M. P. Wilson, K. Narayanan, H. D. Pfister, A. Sprintson, "Joint Physical Layer Coding and Network Coding for Bidirectional Relaying," IEEE Transactions on Information Theory, Vol. 56, No. 11, pp. 5641–5654, Nov. 2010.
S. Zhang and S-C. Liew, "Channel coding and decoding in a relay system operated with physical-layer network coding," IEEE Journal on Selected Areas in Communications, vol.27, no.5, pp.788–796, June 2009.

# Encoder-Decoder Setup



# Results



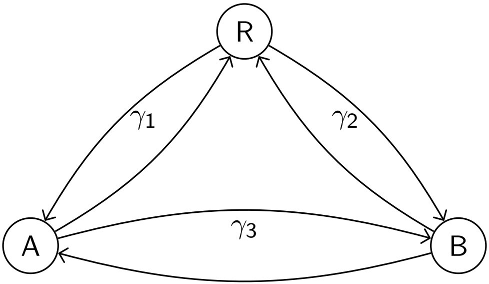

- $\gamma_1 = 5.45$ dB, $\gamma_2 = 7$ dB
- Close to capacity bound

# Summary

- Two-way relaying without direct link
  - Capacity region outer bounds
  - Achievable rate regions
  - Use of side information in broadcast & compute and forward
- Code designs
  - Code constructions closer to outer bound
  - Use of nested codes
  - Both symmetric and asymmetric channels

Two-way relaying with direct link

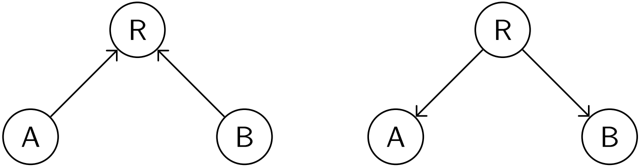
# Gaussian Two-way Relay Channel




- Nodes $A$ and $B$ want to communicate with each other
- Half-duplex nodes
- Gaussian links, Reciprocal, SNRs: $\gamma_1, \gamma_2, \gamma_3$
- Rate region: Set of achievable $(R_a, R_b)$

# Components of a Relaying Protocol

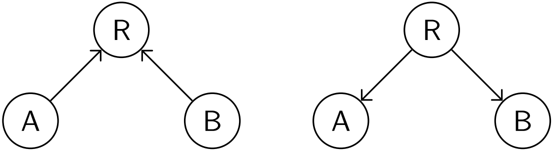
- Sequence of states



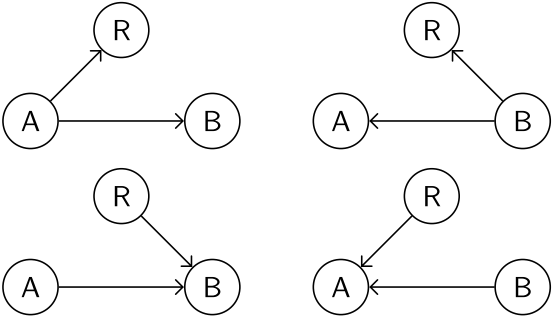

- Coding scheme in each state
  - Encoding at the transmitters
  - Decoding/Processing at the receivers
    - Processing at the Relay – Amplify, Decode, Estimate, Compress, etc.
    - Side information across states

# States

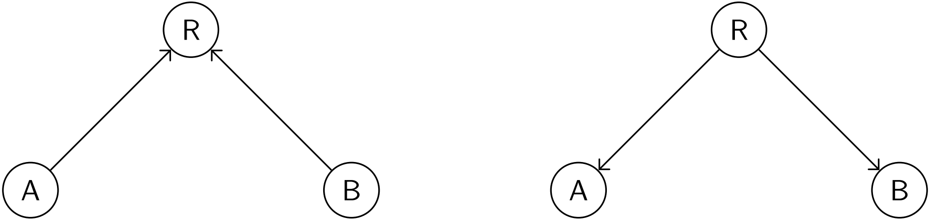
- Without direct link




- With direct link



# MABC protocol [3] [4]

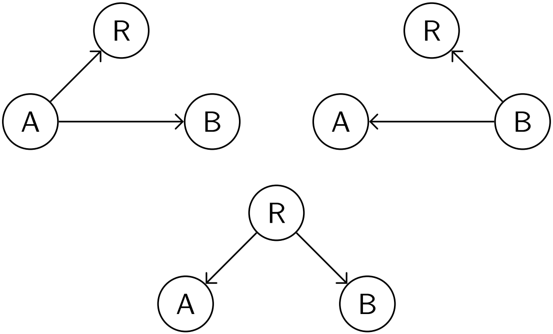



- Direct link not used
- Relaying scheme: Decode, Amplify, Compress, Compute, ...

[3] S. Kim, P. Mitran, and V. Tarokh, Performance bounds for bidirectional coded cooperation protocols, Information Theory, IEE Transactions on, vol. 54, no. 11, pp. 52355241, 2008.

[4] S. Kim, N. Devroye, P. Mitran, and V. Tarokh, Achievable rate regions and performance comparison of half duplex bi-directior relaying protocols, Information Theory, IEEE Transactions on, vol. 57, no. 10, pp. 64056418, 2011.

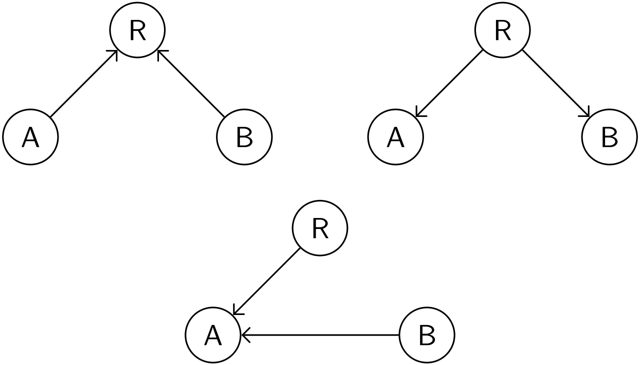
# TDBC and HBC protocols[5]




- TDBC: Time-division transmission to relay, side information at destination used for decoding in last state
- HBC: Also uses multiple access state

[5] S. Kim, P. Mitran, and V. Tarokh, Performance bounds for bidirectional coded cooperation protocols, Information Theory, IEE Transactions on, vol. 54, no. 11, pp. 52355241, 2008.

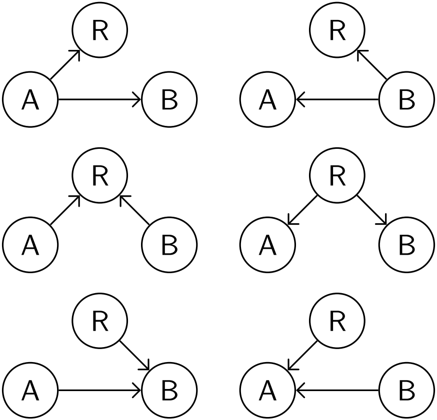
# CoMABC protocol[6]




- MABC with additional MAC transmission state to one of the destinations
- Choice of MAC state depends on channel asymmetry (here $\gamma_2 > \gamma_1$)

[6] Y. Tian, D. Wu, C. Yang, and A. Molisch, Asymmetric two-way relay with doubly nested lattice codes, Wireless Communicati
IEEE Transactions on, no. 99, pp. 19, 2012.

# 6-state protocols[7] [8]




- 6-state protocol: HBC + 2 MAC states
- 6-state DF: No side information across states, DF

[7] C. Gong, G. Yue, and X. Wang, A transmission protocol for a cognitive bidirectional shared relay system, IEEE Journal of Sele Topics in Signal Processing,, vol. 5, no. 1, pp. 160 170, Feb. 2011.

[8] Ishaque Ashar K, Prathyusha V, S. Bhashyam, A. Thangaraj,"Outer Bounds for the Capacity Region of a Gaussian Two-way R Channel, 50$^{th}$ Annual Allerton Conference on Communication, Control, and Computing, Monticello, IL, USA, Oct. 2012.

# Relaying Protocols: Summary

- 2-4 state protocols [9] [10] [11]
  - MABC, TDBC, HBC, CoMABC
- 6 state protocols [12] [13]

[9] S. Kim, P. Mitran, and V. Tarokh, Performance bounds for bidirectional coded cooperation protocols, Information Theory, IEEE Transactions on, vol. 54, no. 11, pp. 52355241, 2008.

[10] S. Kim, N. Devroye, P. Mitran, and V. Tarokh, Achievable rate regions and performance comparison of half duplex bi-directional relaying protocols, Information Theory, IEEE Transactions on, vol. 57, no. 10, pp. 64056418, 2011.

[11] Y. Tian, D. Wu, C. Yang, and A. Molisch, Asymmetric two-way relay with doubly nested lattice codes, Wireless Communications, IEEE Transactions on, no. 99, pp. 19, 2012.

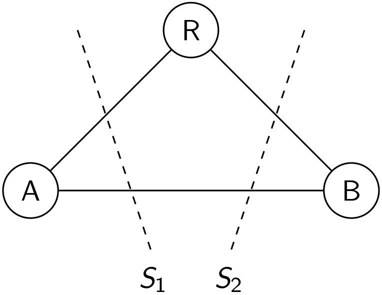
[12] C. Gong, G. Yue, and X. Wang, A transmission protocol for a cognitive bidirectional shared relay system, IEEE Journal of Selected Topics in Signal Processing,, vol. 5, no. 1, pp. 160 170, Feb. 2011.

[13] Ishaque Ashar K, Prathyusha V, S. Bhashyam, A. Thangaraj,"Outer Bounds for the Capacity Region of a Gaussian Two-way Relay Channel, $50^{th}$ Annual Allerton Conference on Communication, Control, and Computing, Monticello, IL, USA, Oct. 2012.

# Rest of the Talk

- Outer bound for any protocol
  - Based on half-duplex cut-set bound
- Linear program formulation: two ways
  - max $R_a$ subject to $R_a = kR_b$
  - max $R_a + kR_b$
- Comparison with known protocols
  - Importance of states
  - Side information across states
  - Relaying scheme
  - Comparison with analytical outer bounds

# Half-Duplex Cut-set Bound[14]




- Depends on time fraction for each state $\{\lambda_i\}$
- Bound flow in both directions for each cut $\Rightarrow$ 4 bounds

$$\begin{aligned} R_a \leq \lambda_1 I\left(X_a; Y_r, Y_b | i = 1\right) &+ \lambda_3 I\left(X_a; Y_r | X_b, i = 3\right) \\ &+ \lambda_5 I\left(X_a; Y_b | X_r, i = 5\right), \\ R_b \leq \lambda_2 I\left(X_b; Y_a | i = 2\right) &+ \lambda_4 I\left(X_r; Y_a | i = 4\right) \\ &+ \lambda_6 I\left(X_b, X_r; Y_a | i = 6\right) \end{aligned}$$

[14] M. Khojastepour, A. Sabharwal, and B. Aazhang, On capacity of gaussian cheap relay channel, in Global Telecommunications Conference, 2003 (GLOBECOM 2003) IEEE, vol. 3., 2003, pp. 1776-1780.

# Outer Bound to Rate Region

- Bound mutual information terms to get linear constraints
- Given $\{\lambda_i\}$, all $(R_a, R_b)$ such that

$$
\begin{aligned}
R_a &\leq \lambda_1 \mathcal{C}(\gamma_1 + \gamma_3) + \lambda_3 \mathcal{C}(\gamma_1) + \lambda_5 \mathcal{C}\left(\gamma_3\right), \\
R_a &\leq \lambda_1 \mathcal{C}(\gamma_3) + \lambda_4 \mathcal{C}(\gamma_2) + \lambda_5 \mathcal{C}\left((\sqrt{\gamma_2} + \sqrt{\gamma_3})^2\right), \\
R_b &\leq \lambda_2 \mathcal{C}(\gamma_2 + \gamma_3) + \lambda_3 \mathcal{C}(\gamma_2) + \lambda_6 \mathcal{C}\left(\gamma_3\right), \\
R_b &\leq \lambda_2 \mathcal{C}(\gamma_3) + \lambda_4 \mathcal{C}(\gamma_1) + \lambda_6 \mathcal{C}\left((\sqrt{\gamma_1} + \sqrt{\gamma_3})^2\right),
\end{aligned}
$$

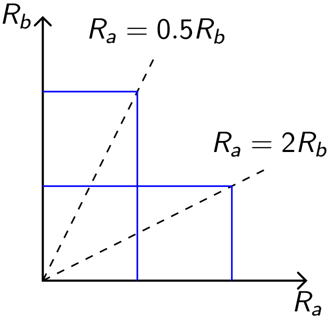
- $\mathcal{C}(\gamma) \triangleq \log_2(1+\gamma)$

# Linear Program to Compute Outer Bound

$$\max_{R_b, \{\lambda_i\}} R_b$$

subject to

- $R_a = kR_b$
- Linear constraints from outer bound
- $\sum_{i=1}^{6} \lambda_i \leq 1, \; \lambda_i \geq 0, \; R_b \geq 0$

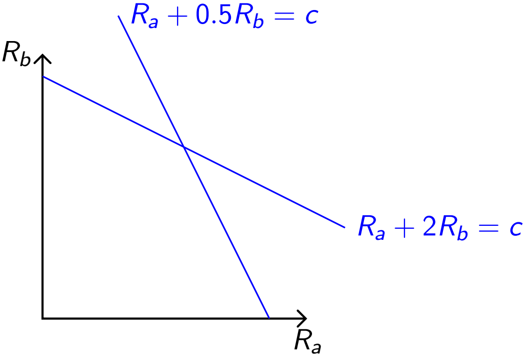


# Alternative Linear Program

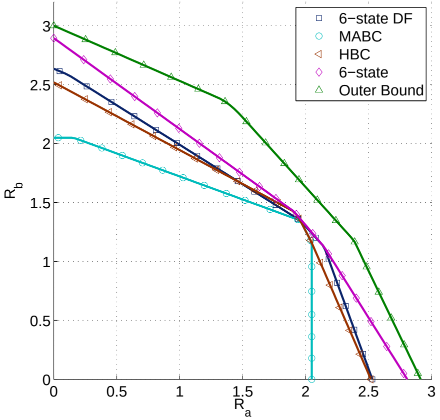
$$\max_{R_a, R_b, \{\lambda_i\}} R_a + kR_b$$

subject to

- Linear constraints from outer bound
- $\sum_{i=1}^{6} \lambda_i \leq 1, \;\; \lambda_i \geq 0, \;\; R_b \geq 0$



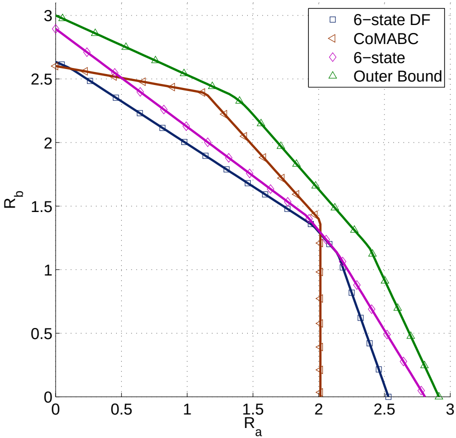
# Comparison: Outer Bound vs. Achievable Rate Regions




$$\gamma_1 = 10 \text{ dB}, \gamma_2 = 15 \text{ dB}, \gamma_3 = 3 \text{ dB}$$

- Importance of using appropriate states
  - MABC: 2 states, HBC: 4 states
- Decoding with side information (6-state vs. 6-state DF)

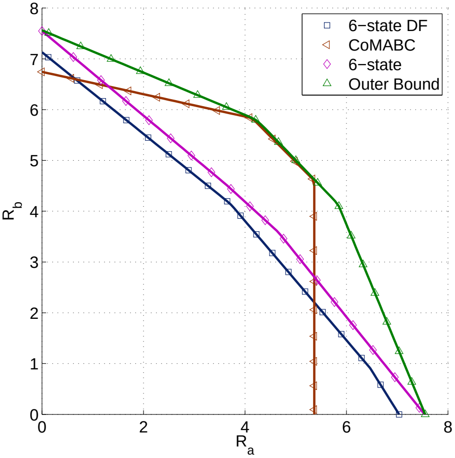
# Comparison: Beyond Decode-and-Forward




$\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB, $\gamma_3 = 3$ dB

- CoMABC: 3 states, estimate XOR
- Optimized for sum rate

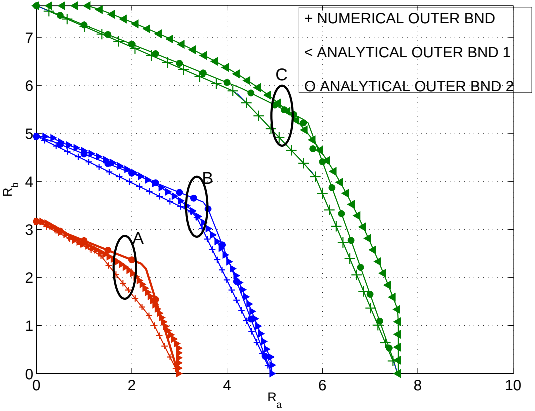
# Comparison: CoMABC at High SNR




$$\gamma_1 = 30 \text{ dB}, \gamma_2 = 35 \text{ dB}, \gamma_3 = 13 \text{ dB}$$

- Close to outer bound in some region

# Comparison of analytical and numerical outer bounds




- Case A: $\gamma_1 = 10$ dB, $\gamma_2 = 15$ dB, $\gamma_3 = 3$ dB
- Case B: $\gamma_1 = 20$ dB, $\gamma_2 = 20$ dB, $\gamma_3 = 8$ dB
- Case C: $\gamma_1 = 30$ dB, $\gamma_2 = 35$ dB, $\gamma_3 = 13$ dB

# Summary

Gaussian Two-Way Relaying with direct link

- Relaying protocols
- Outer bound for any protocol
- Computation of bound and comparisons
  - Two-dimensional rate region
  - Importance of using all states
  - Using side information across states
  - Beyond decode-and-forward
- Analytical bounds

Summary

# Summary

- Two-way relaying as a simple multi-hop multi-flow problem
- Two-way relaying without direct link
  - Relaying protocols
  - Achievable rate regions
  - Outer bounds
  - Code designs
- Two-way relaying with direct link
  - Relaying protocols
  - Achievable rate regions
  - Outer bounds

# Directions for Future Work

- Coding schemes for two-way relaying with direct link
- Capacity for some channel conditions
- Capacity gap
- Multi-antenna nodes
- Multiple relays