Half-Duplex Gaussian Relay Networks with Interference Processing Relays

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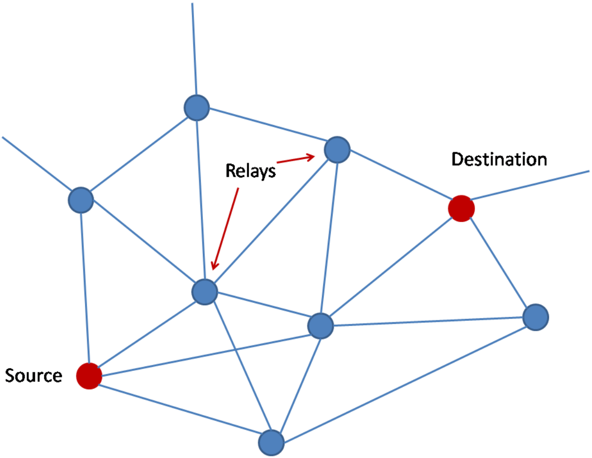
Wireless Relay Networks

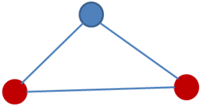


- Single shared resource – wireless channel
- Wireless systems: Time-variations, Interference
- Relaying important in spectrally efficient wireless networks

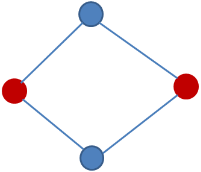
Relaying: What is known/unknown?

Single source-destination pair Gaussian relay networks

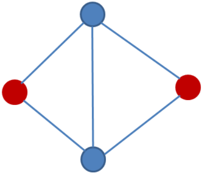
- Capacity unknown for arbitrary topology
- Cut-set upper bound
- Achievable rates for specific protocols and topologies
- Appproximate capacity



Three terminal network



Diamond Channel



Diamond Channel
with Interfering Relays

Wireless Relaying: Assumptions and Results

Duplex	SNR	Cooperation	Topology
Full	Large	MIMO	Arbitrary, Directed
Half	All	Limited	Restricted
		No MIMO	Arbitrary

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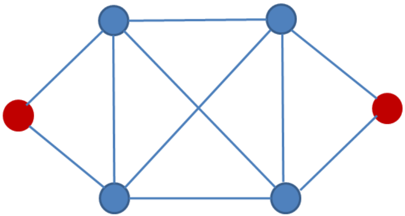
Duplex	SNR	Cooperation	Topology
Full	Large	MIMO	Arbitrary, Directed
Half	All	Limited	Restricted
		No MIMO	Arbitrary

- [Avestimehr et al] Both, Large SNR, MIMO, Arbitrary directed
  - Constant gap to capacity
- [Sreeram et al] Both, Large SNR, MIMO, Arbitrary
  - Diversity-multiplexing trade-off
- [Chang et al] Half duplex, All SNR, Limited, Restricted
  - Rates close to capacity
- [Bagheri et al] Half duplex, All SNR, No MIMO, Restricted
  - Constant gap to capacity

Our Assumptions

- Half-duplex
- All SNR
- No MIMO/Limited cooperation
- Restricted, arbitrary
- Decode-and-Forward

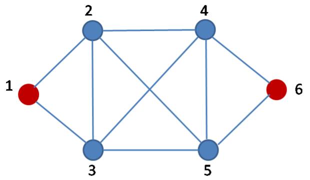


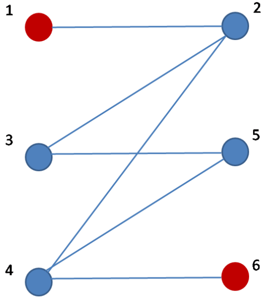
Two-stage relay network

Our Work

- Multistage relaying
- Heuristic scheduling of states + Coding for interference networks
- Flow optimization
- Interference processing vs. Interference avoidance
- Strong and weak interference conditions

States of a Half-Duplex Network





- Each node: Transmit, Receive, or Idle
- Each state is an interference network

Choice of States: Observations

Interference Avoidance

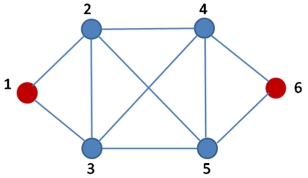
- Only one node can transmit at any time

Interference Processing

- Source should be in transmit mode
- Destination should be in receive mode
- Relays should be in both transmit and receive modes
  - Required for information flow

Atleast two node-disjoint paths required for source to be transmitting in all chosen states

Two-Path Two-State Schedule

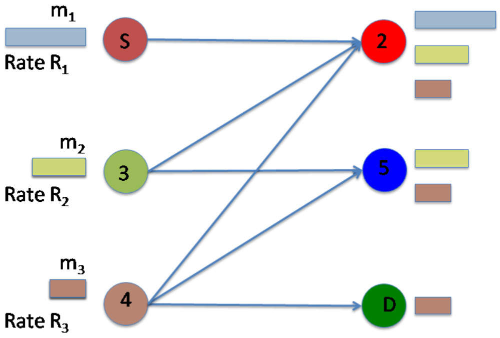


- Shortest (three-hop) paths connecting S and D
  - Path P1: S $\rightarrow$ 2 $\rightarrow$ 4 $\rightarrow$ D
  - Path P2: S $\rightarrow$ 3 $\rightarrow$ 5 $\rightarrow$ D
  - Path P3: S $\rightarrow$ 2 $\rightarrow$ 5 $\rightarrow$ D
  - Path P4: S $\rightarrow$ 3 $\rightarrow$ 4 $\rightarrow$ D.
- Only two pairs of node-disjoint paths: (P1, P2) and (P3, P4).
- States from (P1, P2):
  - State S1: Nodes S, 3, 4 transmit, Nodes 2, 5, D receive
  - State S2: Nodes S, 2, 5 transmit, Nodes 3, 4, D receive

Coding for each state

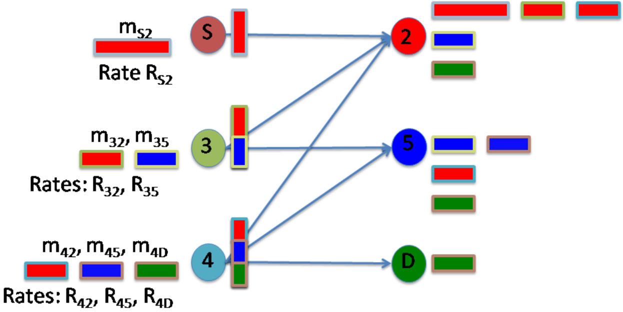
- $M \times N$ interference network [Carleial1978]
- Possible message from each transmitter to each subset of receivers
  - $M(2^N - 1)$ possible rates
- $M-$user Interference channel
  - M possible messages (M rates)
- Achievable rate regions based on
  - Superposition
  - Successive interference cancellation
  - Dirty paper coding

Common Broadcast (CB)



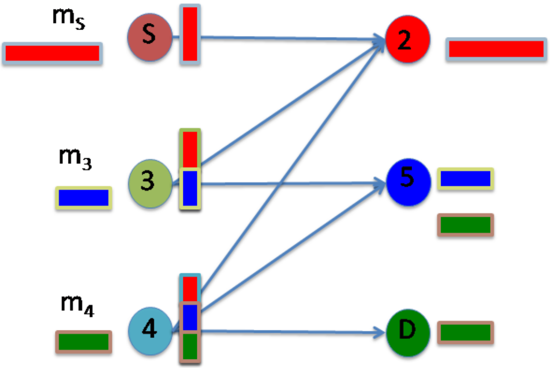
- Rate limited by weakest link
- Receivers employ SIC/MAC decoding

Superposition Coding (SC)



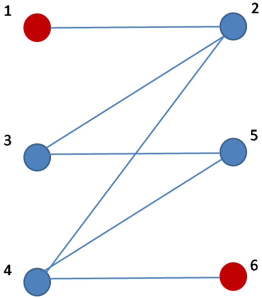
- Transmitters send superposed codewords
- Constraints involve power allocation parameters (non-linear)
- Larger rate region than CB

Dirty paper coding (DPC) at the source



- Source: origin for all messages; knows m_3 and m_4
- Source does DPC to eliminate interference at receiver 2
- Can be combined with CB or SC at other transmitters

Coding for the Two-Stage Relay Example



DPC-SC

- State S1: Nodes S (1), 3, 4 transmit, Nodes 2, 5, D (6) receive
- Node S: Transmit to Node 2 using DPC
- Node 3: Transmit to Node 5
- Node 4: Transmit to Nodes 5 and D using SC

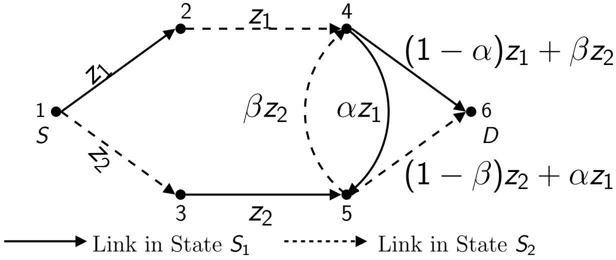
Flow Optimization

- Joint optimization problem

maximize Rate
subject to

- Scheduling constraints
  - State k is ON for λ_k units of time
  - Total transmission time is one unit
- Rate region constraints
  - appropriate rate region depending on the coding scheme
- Flow constraints
  - Total flow in a link $(i, j) = \sum_k$ flow in link (i, j) in state k
  - Outgoing flow from Node i - Incoming flow to Node i = Rate, if i = S,
    -Rate, if i = D,
    0, otherwise.

Two-Stage Relay Flow optimization: DPC-SC



Two-Stage Relay Flow optimization

$$\max_{0 \leq \lambda_1, \lambda_2, \alpha, \beta \leq 1} R = z_1 + z_2,$$

subject to rate constraints

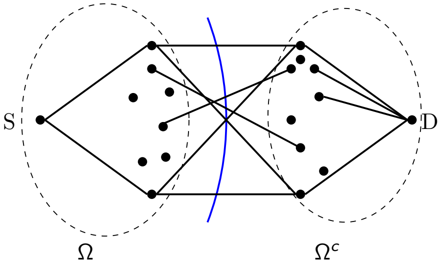
- Flow in each link less than average rate

$$z_1 \leq \lambda_1 R_{S2}, \quad z_1 \leq \lambda_2 R_{24}, \quad z_2 \leq \lambda_2 R_{S3}, \quad z_2 \leq \lambda_1 R_{35},$$
$$(1-\alpha) z_1 + \beta z_2 \leq \lambda_1 R_{4D}, \quad (1-\beta) z_2 + \alpha z_2 \leq \lambda_2 R_{5D},$$
$$\alpha z_1 \leq \lambda_1 R_{45}, \quad \beta z_2 \leq \lambda_2 R_{54},$$

- Scheduling constraint: $0 \leq \lambda_1 + \lambda_2 \leq 1$
- Rates chosen according to rate region of interference network

$$(R_{S2}, R_{35}, R_{45}, R_{4D}) \in \mathcal{R}_1, (R_{S3}, R_{24}, R_{54}, R_{5D}) \in \mathcal{R}_2.$$

Cut-Set Upper Bound



- Full Duplex Network[1]

$$R \leq \min_{\Omega} I(X^{\Omega}; Y^{\Omega^c} | X^{\Omega^c}).$$

- Half Duplex Network [2]

$$R \leq \sup_{\lambda_k} \min_{\Omega} \sum_{k=1}^{\mathscr{M}} \lambda_k I(X^{\Omega}_{(k)}; Y^{\Omega^c}_{(k)} | X^{\Omega^c}_{(k)}).$$

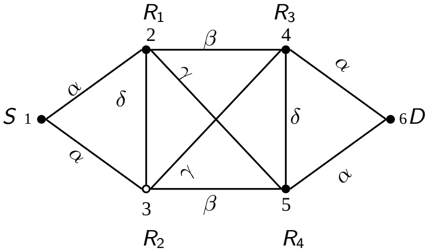
[1] T. Cover *et al*, Elements of information theory, John Wiley

[2] M. Khojestepour,*et al*,,Bounds on achievable rates for general multiterminal n/ws with practical constraints, IPSN, 2003

Numerical Results

Parameters:

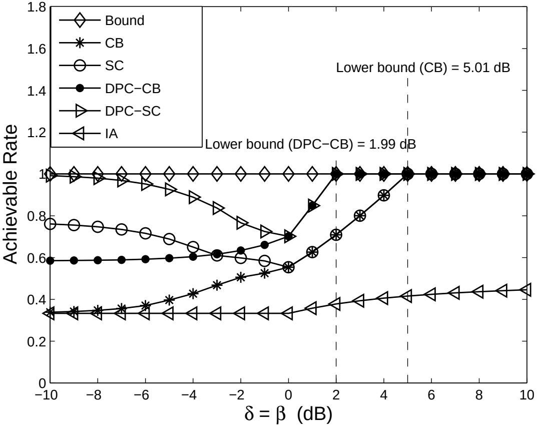
- Tx power, $P = 3$ units
- Noise variance, $\sigma^2 = 1$
- Variable channel gains



Comparison:

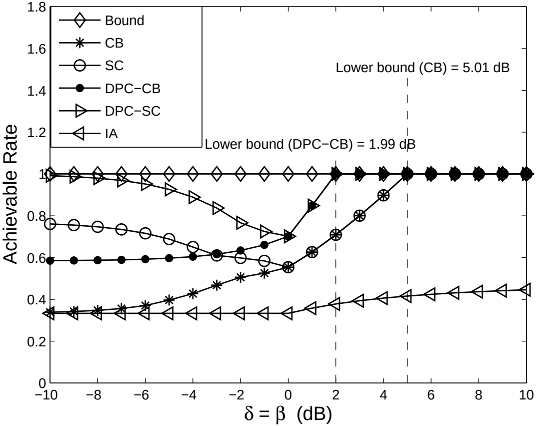
- Interference avoidance throughput - Lower bound
- Upper bound
- Achievable rates of the proposed relaying schemes

Two Stage Relay Network - vary $\beta = \delta$, $\alpha = 1, \gamma = 1$



- Upper bounds
  - Cut-Set bound $= C(2\alpha^2 P) = 1.40$
  - Source rate $\leq C(\alpha^2 P) = 1$

Two Stage Relay Network - vary $\beta = \delta$, $\alpha = 1, \gamma = 1$



- Upper bounds
  - Cut-Set bound $= C(2\alpha^2 P) = 1.40$
  - Source rate $\leq C(\alpha^2 P) = 1$
- Large β
  - Strong interference
  - $S \to R_1 \to R_3 \to D$, $S \to R_2 \to R_4 \to D$

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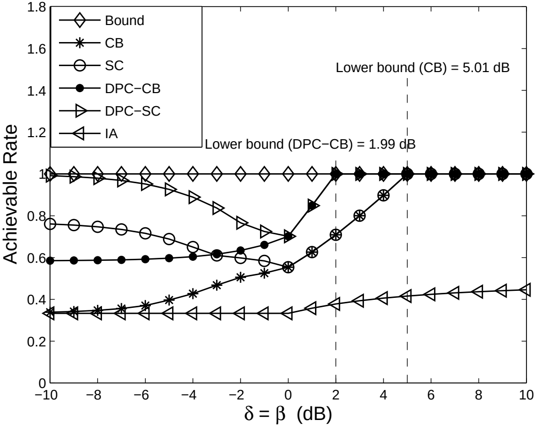

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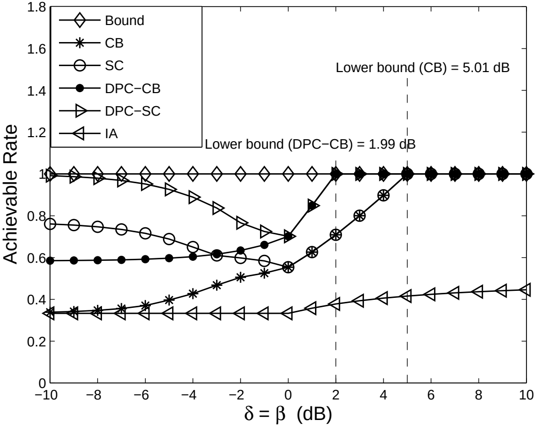

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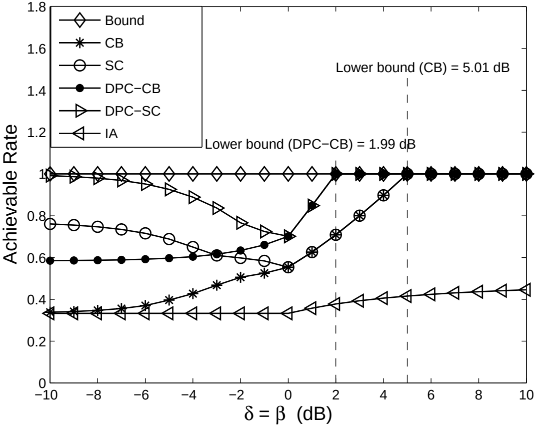
  - $S \to R_1 \to R_4 \to D$, $S \to R_2 \to R_3 \to D$
- $\beta = 0$ dB
  - DPC is better
  - SC and CB are same

Two Stage Relay Network - vary $\beta = \delta, \alpha = 1, \gamma = 1$



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- $\beta = 0$ dB
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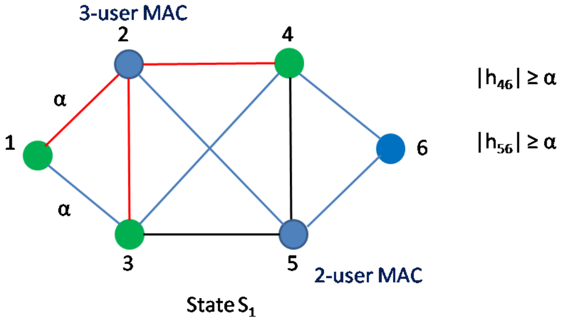
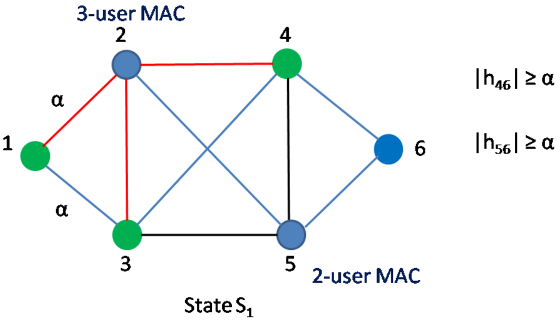
  - SC and CB are same
- HD relaying meets the bound for some channel gains

Strong Interference



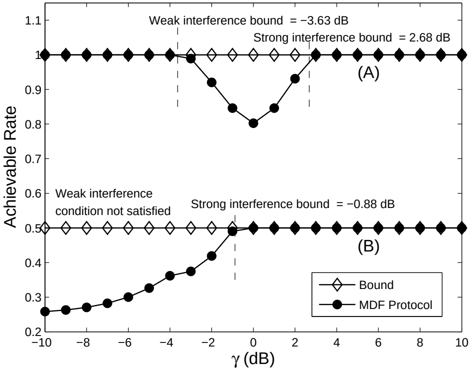
- $|h_{46}| \geq \alpha$, $|h_{56}| \geq \alpha$
- 2-user MAC: $2C(\alpha^2 P) \leq C(2h_{min}^2 P)$ where $h_{min} = \min(|h_{45}|, |h_{35}|)$

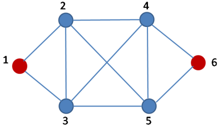
Weak Interference



- $|h_{46}| \geq \alpha$, $|h_{56}| \geq \alpha$
- 2-user MAC: $C(\alpha^2 P) \leq C\left(\frac{h_{35}^2 P}{1+h_{45}^2 P}\right)$

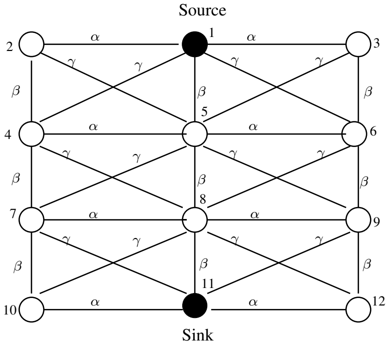
Strong and Weak Interference Bounds



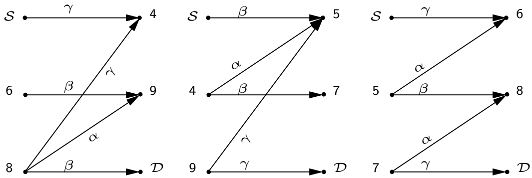


(A): $h_{23} = h_{25} = h_{45} = h_{34} = \gamma,$
$h_{S2} = h_{S3} = h_{5D} = h_{4D} = 1,$
$h_{24} = h_{35} = 1.25$
(B): $h_{23} = h_{25} = h_{45} = h_{34} = h_{5D} = \gamma,$
$h_{S2} = h_{S3} = h_{24} = h_{35} = h_{4D} = 0.58$

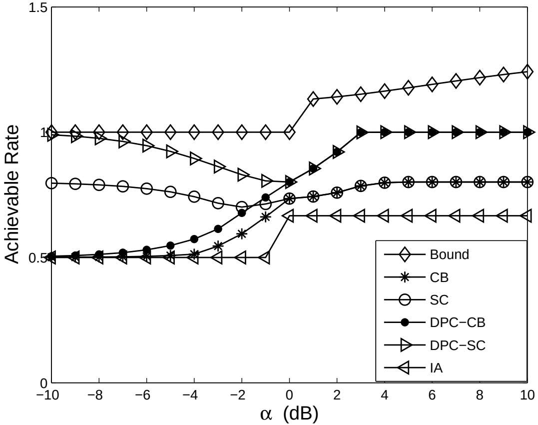
Grid network



- Three states



Performance in Grid Network, $\beta = 1, \gamma = 1$, vary α



Summary

- States of a relay network as interference networks
- Scheduling of states using path heuristic
- Interference processing receivers at the relays
  - No cooperative decoding between relays
- Strong and weak interference conditions on channel gains