Performance of Transmit Antenna Selection with Maximal Ratio Combining in the Presence of Delayed Feedback

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***Abstract*— In this paper, we consider Multiple Input Multiple Output (MIMO) systems with antenna selection at the transmitter and Maximal Ratio Comb should be removed from the URLs mentioned in the references.ining (MRC) at the receiver. Employing antenna subset selection has been shown to reduce the transmitter complexity without any loss in diversity. However, feedback delay leads to selection of the wrong subset resulting in performance degradation. In this paper, we evaluate the outage probability of a single transmit antenna selection scheme in a MIMO system under conditions of delayed Channel State Information (CSI).**

I. Introduction

Multiple antenna arrays have been shown to provide tremendous improvement in the performance of a wireless communication system [1]. However, mobile devices can accommodate only one or two antennas because of the size and power limitations. Therefore, in order to decrease the probability of outage at a given rate, large number of antennas have to be used at the transmitter. The major bottleneck in deploying large number of antennas is the cost of the RF chain hardware associated with each antenna. The diversity order, however, has been shown to depend only on the total number of antenna elements irrespective of the number of antennas that are used simultaneously [2]. Therefore, employing more antenna elements at the transmitter and using only the "best" subset of them reduces transmitter complexity and leads to lower feedback bandwidth while preserving the diversity benefits. The outage performance of the Multiple Input Multiple Output (MIMO) transmit antenna selection scheme with maximal ratio combining at the receiver is presented in [3] and the outage probability of the transmit antenna selection system employing space time coding is presented in [4]. It has been shown in [3] and [4] that antenna selection preserves the diversity benefit.

Transmit antenna selection techniques require CSI to select the "best" set of antennas. Antenna subset selection is performed at the receiver and the selected index is fedback to the transmitter. Estimation errors and channel variations due to feedback delay may lead to erroneous selection. The degradation in the performance of the MISO beamforming system due to feedback delay is presented in [7]. The BER performance of the beamforming system in the presence of feedback delay is analysed in [8]. The loss in the achievable capacity of a hybrid selection - maximal ratio transmission system due to channel estimation errors is studied in [5] using simulations. In [10], the authors show that the asymptotic diversity order of the antenna selection system depends upon the ordinal number of the selected antenna. The effect of the errors in the feedback channel is discussed in [9].

In this paper, we analyze the outage probability of the antenna selection/MRC scheme in the presence of delayed CSI when one out of the multiple transmit antennas is selected for transmission. Analysis on outage probability is performed as a function of ρ, the correlation coefficient between the actual and the delayed CSI. Results are shown for any $N_t \times N_r$ system. Analytical results show that for values of $\rho < 1$, the diversity gain becomes equal to the number of receive antennas. The rest of the paper is organised as follows: section II describes the system model. Section III presents the outage analysis of the system in the presence of perfect antenna selection. Section IV presents the outage analysis of the system in the presence of delayed CSI. Section V presents the numerical results and conclusions are drawn in section VI.

II. System Model

The system model is depicted in Fig 1. A MIMO system with N_t transmit antennas and N_r receive antennas is considered. Antenna selection is performed at the transmitter. The channel between the transmitter and receiver is assumed to be frequency flat. The received vector at time index k is therefore, represented as:

$$\mathbf{y}(k) = \sqrt{P}\mathbf{h}_{sel}(k)x(k) + \mathbf{n}(k), \tag{1}$$

where $x(k)$ represents the transmit symbol at time k, P is the transmit power, $\mathbf{n}(k) \sim \mathcal{CN}\left(0, \sigma_n^2 \mathbf{I}_{N_r}\right)$ is the vector of Additive White Gaussian Noise (AWGN) and $\mathbf{h}_{sel}(k)$ is the selected channel vector. $\mathbf{h}_{sel}(k)$ is one of the columns of the $N_r \times N_t$ channel matrix $\mathbf{H}(k)$ at instant k. The elements of $\mathbf{H}(k)$ are assumed to be i.i.d $\mathcal{CN}(0, 1)$. A block fading model is considered, where the elements of $\mathbf{H}$ are assumed to be constant over a block and correlated across blocks, the correlation coefficient being dependent on the Doppler frequency and the block duration. The channel vector corresponding to the i^{th} and $(i-d)^{th}$ block are related through a first order AR model as [7]:

$$\mathbf{H}_{old} = \rho\mathbf{H} + \sqrt{1-\rho^2}\mathbf{W}, \quad (2)$$

where $\mathbf{H}$ and $\mathbf{H}_{old}$ represent channel matrices corresponding to the blocks i and $(i-d)$, and $\mathbf{W}$ is the matrix of i.i.d. zero mean unit variance Gaussian random variables. The parameter ρ is the correlation between entries of $\mathbf{H}$ and $\mathbf{H}_{old}$. From Jakes' model, $\rho = J_0(2\pi f_d T d)$, where f_d is the Doppler frequency and T is the block duration.

Antenna selection is performed at the receiver and the selected index is fedback to the transmitter. Due to feedback delay, the information on the selected antenna index is used at the transmitter only a few block after the selection is performed.

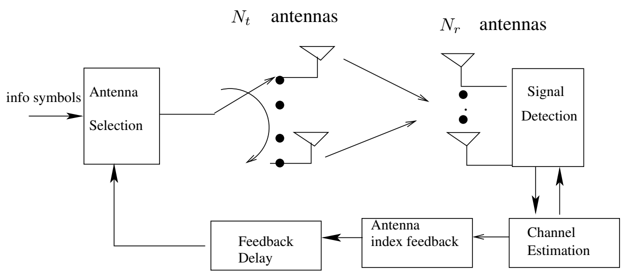

Fig. 1: System Model

III. Outage analysis with perfect antenna selection

In the case of perfect antenna selection, the selected antenna corresponds to that of the maximum gain. Therefore, $||\mathbf{h}_{sel}||^2 = \max_{i=1,2,\cdots,N_t} ||\mathbf{h}_i(k)||^2$. Since the channel coefficients are assumed to be i.i.d. complex Gaussian, the outage probability with perfect antenna selection is given as [3]:

$$P(\text{outage}) = \left[\gamma_{N_r}(\beta)\right]^{N_t} \quad (3)$$

where $\beta = \frac{e^R-1}{SNR}$ and R is the required rate in nats/sec/Hz. $SNR = \frac{P_d}{\sigma_n^2}$, where P_d is the transmit data power. $\gamma_i(x)$ is the lower incomplete Gamma function of order i. $\gamma_i()$ is monotonically decreasing with slope i. Therefore, $(\gamma_{N_r}())^{N_t}$ falls with the rate N_tN_r, which implies that the system with perfect antenna selection achieves a diversity order of N_tN_r [3].

IV. Outage analysis of antenna selection with delayed CSI

In this section, the degradation due to the feedback delay is studied. Due to feedback delay, antenna selection is done based on past channel coefficients instead of current coefficients i.e., $||\mathbf{h}_{sel}(k)||^2 = ||\mathbf{h}_j(k)||^2$, where $j = \arg\max_{i=1,2,\cdots,N_t} ||\mathbf{h}_{old_i}||^2$. Outage occurs when the selected antenna falls into outage. Therefore,

$$P(\text{outage}) = \sum_{i=1}^{N_t} P\left(\text{Antenna i is selected}, ||\mathbf{h}_i||^2 < \beta\right) \quad (4)$$

$$= \sum_{i=1}^{N_t} P\left(||\mathbf{h}_i||^2 < \beta, ||\mathbf{h}_{old_i}||^2 > \max_{j=1,2,\cdots,N_t, j\neq i} ||\mathbf{h}_{old_j}||^2\right).$$

Using $A_i = ||\mathbf{h}_i||^2$, $\alpha_i = ||\mathbf{h}_{old_i}||^2$ and $Z_i = \max_{j=1,2,\cdots,N_t,j\neq i} ||\mathbf{h}_{old_j}||^2$ we get,

$$P(\text{outage}) = \sum_{i=1}^{N_t} P\left(A_i < \beta, \alpha_i > Z_i\right) \quad (5)$$

Using the relation in equation (2), we get

$$\alpha_i = \sum_{j=1}^{N_r} \left|\rho h_{ji} + \sqrt{1-\rho^2} w_{ji}\right|^2 \quad (6)$$

Since $w_{ji} \sim \mathcal{CN}(0,1)$, α_i is noncentral chisquare distributed with $2N_r$ degrees of freedom, for a given A_i. The non centrality parameter is $\delta_i = \rho^2 A_i$. Since h_{ji}'s and w_{ji}'s are i.i.d., α_i's and A_i's are also i.i.d. Therefore, by symmetry, we have

$$P(\text{outage}) = N_t P\left(A_1 < \beta, \alpha_1 > Z_1\right) \quad (7)$$

By definition, Z_1 depends on $||\mathbf{h}_i||^2, i = 2, 3, \cdots N_t$ and $A_1 = ||\mathbf{h}_1||^2$. Therefore, Z_1 and A_1 are independent and we get,

$$P(\text{outage}) = N_t \int_0^\infty \int_0^\beta P(\alpha_1 > z/A_1, z) f_{A_1}(a_1) f_Z(z) da_1 dz \quad (8)$$

Since α_1, given A_1 is noncentral chisquare distributed with $2N_r$ degrees of freedom, we have

$$P(\alpha_1 > z/A_1, z) = 1 - F_{(nc-\chi^2, 2N_r, \delta_1)}(z), \quad (9)$$

where $F_{(nc-\chi^2,2N_r,\delta 1)}()$ is the CDF of the non-central chi squared random variable with $2N_r$ degrees of freedom and noncentrality parameter δ_1. Since $A_1 = ||\mathbf{h}_1||^2$, the pdf of A_1 is given by:

$$f_{A_1}(a_1) = \frac{a_1^{N_r-1} e^{-a_1}}{(N_r-1)!} \quad (10)$$

After integrating over A_1, we get

$$P(\alpha_1 > z/z) = \sum_{j=0}^{\infty} \binom{j+N_r-1}{j} \frac{\mu^j}{(1+\mu)^{j+N_r}}$$
$$\gamma_{j+N_r}\left(\beta(1+\mu)\right) \Gamma_{j+N_r}\left(z(1+\mu)\right), \quad (11)$$

where $\gamma_i(x)$ and $\Gamma_i(x)$ are the lower and upper incomplete Gamma functions of order i respectively and $\mu = \frac{\rho^2}{1-\rho^2}$. It can be seen that Z denotes the maximum of $N_t - 1$ i.i.d. chisquare random variables each of degree $2N_r$. The pdf of

Z is therefore, given by:

$$f_Z(z) = (N_t-1)\left[1-e^{-z}\sum_{i=0}^{N_r-1}\frac{z^i}{i!}\right]^{N_t-2}\frac{z^{N_r-1}}{(N_r-1)!}e^{-z}$$
$$= (N_t-1)\sum_{i=0}^{N_t-2}\binom{N_t-2}{i}(-1)^i e^{-z(i+1)}\left(\sum_{l=0}^{N_r-1}\frac{z^l}{l!}\right)^i$$
$$\frac{z^{N_r-1}}{(N_r-1)!} \tag{12}$$

Using multinomial theorem,

$$f_Z(z) = \frac{N_t-1}{(N_r-1)!}\sum_{i=0}^{N_t-2}\binom{N_t-2}{i}(-1)^i e^{-z(i+1)}z^{N_r-1}$$
$$\sum_{l_1 l_2\cdots l_{N_r}}^{i}\left(\frac{i!}{l_1!l_2!\cdots l_{N_r}!}\right)1^{l_1}\left(\frac{z}{1!}\right)^{l_2}\cdots\left(\frac{z^{N_r-1}}{(N_r-1)!}\right)^{l_{N_r}} \tag{13}$$

Using equation (13) and equation (11), the outage probability is calculated as:

$$P(\text{outage}) = \sum_{i=2}^{N_t}\binom{N_t}{i}(-1)^i\sum_{l_1 l_2\cdots l_{N_r}}^{i-2}\frac{(i-2)!}{l_1!l_2!\cdots l_{N_r}!}$$
$$\frac{(L+N_r-1)!}{(0!)^{l_1}(1!)^{l_2}\cdots((N_r-1)!)^{l_{N_r}}(N_r-1)!}$$
$$\Bigg[\frac{i}{(i-1)^{L+N_r-1}}\gamma_{N_r}(\beta)-\left(\frac{1}{i(\mu+i)}\right)^{N_r-1}\left(\frac{1}{\mu+i}\right)^L$$
$$\sum_{p=0}^{L+N_r-1}\left(\frac{1+\mu}{i-1}\right)^p\sum_{k=0}^{L+N_r-1-p}\binom{L+2N_r-1}{N_r+k+p}$$
$$\binom{k+N_r-1}{k}\left(\frac{\mu}{i}\right)^k\gamma_{k+N_r}\left(\frac{i\beta(1+\mu)}{i+\mu}\right)\Bigg], \tag{14}$$

where the integers $[l_1 l_2\cdots l_{N_r}]$ are such that $\sum_{t=1}^{N_r} l_t = i-2$ and $L = \sum_{t=0}^{N_r-1} t l_{t+1}$. For the sake of simplicity, we present only the derivation of the outage probability for $N_t\times 1$, $N_t\times 2$ and $2\times N_r$ systems in the appendix and the expressions of the outage probability of the above systems are given in the following subsections.

A. $N_t \times 1$ System

In the case of $N_r = 1$, the expression for outage probability can be obtained as (details of the derivation are in Appendix VI):

$$P(\text{outage}) = 1+\sum_{i=1}^{N_t}\binom{N_t}{i}(-1)^i e^{-\left(\frac{\mu+1}{\mu+i}\right)i\beta}. \tag{15}$$

Using Taylor's series expansion for e^{-x} at high SNR's, the asymptotic diversity gain is found to be 1 for $0<\rho<1$. Therefore, we can see that delayed feedback reduces the asymptotic diversity order of the system.

B. $N_t \times 2$ System

In the case of $N_r = 2$, the expression for outage probability can be obtained as (details are in Appendix VI):

$$P(\text{outage}) = \sum_{i=2}^{N_t}\binom{N_t}{i}(-1)^i i\sum_{l=0}^{i-2}\binom{i-2}{l}(l+1)!$$
$$\left(\frac{1}{i-1}\right)^{l+1}\gamma_2(\beta)-\sum_{i=2}^{N_t}\binom{N_t}{i}\frac{(-1)^i}{(\mu+i)i}\sum_{l=0}^{i-2}\binom{i-2}{l}$$
$$\left(\frac{1}{\mu+i}\right)^l(l+1)!\sum_{p=0}^{l+1}\left(\frac{1+\mu}{i-1}\right)^p$$
$$\sum_{k=0}^{l+1-p}\binom{l+3}{p+k+2}\binom{k+1}{k}\left(\frac{\mu}{i}\right)^k\gamma_{k+2}\left(\frac{i\beta(1+\mu)}{i+\mu}\right) \tag{16}$$

C. $2 \times N_r$ System

In this case, the outage probability is given by (details in appendix VI):

$$P(\text{outage}) = 2\gamma_{N_r}(\beta)-\left(\frac{1}{2(\mu+2)}\right)^{N_r-1}\sum_{t=0}^{N_r-1}(1+\mu)^t$$
$$\sum_{k=0}^{N_r-1-t}\binom{2N_r-1}{N_r+t+k}\binom{K+N_r-1}{k}\left(\frac{\mu}{2}\right)^k$$
$$\gamma_{k+N_r}\left(2\beta\frac{1+\mu}{2+\mu}\right) \tag{17}$$

V. RESULTS

MIMO systems with different N_t and N_r combinations are considered. The desired rate R is 2 nats/sec/Hz. Correlation coefficient (ρ) of 0.97 is considered, which corresponds to a normalized Doppler of 0.06. For example, Doppler frequency of 30Hz and time delay of 2 msec lead to a normalized Doppler of 0.06. The outage probability of the system without feedback is also considered for the sake of comparison.

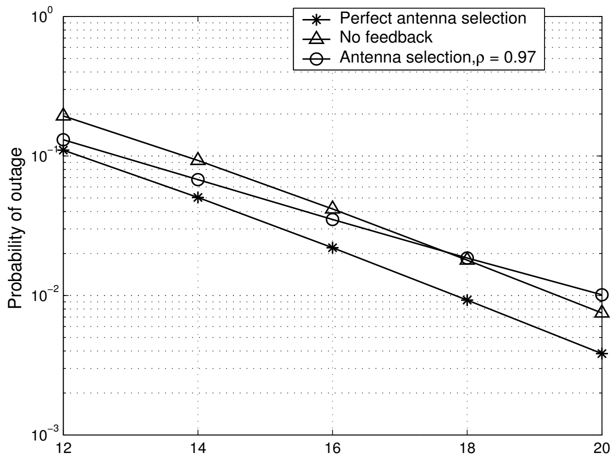

Fig. 2: SNR vs. Probability of outage for 2×1, normalized Doppler = 0.06

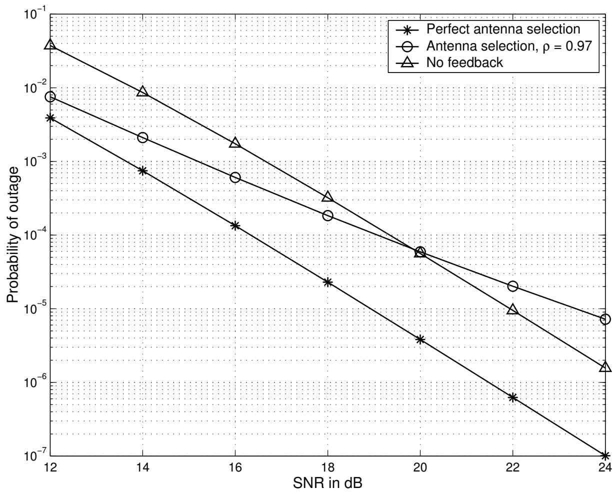

Fig. 3: SNR vs. Probability of outage for 2×2 system, normalized Doppler = 0.06

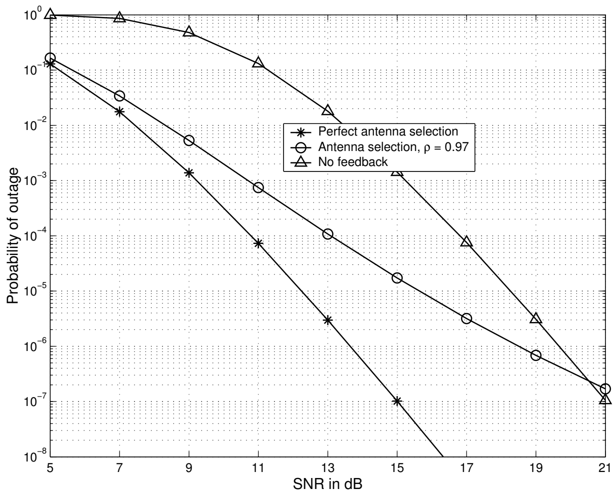

Fig. 4: SNR vs. Probability of outage for 4×2 system, normalized Doppler = 0.06

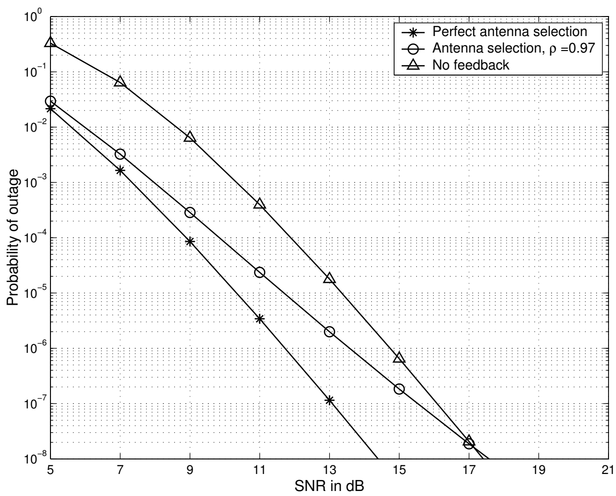

Fig. 5: SNR vs. Probability of outage for 2×4 system, normalized Doppler = 0.06

Fig. 2 depicts the outage performance of the 2×1 system with delayed CSI. It is clear from the graph that delayed CSI leads to a diversity order of 1 instead of 2. However, the diversity order of the open loop system is 2. Therefore at high SNRs the antenna selection scheme with delayed CSI becomes worse compared to the open loop scheme. This shows that the gain in adaptation vanishes due to delayed CSI. Fig. 3 depicts the outage performance of the 2×2 system with delayed CSI. It can be seen that the diversity order of the system is 2 instead of 4. Here again, the outage probability of the open loop system becomes better compared to the antenna selection scheme with delayed CSI at high SNRs. Fig. 4 depicts the outage performance of the 4×2 system. In this case also it can be seen that delayed CSI causes antenna selection to become worse than the open loop scheme at the SNR of about 20dB. Fig. 5 depicts the performance of the 2×4 system. In this case also, the performance of the antenna selection system with delayed CSI becomes worse than the open loop scheme at SNR of about 17dB.

VI. Conclusions

In this paper, the outage performance of the MIMO antenna selection/MRC system in the presence of delayed CSI is studied. Expressions for the outage probability of a $N_t \times N_r$ system is obtained. Results show that feedback delay leads to significant degradation in performance. The asymptotic diversity gain tends to N_r, the number of receive antennas in the presence of delay. At high SNRs, the performance of the antenna selection system with delayed feedback becomes worse than the open loop scheme. Channel prediction techniques can be used to increase the crossover SNR, i.e., the SNR at which the antenna selection scheme falls behind the open loop scheme as in [11] where prediction has been used in adaptive beamforming systems with delayed feedback.

Appendix I: Derivation of the outage probability of $N_t \times 1$ system

For $N_r = 1$, equation (11) reduces to:

$$P(\alpha_1 > z/z) = \sum_{j=0}^{\infty} \frac{\mu^j}{(1+\mu)^{j+1}} \gamma_{j+1}\left(\beta(1+\mu)\right) \Gamma_{j+1}\left(z(1+\mu)\right), \tag{18}$$

The pdf of z reduces to:

$$f_Z(z) = (N_t - 1) \sum_{i=0}^{N_t-2} \binom{N_t-2}{i} (-1)^i \exp^{-z(i+1)} \tag{19}$$

Using equation (18) and equation (19), equation (8) can be written as:

$$P(\text{outage}) = N_t \int_0^{\infty} P(\alpha_1 > z/z) f_Z(z) dz \tag{20}$$

Upon integrating, we get,

$$P(\text{outage}) = N_t(N_t-1)\sum_{i=0}^{N_t-2}\binom{N_t-2}{i}\frac{(-1)^i}{(2+\mu+i)}$$

$$\sum_{j=0}^{\infty}\frac{\mu^j}{(1+\mu)^{j+1}}\gamma_{j+1}\left(\beta(1+\mu)\right)\sum_{k=0}^{j}\left(\frac{1+\mu}{2+\mu+i}\right)^k \quad (21)$$

Upon simplifying, we get

$$P(\text{outage}) = 1+\sum_{i=1}^{N_t}\binom{N_t}{i}(-1)^i e^{-\left(\frac{\mu+1}{\mu+i}\right)i\beta}. \quad (22)$$

Appendix II :Derivation of the outage probability of $N_t \times 2$ system

For $N_r = 2$, equation (11) reduces to:

$$P(\alpha_1 > z/z) = \sum_{j=0}^{\infty}(j+1)\frac{\mu^j}{(1+\mu)^{j+2}}$$

$$\gamma_{j+2}\left(\beta(1+\mu)\right)\Gamma_{j+2}\left(z(1+\mu)\right) \quad (23)$$

The pdf of Z is :

$$f_Z(z) = (N_t-1)\sum_{i=0}^{N_t-2}\binom{N_t-2}{i}(-1)^i e^{-z(i+1)}\sum_{l=0}^{i}\binom{i}{l}z^{l+1} \quad (24)$$

Upon integrating equation (23) with respect to z, we get,

$$P(\text{outage}) = N_t(N_t-1)\sum_{i=0}^{N_t-2}\binom{N_t-2}{i}\frac{(-1)^i}{(2+\mu+i)^2}$$

$$\sum_{l=0}^{i}\binom{i}{l}\left(\frac{1}{2+\mu+i}\right)^l\sum_{j=0}^{\infty}(j+1)\frac{\mu^j}{(1+\mu)^{j+2}}$$

$$\gamma_{j+2}\left(\beta(1+\mu)\right)\sum_{k=0}^{j+1}\left(\frac{1+\mu}{2+\mu+i}\right)^k\frac{(k+l+1)!}{k!} \quad (25)$$

However,

$$\sum_{k=0}^{j+1}\left(\frac{1+\mu}{2+\mu+i}\right)^k\frac{(k+l+1)!}{k!} = (l+1)!\left[\left(\frac{2+\mu+i}{1+i}\right)^{l+2} - \right.$$

$$\left.\left(\frac{1+\mu}{2+\mu+i}\right)^{j+1}\sum_{t=1}^{l+2}\binom{l+j+3}{t+j+1}\left(\frac{1+\mu}{1+i}\right)^t\right] \quad (26)$$

Substituting equation(26) in equation (25) and simplifying we get,

$$P(\text{outage}) = \sum_{i=2}^{N_t}\binom{N_t}{i}(-1)^i i\sum_{l=0}^{i-2}\binom{i-2}{l}(l+1)!$$

$$\left(\frac{1}{i-1}\right)^{l+1}\gamma_2(\beta) - \sum_{i=2}^{N_t}\binom{N_t}{i}\frac{(-1)^i}{(\mu+i)i}\sum_{l=0}^{i-2}\binom{i-2}{l}$$

$$\left(\frac{1}{\mu+i}\right)^l(l+1)!\sum_{p=0}^{l+1}\left(\frac{1+\mu}{i-1}\right)^p$$

$$\sum_{k=0}^{l+1-p}\binom{l+3}{p+k+2}\binom{k+1}{k}\left(\frac{\mu}{i}\right)^k\gamma_{k+2}\left(\frac{i\beta(1+\mu)}{i+\mu}\right) \quad (27)$$

Appendix III :Derivation of the outage probability of $2 \times N_r$ system

For $N_t = 2$, the pdf of Z is given by: $f_Z(z) = \frac{z^{N_r-1}}{(N_r-1)!}e^{-z}$. Upon integrating equation (11) with respect to z, we get,

$$P(\text{outage}) = \frac{2}{(1+\mu)^{N_r}(2+\mu)^{N_r}}$$

$$\sum_{j=0}^{\infty}\binom{j+N_r-1}{j}\left(\frac{\mu}{1+\mu}\right)^j\gamma_{j+N_r}\left(\beta(1+\mu)\right)$$

$$\sum_{k=0}^{j+N_r-1}\binom{k+N_r-1}{k}\left(\frac{1+\mu}{2+\mu}\right)^k \quad (28)$$

Following the approach as in appendix VI, we get

$$P(\text{outage}) = 2\gamma_{N_r}(\beta) - \left(\frac{1}{2(\mu+2)}\right)^{N_r-1}\sum_{t=0}^{N_r-1}(1+\mu)^t$$

$$\sum_{k=0}^{N_r-1-t}\binom{2N_r-1}{N_r+t+k}\binom{K+N_r-1}{k}\left(\frac{\mu}{2}\right)^k$$

$$\gamma_{k+N_r}\left(2\beta\frac{1+\mu}{2+\mu}\right) \quad (29)$$

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