Co-ordinate Interleaved Non-orthogonal Amplify and Forward Relaying Protocol

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***Abstract*— In this paper, we propose and analyze a coordinate interleaved non-orthogonal amplify and forward (CINAF) protocol for the single antenna half-duplex relay channel. Compared to a conventional non-orthogonal amplify and forward (NAF) protocol, the proposed protocol achieves full diversity for all transmitted symbols. The proposed precoding scheme also has the advantage of a low complexity zero forcing successive interference cancellation at the receiver compared to joint maximum likelihood decoding for an existing precoding scheme. In a cooperation frame having N symbol intervals, the conventional NAF protocol transmits N symbols. Out of these N symbols only $\frac{N}{2}$ symbols get the full diversity gain provided by the cooperative network. The remaining $\frac{N}{2}$ symbols do not have any diversity advantage. In order to achieve full diversity gain, we propose precoded transmission using coordinate interleaving. We prove that the proposed scheme achieves full diversity for all symbols. We also show using simulations that the proposed scheme is better in terms of performance compared to an existing precoding scheme.**

I. Introduction

The classical relay channel consists of three nodes, a $source$ that transmits information, a $destination$ that receives the information and a $relay$ that both receives and transmits the information. Relaying is the process when a terminal ($relay$) receives a signal from another terminal ($source$) and transmits it to a third one ($destination$). Various relaying strategies are discussed in [1]. We consider Amplify and Forward relaying where the relays amplify the received signal and forward the amplified signal to destination [2]. Depending on whether the transmission from source and relay interfere at the destination or not, the relay protocols can be classified as either orthogonal or non-orthogonal. In non-orthogonal relay protocols, the source terminal transmits new information to the destination while the relay forwards information it received from the source in the previous time slot [3]. Non-orthogonal protocols achieve higher spectral efficiency than the orthogonal ones.

We consider half duplex relaying. In a conventional non-orthogonal AF (NAF) protocol, N symbols are received in N symbol intervals. Transmissions from the source and relay interfere in $\frac{N}{2}$ symbol intervals. Though this NAF protocol achieves a spectral efficiency of 1 symbol/channel use, only half the symbols transmitted achieve full diversity (received through both the source-destination and relay-destination paths), and the other half of the symbols achieve no diversity advantage (received only through source-destination path).

Precoding symbols in an NAF protocol can be used to achieve full diversity gain for all the symbols transmitted. In [4], unitary precoders that achieve full diversity gain are introduced. Coding gain is also optimized within a class of unitary precoders. In this paper, we propose a new coordinate interleaved NAF (CINAF) protocol that achieves full diversity and performs better than the precoder in [4] in terms of bit error rate. The diversity gain is proved using analysis of the pariwise error probability. Simulation results are used to show that the proposed protocol performs significantly better over the coding gain optimized precoder introduced in [4]. We also achieve this performance with a lower complexity zero-forcing successive interference cancellation (ZF-SIC) receiver at the destination.

We achieve full diversity by employing rotated QAM constellations and interleaving the coordinates of the symbols in such a way that at least one of the co-ordinates gets full diversity advantage. Co-ordinate interleaving for spatial multiplexing was introduced in [5]. Recently, [6] and [7] presented cooperative relay algorithms based on rotated constellations. While [6] proposed distributed space-time code design when the relay terminals have two antennas, [7] investigates decode-and-forward orthogonal relaying based on signal space diversity. In order to enable the ZF-SIC receiver, we transmit N symbols in $N+2$ symbol intervals. While the symbol rate is slightly less than 1 symbol/channel use, the rate approaches 1 as N increases.

Notations: Bold upper- and lower case letters are used to denote matrices and column vectors respectively. Superscripts T and $\dagger$ denotes their transposition and conjugate transpose respectively. $\mathbb{P}$ and $\mathbb{E}$ denote, respectively, probability and expectation operator. $\Re$, $\Im$ and $\mathcal{CN}(0, \sigma^2)$ denote real and imaginary parts of a complex number and circularly symmetric complex gausssian distribution with variance σ^2 respectively.

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II. System Model

Consider the relay channel where the relay terminal R is half duplex and source S, relay R and destination D are assumed to be single antenna. Let h_{ij} denote the channel gain between i^{th} and j^{th} terminal, $i \in (S, R)$ and $j \in (R, D)$. It is assumed that h_{SR} is known at relay R and h_{SR}, h_{RD} and h_{SD} are known at destination D. Consider conventional non-orthogonal amplify and forward (NAF) relaying. A co-operation frame consists of two symbol intervals. During the first symbol interval, the source will broadcast to both relay and destination. During the second symbol interval, the relay will amplify and forward the received symbol during the first symbol interval and the source transmits a new symbol to the destination. Therefore the symbols sent in the first symbol interval of a cooperation frame are received through two independent fading paths ($S \rightarrow D$ and $S \rightarrow R \rightarrow D$), and the symbols sent in the second interval of a cooperation frame are received through only one path ($S \rightarrow D$). Thus, in this NAF protocol, we send N symols in N symbol intervals. Out of these, only $\frac{N}{2}$ symbols get the full diversity provided by the cooperative network. Let x_{2i-1} be the symbol sent and y_{2i-1} be the received symbol in the first symbol interval of i^{th} cooperation frame. Let x_{2i} be the symbol sent and y_{2i} be the received symbol in the second symbol interval of i^{th} cooperation frame, $i \in \mathbb{N}$.

$$y_{D,2i-1} = h_{SD}\sqrt{P_S}x_{2i-1} + n_{D,2i-1} \quad (1)$$

$$y_{R,2i-1} = h_{SR}\sqrt{P_S}x_{2i-1} + n_{R,2i-1}, \quad (2)$$

where $y_{j,k}$ and $n_{j,k}$ denote the received signal and additive noise at terminal $j \in \{R, D\}$ respectively at the end of k^{th} symbol interval, $n_{j,k} \sim \mathcal{CN}(0, \sigma_j^2)$. P_S is the transmit power of the source. During the next symbol interval of the cooperation frame, the source terminal transmits a new symbol x_{2i} and the relay forwards an amplified version of $y_{R,2i-1}$ to the destination simultaneously. Thus, in the second symbol interval, the destination receives

$$y_{D,2i} = h_{SD}\sqrt{P_S}x_{2i} + h_{RD}\sqrt{P_R}(ay_{R,2i-1}) + n_{D,2i}, \quad (3)$$

where $a = \sqrt{\frac{P_R}{P_S+\sigma_R^2}}$ is the scaling employed by the relay to satisfy the *long term* power constraint of P_R. Without loss of generality, we will take $P_R = P_S = 1$. The functioning of the above protocol can be expressed by a single matrix equation, a multiple-input multiple-output (MIMO) channel, as follows.

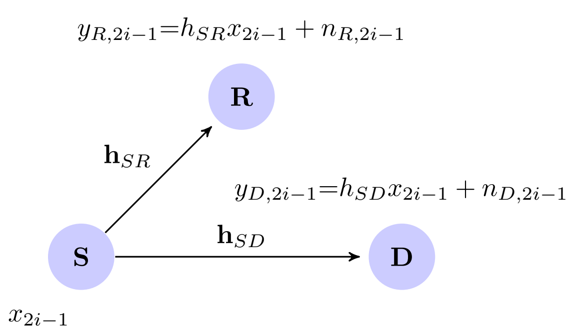

Fig. 1. First symbol interval of i^{th} cooperation frame

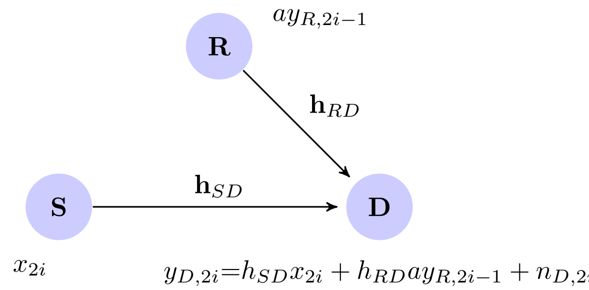

Fig. 2. Second symbol interval of i^{th} cooperation frame

$$\underbrace{\begin{bmatrix} y_{D,2i-1} \\ y_{D,2i} \end{bmatrix}}_{\mathbf{y_i}} = \underbrace{\begin{bmatrix} h_{SD} & 0 \\ ah_{SR}h_{RD} & h_{SD} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_{2i-1} \\ x_{2i} \end{bmatrix}}_{\mathbf{x_i}} + \underbrace{\begin{bmatrix} w_{2i-1} \\ w_{2i} \end{bmatrix}}_{\mathbf{w_i}} \quad (4)$$

Now, we can use precoding techniques for this equivalent MIMO model of the relaying protocol.

III. CINAF protocol

In the proposed CINAF protocol, the input symbols are chosen from a unit energy rotated QAM constellation, denoted by $x_l \in e^{j\theta}\mathcal{X}$, where $\mathcal{X}$ is a conventional un-rotated QAM constellation.

1) Precoding: Let $\{x_k\}_{k=1}^K$ be the input symbol to be transmitted, chosen from a unit energy rotated QAM signal set. The symbols are precoded to obtain $\{\tilde{x}_k\}_{k=1}^K$ where

$$\tilde{x}_k = \begin{cases} \Re x_k + j\Im x_{k+1} & \text{when } k = 2l-1, l \in \mathbb{N} \\ \Re x_k + j\Im x_{k-1} & \text{otherwise} \end{cases}$$

2) Transmission: The transmit symbol vector during the i^{th} cooperation frame is given by

$$\mathbf{x}_i = \begin{cases} [\tilde{x}_1 \ 0]^T & i = 1 \\ [\tilde{x}_{2i-1} \ \tilde{x}_{2i-2}]^T & i > 1 \end{cases}$$

The symbol vector transmitted over successive cooperation frames can be arranged as the following matrix:

$$\begin{bmatrix} \tilde{x}_1 & \tilde{x}_3 & \dots & \dots \\ 0 & \tilde{x}_2 & \tilde{x}_4 & \dots \end{bmatrix} \quad (5)$$

3) Receiver:

- Compute QR decomposition of $\mathbf{H}$ as $\mathbf{H} = \mathbf{QR}$, and left multiply $[\mathbf{y}_{D,1} \quad \mathbf{y}_{D,2}]^T$ with $\mathbf{Q}^\dagger$ to obtain $\tilde{\mathbf{z}}_{D,1} = [\tilde{z}_{D,1} \quad \tilde{z}_{D,2}]^T$ and $\tilde{\mathbf{z}}_{D,2} = [\tilde{z}_{D,3} \quad \tilde{z}_{D,4}]^T$. ie.,

$$\begin{bmatrix} \tilde{z}_{D,1} & \tilde{z}_{D,3} \\ \tilde{z}_{D,2} & \tilde{z}_{D,4} \end{bmatrix} = \underbrace{\begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} \tilde{x}_1 & \tilde{x}_3 \\ 0 & \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} w_{D,1} & w_{D,3} \\ w_{D,2} & w_{D,4} \end{bmatrix}, \quad (6)$$

where

$$\left[w_{D,1} \quad w_{D,2}\right]^T = \mathbf{Q}^\dagger \left[n_{D,1} \quad (n_{D,2} + ah_{RD}n_{R,1})\right]^T$$

and $\begin{bmatrix} w_{D,3} & w_{D,4} \end{bmatrix}^T = \mathbf{Q}^\dagger \begin{bmatrix} n_{D,3} & (n_{D,4} + ah_{RD}n_{R,3}) \end{bmatrix}^T$.

- De-interleave $\tilde{z}_{D,1}$ and $\tilde{z}_{D,4}$ to obtain $z_{D,1}$ and $z_{D,4}$:

$$\begin{aligned} z_{D,1} &= \Re\tilde{z}_{D,1} + j\Im\tilde{z}_{D,4} \\ &= r_{11}\Re x_1 + jr_{22}\Im x_1 + \Re w_{D,1} + j\Im w_{D,4} \end{aligned} \quad (7)$$

$$\begin{aligned} z_{D,4} &= \Re\tilde{z}_{D,4} + j\Im\tilde{z}_{D,1} \\ &= r_{22}\Re x_2 + jr_{11}\Im x_2 + \Re w_{D,4} + j\Im w_{D,1} \end{aligned} \quad (8)$$

Let $\mathbf{w} = \begin{bmatrix} w_{D,1} & w_{D,3} \\ w_{D,2} & w_{D,4} \end{bmatrix}$ and $\tilde{\mathbf{w}} = \begin{bmatrix} \tilde{w}_{D,1} & \tilde{w}_{D,3} \\ \tilde{w}_{D,2} & \tilde{w}_{D,4} \end{bmatrix} = \mathbf{Q}^\dagger \begin{bmatrix} w_{D,1} & w_{D,3} \\ w_{D,2} & w_{D,4}. \end{bmatrix}$. We have $w_{D,1} = n_{D,1}$ and $w_{D,4} = n_{D,4} + ah_{RD}n_{R,3}$, where $n_{D,i}, n_{R,i} \sim \mathcal{CN}(0, \sigma^2)$ and $h_{RD} \sim \mathcal{CN}(0,1)$. Thus, taking each column of $\tilde{\mathbf{w}}$ we get,

$$\underbrace{\begin{bmatrix} \tilde{w_{D,1}} \\ \tilde{w_{D,2}} \end{bmatrix}}_{\mathbf{w}_1} = \mathbf{Q}^\dagger \underbrace{\begin{bmatrix} n_{D,1} \\ n_{D,2} + ah_{RD}n_{R,1} \end{bmatrix}}_{\mathbf{n}_1}$$

$$\underbrace{\begin{bmatrix} \tilde{w_{D,3}} \\ \tilde{w_{D,4}} \end{bmatrix}}_{\mathbf{w}_2} = \mathbf{Q}^\dagger \underbrace{\begin{bmatrix} n_{D,3} \\ n_{D,4} + ah_{RD}n_{R,3} \end{bmatrix}}_{\mathbf{n}_2},$$

where $\mathbf{n}_1, \mathbf{n}_2 \sim \mathcal{CN}(\mathbf{0}, \mathbf{C})$ and $\mathbf{w}_1, \mathbf{w}_2 \sim \mathcal{CN}(\mathbf{0}, \mathbf{K})$,

$$\mathbf{C} = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 + a^2|h_{RD}|^2\sigma_R^2 \end{bmatrix}, \text{ and } \mathbf{K} = \mathbf{Q}^\dagger\mathbf{C}\mathbf{Q}.$$

$$\text{Let } \mathbf{K} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, \mathbf{J} \triangleq \begin{bmatrix} \frac{K_{11}}{2} & 0 \\ 0 & \frac{K_{22}}{2}, \end{bmatrix}^{-1},$$

and $\mathbf{M} \triangleq \begin{bmatrix} r_{11} & 0 \\ 0 & r_{22}. \end{bmatrix}$. Thus, $\Re\tilde{w}_{D,1} \sim \mathcal{CN}(0, \frac{K_{11}}{2})$ and $\Im\tilde{w}_{D,4} \sim \mathcal{CN}(0, \frac{K_{22}}{2})$.

- Let $\mathbf{y}_1 \triangleq \begin{bmatrix} \Re z_{D,1} & \Im z_{D,1} \end{bmatrix}^T, \mathbf{x}_1 \triangleq \begin{bmatrix} x_{qr} & x_{qi} \end{bmatrix}^T$ and $\hat{\mathbf{x}}_1 \triangleq \begin{bmatrix} \hat{x}_{qr} & \hat{x}_{qi} \end{bmatrix}^T$, where $x_{qr} = \Re\mathcal{X}$ and $x_{qi} = \Im\mathcal{X}$. Then, the ML decoder becomes

$$\hat{\mathbf{x}}_1 = \arg\min_{\mathbf{x}_1 \in \mathcal{S}_1} \left\{ (\mathbf{y}_1 - \mathbf{M}\mathbf{x}_1)^T \mathbf{J} (\mathbf{y}_1 - \mathbf{M}\mathbf{x}_1) \right\} \quad (9)$$

where $\mathcal{S}_1 = [\Re\mathcal{X} \ \ \Im\mathcal{X}]^T$. Similarly, $\Im\tilde{w}_{D,1} \sim \mathcal{CN}(0, \frac{K_{11}}{2})$ and $\Re\tilde{w}_{D,4} \sim \mathcal{CN}(0, \frac{K_{22}}{2})$. Let $\mathbf{y}_4 \triangleq \begin{bmatrix} \Im z_{D,4} & \Re z_{D,4} \end{bmatrix}^T$, $\mathbf{x}_4 \triangleq \begin{bmatrix} x_{qi} & x_{qr} \end{bmatrix}^T$ and $\hat{\mathbf{x}}_4 \triangleq \begin{bmatrix} \hat{x}_{qi} & \hat{x}_{qr} \end{bmatrix}^T$. Then, the ML decoder becomes

$$\hat{\mathbf{x}}_4 = \arg\min_{\mathbf{x}_4 \in \mathcal{S}_2} \left\{ (\mathbf{y}_4 - \mathbf{M}\mathbf{x}_4)^T \mathbf{J} (\mathbf{y}_4 - \mathbf{M}\mathbf{x}_4) \right\} \quad (10)$$

where $\mathcal{S}_2 = [\Im\mathcal{X} \ \ \Re\mathcal{X}]^T$. Thus, we get $\hat{x}_1 = \hat{\mathbf{x}}_1(1,1) + j\hat{\mathbf{x}}_1(1,2)$ and $\hat{x}_2 = \hat{\mathbf{x}}_4(1,2) + j\hat{\mathbf{x}}_4(1,1)$.

- Now, generate $\tilde{\hat{x}}_2 = \Re\hat{x}_2 + \Im\hat{x}_1$ and subtract the interference from x_2 in $\tilde{z}_{D,3}$. Assuming $\tilde{\hat{x}}_2 = \hat{x}_2$, we get

$$\tilde{z}_{D,3} = r_{11}\tilde{x}_3 + w_{D,3}. \quad (11)$$

From the subsequent received vector $\mathbf{y}_{D,3}$, we have

$$\tilde{z}_{D,5} = r_{1,1}\tilde{x}_5 + r_{1,2}\tilde{x}_4 + w_{D,5}, \quad (12)$$

$$\tilde{z}_{D,6} = r_{2,2}\tilde{x}_4 + w_{D,6}. \quad (13)$$

The set of equations (11), (12) and (13) are similar to (6). Hence, decoding is continued in a similar manner as above.

IV. Performance Analysis

In this Section, we analyze the pairwise error probability for the CINAF protocol and show that full diversity of two is achieved for all symbols. The analysis is different from that of a standard MIMO system since the noise at the destination is also dependent on the relay-destination channel h_{RD}. Therefore, the analysis proceeds in 2 steps: (1) Analysis for a given h_{RD}, followed by (2) averaging over h_{RD}.

$$\text{Let} \qquad \mathbf{J} = \begin{bmatrix} J_{11} & 0 \\ 0 & J_{22} \end{bmatrix}. \quad (14)$$

Then, we have

$$\begin{aligned} &(\mathbf{y}_1 - \mathbf{M}\mathbf{x}_1)^T \mathbf{J} (\mathbf{y}_1 - \mathbf{M}\mathbf{x}_1) \\ &= J_{11}(\Re z_{D,1} - r_{11}x_{qr})^2 + J_{22}(\Im z_{D,1} - r_{22}x_{qi})^2. \end{aligned} \quad (15)$$

Pairwise error probability bound:
Let $\mathbf{u}_i = [u_{iR} \ u_{iI}]^T$, where $u_{iR} = \Re\mathbf{u}_i$ and $u_{iI} = \Im\mathbf{u}_i$, and $\mathbf{u}_j = [u_{jR} \ u_{jI}]^T$, where $u_{jR} = \Re\mathbf{u}_j$ and $u_{jI} = \Im\mathbf{u}_j$. Let $u_{iR} - u_{jR} = \alpha_{ij}$ and $u_{iI} - u_{jI} = \beta_{ij}$, where $\alpha_{ij}^2 > 0$ and $\beta_{ij}^2 > 0$. Then, we have

$$\begin{aligned} \mathbb{P}(\mathbf{u}_i \to \mathbf{u}_j) &= \mathbb{P}\begin{bmatrix} (\mathbf{y}_1 - \mathbf{M}\mathbf{u}_i)^T \mathbf{J} (\mathbf{y}_1 - \mathbf{M}\mathbf{u}_i) \\ > (\mathbf{y}_1 - \mathbf{M}\mathbf{u}_j)^T \mathbf{J} (\mathbf{y}_1 - \mathbf{M}\mathbf{u}_j) \end{bmatrix} \quad (16) \\ &= \mathbb{P}\begin{bmatrix} 2J_{11}\Re w_{D,1} r_{11}\alpha_{ij} + 2J_{22}\Im w_{D,4} r_{22}\beta_{ij} \\ < -(J_{11}r_{11}^2\alpha_{ij}^2 + J_{22}r_{22}^2\beta_{ij}^2) \end{bmatrix}. \end{aligned}$$

Also, $\mathrm{Var}(J_{11}\Re w_{D,1} r_{11}\alpha_{ij} + J_{22}\Im w_{D,4} r_{22}\beta_{ij})$
$= J_{11}^2 \mathrm{Var}(\Re w_{D,1}) r_{11}^2\alpha_{ij}^2 + J_{22}^2 \mathrm{Var}(\Im w_{D,4}) r_{22}^2\beta_{ij}^2$,

$$\mathrm{Var}(\Re w_{D,1}) > \frac{\sigma_D^2}{2} \text{ and } \mathrm{Var}(\Im w_{D,4}) < \frac{\sigma_D^2}{2} + \frac{|h_{RD}|^2\sigma_R^2}{2(1+\sigma_R^2)}.$$

$$\text{Let } \sigma^2 = \frac{1}{2}\left(\sigma_D^2 + \frac{|h_{RD}|^2\sigma_R^2}{(1+\sigma_R^2)}\right) = \kappa_1\sigma_D^2 \quad (17)$$

$$\text{where } \kappa_1 = \frac{1}{2}\left(1 + \frac{\sigma_R^2|h_{RD}|^2}{(1+\sigma_R^2)\sigma_D^2}\right).$$

Therefore, $\mathrm{Var}(J_{11}\Re w_{D,1} r_{11}\alpha_{ij} + J_{22}\Im w_{D,4} r_{22}\beta_{ij})$
$< (J_{11}^2 r_{11}^2\alpha_{ij}^2 + J_{22}^2 r_{22}^2\beta_{ij}^2)\sigma^2$.

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Pairwise error probability bound given h_{RD}:
From (16), we have $\mathbb{P}(\mathbf{u}_i \to \mathbf{u}_j | h_{RD})$

$$< \mathbb{E}_{r_{11}, r_{22}}\left[Q\left(\frac{J_{11}r_{11}^2\alpha_{ij}^2 + J_{22}r_{22}^2\beta_{ij}^2}{\sqrt{(J_{11}^2 r_{11}^2\alpha_{ij}^2 + J_{22}^2 r_{22}^2\beta_{ij}^2)2\sigma}}\right)\right]$$

Let $\mathrm{J}_{min} = \min(J_{11}, J_{22})$ and $\mathrm{J}_{max} = \max(J_{11}, J_{22})$. Thus, $\mathbb{P}(\mathbf{u}_i \to \mathbf{u}_j | h_{RD})$

$$< \mathbb{E}_{r_{11}, r_{22}}\left[Q\left(\frac{\mathrm{J}_{max}\left(r_{11}^2\alpha_{ij}^2 + r_{22}^2\beta_{ij}^2\right)}{\mathrm{J}_{min}\left(\sqrt{(r_{11}^2\alpha_{ij}^2 + r_{22}^2\beta_{ij}^2)}2\sigma\right)}\right)\right].$$

Also $J_{11} = \dfrac{2}{K_{11}}$, $J_{22} = \dfrac{2}{K_{22}}$, and $\dfrac{\sigma_D^2}{2} < K_{11}, K_{22} < \kappa_1 \sigma_D^2$.

$$\text{Hence,} \quad \frac{\mathrm{J}_{max}}{\mathrm{J}_{min}} > 1.$$

$$\begin{aligned}
\text{Thus,} \quad & \mathbb{P}(\mathbf{u}_i \to \mathbf{u}_j | h_{RD}) \\
< \quad & \mathbb{E}_{r_{11}, r_{22}} \left[Q \left(\frac{\sqrt{\left(r_{11}^2 \alpha_{ij}^2 + r_{22}^2 \beta_{ij}^2\right)}}{2\sigma} \right) \right] \\
< \quad & \mathbb{E}_{r_{11}, r_{22}} \left[Q \left(\sqrt{\frac{\left(r_{11}^2 \alpha_{ij}^2 + r_{22}^2 \beta_{ij}^2\right) \text{SNR}}{\kappa}} \right) \right],
\end{aligned}$$

$$\text{where} \quad \text{SNR} = \frac{2}{\sigma_D^2} \quad \text{and } \kappa = 8\kappa_1.$$

Let the first column of $\mathbf{H}$ be $\mathbf{h}_1$ and second column be $\mathbf{h}_2$, where $\mathbf{h}_1 = [h_{SD} \quad a h_{SR} h_{RD}]^T$ and $\mathbf{h}_2 = [0 \quad h_{SD}]^T$. Also, $a^2 = \frac{1}{1+\sigma_R^2}$ with $P_S = P_R = 1$. At high SNR, $\sigma_R^2 \ll 1$ and hence $a^2 \approx 1$. Since $r_{11}^2 = \|\mathbf{h}_1\|^2$, for a given h_{RD}, $r_{11}^2 \sim \chi_4^2$. Also, as $r_{22} = \|\mathbf{h}_2 \perp \mathbf{h}_1\|^2$ and $r_{22}^2 \le r_{11}^2$, we can write

$$r_{11}^2 \alpha_{ij}^2 \text{SNR} < (r_{11}^2 \alpha_{ij}^2 + r_{22}^2 \beta_{ij}^2) \text{SNR} < r_{11}^2 (\alpha_{ij}^2 + \beta_{ij}^2) \text{SNR}.$$

Therefore, from above we can write (18),

$$\text{where} \quad C_L = \frac{\alpha_{ij}^2 + \beta_{ij}^2}{\kappa} \quad \text{and} \quad C_U = \frac{\alpha_{ij}^2}{\kappa}.$$

Since $r_{11}^2 \sim \chi_4^2$ given h_{RD}, its pdf is given by $f_{r_{11}^2}(\gamma) = \frac{1}{4}\gamma e^{-\gamma/2}$. For $\gamma \to 0^+$, $f_{r_{11}^2}(\gamma)$ can be approximated by

$$f_{r_{11}^2}(\gamma) = a\gamma^\nu + o(\gamma^{\nu+\epsilon}),$$

where $a = 1/4$ and $\nu = 1$ and $\epsilon > 0$. We know from [8] that for the pdf of a non-negative random variable ζ, for $\zeta \to 0^+$, can be approximated by $f(\zeta) = a\zeta^\nu + o(\zeta^{\nu+\epsilon})$, where $\epsilon > 0$ and a is positive constant. Then

$$\int_0^\infty Q\left(\sqrt{c\zeta\bar{\rho}}\right) f(\zeta) \mathrm{d}\zeta = \frac{2^\nu \mathrm{a} \Gamma(\nu + \frac{3}{2})}{\sqrt{\pi}(\nu+1)} (\mathrm{c}\bar{\rho})^{-(\nu+1)} + o(\bar{\rho}^{-(\nu+1)}). \tag{19}$$

Therefore (18) can be written as (20), where

$$C_{ij}^{LB} = \frac{\kappa^2 \Gamma(\frac{5}{2})}{(\alpha_{ij}^2 + \beta_{ij}^2)^2 4\sqrt{\pi}} \text{ and } C_{ij}^{UB} = \frac{\kappa^2 \Gamma(\frac{5}{2})}{\alpha_{ij}^4 4\sqrt{\pi}}.$$

Averaging over h_{RD}: To find $\mathbb{P}(\mathbf{u}_i \to \mathbf{u}_j)$, we need to find

$$\mathbb{E}_{|h_{RD}|^2} \left\{ \mathbb{E}_{r_{11}, r_{22}} \left[Q \left(\sqrt{\frac{(r_{11}^2 \alpha_{ij}^2 + r_{22}^2 \beta_{ij}^2) \text{SNR}}{\kappa}} \right) \right] \right\}.$$

To find the above expectation, let us consider the expectation of lower and upper bound in (20) over $|h_{RD}|^2$. We consider the channel to be Rayleigh faded. i.e. $h_{RD} \sim \mathcal{CN}(0,1)$. Therefore $|h_{RD}|^2 \sim \exp(1)$.

$$\begin{aligned}
\mathbb{E}_{|h_{RD}|^2}\left[C_{ij}^{LB} \text{SNR}^{-2}\right] &= \mathbb{E}_{|h_{RD}|^2}\left[\frac{\kappa^2 \Gamma(\frac{5}{2})}{(\alpha_{ij}^2 + \beta_{ij}^2) 4\sqrt{\pi}} \text{SNR}^{-2}\right] \\
&= \mathbb{E}_{|h_{RD}|^2}\left[p(1+q\gamma)^2 \text{SNR}^{-2}\right] \\
&= C_{LB} \text{ SNR}^{-2},
\end{aligned}$$

where $p = \dfrac{4\Gamma(\frac{5}{2})}{(\alpha_{ij}^2 + \beta_{ij}^2)\sqrt{\pi}}$, $q = \left(1 + \dfrac{\sigma_R^2}{(1+\sigma_R^2)\sigma_D^2}\right)^2$, and $\gamma = |h_{RD}|^2$. Similarly, we have

$$\mathbb{E}_{|h_{RD}|^2}\left[C_{ij}^{UB} \text{SNR}^{-2}\right] = C_{UB} \text{ SNR}^{-2}.$$

Thus, we get (21). Hence,

$$\mathbb{E}_{|h_{RD}|^2} \left\{ \mathbb{E}_{r_{11}, r_{22}} \left[Q \left(\sqrt{\frac{(r_{11}^2 \alpha_{ij}^2 + r_{22}^2 \beta_{ij}^2) \text{SNR}}{\kappa}} \right) \right] \right\}$$

$$= C \text{ SNR}^{-2},$$

where C is the coding gain. Similarly, proceeding this way, we can show that second symbol in the cooperation frame also has the same exponent for SNR. Hence, both the symbols transmitted in each cooperation frame are received with atleast second order diversity, the maximum available diversity for the model. In the above analysis, error propagation in the SIC step is neglected. However, this is considered in the simulation results presented in the next section.

V. Simulation Results

We have evaluated the symbol error rate (SER) and bit error rate (BER) of the proposed CINAF protocol through Monte Carlo simulations. We have also obtained the performance of the coding gain optimized precoding scheme proposed in [4] (denoted by F_{oc}), and the X-codes proposed by Mohammed *et al.* [9]. We note that, though X-codes are developed for MIMO channels, they can be applied in the context of NAF relaying, as the later can be modeled as a MIMO channel as shown by Eqn.(4). We consider *uncoded* transmission of 4-QAM signals. The channel gains $h_{ij}, i \in \{S, R\}, j \in \{R, D\}$, remain constant for a block length of ten cooperation frames and vary independently from frame to frame. Also, we consider $P_R = P_S = 1$ and $\sigma_D^2 = \sigma_R^2$. The 4-QAM is rotated by an angle of $28.5°$ in case of CINAF and by $27.9°$ in case of X-codes [9].

Fig. 3 shows that, with respect to SER, CINAF outperforms the other two schemes. In case of F_{oc}, the symbols are decoded through joint ML decoding, which becomes impractical to implement for signal sets of higher cardinality. As discussed previously, CINAF works with a linear-complexity ZF-SIC receiver. Thus, CINAF offers better error rate performance with a low complexity receiver. Fig. 4 compares the BER of CINAF with the other two schemes. As can be observed, CINAF performs better than F_{oc} and equally well as X-codes.

However, it should be noted that, in a frame length of N, CINAF transmits only $N-2$ symbols, while the other two schemes transmit N symbols. For higher values of N, the loss in spectral efficiency of CINAF becomes negligible.

$$\mathbb{E}_{r_{11}}\left[Q\left(\sqrt{C_L r_{11}^2 \mathrm{SNR}}\right)\right] < \mathbb{E}_{r_{11},r_{22}}\left[Q\left(\sqrt{\frac{(r_{11}^2\alpha_{ij}^2 + r_{22}^2\beta_{ij}^2)\mathrm{SNR}}{\kappa}}\right)\right] < \mathbb{E}_{r_{11}}\left[Q\left(\sqrt{C_U r_{11}^2 \mathrm{SNR}}\right)\right] \tag{18}$$

$$C_{ij}^{LB}\mathrm{SNR}^{-2} < \mathbb{E}_{r_{11},r_{22}}\left[Q\left(\sqrt{\frac{(r_{11}^2\alpha_{ij}^2 + r_{22}^2\beta_{ij}^2)\mathrm{SNR}}{\kappa}}\right)\right] < C_{ij}^{UB}\mathrm{SNR}^{-2} \tag{20}$$

$$C_{LB}\ \mathrm{SNR}^{-2} < \mathbb{E}_{|h_{RD}|^2}\left\{\mathbb{E}_{r_{11},r_{22}}\left[Q\left(\sqrt{\frac{(r_{11}^2\alpha_{ij}^2 + r_{22}^2\beta_{ij}^2)\mathrm{SNR}}{\kappa}}\right)\right]\right\} < C_{UB}\ \mathrm{SNR}^{-2} \tag{21}$$

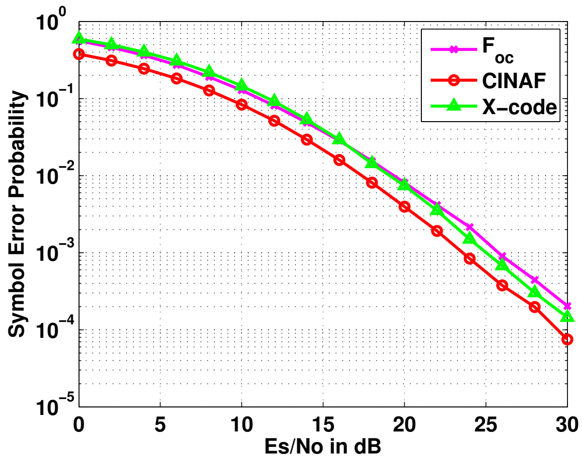

Fig. 3. Symbol error probability of CINAF, X-coded NAF and F_{oc} NAF

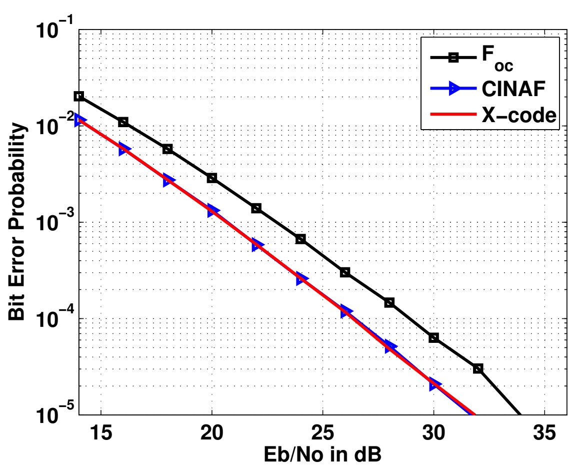

Fig. 4. Bit error probability of CINAF, X-coded NAF and F_{oc} NAF

VI. Conclusion

In a conventional non-orthogonal amplify and forward protocol, only half the transmitted symbols enjoy the full diversity gain offered by the cooperative relay channel. In this paper, we have proposed coordinate interleaved non-orthogonal amplify and forward protocol (CINAF) that achieves full diversity gain for *all* the transmitted symbols, with a ZF-SIC receiver. CINAF employs rotated QAM constellations and interleaves the coordinates of the transmitted symbols such that atleast one component of every symbol experiences full diversity gain. By analysing the pairwise error probability, CINAF is shown to achieve full dievsrity gain. The Monte Carlo simulations confirm the analytical results and also show that the proposed protocol outperforms an optimized precoding scheme proposed in [4]. Thus, CINAF offers improved error rate performance with a low complexity receiver.

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