EE613: Estimation Theory
Problem Set 8

1. Consider the estimation of DC in zero-mean Gaussian noise, and assume that the noise samples are correlated with the covariance matrix

$$\mathbf{C} = \sigma^2 \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 & 0 \\ \rho & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \rho & \cdots & 0 & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & \rho \\ 0 & 0 & 0 & 0 & \cdots & \rho & 1 \end{bmatrix},$$

   where $|\rho| < 1$ and N, the dimension of the matrix, is assumed to be even. $\mathbf{C}$ is a block-diagonal matrix and so is easily inverted. Find the BLUE and its variance and interpret your results.

2. Assume that $x[n] = As[n] + w[n]$ for $n = 0, 1, \ldots, N-1$ are observed, where $w[n]$ is zero mean noise with covariance matrix $\mathbf{C}$ and $s[n]$ is a known signal. The amplitude A is to be estimated using a BLUE. Find the BLUE and discuss what happens if $\mathbf{s} = [s[0]\ s[1]\ \ldots\ s[N-1]]^T$ is an eigenvector of $\mathbf{C}$. Also, find the minimum variance.