EE613: Estimation Theory
Problem Set 3

1. If $x[n]$ for $n = 0, 1, \cdots, N-1$ are IID according to $\mathcal{U}[0, \theta]$, show that the regularity condition does not hold, i.e.,

$$E\left[\frac{\partial \ln p(\mathbf{x}; \theta)}{\partial \theta}\right] \neq 0 \quad \text{for all } \theta > 0.$$

Hence, the CRLB cannot be applied to this problem.

2. If a single sample $x[0] = A + w[0]$ is observed and $w[0]$ has the PDF $p(w[0])$ which can be arbitrary, show that the CRLB for A is

$$\operatorname{var}(\hat{A}) \geq \left[\int_{-\infty}^{\infty} \frac{\left(\frac{dp(u)}{du}\right)^2}{p(u)} du\right]^{-1}.$$

Evaluate this for the Laplacian PDF

$$p(w[0]) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|w[0]|}{\sigma}\right)$$

and compare the result to the Gaussian case.

3. We observe two samples of a DC level in correlated Gaussian noise

$$x[0] = A + w[0]$$
$$x[1] = A + w[1]$$

where $\mathbf{w} = [w[0] \;\; w[1]]^T$ is zero mean with covariance matrix

$$\mathbf{C} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

The parameter ρ is the correlation coefficient between $w[0]$ and $w[1]$. Compute the CRLB for A and compare it to the case when $w[n]$ is WGN or $\rho = 0$. Also, explain what happens when $\rho \to \pm 1$.

4. For a 2×2 Fisher information matrix

$$\mathbf{I}(\boldsymbol{\theta}) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

which is positive definite. Show that

$$\left[\mathbf{I}^{-1}(\boldsymbol{\theta})\right]_{11} > \frac{1}{[\mathbf{I}(\boldsymbol{\theta})]_{11}}.$$

What does this say about estimating a parameter when a second parameter is either known or unknown? When does equality hold and why?