EE613: Estimation Theory
Problem Set 11

1. The data model is $x[n] = a\cos(2\pi f_0 n) + b\sin(2\pi f_0 n) + w[n]$ for $n = 0, 1, \ldots, N-1$, where f_0 is a multiple of $1/N$, excepting 0 or $1/2$ (for which $\sin(2\pi f_0 n)$ is identically zero), and $w[n]$ is WGN with variance σ^2. It is desired to estimate $\boldsymbol{\theta} = [a \ b]^T$. a and b are random variable with prior PDF $\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, \sigma_\theta^2 \mathbf{I})$, and $\boldsymbol{\theta}$ is independent of $w[n]$. Find the MMSE estimate of $\boldsymbol{\theta}$.

2. For the posterior PDF
$$p(\theta|x) = \begin{cases} \exp\left[-(\theta - x)\right] & \theta > x \\ 0 & \theta < x \end{cases}$$
find the MMSE and MAP estimators.

3. The data $x[n] = A + w[n]$ for $n = 0, 1, \ldots, N-1$ are observed. The unknown parameter A is assumed to have the prior PDF
$$p(A) = \begin{cases} \lambda \exp\left(-\lambda A\right) & A > 0 \\ 0 & A < 0 \end{cases}$$
where $\lambda > 0$, and $w[n]$ is WGN with variance σ^2 and is independent of A. Find the MAP estimator of A.

4. In fitting a line through experimental data we assume the model
$$x[n] = A + Bn + w[n] \qquad -M \leq n \leq M$$
where $w[n]$ is WGN with variance σ^2. If we have some prior knowledge of the slope B and intercept A such as
$$\begin{bmatrix} A \\ B \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} A_0 \\ B_0 \end{bmatrix}, \begin{bmatrix} \sigma_A^2 & 0 \\ 0 & \sigma_B^2 \end{bmatrix}\right)$$
find the MMSE estimator of A and B as well as the minimum MSE. Assume that A, B are independent of $w[n]$.