EE5160: Error Control Coding
Problem Set 8

1. Show that for a graph $\mathcal{G} = (V, E)$ with m edges and n vertices, v_1, v_2, ..., v_n, the sum of degrees of vertices is equal to $2m$, i.e.,

$$\sum_{i=1}^{n} d(v_i) = 2m.$$

   Using the above result, show that the number of vertices with odd degree is even.

2. Show that if a bipartite graph contains cycles, then all cycles have even lengths.

3. Draw the Tanner graph corresponding the parity check matrix given below.

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

   Find the total number of edges in the graph.

4. Let a be a binary r.v. and let $p_1 = Pr(a = 1)$. Show that

$$p_1 = \frac{1}{1 + \exp(-L(a))}$$

   and

$$p_1 = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{L(a)}{2}\right)$$

   where $L(a) = \ln(p_0/p_1)$.

5. (Simulation) Simulate SPA decoding of the $(7, 4)$ Hamming code on the binary-input AWGN channel and plot the bit error probability P_b versus E_b/N_0 (dB). Use the following $\mathbf{H}$ matrix for the $(7, 4)$ Hamming code to design your SPA decoder:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

   Simulate an exhaustive-search maximum-likelihood decoder (i.e., a minimum Euclidean-distance decoder) on the binary-input AWGN channel and compare its P_b versus E_b/N_0 curve with that of the SPA decoder.