EE 511 Problem Set 2

Due on 3 Sep 2007

1. A and B are two events. Which is larger, $P[AB|A]$ or $P[AB|(A+B)]$?

2. Events A and B are mutually exclusive. Can they be independent?

3. Consider a random variable X with the property that $a \leq X(s) \leq b$ for all s belonging to the sample space S. Which statement(s) is(are) true? (a) $F_X(x) = 0$ for $x < a$, (b) $F_X(x) = 0$ for $x > b$, (c) $F_X(x) = 1$ for $x > a$, and (d) $F_X(x) = 1$ for $x > b$.

4. Consider a random variable X with pdf $f_X(x)$ given by

$$f_X(x) = \begin{cases} c(1-x^2) & -1 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

   (i) Find c such that $f_X(x)$ is a valid pdf, and (ii) calculate $P[X > 0]$, $P[X < 1/2]$ and $P[|X| > 0.75]$.

5. Which of the following functions can be the probability density function of a random variable? (i) $f(x) = x$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise, (ii)$f(x) = |x|$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise.

6. Two random variables X and Y have the following joint CDF:

$$F_{X,Y}(x,y) = \begin{cases} 1 & \{x \in [1,\infty) \cap y \in [1,\infty)\} \\ 1/2 & \{x \in [0,1) \cap y \in [1,\infty)\} \\ 1/2 & \{x \in [1,\infty) \cap y \in [0,1)\} \\ 1/4 & \{x \in [0,1) \cap y \in [0,1)\} \\ 0 & \text{else} \end{cases}$$

   Determine and sketch $F_X(x)$ and $F_Y(y)$.

7. Let X and Y be two random variables such that $X(s) \leq Y(s)$ for all $s \in S$. Show that their CDF's satisfy $F_X(\alpha) \geq F_Y(\alpha)$ for all α.